NANYANG TECHNOLOGICAL UNIVERSITY

INVENTORY MANAGEMENT IN FIRMS THAT MARKET PRODUCTS IN MULTIPLE CHANNELS

ZHOU ZIHAN

NANYANG BUSINESS SCHOOL

2017

INVENTORY MANAGEMENT IN FIRMS THAT MARKET PRODUCTS IN MULTIPLE CHANNLES

ZHOU ZIHAN

2017

INVENTORY MANAGEMENT IN FIRMS THAT MARKET PRODUCTS IN MULTIPLE CHANNELS

ZHOU ZIHAN

ZHOU ZIHAN

NANYANG BUSINESS SCHOOL

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2017

Abstract

In addition to traditional channels such as catalogs and brick-and-mortar stores, merchants are now offering touch points like web storefronts and mobile apps. The introduction of new channels revolutionizes shopping experience and has made managers revise supply chain design. We refer to marketing a product in more than one channel as multi-channel marketing.

Chapter 1 reviews the latest literature on managing a supply chain in the context of multi-channel marketing. This is to develop a framework on which our work is based. Both theoretical findings and practical executions are included.

Chapter 2 compares end-of-period with continuous cost accounting of inventory in serial systems. End-of-period accounting calculates inventory-related costs when inventory is received or shipped whereas continuous accounting calculates inventory-related costs continuously. An approach to calculating inventory-related costs installation by installation is provided. By taking this approach, the switch from end-of-period accounting to continuous accounting is easy to handle. It is proved that the deployment of end-of-period accounting overshoots the optimal reorder points when inventory-related costs are incurred continuously. The effect of the overshot reorders on the inventory-related costs is substantial when the cost parameters are large, customer demand is high and reorder intervals are long. Numerical results

show that continuous accounting takes longer computation time than end-of-period accounting. Furthermore, the computation time difference grows exponentially as the number of installations increases.

Chapter 3 studies one-warehouse multi-retailer systems where inventory is redistributed every period and the warehouse may also meet customer demand. By considering the assignment of each unit of inventory, an approach to determining the optimal redistribution is provided. The inventory-related cost is proved to be a convex function of the order-up-to level, which facilitates the search for the optimal order-up-to level for the warehouse. Based on a lower bound of the total cost which consists of the cost of ordering and the inventory-related cost, an upper bound of the optimal reorder interval is found. Afterwards, two methods that are aimed at reducing the inventory-related cost during one period are studied. One is referring customer demand to the warehouse who incurs a lower holding cost rate than the retailer. Numerical results show that this method is effective when inventory is sufficient. The other method is stockout-based substitution and this method works well especially when the total supply is close to the total demand.

Chapter 4 discusses management of demand in two channels where the demand in each channel is a general function of the price, the price differential and the substitutability between the products in the two channels. It characterizes the total amount of demand when the price is optimal. Further, if there exists a price differential, demand migrates from the highly priced channel to the relatively lowly priced one. In this situation, the effect of the migration of demand on the total profit is mediated by the profit margins. In a multi-period setting where the price differential in one period affects the customer base in the following period, the optimal policy, if unique, should group the periods during which the customer base is the same.

Acknowledgment

First and foremost, I express my deep gratitude to my supervisor Dr. Wang Qinan for his expert guidance on my research. Before I made a start on doing research for my thesis, he patiently taught me about the fundamentals of inventory management. After I began researching into this subject, he shared in-depth knowledge and great wisdom with me and that inspired the studies in this thesis.

I would like to thank all professors for guiding me through the coursework that was carried out during the first three years. They equipped me with practical skills that I need for the research.

I greatly appreciate the administrative support of PhD office. I am also deeply indebted to NBS PhD Committee for their efforts to make the atmosphere in the office cooperative.

I am grateful for the funding that has been provided by Nanyang Business School. I took that opportunity and presented the research in an overseas conference, which is a valuable experience.

Last but not least, I wish to acknowledge the support of my family and friends. They greatly enrich my life.

Contents

List of Figures

List of Tables

Chapter 1

Impacts of Multi-channel Marketing on Supply Chain Management: a Review of the Latest Literature

1.1 Introduction

Nowadays, customers are reached in multiple ways. In 2013, 87.3 billion direct mails were sent out, 416 commercial emails were received by a user monthly on average and nearly 108 million adults in the U.S. bought something online (DMA, 2014). Meanwhile, many firms are expanding by adding new channels. For example, Zara opened the first web storefront in China at Tmall.com in October 2014 (Inditex, 2014) and Amazon opened the first brick-and-mortar store on the campus of Purdue University in February 2015

(Amazon, 2015).

Multi-channel marketing enables a customer to interact with a merchant in a number of ways, including brick-and-mortar stores, Internet, catalogs, kiosks and so on, from which a customer plans her shopping trajectory and nabs the best deal. A customer may browse the product information online, touch the merchandise in the store, view customer reviews from the mobile app, order at the self-service kiosk and receive delivery at home.

Many firms are adopting multi-channel marketing to satisfy the needs of their customers and maintain competitive advantages. For example, Marks & Spencer (M&S) offers four ways to shop, i.e., online, mobile, in store and over the phone. These channels are linked such that product codes are identifiable across channels and items bought online can be returned to or exchanged in any store (M&S, 2015). Another example is Argos who has three ordering channels, i.e., Internet, phone and store. In 2014, 44% of total sales come from the Internet. Meanwhile, the firm claims that the network of 734 stores plays an important role in providing local convenience and facilitating multi-channel sales such as collecting an online order from the store (Argos, 2014).

In this chapter, we review literature on supply chain management in a multi-channel framework based on theoretical findings and practical executions, with a focus on the most recent development in the field. We refer the reader to Rangaswamy and Van Bruggen (2005), Neslin and Shankar (2009) and Zhang et al. (2010) for excellent reviews of past literature (covered up to 2010). Our contribution is tripartite. Firstly, we communicate the latest (mainly from 2009 to 2014) findings on multi-channel marketing. Secondly, we look upon both the demand side (customer behavior) as well as the supply side to catch the interaction between them. Thirdly, we connect theory and practice by using instances from industry practice to illustrate theoretical findings.

The remaining sections are organized as follows. We first recapitulate results on customer behavior, such as channel preference and customer satisfaction, towards multi-channel marketing. Then we synthesize findings on whether to be multi-channel and how to coordinate multiple channels. Afterwards, inventory models in multi-channel marketing are reviewed and limitations of existing methods are pointed out. We conclude the chapter in Section 1.6.

1.2 Impacts of Channel Strategy on Customer Behavior

Concerns about customer demand are foremost on many managers' minds. On the one hand, multi-channel marketing raises awareness about the product. As a result, market penetration is likely to rise. On the other hand, multi-channel marketing intensifies competition for sales as customers have easy access to product information and may compare products among companies. It is necessary to know customer behavior well because customers have a significant impact on operations management of the supply chain (Arnold et al., 2013) . A clear understanding of customer behavior is a prerequisite for a winning strategy.

1.2.1 Which Channels Does a Customer Prefer?

A consumer can use a variety of channels to make purchases, depending on individual preferences. She may be used to buying books from her favorite bookstore, prefer to buy a computer online, or have a collection of T-shirts bought from stores and online as well.

Channel usage intention not only varies across product categories (Kushwaha and Shankar, 2013), but also evolves over stages of purchasing and differs among customers (Frambach et al., 2007). At each stage of purchasing, a customer's intention to use a particular channel is influenced by channel attributes (e.g., convenience, risk), her past experiences with that channel and her channel intention at another stage. The relative importance of the three factors varies across stages (Gensler et al., 2012b).

Single-channel or Multi-channel

Compared with single-channel marketing, multi-channel marketing provides customers with more touch points to acquire production information and larger product range to choose from. It benefits the customers who are seeking that provision.

First of all, customers have multiple ways to find out product information with multi-channel marketing. They can search the Internet and inquire further in the store. However, a multi-channel framework does not fit in with each market. A customer who has no access to Internet may not think highly of the opening of an online channel.

Secondly, product categories govern effectiveness of multi-channel marketing. Kushwaha and Shankar (2013) identify the most valuable customers (traditional-channel, electronic-channel, or multi-channel) for each product category (utilitarian vs. hedonic; low-risk vs. high-risk). They find that multi-channel customers spend more than single-channel customers only on hedonic products, because hedonic products trigger variety-seeking behavior and customers buy from multiple channels for a wide selection. Traditional-channel customers spend more than others on low-risk products. Electronic-channel customers spend more than others on high-risk/ utilitarian product. These findings help guide marketers' target. A firm offering hedonic products should be dedicated to all channels, whereas one offering high-risk/ utilitarian products targets electronic customers.

Online or Offline

A customer may look for information and compare products on the Internet. To check a product or to secure a payment, she may go offline. Other than that, is a customer apt to visit a web storefront or come to a brick-and-mortar store? How do online and offline customers behave differently?

Although a customer with sufficient Internet experience is inclined to use online channels (Frambach et al., 2007), she is observed to display low loyalty towards one web storefront (Ansari et al., 2008). That is, she makes few subsequent purchases from there. This phenomenon is conjectured to be owing to low switching costs over the Internet and Internet customers' lack of engagement in personal service. To deal with low loyalty, Gilt.com, which is a flash sales site, launched the Gilt Insider Program in 2013. Members earn points as they interact with the site and they can redeem points to get benefit such like discounts and free shipping (Gilt, 2013).

Another difference between online and offline customers is that promotions work differently for the two (Zhang and Wedel, 2009). Loyalty promotions, aiming at retaining customers, are more profitable online than offline, whereas competitive promotions, targeting at acquiring customers, are the opposite. While individual-level customized promotions excel segment- and mass market-level ones remarkably in the promotion-sensitive categories online, the incremental profit of offering individual-level customized promotions over the other two is marginal offline. Low offline redemption rates partially account for those differences.

1.2.2 Cross-channel Effect

Will purchase intentions in one channel spill over into another? If a firm is multi-channel, it can be difficult to tell where a sale originates. For instance, a store sale may stem from an attractive online advertisement or a pleasant TV shopping experience before.

Theoretically, research shows that cross-channel effect is manifold. Firstly, cross-channel effects from online advertising to offline sales and from offline advertising to online sales exist, but the former is stronger than the latter (Dinner et al., 2014). Secondly, offline brand image influences online brand belief and online performance influences offline brand belief. A strong offline brand image minimizes the impact of negative online performance on offline brand belief (Kwon and Lennon, 2009). Thirdly, the intention to search in channel A of firm 1 and purchase in channel B of firm 2 is associated positively with a customer's perceived efficacy of utilizing multiple channels, attractiveness of channel B of firm 2 and negatively with firm 1's lock-in level (Chiu et al., 2011).

These findings suggest that one channel influences customer intention towards another. Hence, channel and communication options should be designed, implemented and managed properly so that sales and brand equity are maximized and synergistic (Keller, 2010).

1.2.3 Customer Satisfaction

Will a customer appreciate a multi-channel firm? How will a customer critique multiple channels holistically?

Multi-channel strategy satisfies a customer via enhanced service outputs and this enhances customer-retailer loyalty (Wallace et al., 2004), regardless of whether the customer conducts business with a single company or competing companies (Larivière et al., 2011). On the other hand, some research cautions a firm against myopic multi-channel strategy. A customer's overall satisfaction depends on service quality of each channel (Montoya-Weiss et al., 2003) and the evaluation of a particular channel is influenced by qualities of other channels as well (Fernández-Sabiote and Román, 2012). Discrepancies in service qualities in different channels weaken the positive associations between Web quality and customer trust and commitment (Liao et al., 2011). Thus, a firm should balance its efforts among channels.

1.2.4 To Right Channel Customers

When and how to direct a customer to the right channel?

A firm should right channel a customer soon after acquisition. At a trial stage when a customer is learning about the product and purchases, her channel choice is influenced by the firm's marketing efforts and the influence is stronger than at a post-trial stage when she has learnt her preference for the product and purchases (Valentini et al., 2011).

Steering customers from one channel to another induces reluctance, but a voluntary or rewarded migration strategy induces less reluctance than a punishment-based or enforcing one (Trampe et al., 2014). For example, DBS Bank rewarded a customer S$5 for downloading and registering the mobile app DBS PayLah! when the app was introduced into Singapore in 2014 (DBS, 2014).

1.3 Is It Beneficial to Be Multi-channel?

Profitability of multi-channel marketing depends on circumstances. Putting up product information online can do more harm than good in that customers make fewer trips to stores and spend less money than before (Van Nierop et al., 2011). On the other hand, Gensler et al. (2012a) find that the opening of an online channel increases revenue and decreases cost. Thus, adoption of multi-channel marketing should be negotiated on a case-by-case basis.

Two multi-channel structures have been recorded in the literature. One is that a retailer sells in multiple channels, e.g., Bust Buy vends at BestBuy.com, the Best Buy app along with Best Buy stores (BestBuy, 2015a).

The other is that a manufacturer sells through an independent retailer as well as in his direct channel. For example, Nike's products are carried by retailers, such as SportsLink and TANGS, and Nike direct stores in Singapore (Nike, 2015). A manufacturer's and a retailer's desirable supply chain structures may be different and both change with market conditions, operational costs and coordination between the two members (Cai, 2010).

1.3.1 Should a Retailer Be Multi-channel?

Impacts of adding clicks to bricks and adding bricks to clicks are documented. It is found that the launch of an online channel does not cannibalize offline sales significantly (Biyalogorsky and Naik, 2003), but improves a firm's performance measures(Xia and Zhang, 2010). In contrast, the opening of the store initially cannibalizes catalog demand and repeat purchase from the direct channel. Over time it increases demand in both channels and brings in more and more first-time customers to the direct channel (Avery et al., 2012).

Practically, it can be strategic to open multiple channels. On the one hand, Internet helps improve a store channel as it is observed that the option of online buying and store pickup decreases online sales but increase store visits and sales (Gallino and Moreno, 2014). On the other hand, a multi-channel service meets the needs of customers. For example, the service of Check & Reserve at Argos enables a customer to reserve a product (via Internet, over a phone, etc.) and get it instantly when she comes to the store. This service shortens in-store searching time for that product and is especially useful if the customer needs it urgently (Argos, 2015).

However, a retailer is not necessarily better off running more channels. Carrillo et al. (2014) show that the customer propensity of online purchases and risk of leftover inventories partially determine the optimal channel mix. When the propensity is extreme and/ or risk is high, a single-channel strategy tends to be optimal. Otherwise, when the propensity is moderate and risk is low, a dual-channel strategy is reasonable. Indeed, eliminating a channel can be beneficial. Konuş et al. (2014) find that eliminating the catalog search channel drives customer traffic away from the telephone channel but towards the Internet. Revenue loss due to reduced purchases is offset by cost savings in not operating that channel.

1.3.2 Should a Manufacturer Be Multi-channel?

General research unfolds the merit of a manufacturer's multi-channel structure and the framework to build it. Establishing a new channel enhances firm value and the value is positively related to competitive intensity and industry turbulence, which implies the effectiveness of a multi-channel strategy in spreading risk in a volatile market (Homburg et al., 2014). Sharma and Mehrotra (2007) come up with a six-stage framework for developing the optimal channel mix. It starts from examining each channel and then all the channels as a whole. This developing process starts over again, after observing feedback from customers and channels, for continued improvement.

In this section, we are to address specific questions, such like should a manufacturer sell through a retailer? Should he open a direct channel?

Should he do both?

How to construct a promising channel structure? When demand variability is low, it is profitable for the manufacturer to open a direct channel (Aussadavut et al., 2008). Moreover, the dual-channel strategy profits the manufacturer more than the single-channel (either retail channel only or direct channel only) strategy and induces a higher service level in the direct channel than the direct-channel-only strategy (Chen et al., 2008). Besides, a manufacturer profits from having multiple retailers, even if they place its direct channel in an unfavorable condition (David and Adida, 2014).

In addition, a manufacturer can benefit even more. When decision making is coordinated and centralized, opening a direct channel increases the total profit (Aussadavut et al., 2008) and a dual-channel strategy that allows a customer to search and buy from the other channel in case of stockouts in her preferred channel can reduce inventory costs substantially (Chiang and Monahan, 2005).

However, multi-channel marketing does not always pay off and adoption should be negotiated on a case-by-case basis. Firstly, revenues should offset expenditures of running multi-channel and it is not an unusual practice to close unprofitable business. In 2012, PUMA introduced the Transformation and Cost Reduction Program and had closed 73 stores by the end of 2013 (PUMA, 2013). Secondly, when manufacturers are competing, a stronger customer preference for direct (indirect) channel does not necessarily lead to more openings of direct (indirect) channels (Hsiao and Chen, 2013). A manufacturer may choose to differentiate its channels from the mainstream so as to avert fierce competition. Thirdly, introducing an Internet channel does not necessarily increase profits (Yoo and Lee, 2011; Lu and Liu, 2015). Impacts of the introduction depend on channel structures and market conditions (Yoo and Lee, 2011; Hsiao and Chen, 2014). Fourthly, a multi-channel manufacturer may have difficulty in obtaining a single customer view that is essential for analyzing past shopping behaviors and personalizing future offerings. This is partially because sales records in different channels are neither consistent nor unified. To tackle this problem, Clarins rewards customers for recording their offline purchases online at Club Clarins (Clarins, 2015). Fifthly, conflicts may arise while channels are competing for customers.

1.4 Coordination Strategy

Why should members of a multi-channel supply chain coordinate? Firstly, Balakrishnan et al. (2014) report a case where two players compete fiercely. When customers have the option to browse products in a brick-and-mortar store and switch to purchase them from an e-tailer, intense price competition between the two emerges and lowers equilibrium payoffs to both. Secondly, decentralization sets the retail price low in the direct channel and high in the retail channel and cuts the total profit compared with centralization (Xu et al., 2014). Thirdly, decentralization fails to take advantage of risk pooling, as Chiang (2010) show that increasing the proportion of customers who substitute the in-stock channel for the out-of-stock one can either increase or decrease the total profit if activities are decentralized. That horizontal substitution and the vertical double marginalization induce each member to stock more or less goods than are needed (Boyaci, 2005). On the other hand, integrating resources and operations across multiple channels protects a firm's short-term and long-term interests (Oh et al., 2012). Thus, coordination is sought-after.

However, members do not naturally coordinate. While coordination is beneficial to the supply chain system as a whole, it does not necessarily benefit each member equitably. For example, having access to the retailer's information increases a manufacturer's profit in the Stackelberg competition led by him, because knowledge of the retailer's response aids him in deciding the wholesale price and the retail price in his direct channel. However, whether the manufacturer shares his information with the retailer or not does not impact the retailer, because the retailer can observe the manufacturer's information via the wholesale price (Yan and Pei, 2011). Therefore, incentive to coordinate is needed.

In this section, we summarize incentive schemes to boost supply chain performance based on theoretical knowledge and practical experience. Some of them are pure strategies, e.g., decreasing the wholesale price only. A pure strategy may benefit all members but not necessarily to the same extent as centralization. As Boyaci (2005) observed, many simple contracts that prompt members to coordinate in a single-channel supply chain do not do the job in a multi-channel one because of the coexistence of substitution and double marginalization. That is why a lot of hybrid schemes have been devised, e.g., product differentiation accompanied by profit sharing in Yan (2011) and adjustments to wholesale and retail prices in Cai (2010).

1.4.1 Adjusting the Wholesale Price

Adjustments to wholesale prices feed into retail prices and the latter affect sales. Adjusting the wholesale price in a dual-channel system where the manufacturer sells in its direct channel and via retailers mitigates double marginalization in a retailer-channel-only structure. It can help cultivate coordination and resolve channel conflict as well (Tsay and Agrawal, 2004). Specifically, the introduction of a direct channel will be accompanied by a wholesale price reduction (Arya et al., 2007) that prompts the retailer to cut the retail price and that stimulates demand at the retailer. Both the manufacturer and the retailer profit from the adjustment (Chiang et al., 2003). Especially, a linear quantity discount on the wholesale price improves the overall efficiency (David and Adida, 2014).

1.4.2 Adjusting Retail Prices

Should retail prices in different channels be equal? In practice, we see both equal and unequal prices. While the conventional wisdom is that deals online are cheaper than offline, BestBuy.com declares that "we'll match the product prices of key online and local competitors" (BestBuy, 2015b).

How to set retail prices in multiple channels strategically? First of all, if the retail price in the direct channel (p) is to maximize the total profit, then either keeping the retail price at the retailer unchanged or adjusting it to p undershoots the global optimal value by no more than 4% in many cases (Huang and Swaminathan, 2009). This research partially justifies the existence of both equal and unequal prices. Secondly, unequal prices direct customer flow. Some manufacturer opens a direct store downtown, showcasing his products and/ or educating customers. Bernstein et al. (2009) shows that if customer valuation increases after visits, then the direct store should set a higher retail price than retailers. In doing so, the manufacturer is directing customer traffic to retailers and averts the high cost of carrying inventories downtown. Zhang (2009) shows that when the online margin is low, the multi-channel retailer benefits from reducing the offline price and advertising that reduced price online. That practice attracts customers to offline offers where the margin is relatively large. Thirdly, if a manufacturer sells through a retailer and an e-tailer, then the retailer tends to set a low margin whereas the e-tailer is inclined to set a high margin in equilibrium. Consequently, the retailer is expected to retain grocery shoppers and grab some Internet shoppers, whereas the e-tailer aims the Internet shoppers who place a premium on it (Hsiao and Chen, 2014).

However, coordination strategy should be revised as market conditions change over time. Otherwise, a player may fall back into maximizing his own profit. For example, when the direct channel is inconvenient and costly relative to the retail channel, it is reasonable for the manufacturer to match his retail price to the retailer's. As inconveniency and costs decrease, profits at both the manufacturer and the retailer grow. Nevertheless, the manufacturer can be better-off if he sets a lower retail price than the retailer, at the expense of the retailer's profit (Cattani et al., 2006).

1.4.3 Profit or Revenue Sharing

Profit or revenue sharing prompts members to set prices wisely and share resources such as information, sales effort and inventory so that the optimal total profit is achieved. Xu et al. (2014) propose a two-way revenue sharing contract and show that it coordinates a risk-averse dual-channel supply chain if a condition on the wholesale price and the two fractions of revenue share is fulfilled. Yan and Pei (2011) suggest profit sharing to facilitate information sharing. The split in the additional profits is obtained by solving the Nash bargaining problem. To improve the situation where a retailer's sales effort is frustrated by an e-tailer who free rides that effort and offers the product at a lower price than the retailer, Xing and Liu (2012) devise a coordination scheme where the manufacturer compensates the retailer for each item that he has sold at that lower price at customers' request. Boyaci (2005) construct a two-part compensation-commission contract where the retailer receives compensation from the manufacturer for each residue of the stock below the target and earns commission on each sale above that target. The optimal stock levels are then achieved as well.

1.4.4 Mutually Beneficial Arrangement

While the above-mentioned strategies are detailed instructions on how to coordinate, there are some general guidelines to follow.

Firstly, products in different channels are differentiated (Vinhas and Anderson, 2005). For example, Watsons Singapore sells Colgate toothbrush twin packs in stores whereas its web storefront offers single and triple packs only (Watsons, 2015). Other than changes in packs, products can be differentiated with value-added services. For example, Topshop provides in-store Personal Shopper service. During an appointment, a Personal Shopper gives advice and recommends garments to a customer, so that the customer is brought up-to-date with fashion (Topshop, 2015). Differentiation promotes profit in each channel. It makes a more valuable contribution to profit when customers are less sensitive to the price and/or the market is larger (Melewar et al., 2010). However, differentiation does not naturally maximize profits. While accompanied by a certain coordination scheme such as profit sharing, differentiation boosts profit at each member (Yan, 2011; Yan et al., 2011).

Secondly, each channel that participates in a sale is compensated, regardless of whether the channel is used in the pre-purchase or the purchase stage (Vinhas and Anderson, 2005). This is the practice in Evans Cycles. If a sale like click and collect involves EvansCycles.com and a store, then both channels are rewarded (Charlton, 2013).

Thirdly, order ownership is clarified (Vinhas and Anderson, 2005) for practices such as referring all demand to the direct channel and rendering all demand to the retailer (Tsay and Agrawal, 2004). For example, store managers in John Lewis are responsible for both store and online sales in their catchment area (Ruddick, 2013).

Fourthly, sharing real-time information throughout the supply chain reduces costs (Mahar and Wright, 2009) (Mahar et al., 2009) (Mahar et al., 2012). It helps determine which site should handle an online order so that stocks at all sites are delicately balanced. Liu et al. (2010) tackle the problem of delivering online orders from warehouses that also satisfy in-store demands.

They observe that the warehouse with high in-store demand uncertainty and low unit transportation cost from the distribution center through that warehouse to a region is a good candidate to deliver online orders originating from that region. Besides, the sharing also polishes up online shopping experiences. For example, M&S.com updates stock availability every 15 minutes and refreshes it at the checkout so that each item in the "bag" belongs to the customer and the dedicated item is delivered on time (M&S, 2014).

1.5 Inventory Models for Multi-channel Firms

Previous sections were judgments about the multi-channel strategy. It was pointed out that a multi-channel strategy has an effect on demand characteristics and supply patterns and causes channels to compete against each other. Thereafter, various coordination mechanisms were suggested so as to mitigate conflicts. This section will discuss the physical distribution aspect of supply chains for multi-channel firms. Physical distribution is the movement and storage of finished goods from the end of production to the customer (Arnold et al., 2013). Customers are acquired from multiple channels and there can be one or more sites fulfilling customer demand. Physical distribution systems are rich in literature and they prove adaptable to solving multi-channel inventory management issues.

1.5.1 A Single Fulfillment Site

A multi-channel firm can have exactly one site (belonging to one channel) to fulfill customer demand and other sites to serve purposes rather than demand fulfillment. For example, the manufacturer advertises products online and refers customers to its brick-and-mortar store for purchase. Another example is that customers can place orders online and their orders are shipped from the brick-and-mortar store which also meets in-store demand. In both examples, the online channel motivates sales at the store and all customer demand is fulfilled by the store.

The distribution inventory system for such firms can be modeled by a serial system where each stock point has at most one immediate successor. Products are transferred from a chain of warehouses to the store to meet customer demand there. If the store replenishes stock from the manufacturer directly, the distribution inventory system is reduced to a single store. For both multi-echelon serial systems and single-echelon systems, various inventory control policies have been developed to guide material flow through and properties of the optimal policy parameters that minimize cost have been studied.

The following costs are commonly used for inventory management decisions: the costs associated with order placement and excess and shortage of inventory (Axsäter, 2007). The cost of placing an order is incurred regardless of the size of an order and covers order preparation, follow-up, etc. Carrying costs or holding costs include capital costs, storage costs and risk costs. Shortage costs are incurred when there is a stockout. The sum of the latter two is referred to as inventory-related costs (Rudi et al., 2009). While literature on multi-echelon systems mainly uses end-of-period accounting for inventory-related costs, Chapter 2 discusses echelon (r,nQ,T) policies in serial systems where inventory-related costs are incurred continuously.

1.5.2 Multiple Fulfillment Sites

When demand is huge, a multi-channel firm can establish multiple fulfillment sites to meet demand. For example, one or more stores meet in-store demand while another site is set up to fulfill online orders. Another example is that a warehouse ships online orders and replenishes a brick-and-mortar store as well. Those are examples of one-warehouse multi-retailer systems.

In modeling one-warehouse multi-retailer systems, the warehouse usually plays one of the following three roles. Firstly, the warehouse is an outside supplier or goods stay there for only a short time, so that inventory costs are not charged at the warehouse (such systems are called R-systems, see e.g. Gallego and Simchi-Levi (1990)). Second, the warehouse is a storage facility that receives large-volume goods and replenishes retailers' stocks (see e.g. Gallego et al. (2007)). Thirdly, in the modern multi-channel marketing, a warehouse is also equipped to ship online orders. In that situation, the warehouse not only supplies retailers but also meets customer demand directly (such systems are called dual-channel systems, see e.g. Chiang and Monahan (2005)).

In order to meet customer demand at multiple retailers (in case of dual-channel systems, at the warehouse as well) at the minimum costs, inventory should be distributed among all stock points properly. In continuous review systems where the stock is monitored at all times, inventory is usually controlled by (R,nQ) policies such that whenever the stock drops to R or below, a minimum number of batches of size Q are ordered at upstream to raise stock above R. Such one-warehouse multi-retailer systems have been evaluated and optimized in Chen and Zheng (1997), Axsäter (2000) and Axsäter (2003) among others. With continuous review (R,nQ) policies, there are hardly any instances where two or more retailers place orders at the warehouse simultaneously. On the other hand, in periodic review systems where the stock is inspected at predetermined time instants, review intervals are usually multiples of a base period. Thus it is not rare to observe several retailers placing orders at the warehouse simultaneously. Indeed, such incident provides an opportunity to redistribute stock among these installations. While literature on redistribution of inventory mainly assumes that the length of the order cycle is pre-determined, Chapter 3 develops an method to determine the length of the order cycle in view of a fixed cost for each order.

1.5.3 Demand Modeling

Inventory decisions are on how to match supply with demand in a cost-effective way. While Chapters 2 and 3 discuss the supply part of the supply chain, Chapter 4 discusses the demand part. While most literature assumes demand to be a linear function of a number of factors, Chapter 4 considers a general form of demand.

1.6 Conclusions

In this chapter, we review literature on supply chain management in a multi-channel framework based on theoretical findings and practical executions, with a focus on the most recent development in the field.

Meanwhile, we observe a relative lack of explorations of after-sales service in concurrent channels. Which channels are preferable for interacting with the merchant after sales? How should multiple channels collectively handle returns and exchanges? These explorations will broaden our knowledge of customer behavior and instruct supply chain managers in resource allocation.

Chapter 2

A New Method to Evaluate Serial Systems Implementing Echelon (r,nQ,T) Policies

2.1 Introduction

End-of-period and continuous accounting are two schemes in which inventory-related costs are assessed. The end-of-period accounting evaluates inventory at the end of each predetermined period such as a replenishment interval or a period of one unit time. It is a conventional method to assess cost and has the advantage of being mathematically tractable (Rudi et al., 2009). The continuous accounting assesses cost in an ongoing manner and is applicable to the cost that is incurred continuously over time. Back order cost is such an example because it depends on the duration of stockouts. The inventory-related costs can be huge, since the inventory on hand alone financially represents from 20% to 60% of the total assets on the balance sheet in manufacturing companies (Arnold et al., 2013). Thus a proper evaluation of inventory on which decision is based is essential for effective operations management.

Both accounting schemes have been used frequently to model the inventory-related costs incurred in holding or back-ordering an item in single-installation inventory systems. Liu and Song (2012) and Ang et al. (2013) establish the properties of a periodic-review order-up-to (S,T) policy and develop the techniques for searching for the optimal parameters that minimize the cost with end-of-period accounting. Rao (2003) investigates the properties of an (S,T) policy with continuous accounting based on the results in Hadley and Whitin (1963). When the reorder level is also a policy parameter, Lagodimos et al. (2012) propose a decomposition approach to determining the optimal (s,S,T) policy that minimizes the total average cost with continuous accounting. Recently, Rudi et al. (2009) compare the two accounting schemes in a setting where cost is incurred continuously. They point out that deploying the end-of-period accounting always overshoots the order-up-to level and results in a rise in the inventory-related costs.

While assessing the inventory-related costs in the multi-echelon inventory systems where inventory transfers from an upstream installation to the downstream echelon, the majority of literature uses end-of-period accounting. Moreover, the evaluation is based on the echelon stock that was introduced in Clark and Scarf (1960). Chen and Zheng (1994) get the probability mass functions (pmf) for inventory levels in each echelon and obtain the expected cost through pmf. Van Houtum et al. (2007) and Shang and Zhou (2010) recursively calculate the average inventory-related costs for each echelon and finally obtain the total cost. To our knowledge, Wang and Wan (2016) is the only work that uses continuous accounting to assess the inventory-related costs in multi-echelon inventory systems.

Herein, we study a serial inventory system that uses echelon (r,nQ,T) policies. With such a policy, inventory in each echelon is reviewed periodically. When it drops to the reorder point or below, the installation places an order for multiple base order quantities at its immediate upstream installation. Our contribution is the following: (1) we develop a method of assessing the inventory-related costs when the cost is incurred continuously in this serial inventory system; (2) our evaluation method enables installations to calculate inventory-related costs independently of each other; (3) we prove that deploying end-of-period accounting for continuously accumulated cost results in overshot reorder points in all echelons; (4) numerical tests show that continuous accounting has disadvantages in terms of computation time; and (5) given the advantages in accuracy and the disadvantages in speed, continuous accounting is appropriate for the supply chains with a small number of installations.

2.2 Model

In this chapter, we consider a serial inventory system that consists of N installations and inventory transfers through installations $N, N-1, ..., 1$ in sequence (Figure 2.1). For ease of description, the supplier that replenishes installation N is referred to as installation $N+1$. For $j \in \{1, 2, ..., N\}$, installation j and downstream constitute echelon j. Installation j is replenished according to an echelon (r,nQ,T) policy. With this policy, every T_j unit time, installation j reviews nominal echelon j inventory position (shortened to NIP_j from this point onwards) which equals the orders placed but not yet received by installation j plus the inventory at installation j plus the inventory in transit to or at installation $j-1, j-2, ..., 1$ minus the back orders at installation 1. Equivalently, NIP_j equals the orders placed but not yet received by installation j, plus all inventory in echelon j, minus the back orders at installation 1. If NIP_j is less than or equal to the reorder point r_j, installation j orders a multiple of Q_j units from installation $j+1$ to raise NIP_j to be in $\{r_j+1, r_j+2, ..., r_j+Q_j\}$. Once receiving the request by installation j, installation $j+1$ ships out the supplies immediately. If installation j requests more inventory than is available, installation $j+1$ ships out all available inventory. The model assumes that installation $N+1$ has ample stock. The transportation lead time from installations $j+1$ to j is L_j. Customers request goods from installation 1 and the customer demands during non-overlapping intervals of equal length are independent, identically distributed and non-negative integers. Unsatisfied demand units are backordered.

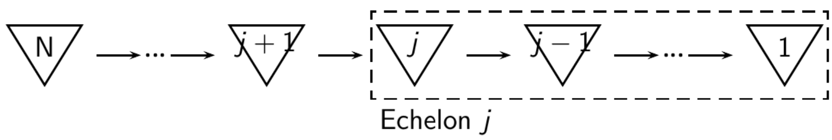

Figure 2.1: A serial system consisting of N installations of which each installation is denoted by an inverted triangle

It is assumed that (1) the review epochs are organised: a review at echelon

$j, j \in \{2, 3, ..., N\}$, triggers a review at echelon $j-1$ after L_j so that the inventory that just arrives at installation j has an opportunity to be shipped out immediately (Chao and Zhou, 2009); (2) initially installation j keeps a supply of multiple Q_{j-1} units on hand to avert immovable residue of the stock (Chen and Zheng, 1994); and (3) for coordination purposes, Q_js and T_js follow the integer-ratio rule with Q_j divisible by Q_{j-1} and T_j divisible by T_{j-1}.

Continuous accounting assesses inventory-related costs at all times. End-of-period accounting evaluates inventory-related costs just before replenishment is received from upstream and just before an order is shipped downstream. The inventory-related costs consist of holding cost and back order cost. The holding cost is charged at h'_j per unit time for each unit of inventory on hand at installation $j, j \in \{1, 2, ..., N\}$, or in transit to installation $j-1, j \in \{2, 3, ..., N\}$. The back order cost is only applicable to the unsatisfied demand at installation 1 and is charged at b per unit per unit time.

To this end, we introduce the following notation. IP_j represents echelon j inventory position which equals the inventory in transit to or at installations $1, 2, ..., j$ minus the back orders at installation 1. Installation 1 inventory position is the same as $IP_1.IL'_j$ represents installation j inventory level which equals inventory on hand minus back orders at installation $j.IP_j(t)$ and $IL'_j(t)$ represent the realization of IP_j and IL'_j at t respectively. t^+ or $(t)^+$ represents the instant immediately after the replenishment activities (ordering, shipping and receiving) at $t.[x]^+ = \max\{x, 0\}.g(d, t)$ represents the probability of having d units of customer demand in t unit time. $G(d, t)$ represents the probability of having at most d units of customer demand in t unit time. To condense mathematical expressions, we define $\tau_{j,p} = L_j + pT_{j-1}, p \in \{0, 1, ..., T_j/T_{j-1} - 1\}$.

2.3 Cost Calculation

In this section, we describe the approach to assessing inventory-related costs. The inventory-related costs at echelon $j, j \in \{2, 3, ..., N\}$, consist of the holding cost of the inventory on hand at installation j, the holding cost of the inventory transferring from installation j to installation $j-1$ and the inventory-related costs at echelon $j-1$. The inventory-related costs at echelon 1 are the holding cost and the back order cost at installation 1. We first show that the holding cost of in-transit inventory is constant and then develop a top-down and recursive method to calculate inventory-related costs.

Let λ denote the mean demand per unit time. The holding cost of in-transit inventory is calculated in the following way. Installation $j, j \in \{2, 3, ..., N\}$, on average supplies installation $j-1$ at a rate of λ per unit time. It takes L_{j-1} unit time to move from installation j to installation $j-1$. By Little's law (Little, 2011), the average amount of inventory on its way to installation $j-1$ is λL_{j-1}. The holding cost of that inventory is $h'_j \lambda L_{j-1}$ per unit time.

Next, we detail the recursive routine to calculate the inventory-related costs at echelon $j, j \in \{2, 3, ..., N\}$. Let t denote a review epoch of installation j. In other words, installation j reviews NIP_j at time t. Let $R, R \in \mathbb{Z}$, be the realization of IP_j at t^+, i.e., $IP_j(t^+) = R$.

Lemma 2.3.1. *(Key Observation) The realization of the customer demand* *in* $(t, t+\tau_{j,p}]$ *can be written in the form of* $D(t, t+\tau_{j,p}] = R - r_{j-1} - i - mQ_{j-1}, i \in \{1, 2, ..., Q_{j-1}\}, m \in \mathbb{Z}$. *Moreover,*

$$IL'_j\big((t+\tau_{j,p})^+\big) = mQ_{j-1}, \tag{2.1}$$

$$IP_{j-1}\big((t+\tau_{j,p})^+\big) = \begin{cases} r_{j-1} + i, & m > 0, \\ r_{j-1} + i + mQ_{j-1}, & m \leq 0, \end{cases} \tag{2.2}$$

Lemma 2.3.1 governs the inventory flow from installation j to installation $j-1.IL'_j$ is always a multiple of Q_{j-1}. At time $(t+\tau_{j,p})^+$, any inventory on hand at installation $j(m > 0)$ indicates that IP_{j-1} has been raised above the reorder point and equals NIP_{j-1}. On the other hand, a shortage of inventory at installation $j(m \leq 0)$ indicates that IP_{j-1} falls short of NIP_{j-1} by $-mQ_{j-1}$ units.

The method calculates the holding cost of the inventory at installation j over T_j interval by interval. IL'_j remains unchanged over $(t+\tau_{j,p}, t+\tau_{j,p+1}]$ which is the time interval between successive replenishment activities at installation j, so end-of-period accounting and continuous accounting will report the same installation j holding cost. Installation j holding cost over $(t+L_j, t+L_j+T_j]$ equals $\sum_{p=0}^{T_j/T_{j-1}-1}\sum_{m=1}^{\infty}\sum_{i=1}^{Q_{j-1}} h'_j T_{j-1} m Q_{j-1} \cdot g(R - r_{j-1} - i - mQ_{j-1}, \tau_{j,p})$. It is simplified into $\sum_{p=0}^{T_j/T_{j-1}-1}\sum_{m=1}^{\infty} h'_j T_{j-1} Q_{j-1} \cdot G(R - r_{j-1} - i - mQ_{j-1}, \tau_{j,p})$.

Let $\pi_j(R)$ denote the inventory-related costs at echelon j over T_j given that the value of IP_j immediately after an instant of installation j ordering is R. The holding costs of the inventory at installation j and the inventory in transit to installation $j-1$ have been calculated above. The inventory-related costs at echelon $j-1$ depend on the realization of IP_{j-1} which is shown in Lemma 2.3.1. Based on these results,

$$\pi_j(R) = \sum_{p=0}^{T_j/T_{j-1}-1} \Bigg[\sum_{m=1}^{\infty} h'_j T_{j-1} Q_{j-1} G(R - r_{j-1} - 1 - mQ_{j-1}, \tau_{j,p})$$
$$+ \sum_{m=1}^{\infty} \sum_{i=1}^{Q_{j-1}} \pi_{j-1}(r_{j-1}+i) g(R - r_{j-1} - i - mQ_{j-1}, \tau_{j,p})$$
$$+ \sum_{m=-\infty}^{0} \sum_{i=1}^{Q_{j-1}} \pi_{j-1}(r_{j-1} + i + mQ_{j-1}) g(R - r_{j-1} - i - mQ_{j-1}, \tau_{j,p}) \Bigg]$$
$$+ h'_j \lambda L_{j-1} T_j. \tag{2.3}$$

Hadley and Whitin Hadley and Whitin (1963) show that NIP_N immediately after a review is uniformly distributed in $\{r_N+1, ..., r_N+Q_N\}$ as long as the demands in different periods are independent. Moreover, since installation $N+1$ has ample stock, $IP_N = NIP_N$. Therefore, the expected total inventory-related costs per unit time is

$$C(r_1, r_2, ..., r_N) = \frac{1}{T_N Q_N} \sum_{i=1}^{Q_N} \pi_N(r_N + i), \tag{2.4}$$

where the function $\pi_j, j \in \{2, 3, ..., N\}$, are recursively calculated as Eq. (2.3).

The function π_1 varies according to the accounting scheme. In the case of continuous accounting, the inventory-related costs at echelon 1 over T_1, given that the value of IP_1 immediately after an instant of installation 1 ordering is $R, R \in \mathbb{Z}$, are shown in Rao (2003) to be

$$\begin{aligned}\pi_{c,1}(R) &= \mathbb{E}\Big[\int_0^{T_1} h_1'\big[R - D(0, L_1 + t]\big]^+ + b\big[D(0, L_1 + t] - R\big]^+ dt\Big] \\ &= (h_1' + b)\sum_{d=0}^{R-1}\int_0^{T_1} G(d, L_1 + t)dt + bT_1\big[\lambda(T_1/2 + L_1) - R\big].\end{aligned} \tag{2.5}$$

In the case of end-of-period accounting, the corresponding costs are shown in Liu and Song (2012) to be

$$\begin{aligned}\pi_{e,1}(R) &= \mathbb{E}\Big[h_1'T_1\big[R - D(0, L_1 + T_1]\big]^+ + bT_1\big[D(0, L_1 + T_1] - R\big]^+\Big] \\ &= (h_1' + b)\sum_{d=0}^{R-1} T_1 \cdot G(d, L_1 + T_1) + bT_1\big[\lambda(T_1 + L_1) - R\big].\end{aligned} \tag{2.6}$$

It is observed from Equation (2.3) that the function π_j needs to call the function π_{j-1} a number of times, the function π_{j-1} needs to call the function π_{j-2} a number of times etc. Each time the function is called, it calculates the holding cost for the inventory that is on its way to the installation downstream. It has been mentioned that the average inventory in transit is constant, so the holding cost for that inventory does not depend on the input into the function. To make the method efficient from a computational point of view, we eliminate the last term $h_j'\lambda L_{j-1}T_j$ from $\pi_j(R)$ in Equation (2.3) and denote the remaining by $\tilde{\pi}_j(R)$. That is, $\forall j = 2, ..., N$,

$$\tilde{\pi}_j(R) = \sum_{p=0}^{T_j/T_{j-1}-1} \left[\sum_{m=1}^{\infty} h'_j T_{j-1} Q_{j-1} G(R - r_{j-1} - 1 - mQ_{j-1}, \tau_{j,p}) \right.$$
$$+ \sum_{m=1}^{\infty} \sum_{i=1}^{Q_{j-1}} \tilde{\pi}_{j-1}(r_{j-1} + i) g(R - r_{j-1} - i - mQ_{j-1}, \tau_{j,p})$$
$$\left. + \sum_{m=-\infty}^{0} \sum_{i=1}^{Q_{j-1}} \tilde{\pi}_{j-1}(r_{j-1} + i + mQ_{j-1}) g(R - r_{j-1} - i - mQ_{j-1}, \tau_{j,p}) \right], \tag{2.7}$$

and $\tilde{\pi}_1(R) = \pi_1(R)$.

It is noted that $\pi_j(R)$ denotes the sum of the inventory-related costs at installations $1, ..., j$ and the holding cost for the inventory in transit. On the other hand, $\tilde{\pi}_j(R)$ represents the sum of the inventory-related costs at installations $1, ..., j$ only. Indeed, $\pi_j(R) - \tilde{\pi}_j(R) = \sum_{k=2}^{j} h'_k \lambda L_{k-1} T_j$. Rewrite $C(r_1, r_2, ..., r_N)$ in Equation (2.4) by using $\tilde{\pi}$, we obtain

$$C(r_1, r_2, ..., r_N) = \frac{1}{T_N Q_N} \sum_{i=1}^{Q_N} \tilde{\pi}_N(r_N + i) + \sum_{k=2}^{N} h'_k \lambda L_{k-1}. \tag{2.8}$$

2.4 Analysis and Discussion

Let $\mathbf{r}$ denote $r_1, r_2, ..., r_N$. We start with the necessary condition for $\mathbf{r}$ to be optimal. Then the necessary condition is proved to be sufficient as well. Based on that result, we specify how to find the optimal reorder points and the procedure is common to both accounting schemes. After that, we compare and contrast the two accounting schemes in terms of inventory-related costs and the optimal reorder points.

Theorem 2.4.1. *Let* $j \in \{1, 2, ..., N\}$*. Given* $\{\mathbf{r}\} \backslash \{r_j\}$*, if* $r_j = r_j^*$ *minimizes Eq. (2.4), then it satisfies*

$CD_1(j) : \pi_j(r_j^* + 1 + Q_j) - \pi_j(r_j^* + 1) \geq h'_{j+1} T_j Q_j$*, and*

$CD_2(j) : \pi_j(r_j^* + Q_j) - \pi_j(r_j^*) \leq h'_{j+1} T_j Q_j$.

Proof. Note that r_j affects the inventory-related costs at installation $j + 1$ and downstream but not upstream.

$$\pi_{j+1}(R|r_j + 1) - \pi_{j+1}(R|r_j) = \left[\pi_j(r_j+1+Q_j)-\pi_j(r_j+1)-h'_{j+1}T_jQ_j\right] \cdot \sum_{p=0}^{T_{j+1}/T_j-1} \sum_{m=1}^{+\infty} g(R-r_j-1-mQ_j, \tau_{j+1,p}). \tag{2.9}$$

Since $C(r_1, ..., r_{j-1}, r_j^*+1, r_{j+1}, ..., r_N) \geq C(r_1, ..., r_{j-1}, r_j^*, r_{j+1}, ..., r_N)$, $\pi_j(r_j^* + 1 + Q_j) \geq \pi_j(r_j^* + 1) + h'_{j+1} T_j Q_j$. Since $C(r_1, ..., r_{j-1}, r_j^*, r_{j+1}, ..., r_N) \leq C(r_1, ..., r_{j-1}, r_j^* - 1, r_{j+1}, ..., r_N)$, $\pi_j(r_j^* + Q_j) \leq \pi_j(r_j^*) + h'_{j+1} T_j Q_j$.

□

Eq. (2.9) derives from the mathematical calculation based on Eq. (2.3) and a deeper understanding of Eq. (2.9) is as follows. At the time of installation j placing an order, if the net inventory in echelon $j + 1$ is $r_j + 1 + mQ_j, m \in \{1, 2, ...\}$, (1) given the reorder point of $r_j + 1, IP_j$ is raised to $r_j + 1 + Q_j$; and (2) given the reorder point of r_j, IP_j is raised to $r_j + 1$ and the batch of size Q_j is stored at installation $j + 1$. If the net inventory in echelon $j + 1$ is any quantity other than $r_j + 1 + mQ_j, m \in \{1, 2, ...\}$, then IP_j immediately after the placement of the order remains unchanged by the increase in the reorder point and thus inventory-related costs remain unchanged.

Lemma 2.4.2. *Let $j \in \{1, 2, ..., N\}$. Given that r_i satisfies $CD_2(i), \forall i = 1, ..., j-1$, then $\pi_j(R + Q_j) - \pi_j(R)$ is non-decreasing with R.*

Proof. Prove by induction. Since $\pi_1(R)$ is convex, $\pi_1(R + Q_1) - \pi_1(R)$ is non-decreasing with R.

Assume that $\pi_j(R+Q_j) - \pi_j(R)$ is non-decreasing with $R, \forall j = 1, 2, ..., k$ where $k \in \{1, 2, ..., N-1\}$.

For $j = k + 1$,

$$
\begin{aligned}
&\big[\pi_{k+1}(R+1+Q_k) - \pi_{k+1}(R+1)\big] - \big[\pi_{k+1}(R+Q_k) - \pi_{k+1}(R)\big] \\
= &\sum_{p=0}^{T_{k+1}/T_k-1} \Big\{h'_{k+1}T_kQ_k - \big[\pi_k(r_k+Q_k) - \pi_k(r_k)\big]\Big\} \cdot g(R - r_k, \tau_{k+1,p}) \\
+ &\sum_{p=0}^{T_{k+1}/T_k-1} \sum_{d=R+1-r_k}^{+\infty} \\
&\Big\{\big[\pi_k(R+1+Q_k-d) - \pi_k(R+1-d)\big] - \big[\pi_k(R+Q_k-d) - \pi_k(R-d)\big]\Big\} \\
&\cdot g(d, \tau_{k+1,p}).
\end{aligned}
\tag{2.10}
$$

Since $\pi_k(R+Q_k) - \pi_k(R)$ is non-decreasing with R by assumption, $\big[\pi_{k+1}(R+1+Q_k) - \pi_{k+1}(R+1)\big] - \big[\pi_{k+1}(R+Q_k) - \pi_{k+1}(R)\big] \geq 0$. In addition, r_k satisfies $CD_2(k)$. Thus Eq. (2.10)≥ 0 and $\pi_{k+1}(R+Q_k) - \pi_{k+1}(R)$ is non-decreasing with R. Consequently, $\pi_{k+1}(R+Q_{k+1}) - \pi_{k+1}(R)$ is non-decreasing with R.

□

Lemma 2.4.2 states that when one more batch is shipped over from upstream, the marginal cost at this echelon increases as the inventory there goes high. It is because as inventory increases, a decreasing proportion of that batch of replenishment fulfills demand and an increasing proportion remains idle. Moreover, when inventory is low, one more batch of replenishment reduces back orders and back order cost, so $\pi_j(R+Q_j) - \pi_j(R) < 0$. When inventory is high and one more batch is added, holding cost increases and $\pi_j(R+Q_j) - \pi_j(R) > 0$.

Theorem 2.4.3. *Given* $\{\mathbf{r}\}\backslash\{r_j\}$ *such that* r_i *satisfies* $CD_2(i), \forall i = 1, 2, ..., j-1$. *If* $r_j = r_j^*$ *satisfies* $CD_1(j)$ *and* $CD_2(j)$, *then* $r_j = r_j^*$ *minimizes* $C(\mathbf{r})$.

Proof. If r_i satisfies $CD_2(i), \forall i = 1, 2, ..., j-1$, then $\pi_j(R+Q_j) - \pi_j(R)$ is non-decreasing with R by Lemma 2.4.2. In addition, r_j^* satisfies $CD_2(j)$, so

$$\forall r_j < r_j^*, \pi_j(r_j + 1 + Q_j) - \pi_j(r_j + 1) \leq h'_{j+1} T_j Q_j \tag{2.11}$$

and thus $C(r_1, ..., r_{j-1}, r_j + 1, r_{j+1}, ..., r_N) \leq C(r_1, ..., r_{j-1}, r_j, r_{j+1}, ..., r_N)$; r_j^* satisfies $CD_1(j)$, so

$$\forall r_j \geq r_j^*, \pi_j(r_j + 1 + Q_j) - \pi_j(r_j + 1) \geq h'_{j+1} T_j Q_j \tag{2.12}$$

and thus $C(r_1, ..., r_{j-1}, r_j + 1, r_{j+1}, ..., r_N) \geq C(r_1, ..., r_{j-1}, r_j, r_{j+1}, ..., r_N)$. So $C(\mathbf{r})$ is non-increasing with $r_j \in (-\infty, r_j^*]$ and non-decreasing with $r_j \in [r_j^*, +\infty)$ and thus $r_j = r_j^*$ is optimal.

□

Theorems 2.4.1 and 2.4.3 show that $CD_1(j)$ and $CD_2(j)$ are jointly necessary and sufficient conditions for r_j^* to be optimal. Based on this result, the procedure for searching for the optimal reorder points is as follows. Start with searching for r_1^* that satisfies $CD_1(1)$ and $CD_2(1)$ and set $r_1 = r_1^*$. Then search for r_2^* that satisfies $CD_1(2)$ and $CD_2(2)$ and set $r_2 = r_2^*$, etc. After this procedure completes, we will get $\mathbf{r} = \mathbf{r}^*$ that minimizes $C(\mathbf{r})$.

2.4.1 Analytical Comparison

In this section, we compare $\mathbf{r}_e^*$ that minimizes the inventory-related costs assessed by end-of-period accounting (denoted by $C_e(\mathbf{r})$) with $\mathbf{r}_c^*$ that minimizes the inventory-related costs assessed by continuous accounting (denoted by $C_c(\mathbf{r})$). We prove that $r_{e,j}^*$ is at least $r_{c,j}^*$ for all $j = 1, 2, ..., N$. Thus mistakenly using $\mathbf{r}_e^*$ as the reorder point of inventory whose costs accumulate continuously poses a risk of overstocks.

Define $f(R)$ as the difference between installation 1 inventory-related costs by the two accounting schemes. That is,

$$\begin{aligned}
&f(R)\\
=&\pi_{c,1}(R) - \pi_{e,1}(R)\\
=&(h_1' + b)\int_{L_1}^{L_1+T_1}\sum_{d=0}^{R-1} G(d,t)dt - (h_1' + b)T_1\sum_{d=0}^{R-1} G(d, L_1 + T_1) - \frac{1}{2}b\lambda T_1^2.
\end{aligned} \tag{2.13}$$

Lemma 2.4.4. *(1)* $\forall R \leq 0, f(R) = -\frac{1}{2}b\lambda T_1^2$;

(2) $\lim_{R\to+\infty} f(R) = \frac{1}{2}h_1'\lambda T_1^2$.

Proof. (1) is a straightforward calculation.

To prove (2), consider a situation in which installation 1 always has inventory on hand to meet demand. The demand during a period of T_1 is expected to be λT_1. Consider the stock that is withdrawn from inventory to meet that demand and the expected holding cost of that stock. Continuous accounting calculates cost on a continuous basis and the expected holding cost under consideration is found to be $\frac{1}{2}h'_1\lambda T_1^2$. End-of-period accounting assesses inventory only at the end of the period when the stock has been withdrawn, so the expected holding cost under consideration is zero. Thus, $f(R)$ is $\frac{1}{2}h'_1\lambda T_1^2$.

□

Lemma 2.4.4 shows the limiting behavior of the difference between installation 1 inventory-related costs by the two accounting schemes. When there is no inventory at installation 1, the conversion from end-of-period accounting to continuous accounting reduces inventory-related costs by $\frac{1}{2}b\lambda T_1^2$. On the other hand, when there is sufficient inventory to prevent stockouts from occurring, staying with continuous accounting rather than converting to end-of-period accounting increases the inventory-related costs by $\frac{1}{2}h'_1\lambda T_1^2$.

Lemma 2.4.5. *$f(R)$ increases with R.*

Proof. The first order difference of $f(R)$ in R is

$$\begin{aligned}&f(R+1)-f(R)\\=&(h'_1+b)\left[\int_{L_1}^{L_1+T_1} G(R,t)dt - T_1G(R,L_1+T_1)\right] \qquad (2.14)\\\geq&0.\end{aligned}$$

□

Lemma 2.4.5 shows that the difference between installation 1 inventory-related costs by the two accounting schemes is an increasing function of installation 1 inventory position. Moreover, as shown in Lemma 2.4.4, that difference is bounded by $-\frac{1}{2}b\lambda T_1^2$ below and by $\frac{1}{2}h_1'\lambda T_1^2$ above. Figure 2.2 depicts the shape of $f(R)$.

The shape of $f(R)$ indicates the situations in which $f(R)$ is substantial or negligible. It is noticed that $f(R)$ attains the lower bound when $R \leq 0$ and $f(R)$ gets closer and closer to the upper bound as R gets closer to infinity. When $b\lambda T_1^2$ is large, the lower bound $-\frac{1}{2}b\lambda T_1^2$ is located far below zero. In this case, a small R makes $f(R)$ sufficiently negative and thus the difference between the two accounting schemes is substantial. When $h_1'\lambda T_1^2$ is large, the upper bound $\frac{1}{2}h_1'\lambda T_1^2$ is located far above zero. In this case, a large R makes $f(R)$ sufficiently positive and thus the difference between the two accounting schemes is also substantial. On the other hand, when $b\lambda T_1^2$ and $h_1'\lambda T_1^2$ are small and/or the inventory is at an intermediate level, the difference between the two accounting schemes is negligible.

Lemma 2.4.6. $\forall j = 1, ..., N$, $\pi_{c,j}(R+Q_j|\mathbf{r}_c^*) - \pi_{c,j}(R|\mathbf{r}_c^*) \geq \pi_{e,j}(R+Q_j|\mathbf{r}_e^*) - \pi_{e,j}(R|\mathbf{r}_e^*)$.

Proof. Prove by induction.

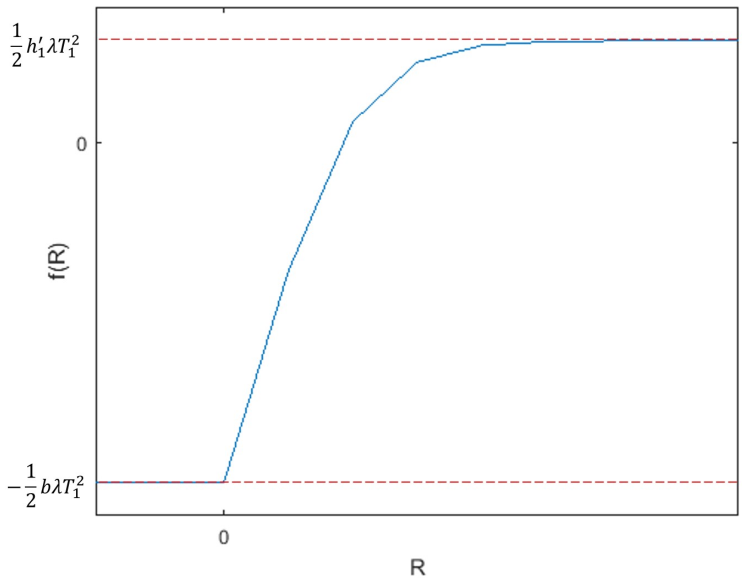

Figure 2.2: $f(R)$ in Equation (2.13) is an increasing function of R and bounded by $-\frac{1}{2}b\lambda T_1^2$ below and by $\frac{1}{2}h_1'\lambda T_1^2$ above

When $j = 1$,

$$\begin{aligned}
&\left[\pi_{c,1}(R+Q_1|\mathbf{r}_c^*) - \pi_{c,1}(R|\mathbf{r}_c^*)\right] - \left[\pi_{e,1}(R+Q_1|\mathbf{r}_e^*) - \pi_{e,1}(R|\mathbf{r}_e^*)\right] \\
=&\left[\pi_{c,1}(R+Q_1|\mathbf{r}_c^*) - \pi_{e,1}(R+Q_1|\mathbf{r}_e^*)\right] - \left[\pi_{c,1}(R|\mathbf{r}_c^*) - \pi_{e,1}(R|\mathbf{r}_e^*)\right] \\
=&f(R+Q_1) - f(R) \\
\geq&0,
\end{aligned} \tag{2.15}$$

where the last inequality follows from Lemma 2.4.5.

Assume $\forall j = 1, ..., k, \pi_{c,j}(R+Q_j|\mathbf{r}_c^*) - \pi_{c,j}(R|\mathbf{r}_c^*) \geq \pi_{e,j}(R+Q_j|\mathbf{r}_e^*) - \pi_{e,j}(R|\mathbf{r}_e^*)$.

When $j = k + 1$,

$$
\begin{aligned}
&\pi_{k+1,e}(R + Q_k|\mathbf{r}_e^*) - \pi_{k+1,e}(R|\mathbf{r}_e^*) \\
=& \sum_{p=0}^{T_{k+1}/T_k - 1} \Big\{ h'_{k+1} T_k Q_k \cdot G(R - r_{e,k}^* - 1, \tau_{k+1,p}) \\
&\quad + \sum_{d=R-r_{e,k}^*}^{+\infty} \big[\pi_{e,k}(R + Q_k - d|\mathbf{r}_e^*) - \pi_{e,k}(R - d|\mathbf{r}_e^*)\big] \cdot g(d, \tau_{k+1,p}) \Big\} \\
\leq& \sum_{p=0}^{T_{k+1}/T_k - 1} \Big\{ h'_{k+1} T_k Q_k \cdot G(R - r_{c,k}^* - 1, \tau_{k+1,p}) \\
&\quad + \sum_{d=R-r_{c,k}^*}^{+\infty} \big[\pi_{e,k}(R + Q_k - d|\mathbf{r}_e^*) - \pi_{e,k}(R - d|\mathbf{r}_e^*)\big] \cdot g(d, \tau_{k+1,p}) \Big\} \\
\leq& \sum_{p=0}^{T_{k+1}/T_k - 1} \Big\{ h'_{k+1} T_k Q_k \cdot G(R - r_{c,k}^* - 1, \tau_{k+1,p}) \\
&\quad + \sum_{d=R-r_{c,k}^*}^{+\infty} \big[\pi_{c,k}(R + Q_k - d|\mathbf{r}_c^*) - \pi_{c,k}(R - d|\mathbf{r}_c^*)\big] \cdot g(d, \tau_{k+1,p}) \Big\} \\
=& \pi_{k+1,c}(R + Q_k|\mathbf{r}_c^*) - \pi_{k+1,c}(R|\mathbf{r}_c^*),
\end{aligned}
\tag{2.16}
$$

where the first inequality results from $\forall S \leq r_{e,k}^*, \pi_{e,k}(S+Q_k|\mathbf{r}_e^*) - \pi_{e,k}(S|\mathbf{r}_e^*) \leq h'_{k+1} T_k Q_k$; the second inequality is the assumption for $j = k$. Consequently, $\pi_{k+1,c}(R + Q_{k+1}|\mathbf{r}_c^*) - \pi_{k+1,c}(R|\mathbf{r}_c^*) \geq \pi_{k+1,e}(R + Q_{k+1}|\mathbf{r}_e^*) - \pi_{k+1,e}(R|\mathbf{r}_e^*)$.

□

Theorem 2.4.7. $\mathbf{r}_e^* \geq \mathbf{r}_c^*$.

Proof. $\forall j = 1, ..., N$,

$$\begin{aligned}
&\pi_{e,j}(r^*_{c,j} + Q_j|\mathbf{r}^*_e) - \pi_{e,j}(r^*_{c,j}|\mathbf{r}^*_e) \\
\leq&\pi_{c,j}(r^*_{c,j} + Q_j|\mathbf{r}^*_c) - \pi_{c,j}(r^*_{c,j}|\mathbf{r}^*_c) \\
\leq&h'_{j+1}T_jQ_j,
\end{aligned} \tag{2.17}$$

where the first inequality results from Lemma 2.4.6. Thus $r^*_{e,j} \geq r^*_{c,j}$.

□

Theorem 2.4.7 states that the reorder points minimizing the continuously accrued inventory-related costs are no higher than those minimizing the periodically accumulated inventory-related costs. That can be plainly explained as follows. The deployment of $\mathbf{r}^*_e$ on average balances the holding cost and the back order cost of the ending inventory which is the lowest during a period. For the same reorder points, continuous accounting gives a higher holding cost and a lower back order cost than end-of-period accounting. Thus the holding cost is outstripping the back order cost, so the reorder points should be decreased to redress the balance.

2.4.2 Numerical Tests

Section 2.3 enunciates that the two accounting schemes differ in calculating the inventory-related costs at installation 1 only. The difference, which is proved to be bounded, causes end-of-period accounting to overshoot reorder points for all echelons, so continuous accounting enjoys advantages in terms of accuracy. This section does numerical tests and shows that continuous accounting suffers disadvantages in computation time. It also discusses how the advantages and disadvantages of using continuous accounting are magnified or attenuated by the measurement of $h_1', \lambda, T_1, L_1, Q_1$ and N with b normalized to be 1.

The customer demand is a Poisson process with rate λ. The details of the base for the numerical tests are $h_1' = 0.3; b = 1; \lambda = 10; T_1 = 1; L_1 = 0.3; Q_1 = 1; N = 1$. While the study of a particular parameter is carried out, the measurement of that parameter varies from test to test and others remain the same as the base. The design of the numerical tests is shown in Table 2.1. Each numerical test records (1) the inventory-related costs $C_c(\mathbf{r}_e^*)$ and $C_c(\mathbf{r}_c^*)$; (2) the amounts of time to calculate $C_e(\mathbf{r}_c^*)$ and $C_c(\mathbf{r}_c^*)$, which are denoted by $t(C_e(\mathbf{r}_c^*))$ and $t(C_c(\mathbf{r}_c^*))$ respectively; (3) the cost difference $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$; and (4) the time difference $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$. The cost differences and time differences are plotted on Figures 2.3, 2.4, 2.5, 2.6, 2.7 and 2.8.

Parameter	Range	Step-size
h_1'	$[0.3, 0.6]$	0.03
λ	$[10, 20]$	1
T_1	$[1, 2]$	0.1
L_1	$[0.3, 0.6]$	0.03
Q_1	$[1, 10]$	1
N	$[1, 5]$ with $h_j' = 0.3 - (j-1)(0.3/N), T_j = 1, L_j = 0.3/N, Q_j = 1, j \in \{1, 2, ..., N\}$	1

Table 2.1: Design of the numerical tests

All figures show that the time difference $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*)) > 0$ and thus continuous accounting has disadvantages in computation time. The time difference is big when the holding cost rate is low, customer demand is high, reorder intervals are long, lead times are long, batch sizes are large or the number of installations is large. Those are the situations in which installation 1 inventory position is high. A high value of R causes $\pi_{c,1}(R)$ in Equation (2.5) to incur far more computation time than $\pi_{e,1}(R)$ in Equation (2.6).

It is worth noticing that while the time difference varies linearly with most parameters, it increases exponentially with the number of installations N. When the number of installations increases from j to $j+1$, the function C in Equation (2.8) calls the function $\tilde{\pi}_{j+1}$ in Equation (2.3) Q_{j+1} times. Each time the function $\tilde{\pi}_{j+1}$ is called, it calls the function $\tilde{\pi}_j$ a number of times etc. In other words, lengthening the serial inventory system by one installation causes several more "function trees" to be called. As end-of-period accounting spends shorter computation time on each leaf (i.e., the function $\tilde{\pi}_1$), it spends substantially shorter computation time on the function trees than continuous accounting.

All figures show that the cost difference $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*) > 0$. They demonstrate that continuous accounting has the merit of accurately arriving at the optimal reorder points. Further, the cost difference generally increases with h_1', λ and T_1. When these parameters are small, $f(R)$ in Equation (2.13) is close to zero. In that situation, the two accounting schemes produce similar inventory-related costs and thus $\mathbf{r}_e^*$ almost equals $\mathbf{r}_c^*$. As a result, the cost difference is small. However, when h_1', λ or T_1 is large, $f(R)$ can be large, which gives rise to substantially dissimilar reorder points. Therefore, Figures 2.3, 2.4 and 2.5 show an upward trend in the cost difference.

In comparison, the cost difference does not exhibit a clear trend as L_1

and Q_1 increase. While L_1 affects the rate of increase in $f(R)$ and Q_1 has indirect effects on the value of $R, f(R)$ is determined by h'_1, b, λ and T_1 for the most part. With respect to N, since $T_j = 1$ for all $j \in \{1, 2, ..., N\}$, it can be proved that when reorder points are optimized, $\tilde{\pi}_N(R)$ has the same value as $\pi_1(R)$ in a single-installation inventory system with the lead time equal to $\sum_{j=1}^{N} L_j (= 0.3)$. The inventory-related costs also consist of the holding cost of the inventory in transit and that holding costs calculated by the two accounting schemes are equal. Therefore, the cost difference does not relate to N.

In summary, continuous accounting is appropriate for inventory systems with a small number of installations. It helps to achieve substantial cost-savings when the cost parameters are large, customer demand is high or reorder intervals are long.

2.5 Conclusions

This chapter develops a method to evaluate the inventory-related costs that are incurred continuously for serial inventory systems. The method facilitates searching for the optimal reorder points that are otherwise overshot by mistakenly using end-of-period accounting. Future study can extend this top-down and recursive method to evaluate other multi-echelon inventory systems, such as one-warehouse multi-retailer systems, where continuous accounting of inventory-related costs are needed. Furthermore, the method deals with installations separately and can be deployed in those systems of which each installation defines its own accounting scheme.

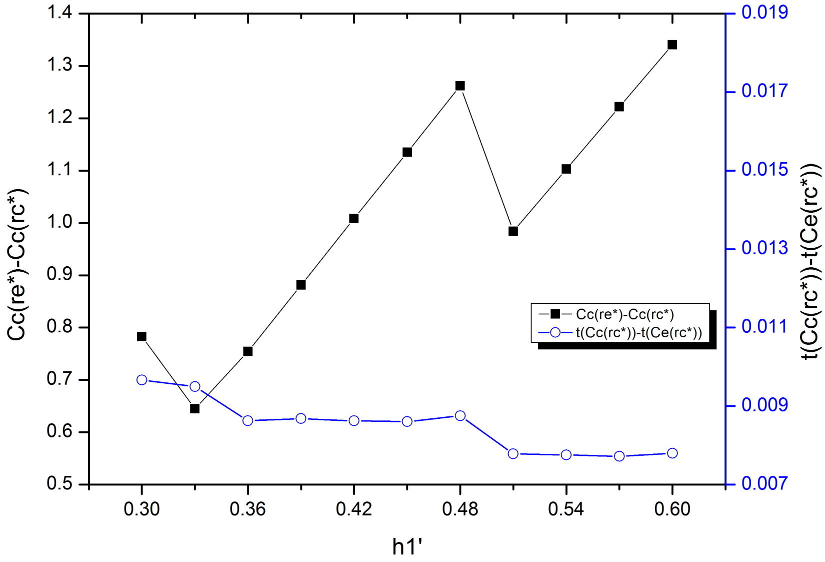

Figure 2.3: $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$ generally increases with h_1' and $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$ decreases linearly with h_1'

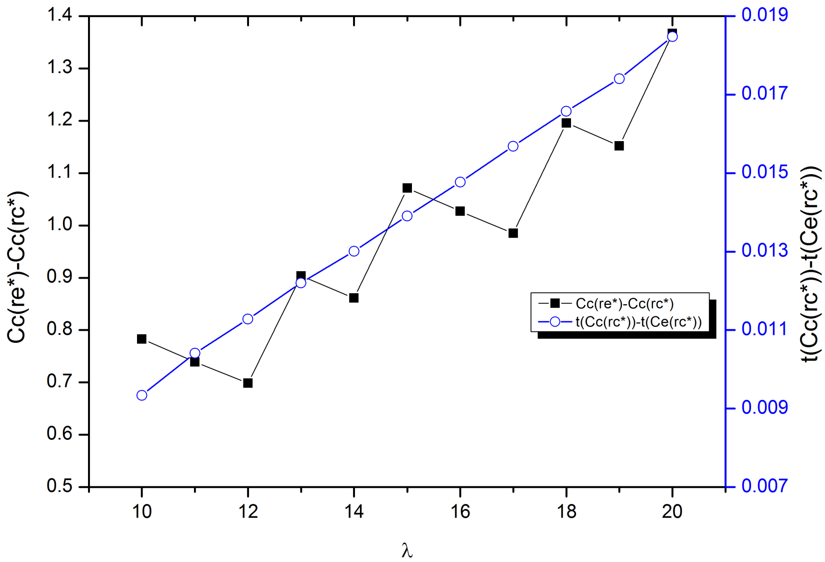

Figure 2.4: $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$ generally increases with λ and $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$ increases linearly with λ

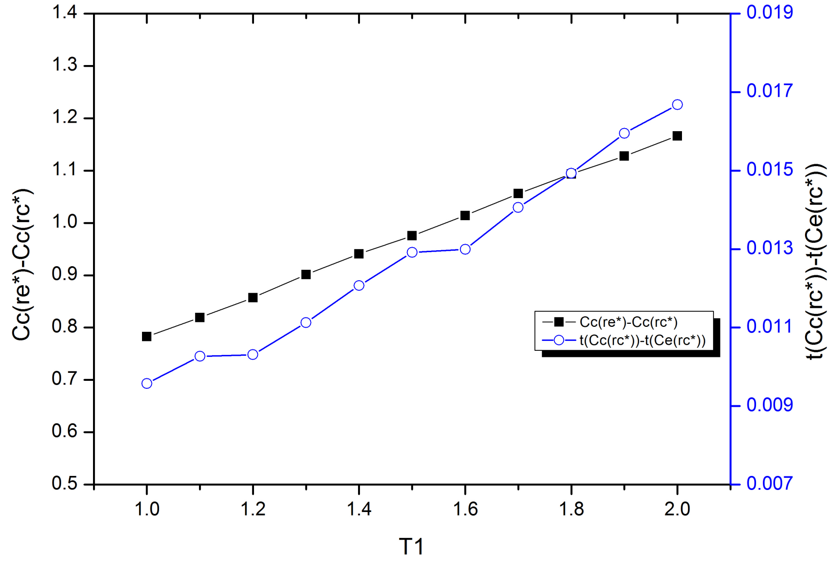

Figure 2.5: $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$ generally increases with T_1 and $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$ increases linearly with T_1

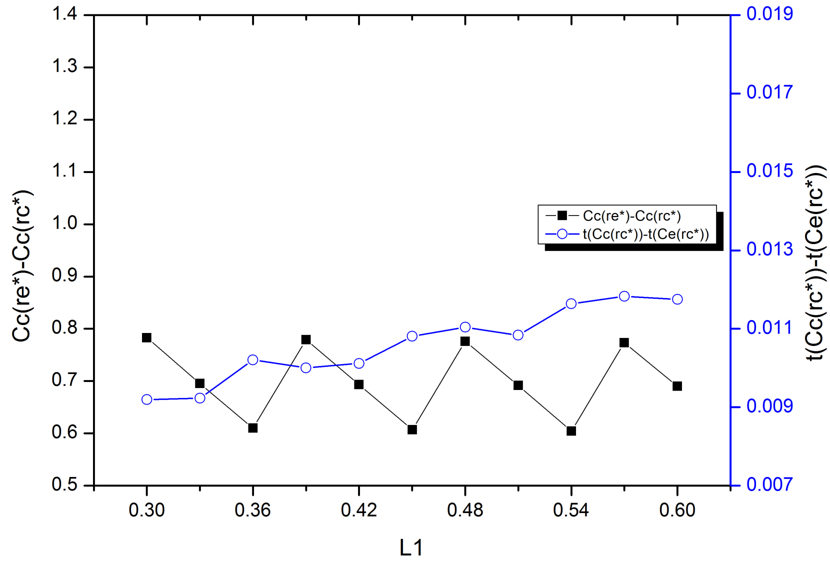

Figure 2.6: $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$ does not exhibit a clear trend and $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$ increases linearly with L_1

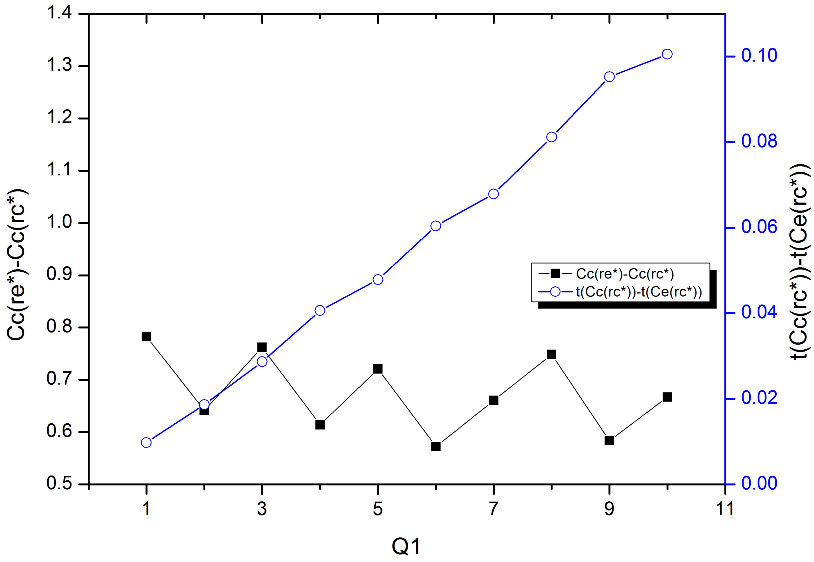

Figure 2.7: $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$ does not exhibit a clear trend and $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$ increases linearly with Q_1

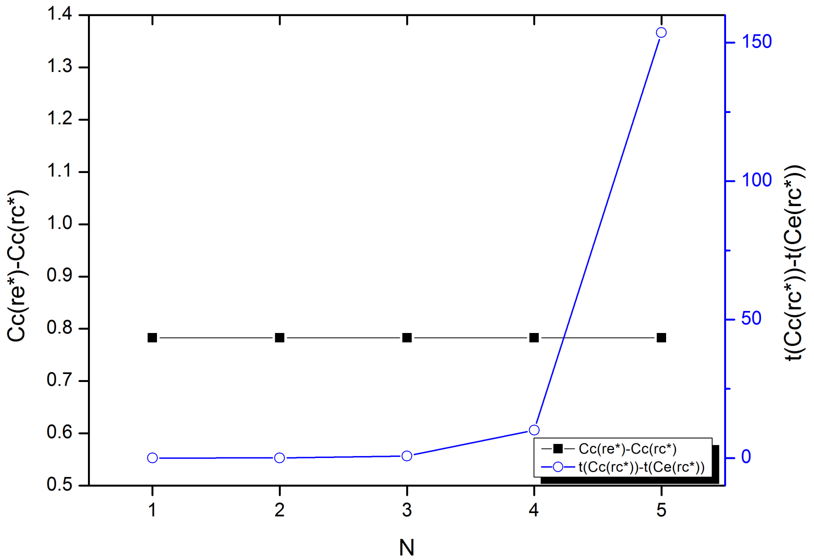

Figure 2.8: $C_c(\mathbf{r}_e^*) - C_c(\mathbf{r}_c^*)$ does not relate to N and $t(C_c(\mathbf{r}_c^*)) - t(C_e(\mathbf{r}_c^*))$ increases exponentially with N

The chapter treats reorder points as decision variables and assesses the impact of accounting schemes on reorder points and inventory-related costs. Future study can also incorporate the cost of orders into the model, treat reorder intervals and batch sizes as decision variables and study how these decision variables are affected by accounting schemes.

Chapter 3

Replenishment and Redistribution of Inventory in a Firm That Sells in Two Channels

3.1 Introduction

Finished goods are moved from the end of production to customers through a distribution inventory system. In these systems, distribution inventory usually accounts for 25% to 30% of distribution costs (Arnold et al., 2013). Therefore, improvement in the efficiency of distribution inventory management, which is typically excised by deciding the order size and the reorder interval for new supplies and the distribution of the stock among stock points, represents considerable savings in distribution costs.

While planning the distribution of the stock, some distribution inventory systems allow transferring goods between the retailers as well as shipping from the supplier. Such examples include Footlocker, Macy's, and a group of chip manufacturers (NEC, Toshiba) sharing a common supplier, ASML (Lien et al., 2011). The movement of goods between the locations in the same echelon of a distribution inventory system is called lateral transshipment (Glazebrook et al., 2015).

Lateral transshipment is a remedy for the situation where one retailer is at the risk of facing stockouts while another has a high level of stock. If lateral transshipment is permitted, the former retailer (the requesting retailer) can get goods from the latter (the requested retailer) rather than wait for the supplier to replenish its stock some time later. Consequently, the cost associated with stockouts at the requesting retailer and the holding cost at the requested retailer are reduced. Lateral transshipment helps realize not only the pooling of inventory but also the pooling of storage capacity (Qiu and Huang, 2011).

Reactive transshipment and proactive transshipment are two common types of lateral transshipment and they differ in the triggering mechanisms and purposes. Reactive transshipment (also called emergency transshipment) is triggered when a stockout occurs and meant to cover the shortfall in supply. Interested readers may refer to Paterson et al. (2011) for a review. Proactive transshipment (also called preventive transshipment) is scheduled for the purpose of balancing stocks among the retailers so as to meet future demand (Tiacci and Saetta, 2011). There are other types of lateral transshipment, such as the hybrid lateral transshipment in Paterson et al. (2012) and

Glazebrook et al. (2015), combining the features of those two common types. Seidscher and Minner (2013) compare the two types of lateral transshipment and show that proactive transshipment has a big advantage over reactive transshipment in terms of the cost efficiency when the inventory pooling opportunity is intermediate. Furthermore, proactive transshipment is planned to satisfy demand over a number of periods, so it is naturally employed in systems under periodic reviews (Paterson et al., 2011).

In distribution inventory systems with proactive transshipment, a policy is often selected to meet a service level or to minimize costs. Given a service-level target, Jönsson and Silver (1987) study one-warehouse multi-retailer systems and make a comparison between the system with and without transshipment. Numerical results show that the former is more advantageous than the latter when demand is dispersed, the reorder interval is long, the number of retailers is large, the service level is high or lead times are short. Tagaras and Vlachos (2002) investigate the benefits of transshipment when the model consists of only two retailers. They conclude that the benefits are substantial only when demand is dispersed.

When a policy is selected to minimize costs, literature often builds models by formulating stochastic programming problems. The objective function is the total cost over a predetermined number of periods and the decision variables are related to each individual replenishment and transshipment. Das (1975) is one of the first papers that consider proactive transshipment in the modeling of distribution inventory systems. The optimal ordering and transfer rules are characterized for the single period model. Özdemir et al. (2006) consider capacitated transshipment between stock points in one period. They represent the problem of transshipment as a capacitated network flow problem and determine order-up-to levels by a numerical solution procedure based on simulation. Lien et al. (2011) study the design for one-period transshipment networks and demonstrate the cost efficiency of the chain configurations (i.e., all retailers are linked in one connected loop). Abouee-Mehrizi et al. (2015) study a multi-period inventory system with lost sales and prove that the optimal replenishment and transshipment polices can be determined from four switching curves. The papers above assume that the instant in time when the transshipment takes place is predetermined. Nevertheless, Agrawal et al. (2004) develop a model where the timing of transshipment is a decision variable. Numerical results show that the benefits of determining the timing according to demand are substantial in a system with balanced starting inventories or high service levels.

However, while investigating inventory control policies for distribution inventory systems with proactive transshipment, literature often implicitly assumes that retailers' orders are not consolidated and the supplier replenishes retailers' stocks directly. This assumption is observable in the modeling where each retailer places an order at the supplier to raise its inventory position to a certain level. As a result, the pooling of demand during replenishment lead time and its associated cost-savings are ignored.

More importantly, literature mainly predetermines system's reorder interval to be one period or a number of periods. To the best of my knowledge, there is no literature in the field of proactive transshipment treating the reorder interval as a decision variable. However, when the system incurs a high fixed cost for each order it places at the supplier, it is necessary to optimize the reorder interval as well. Furthermore, optimizing the reorder intervals for distribution inventory systems has attracted a great deal of attention of academia. Interested readers may refer to Shang et al. (2015) for a review.

This chapter studies inventory control policies in a distribution inventory system with proactive transshipment. New supplies are ordered periodically from an exogenous supplier and the replenishment arrives at the system after a fixed replenishment lead time. Inventory is redistributed among all stock points in every period. The contribution of this chapter is threefold. First, the pooling of demand during the replenishment lead time is modeled. Second, when the optimal proactive transshipment takes place in each period, the inventory-related cost during a reorder interval proves to be convex. Third, the method to determine the optimal reorder interval is developed.

3.2 Model

This chapter considers a one-warehouse multi-retailer system where there are $N-1$ retailers meeting customer demand and the warehouse may also meet customer demand (e.g., ship online orders). The warehouse orders from an exogenous supplier at a cost of K per order. At each instant of ordering, the system inventory position (=inventory on order +inventory on hand -back orders) is raised to the order-up-to level S. The warehouse orders periodically and the length of the reorder interval is n periods (to be determined) in which each period is of length T. The lead time between an order placement at the supplier and the corresponding order receipt at the warehouse is L. At the time when the order is received at the warehouse, all locations review their respective inventories and redistribute the total inventory among themselves. The redistribution of inventory takes place in each period. See Figure 3.1 for the timeline for the events.

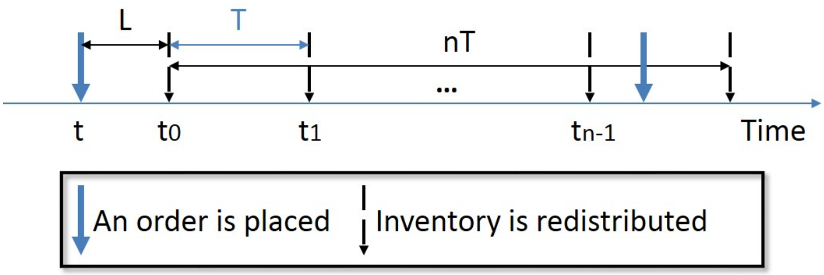

Figure 3.1: The timeline for the events

The warehouse is named location 0 and the retailers are numbered from 1 to $N-1$. The inventory on hand in location $i, i = 0, 1, ..., N-1$, is charged at h_i per unit per unit time. Unsatisfied demand units are back-ordered and back orders are charged at b per unit per unit time. Customer demands during non-overlapping intervals of equal length are independent, identically distributed and non-negative integers. The rate of customer demand in location i is λ_i.

The objective is to minimize the average total cost per period. Decision variables are the reorder interval, the order-up-to level and the redistributed inventories in each period.

It is assumed that the proactive transshipment is instant. This assumption is appropriate for the situations where the warehouse and the retailers are located in close proximity or the deliveries take place during close hours (e.g., overnight). Moreover, shipping and handling fees associated with the proactive transshipment are negligible compared with the cost of ordering. For example, when the warehouse orders from overseas, it spends much on cross-border transportation, handling, documentation and administrative overhead. That creates a situation in which the ordering costs incurred are substantially higher than the proactive transshipment costs.

3.3 The Optimal Solution

This section provides an algorithm on how to compute the optimal redistribution of inventory. When the optimal redistribution takes place in each period, the system inventory-related cost during n periods which is a function of the order-up-to level proves to be convex. Finally an upper bound of the optimal n is developed.

3.3.1 The Redistribution of Inventory

At the time of the redistribution, back orders are fulfilled and what remains of the inventory on hand, i.e., the net inventory, is redistributed among the N locations. If the total of back orders exceeds that of inventory on hand, surplus back orders remain unmet. The back order cost per unit per unit time is the same across the N locations, so the sequence in which back orders are fulfilled (first come, first served; last come, first served etc) does not matter.

First, the inventory-related cost in one location and its convexity, both of which are well-established in the literature, are revisited. Let I_i be the realization of the redistributed inventory level (=inventory on hand - back orders) in location i at the beginning of a period (say, at time t_p). The customer demand there during $(t_p, t_p + \delta]$ only relates to δ and is a random variable denoted by $D_i(\delta)$. In addition, there is no redistribution of inventory during $(t_p, t_p + \delta] \subset (t_p, t_p + T]$. Thus, when $\delta < T$, the inventory level in location i at time $t_p + \delta$ is $I_i - D_i(\delta)$. If $I_i - D_i(\delta) > 0$, the inventory-related cost in location i at time $t_p + \delta$ is $h_i(I_i - D_i(\delta))$. If $I_i - D_i(\delta) \leq 0$, the inventory-related cost in location i at time $t_p + \delta$ is $-b(I_i - D_i(\delta))$. Let $[x]^+$ denote $\max\{x, 0\}$ and $[x]^-$ denote $\max\{-x, 0\}$. Then the inventory-related cost in location i during $(t_p, t_p + T]$ is a function of I_i and

$$
\begin{aligned}
&\mathfrak{C}_i(I_i) \\
=&\mathbb{E}\Big[\int_0^T h_i\Big[I_i - D_i(\delta)\Big]^+ + b\Big[I_i - D_i(\delta)\Big]^- d\delta\Big] \\
=&(h_i + b)\int_0^T \sum_{d=0}^{I_i - 1} G_i(d, \delta) d\delta + \frac{1}{2} b\lambda_i T^2 - bI_iT
\end{aligned}
\tag{3.1}
$$

where $G_i(d, \delta)$ is the probability of having at most d units of customer demand in location i in δ unit time.

Lemma 3.3.1. $\mathfrak{C}_i(I_i)$ *is convex.*

Proof. The second order difference of $\mathfrak{C}_i(I_i)$ is $\big[\mathfrak{C}_i(I_i + 2) - \mathfrak{C}_i(I_i + 1)\big] - \big[\mathfrak{C}_i(I_i + 1) - \mathfrak{C}_i(I_i)\big] = (h_i + b)\int_0^T g_i(I_i + 1, \delta) d\delta$ where $g_i(I_i + 1, \delta)$ is the probability of having $I_i + 1$ units of customer demand in location i in δ unit time. Consequently, the second order difference is non-negative and thus $\mathfrak{C}_i(I_i)$ is convex.

□

Next, the inventory-related cost in two locations and the redistribution of inventory between the two locations are studied. The inventory-related cost in locations i and j during $(t_p, t_p + T]$ is denoted by $\mathfrak{C}_{i,j}(I_i, I_j)$ and $\mathfrak{C}_{i,j}(I_i, I_j) = \mathfrak{C}_i(I_i) + \mathfrak{C}_j(I_j)$. It is noted that (I_i, I_j) is one way to redistribute a total of $I_i + I_j$ units of inventory between the two locations. There are others ways to do that such as $(I_i + 1, I_j - 1)$ and $(I_i - 1, I_j + 1)$.

Theorem 3.3.2. *(I_i, I_j) is the optimal redistribution of $I_i + I_j$ units of inventory between two locations i and j if and only if*

$$\begin{cases} -(h_i + b)\int_0^T G_i(I_i + I_j - I'_j - 1, \delta)d\delta + (h_j + b)\int_0^T G_j(I'_j, \delta)d\delta \leq 0, & \forall I'_j < I_j, \\ -(h_i + b)\int_0^T G_i(I_i + I_j - I'_j - 1, \delta)d\delta + (h_j + b)\int_0^T G_j(I'_j, \delta)d\delta \geq 0, & \forall I'_j \geq I_j. \end{cases} \tag{3.2}$$

Proof. When the two locations have a total of $I_i + I_j$ units of inventory and the redistributed inventory level in location j is I'_j, the redistributed inventory level in location i is $I_i + I_j - I'_j$. The inventory-related cost in locations i and j during $(t_p, t_p + T]$ is $\mathfrak{C}_{i,j}(I_i + I_j - I'_j, I'_j)$ which is a function of I'_j. The first order difference of $\mathfrak{C}_{i,j}(I_i + I_j - I'_j, I'_j)$ is

$$\begin{aligned} &\mathfrak{C}_{i,j}(I_i + I_j - (I'_j + 1), I'_j + 1) - \mathfrak{C}_{i,j}(I_i + I_j - I'_j, I'_j) \\ =&- (h_i + b)\int_0^T G_i(I_i + I_j - I'_j - 1, \delta)d\delta \\ &+(h_j + b)\int_0^T G_j(I'_j, \delta)d\delta. \end{aligned} \tag{3.3}$$

The first order difference increases with I'_j, so $\mathfrak{C}_{i,j}(I_i + I_j - I'_j, I'_j)$ is convex and the theorem follows.

□

Inequality (3.2) is the necessary and sufficient condition for (I_i, I_j) to be the redistributed inventory levels at which the inventory-related cost in locations i and j is minimized given that the two locations have a total of $I_i + I_j$ units of inventory. Specifically, when the redistributed inventory level in location j is I'_j and the first inequality in Inequality (3.2) holds, $\mathfrak{C}_{i,j}(I_i+I_j-(I'_j+1), I'_j+1) \leq \mathfrak{C}_{i,j}(I_i+I_j-I'_j, I'_j)$ according to Equation (3.3). Thus, shifting one unit from locations i to j does not push up the inventory-related cost and I'_j is now increased by 1. The value of I'_j keeps increasing until it reaches a point (i.e., I_j which is the optimal redistributed inventory level) where the second inequality in Inequality (3.2) holds. In this case, $\mathfrak{C}_{i,j}(I_i+I_j-(I'_j+1), I'_j+1) \geq \mathfrak{C}_{i,j}(I_i+I_j-I'_j, I'_j)$ according to Equation (3.3). Consequently, if inventory shifts from locations i to j, it will no longer brings down the inventory-related cost.

Finally, the last part of this section discusses the redistribution of inventory among N locations. Represent the system inventory-related cost during $(t_p, t_p + T]$ by $\mathfrak{C}(I_0, I_1, ..., I_{N-1})$. Let I denote the sum of inventory in the N locations at time t_p, i.e., $I = \sum_{i=0}^{N-1} I_i$, and represent the redistributed inventory levels at which $\mathfrak{C}(I_0, I_1, ..., I_{N-1})$ is minimized by $(I_0^*(I), I_1^*(I), ..., I_{N-1}^*(I))$.

Lemma 3.3.3. *The optimal redistribution of* $I + 1$ *units of inventory is to increase some element in* $(I_0^*(I), I_1^*(I), ..., I_{N-1}^*(I))$ *by 1 and leave others unchanged.*

Proof. Prove by contradiction. Assume that the lemma is false. There exist two locations i and j such that $I_i^*(I+1) = I_i^*(I) - x$ and $I_j^*(I+1) = I_j^*(I) + y$ where x and y are positive integers and unequal.

Case 1: $x < y$.

By Theorem 3.3.2, $-(h_i+b)\int_0^T G_i(I_i^*(I)-1,\delta)d\delta+(h_j+b)\int_0^T G_j(I_j^*(I),\delta)d\delta \geq 0$. Since $y-x > 0$, $-(h_i+b)\int_0^T G_i(I_i^*(I)-1,\delta)d\delta+(h_j+b)\int_0^T G_j(I_j^*(I)+(y\text{-}x),\delta)d\delta > 0$. Thus, $I_j^*(I+1) \leq I_j^*(I)+y-x$ where the RHS is less than $I_j^*(I)+y$. Contradiction.

Case 2: $x > y$.

By Theorem 3.3.2, $I_j^*(I+1) = I_j^*(I)+y$ indicates that $-(h_i+b)\int_0^T G_i(I_i^*(I)-x-1+y,\delta)d\delta+(h_j+b)\int_0^T G_j(I_j^*(I),\delta)d\delta \leq 0$. Since $-x+y < 0$, we have $(h_i+b)\int_0^T G_i(I_i^*(I)-x-1+y,\delta)d\delta < (h_i+b)\int_0^T G_i(I_i^*(I)-1,\delta)d\delta$. Thus, $-(h_i+b)\int_0^T G_i(I_i^*(I)-1,\delta)d\delta+(h_j+b)\int_0^T G_j(I_j^*(I),\delta)d\delta < 0$. Thus, $I_j^*(I) \geq I_j^*(I)+1$. Contradiction.

□

Lemma 3.3.3 implies that the redistributed inventory levels in the N locations do not fall as the total inventory rises. When a new unit is added to the total inventory, the inventory that has been allocated prior to the addition remains in the location that it is designated for and the new unit will replenish a certain location's stocks.

Theorem 3.3.4. *Let* $i = \arg\min\{(h_k+b)\int_0^T G_k(I_k^*(I),\delta)d\delta \mid k = 0,1,...,N-1\}$. *Then* $I_i^*(I+1) = I_i^*(I)+1$ *and* $I_j^*(I+1) = I_j^*(I)\forall j \in \{0,1,...,N-1\}\setminus\{i\}$.

Proof. If the new unit has a choice between locations i and j, it will choose location i if and only if $\mathfrak{C}_{i,j}(I_i^*(I), I_j^*(I)+1)-\mathfrak{C}_{i,j}(I_i^*(I)+1, I_j^*(I)) \geq 0$ which is simplified to be $(h_i+b)\int_0^T G_i(I_i^*(I),\delta)d\delta \leq (h_j+b)\int_0^T G_j(I_j^*(I),\delta)d\delta$.

□

According to Theorem 3.3.4, the new unit is supplied to location i if and only if the value of $(h_i + b)\int_0^T G_i(I_i^*(I), \delta)d\delta$ is the lowest among $\{(h_k + b)\int_0^T G_k(I_k^*(I), \delta)d\delta \mid k = 0, 1, ..., N-1\}$. It is the case that h_i is relatively small and/or the probability of having no stockouts over the period $(t_p, t_p+T]$ is relatively low.

In summary, I units of inventory can be redistributed one by one according to Theorem 3.3.4. Specifically, the optimal redistribution $(I_0^*(I), I_1^*(I), ..., I_{N-1}^*(I))$ is first initialized to be zero and the I units of inventory are numbered from 1 to I. The first unit is supplied to the location with the smallest $(h_k + b)\int_0^T G_k(I_k^*(I), \delta)d\delta, k = 0, 1, ..., N-1$, say location i. As a result, $I_i^*(I)$ is increased by one. Then the redistribution of the second unit is handled similarly. When the redistribution of the Ith unit is finished off, $(I_0^*(I), I_1^*(I), ..., I_{N-1}^*(I))$ is the optimal redistribution.

Based on the discussion above, the algorithm that finds the optimal redistribution of I units of inventory among N locations is written below. As it processes the I units one by one, the complexity of the algorithm is $O(I)$.

1. Initialize $I_k^*(I) = 0, k = 0, 1, ..., N-1$, and $s = 1$.

   Compute $(h_k + b)\int_0^T G_k(I_k^*(I), \delta)d\delta, k = 0, 1, ..., N-1$. Go to Step 2.

2. If $s \leq I$, Compute $i = \arg\min\{(h_k + b)\int_0^T G_k(I_k^*(I), \delta)d\delta \mid k = 0, 1, ..., N-1\}$. Go to Step 3.

   Else, $(I_0^*(I), I_1^*(I), ..., I_{N-1}^*(I))$ is the optimal redistribution and STOP.

3. Update $I_i^*(I) := I_i^*(I) + 1, s := s + 1$. Go to Step 4.

4. Update $(h_i + b)\int_0^T G_i(I_i^*(I), \delta)d\delta$. Go to Step 2.

3.3.2 The Order-up-to Level

Each time the warehouse orders new supplies from the supplier, the system inventory position is raised to the order-up-to level S. Let t represent an instant of ordering and t_p denote the instant of the pth redistribution since the order arrives, i.e., $t_p = t + L + pT, p \in \mathbb{N}$.

Given that the total customer demand over $(t, t_p]$ is d, the total amount of inventory at time t_p is $S - d$. After the optimal redistribution takes place, the redistributed inventory levels are $(I_0^*(S-d), I_1^*(S-d), ..., I_{N-1}^*(S-d))$. Consequently, the system inventory-related cost during $(t_p, t_{p+1}]$, conditioned by the total customer demand over $(t, t_p]$, is $\mathfrak{C}(I_0^*(S-d), I_1^*(S-d), ..., I_{N-1}^*(S-d))$ which is a function of S.

Lemma 3.3.5. *$\mathfrak{C}(I_0^*(S-d), I_1^*(S-d), ..., I_{N-1}^*(S-d))$ is convex.*

Proof. The second order difference of $\mathfrak{C}(I_0^*(S-d), I_1^*(S-d), ..., I_{N-1}^*(S-d))$ in S is

$$\begin{aligned}\Big[&\mathfrak{C}(I_0^*(S+2-d), I_1^*(S+2-d), ..., I_{N-1}^*(S+2-d)) \\ &\qquad - \mathfrak{C}(I_0^*(S+1-d), I_1^*(S+1-d), ..., I_{N-1}^*(S+1-d))\Big] \\ &\qquad - \Big[\mathfrak{C}(I_0^*(S+1-d), I_1^*(S+1-d), ..., I_{N-1}^*(S+1-d)) \\ &\qquad\qquad - \mathfrak{C}(I_0^*(S-d), I_1^*(S-d), ..., I_{N-1}^*(S-d))\Big].\end{aligned} \tag{3.4}$$

We shall prove that Equation (3.4) ≥ 0 in any case. Suppose that $I_i^*(S+1-d) = I_i^*(S-d) + 1$.

Case 1: $I_i^*(S+2-d) = I_i^*(S+1-d) + 1$.

Equation (3.4) is simplified to be $\big[\mathfrak{C}_i(I_i^*(S-d)+2) - \mathfrak{C}_i(I_i^*(S-d)+1)\big] -$

$\big[\mathfrak{C}_i(I_i^*(S-d)+1)-\mathfrak{C}_i(I_i^*(S-d))\big]$ which is non-negative by Lemma 3.3.1.

Case 2: $I_j^*(S+2-d)=I_j^*(S+1-d)+1$ where $j\neq i$.

Equation (3.4) is simplified to be $\big[\mathfrak{C}_j(I_j^*(S-d)+1)-\mathfrak{C}_j(I_j^*(S-d))\big]-\big[\mathfrak{C}_i(I_i^*(S-d)+1)-\mathfrak{C}_i(I_i^*(S-d))\big]$. Regroup the terms and it equals $\mathfrak{C}_{i,j}(I_i^*(S-d),I_j^*(S-d)+1)-\mathfrak{C}_{i,j}(I_i^*(S-d)+1,I_j^*(S-d))$. The latter expression is non-negative since $(I_i^*(S-d)+1,I_j^*(S-d))$ is the optimal redistribution of inventory between locations i and j when the N locations have a total of $S-d+1$ units of inventory.

□

$\mathfrak{C}(I_0^*(S-d),I_1^*(S-d),...,I_{N-1}^*(S-d))$ is the system inventory-related cost during the period $(t_p,t_{p+1}]$ conditioned by the past customer demand. The (unconditional) system inventory-related cost during $(t_p,t_{p+1}]$ equals $\sum_{d=0}^{+\infty}\mathfrak{C}(I_0^*(S-d),I_1^*(S-d),...,I_{N-1}^*(S-d))g(d,L+pT)$ where $g(d,L+pT)$ denotes the probability of having a total of d units of customer demand over $L+pT$ unit time. A summation of the latter over $p=0,1,...,n-1$ is the system inventory-related cost during $(t_0,t_0+nT]$ and denoted by $\mathbb{C}(S,n)$. That is, $\mathbb{C}(S,n)=\sum_{p=0}^{n-1}\sum_{d=0}^{+\infty}\mathfrak{C}(I_0^*(S-d),I_1^*(S-d),...,I_{N-1}^*(S-d))g(d,L+pT)$.

Theorem 3.3.6. *Fix n.$\mathbb{C}(S,n)$ is a convex function of S.*

Proof. According to Lemma 3.3.5, the system inventory-related cost conditioned by the past customer demand is a convex function of S. Therefore, the unconditional system inventory-related cost and the sum of those are also convex functions of S.

□

The convexity of the function $\mathbb{C}(S,n)$ facilitates the search for the optimal S. When S is small, one more unit of replenishment reduces back orders and back order cost, so $\mathbb{C}(S,n)$ decreases with S. When S is large and one more unit is ordered, holding cost rises and, as a result, $\mathbb{C}(S,n)$ increases. The point from which $\mathbb{C}(S,n)$ starts increasing is guaranteed to be the optimal S due to Theorem 3.3.6. In other words, $\mathbb{C}(S,n)$ is minimized at $S^*(n) \overset{def}{=} \{S \mid \mathbb{C}(S,n) - \mathbb{C}(S-1,n) \leq 0 \text{ and } \mathbb{C}(S+1,n) - \mathbb{C}(S,n) \geq 0\}$.

3.3.3 The Reorder Interval

Orders are charged at K for each and an order is placed every n periods. The average cost of ordering per period is K/n. The larger the n is, the lower the average cost of ordering per period is incurred. By contrast, a large n widens the variation in the customer demand between order receipts and thus increases the inventory-related cost per period. Hence, there has to be a trade-off between the cost of ordering and the inventory-related cost if we want to keep the total cost low. This section shows how to determine the optimal n (denoted by n^*) at which the average total cost per period $C(n) \overset{def}{=} \frac{K}{n} + \frac{1}{n}\mathbb{C}(S^*(n),n)$ is minimized.

Note that the system inventory-related cost would be reduced if stocks were pooled and kept in the location with the lowest holding cost per unit per unit time, say location m, and other locations' customer demand was aggregated with that of location m. In other words,

$$\mathbb{C}(S,n) \geq \mathbb{E}\Big[\int_0^{nT} h_m[S - D(L+\tau)]^+ + b[S - D(L+\tau)]^- d\tau\Big], \qquad (3.5)$$

where $D(L+\tau) = \sum_{i=0}^{N-1} D_i(L+\tau)$. The properties of $\mathbb{E}\Big[\int_0^{nT} h_m[S-D(L+\tau)]^+ + b[S-D(L+\tau)]^- d\tau\Big]$ have been recorded in the literature on order-up-to and periodic-review (S,T) policies. Based on the well-established properties, an upper bound of n^* is developed next.

Theorem 3.3.7. *(1) Let $LB(x) = \frac{K}{x} + \frac{h_m b\lambda x T^2}{2(h_m+b)}$ be a function of x over positive numbers. Then $C(n) \geq LB(n)$; and*

(2) For any positive integer $n_0, n^* \leq \bar{n} \overset{def}{=} 2C(n_0)(h_m+b)/(h_m b\lambda T^2)$ where $\lambda = \sum_{i=0}^{N-1} \lambda_i$.

Proof. (1) Divide both sides of inequality (3.5) by n and we obtain

$$\frac{1}{n}\mathbb{C}(S,n) \geq \frac{1}{n}\mathbb{E}\big[\int_0^{nT} h_m[S-D(L+\tau)]^+ + b[S-D(L+\tau)]^- d\tau\big] \tag{3.6}$$

Thus,

$$\min_S\{\frac{1}{n}\mathbb{C}(S,n)\} \geq \min_S \big\{\frac{1}{n}\mathbb{E}\big[\int_0^{nT} h_m[S-D(L+\tau)]^+ + b[S-D(L+\tau)]^- d\tau\big]\big\}, \tag{3.7}$$

where the left-hand side of the inequality is $\frac{1}{n}\mathbb{C}(S^*(n),n)$. Moreover, by using the results in Liu and Song (2012) we obtain $\min_S \big\{\mathbb{E}\big[\int_0^{nT} h_m[S-D(L+\tau)]^+ + b[S-D(L+\tau)]^- d\tau\big]\big\} \geq \frac{h_m b\lambda (nT)^2}{2(h_m+b)}$. Thus,

$$\frac{1}{n}\mathbb{C}(S^*(n),n) \geq \frac{h_m b\lambda n T^2}{2(h_m+b)}. \tag{3.8}$$

Add $\frac{K}{n}$ to both sides and $C(n) \geq LB(n)$ is obtained.

(2) Prove by contradiction and assume that $n^* > \bar{n}$. According to (1), $C(n^*) \geq LB(n^*) > \frac{h_m b\lambda n^* T^2}{2(h_m+b)} > \frac{h_m b\lambda \bar{n} T^2}{2(h_m+b)}$. Substitute $2C(n_0)(h_m+b)/(h_m b\lambda T^2)$

for $\bar{n}$ and we obtain $C(n^*) > C(n_0)$. Contradiction. Hence, $n^* \leq \bar{n}$.

□

As the search goes on to find n^* in $[1, \bar{n}]$ by working out $C(n_0)$ at each $n_0 \in [1, \bar{n}]$, a smaller $C(n_0)$ may be found. As shown by Theorem 3.3.7, a small $C(n_0)$ gives rise to a small $\bar{n}$. Therefore, $\bar{n}$ could be lowered alongside the search, with the result that the list of $C(n_0)$s to be calculated is narrowed down further. The determination of $\bar{n}$ is represented graphically on Figure 3.2.

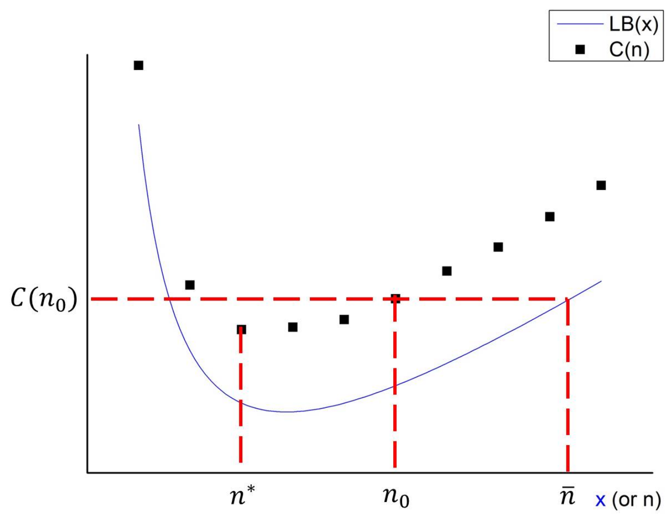

Figure 3.2: The determination of $\bar{n}$

3.4 Cost Effective Ways of Managing Demand

In this section, we consider several cost effective ways of managing demand. They are intended to reduce the inventory-related cost incurred during a period of length $T = 1$. The first is referring in-store purchases to the web, which essentially makes the warehouse satisfy more customer demand. The second is guiding customers to the channel that has stock on hand. Only the inventory-related cost is considered and other costs such as referral cost or shipping cost are not. The reduction in the inventory-related cost represents the maximum cost savings. The customer demand is considered to be a Poisson process.

3.4.1 Referring In-store Purchases to the Web

Sometimes customers' channel intention is flexible. They care what to purchase but not where to purchase. Originally, a customer has planned to purchase goods from the store but he purchases it on-line in the end after talking with a sales representative or receiving an email advertising special offers. M&S is making an effort to familiarize offline customers with the M&S catalog online. It sets up in-store "Browse & order" terminals where a customer can browse products and place orders. We shall see conditions under which referring in-store purchases to the web cuts the inventory-related cost from numerical tests.

In the following numerical tests, $h_0 = 0.1, h_1 = 0.2, b = 1$, the demand rate in location 0 is 10α and that in location 1 is $10(1-\alpha), \alpha = 0, 0.1, ..., 1$. At the beginning of the period, the total stock in the two locations is $I, I = 2, 4, ..., 20$, and Theorem 3.3.4 is used to redistribute these goods to the two locations. In-store demand is supplied by the store while on-line purchases are shipped from the warehouse. For each pair of α and I, we calculate $\mathfrak{C}(I_0^*(I), I_1^*(I))$. Results are shown in Table 3.1.

Table 3.1: The inventory-related cost incurred over one period

α \ I	2	4	6	8	10	12	14	16	18	20
0.0	3.36	2.20	1.50	1.22	1.25	1.45	1.65	1.85	2.05	2.25
0.1	3.40	2.33	1.68	1.33	1.29	1.40	1.60	1.79	1.99	2.19
0.2	3.45	2.37	1.69	1.34	1.27	1.36	1.54	1.74	1.94	2.14
0.3	3.51	2.37	1.68	1.34	1.25	1.32	1.49	1.68	1.88	2.08
0.4	3.47	2.39	1.70	1.34	1.22	1.27	1.44	1.62	1.82	2.02
0.5	3.46	2.36	1.66	1.30	1.19	1.23	1.38	1.56	1.76	1.96
0.6	3.48	2.38	1.63	1.25	1.14	1.18	1.32	1.50	1.70	1.90
0.7	3.47	2.32	1.63	1.22	1.08	1.12	1.26	1.44	1.63	1.83
0.8	3.41	2.34	1.55	1.20	1.02	1.06	1.20	1.37	1.56	1.76
0.9	3.37	2.22	1.53	1.09	0.94	0.98	1.12	1.29	1.49	1.68
1.0	3.33	2.10	1.29	0.86	0.73	0.78	0.92	1.11	1.30	1.50

It is observed that in each row of α, the value of $\mathfrak{C}(I_0^*(I), I_1^*(I))$ first decreases and then increases with I, due to Lemma 3.3.5 which states that $\mathfrak{C}(I_0^*(I), I_1^*(I))$ is convex in I. While each row exhibits convexity, the trend in $\mathfrak{C}(I_0^*(I), I_1^*(I))$ is not uniform among all columns. For all $I \leq 10$, the value is tending upwards in the beginning and tending downwards in the end. For all $I > 10$, the value is decreasing from beginning to end. The reason is that when the total amount of goods is small ($\leq$ expected demand) and it has to be split between two locations, goods are very likely to be out of stock in a location soon. Thus back order cost is the major component of the inventory-related cost. Saving in the former makes the latter decrease considerably. Gathering inventory and demand reduces back order cost to the utmost because demand is met immediately as long as there is an available unit. When the total amount of goods is big ($>$ expected demand), a location tends to overstock. Carrying cost makes up a large proportion of the inventory-related cost. Referring in-store purchases to the web changes the way that the stock is distributed between the two locations. The warehouse carries more and the store carries less than before. Since the stock in the warehouse is charged lower than in the store, carrying cost deceases, and so does the inventory-related cost.

These discoveries may shed some light on deciding whether to refer an in-store purchase to the web. If customers' intention for where to make purchases is highly flexible, awareness about the online channel should be raised. The inventory-related cost reaches the minimum if all customers purchase goods on-line. By contrast, if customers are so reluctant to change the intention that α can be changed just slightly, referring an in-store purchase to the web does not necessarily reduce cost. It may increase that if the stock of goods is insufficient.

3.4.2 Stockout-based Substitution

A customer can back-order an item that is out of stock. A better way to meet demand is shipping from the other location that has stock. If so, a stockout is postponed until a later time and back-ordering cost is reduced. Further, it cuts holding cost in the other location. In practice, the web can be designed to allow "reserve online and pick up in store" should the warehouse have no stock and arrange for personnel to assist customers with placing an order online if the store is facing a stockout. This section is to quantify the reduction in the inventory-related cost resulting from stockout-based substitution. Faced with a stockout in location i and informed of availability in location $1-i$, the percentage of customers that choose to buy from location $1-i$ is β_i. The remaining customers back-order items. Note that the stockout-based substitution with $\beta_i = 1$ is the same as reactive transshipment in which transshipment is triggered by each stockout.

We are to compute the inventory-related cost over the period with stockout-based substitution. After a stock redistribution, location i has $I_{i,0}$ stocks. If $I_{0,0}, I_{1,0} \leq 0$, stockout-based substitution does not have the advantage of reducing the inventory-related cost, because none of the demand can be met. Thus we only assess the situation in which at least one $I_{i,0}$ are positive.

Let X_i represent the length of time when there is no stockout in location i during the period, $H_i(I_{0,0}, I_{1,0})$ denote holding cost at location i and

$B_i(I_{0,0}, I_{1,0})$ be back-ordering cost at location i. The calculations are as follows.

Case 1: $X_0 = 0$.

$$(H_0 + B_0 + H_1 + B_1)(I_{0,0}, I_{1,0}) = \frac{1}{2} b(1 - \beta_0)\lambda_0 T^2 + \mathfrak{C}_1(I_{1,0}). \tag{3.9}$$

The demand rate at location 1 is treated as $\beta_0\lambda_0 + \lambda_1$ when $\mathfrak{C}_1(I_{1,0})$ is calculated.

Case 2: $X_1 = 0$.

Similar to the discussion of Case 1.

Case 3: $0 < X_1 < X_0 \leq T$.

Subcase 3.1: $X_0 < T$.

$$H_0(I_{0,0}, I_{1,0}) = \frac{1}{2} h_0 \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \int_0^T \int_0^{x_0} ((I_{0,0}+I_{0,0}-d_{0,x_1})x_1+(I_{0,0}-d_{0,x_1}+1)(x_0-x_1))$$
$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) gampdf(x_0-x_1, I_{0,0}-d_{0,x_1}, \frac{1}{\beta_1\lambda_1 + \lambda_0}) dx_1 dx_0. \tag{3.10}$$

$$H_1(I_{0,0}, I_{1,0}) = \frac{1}{2} h_1 (I_{1,0} + 1) \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \int_0^T \int_0^{x_0} x_1$$
$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) gampdf(x_0-x_1, I_{0,0}-d_{0,x_1}, \frac{1}{\beta_1\lambda_1 + \lambda_0}) dx_1 dx_0. \tag{3.11}$$

$$B_0(I_{0,0}, I_{1,0}) = \frac{1}{2} b\lambda_0 \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \int_0^T \int_0^{x_0} (T - x_0)^2$$

$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) gampdf(x_0 - x_1, I_{0,0} - d_{0,x_1}, \frac{1}{\beta_1 \lambda_1 + \lambda_0}) dx_1 dx_0. \tag{3.12}$$

$$B_1(I_{0,0}, I_{1,0}) = \frac{1}{2} b\lambda_1 \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \int_0^T \int_0^{x_0} (\beta_1 (T - x_0)^2 + (1 - \beta_1)(T - x_1)^2)$$

$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) gampdf(x_0 - x_1, I_{0,0} - d_{0,x_1}, \frac{1}{\beta_1 \lambda_1 + \lambda_0}) dx_1 dx_0. \tag{3.13}$$

Subcase 3.2: $X_0 = T$.

$$H_0(I_{0,0}, I_{1,0}) = \frac{1}{2} h_0 \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \sum_{d_{0,T-x_1}=0}^{I_{0,0}-d_{0,x_1}-1} \int_0^T ((I_{0,0} + I_{0,0} - d_{0,x_1}) x_1$$

$$+ (I_{0,0} - d_{0,x_1} + I_{0,0} - d_{0,x_1} - d_{0,T-x_1})(T - x_1))$$

$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) poisspdf(d_{0,T-x_1}, (\beta_1 \lambda_1 + \lambda_0)(T - x_1)) dx_1. \tag{3.14}$$

$$H_1(I_{0,0}, I_{1,0}) = \frac{1}{2} h_1 (I_{1,0} + 1) \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \sum_{d_{0,T-x_1}=0}^{I_{0,0}-d_{0,x_1}-1} \int_0^T x_1$$

$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) poisspdf(d_{0,T-x_1}, (\beta_1 \lambda_1 + \lambda_0)(T - x_1)) dx_1. \tag{3.15}$$

$$B_0(I_{0,0}, I_{1,0}) = 0. \tag{3.16}$$

$$B_1(I_{0,0}, I_{1,0}) = \frac{1}{2} b(1-\beta_1)\lambda_1 \sum_{d_{0,x_1}=0}^{I_{0,0}-1} \sum_{d_{0,T-x_1}=0}^{I_{0,0}-d_{0,x_1}-1} \int_0^T (T-x_1)^2$$

$$\cdot gampdf(x_1, I_{1,0}, \frac{1}{\lambda_1}) poisspdf(d_{0,x_1}, \lambda_0 x_1) poisspdf(d_{0,T-x_1}, (\beta_1\lambda_1+\lambda_0)(T-x_1)) dx_1. \tag{3.17}$$

Case 4: $0 < X_0 < X_1 < T$.

Similar to the discussion of Case 3.

Case 5: $X_0 = X_1 = T$.

$$H_0(I_{0,0}, I_{1,0}) = \frac{1}{2} h_0 \sum_{d_{1,T}=0}^{I_{1,0}-1} \sum_{d_{0,T}=0}^{I_{0,0}-1} (I_{0,0} + I_{0,0} - d_{0,T})T$$

$$\cdot\, poisspdf(d_{1,T}, \lambda_1 T) poisspdf(d_{0,T}, \lambda_0 T). \tag{3.18}$$

$$H_1(I_{0,0}, I_{1,0}) = \frac{1}{2} h_1 \sum_{d_{1,T}=0}^{I_{1,0}-1} \sum_{d_{0,T}=0}^{I_{0,0}-1} (I_{1,0} + I_{1,0} - d_{1,T})T$$

$$\cdot\, poisspdf(d_{1,T}, \lambda_1 T) poisspdf(d_{0,T}, \lambda_0 T). \tag{3.19}$$

$$B_0(I_{0,0}, I_{1,0}) = 0. \tag{3.20}$$

$$B_1(I_{0,0}, I_{1,0}) = 0. \tag{3.21}$$

Table 3.2 shows results of the same experiment as Table 3.1. We use $\beta_0 = \beta_1 = 0.5$ and assess the effect of stockout-based substitution on the inventory-related cost. Some interesting observations emerge.

First, all percentage changes are non-positive. It shows that stockout-based substitution reduces the inventory-related cost.

Second, if all customers buy from one location and the other location does not carry goods, carrying out stockout-based substitution is of no avail. It does not reduce the inventory-related cost. Note the 0's in the first 5 cells in the row of $\alpha = 0$ and in the entire row of $\alpha = 1$. When $\alpha = 1$, all customers buy online and the store serves as a showroom. Since carrying goods in the warehouse is cheaper than in the store, all goods are consigned to the warehouse. If the item is out of stock in the warehouse, it is back-ordered. Similarly, when $\alpha = 0$ and I is small, all goods are consigned to the store. An item that is out of stock in the store is back-ordered.

Third, stockout-based substitution reduces the inventory-related cost significantly when the total supply and the expected total demand do not differ widely. When $\alpha = 0(0.1)$, substantial reduction occurs at $I = 14(6)$. When $\alpha = 0.2, 0.3, ..., 0.9$, it occurs at $I = 8$. When the total supply is comparable in size to the expected total demand and is divided up between the two locations, goods are very likely to be mis-located. While the item is back-ordered in one location, it is available in the other. Stockout-based substitution opens up an opportunity to hand over demand to the the location that has stock.

Table 3.2: Percentage change in the inventory-related cost incurred over one period

α \ I	2	4	6	8	10	12	14	16	18	20
0.0	0.00	0.00	0.00	0.00	0.00	-2.87	*-3.96*	-3.70	-3.35	-3.06
0.1	-0.62	-2.94	*-9.86*	-8.25	-8.93	-6.64	-3.87	-3.66	-3.32	-3.02
0.2	-1.44	-6.13	-8.16	*-10.76*	-10.14	-6.93	-3.84	-3.60	-3.26	-2.96
0.3	-2.55	-5.24	-9.70	*-12.62*	-11.21	-7.29	-3.84	-3.51	-3.18	-2.88
0.4	-2.47	-6.75	-11.81	*-14.37*	-12.23	-7.65	-3.85	-3.41	-3.07	-2.77
0.5	-2.14	-5.68	-9.82	*-11.78*	-9.70	-7.99	-3.83	-3.26	-2.91	-2.62
0.6	-2.59	-6.67	-10.34	*-12.17*	-10.02	-8.27	-3.74	-3.05	-2.68	-2.40
0.7	-2.36	-5.30	-10.33	*-12.70*	-10.18	-8.45	-3.55	-2.74	-2.36	-2.10
0.8	-1.33	-6.56	-8.78	*-13.50*	-10.03	-8.38	-3.16	-2.24	-1.87	-1.64
0.9	-0.57	-2.83	-8.05	*-10.36*	-8.98	-7.57	-2.44	-1.43	-1.09	-0.94
1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The Effect of Ship-from-store on the Optimal Redistribution

In the first part of Section 3.4.2, we have quantified cost reductions made by stockout-based substitution. If that substitution is taken into consideration when deciding stock redistribution, it can be even more attractive. In this section, we study stockout-based ship-from-store in which an online order is shipped from a store should the product be out of stock in the warehouse. We shall see how ship-from-store affects redistribution of stock from examples.

This paragraph describes the examples in detail. There are four locations. Location 0 is the warehouse and locations 1, 2 and 3 are stores. If the product is in stock in location 0, then online orders are shipped from location 0. Otherwise online orders are shipped over from location 1 if it has stock. Locations 2 and 3 are not equipped to deal with online orders. The total stock in the four locations is $I = 4$. The back order cost rate is 1. Location 0's, 1's and 2's holding cost rates are 0.1, 0.2 and 0.3 respectively. Location 3's holding cost rate is between 0.4 and 0.9. Four locations' demand rates are either all 1 or all 4. Table 3.3 shows the optimal redistribution of stock when the ship-from-store approach is not/ is adopted. We obtain the former through Theorem 3.3.4 and the latter through exhaustive search.

We make some observations about the effect of ship-from-store on the optimal redistribution from Table 3.3.

When $\lambda = \mathbf{1}$ and the ship-from-store approach is not adopted, if h_3 is small (0.4 or 0.5), each location is given one unit. If h_3 is large (> 0.5), then location 3 is not given any units and location 0 is given two one of which is safety stock kept in case the online demand is greater than expected.

Table 3.3: The optimal redistribution of stock when the ship-from-store approach is not/ is adopted

λ \ h_3	0.4	0.5	0.6	0.7	0.8	0.9	Ship-from-store
1	[1 1 1 1]	[1 1 1 1]	[2 1 1 0]	[2 1 1 0]	[2 1 1 0]	[2 1 1 0]	No
	[1 1 1 1]	[1 1 1 1]	[1 1 1 1]	[1 2 1 0]	[1 2 1 0]	[1 2 1 0]	Yes
4	[1 1 1 1]	[1 1 1 1]	[1 1 1 1]	[1 1 1 1]	[1 1 1 1]	[1 1 1 1]	No
	[0 2 1 1]	[0 2 1 1]	[0 2 1 1]	[0 2 1 1]	[0 2 1 1]	[0 3 1 0]	Yes

Consider that unit. If the ship-from-store model is adopted and h_3 is medium (0.6), that unit will be assigned to location 3. If h_3 is large (> 0.6), that unit will be assigned to location 1 so that it becomes the shared safety stock with location 0.

When $\lambda = \mathbf{4}$, demand is far more than supply. Thus holding cost is insignificant in comparison to back order cost. If the ship-from-store approach is not adopted, each location carries goods. Otherwise inventory is pooled into location 1 to be used for purchases online and from store 1. If h_3 is not large (0.4 to 0.8), stocks in locations 0 and 1 are pooled. If h_3 is large (0.9), stocks in locations 0, 1 and 3 are pooled.

3.5 Conclusions

This chapter studies inventory control policies in one-warehouse multi-retailer systems where the warehouse may also meet customer demand and proactive transshipment is enabled. The proactive transshipment should be made according to the algorithm that solves the optimal redistribution of inventory. The optimal amount of inventory that a system should carry is determined from the first-order condition due to the convexity of the system inventory-related cost. Based on the average total cost per period and a lower bound of it, a dynamic upper bound of the optimal length of the reorder interval is developed.

It is assumed that the proactive transshipment is instant, and shipping and handling fees associated with the proactive transshipment are negligible compared with the cost of ordering for the reasons given in Section 3.2. On the other hand, if the lead time to perform the proactive transshipment is significant or the cost of the proactive transshipment is non-negligible, the optimal solution obtained from this chapter leads to an upper bound of the cost in the new model and the optimal cost obtained from this chapter is a lower bound.

Chapter 4

Demand Management in Multi-channel Marketing

4.1 Introduction

Demand management is a prelude to inventory management, because inventory management is essentially matching supply and demand of products. Therefore, searching for clues about what good demand management looks like is a crucial issue. However, demand management in multi-channel marketing is not an easy task. For one thing, customer demand keeps on changing as customers change their price sensitivity or lose interest in products in any channel. For another, interaction between channels makes demand management even more complex.

Various ways to model demand in multi-channel marketing have been recorded in the literature. A usual method is to consider demand to be a function of price and the demand in each channel is subject to a migration effect. The migration effect captures the demand that migrates from one channel to another to enjoy the benefits of the latter such as monetary advantage or good service. For example, when price varies from channel to channel, the low-price seekers will purchase the product from the relatively lowly priced channel (Perakis and Sood, 2006; Huang and Swaminathan, 2009; Cai et al., 2009). The migration can be also from the online channel to the retailer channel when value is added to the product in the retailer channel (Mukhopadhyay et al., 2008) or when there is a delivery lead time for the orders placed through the online channel (Hua et al., 2010).

Demand can be modeled not only directly in the above-mentioned ways but also through utility functions. The latter stream of literature models an individual customer's utility of purchasing each channel's product. The customer will not make a purchase from any channel if all utilities are non-positive. If some utilities are positive, the customer will purchase the product from the channel that gives the highest utility. The utility is usually considered to be a function of price (Cattani et al., 2006; Yoo and Lee, 2011) or a function of price and service (Aussadavut et al., 2008; Chen et al., 2008; Ofek et al., 2011). Depending on the purpose of the research, other factors such as the store inventory level can also be taken into account (Gao and Su, 2016).

It is noted that regardless of whether demand is modeled directly or through utility functions, price plays an essential role. However, it is generally assumed that demand or utility is a linear function of price. Research findings in the field of nonlinearity are underdeveloped. Moreover, most literature considers decentralized control where channels compete for customers and focuses on providing channels economic incentives to coordinate policy. The issue of centralized control is not addressed adequately. This chapter considers each channel's demand to be a general function of prices, price differentials and substitutabilities between products that are centrally controlled. It characterizes the total amount of demand when prices are chosen optimally. The migration effect due to the price differential is discussed. Finally, it considers the management of multi-period demand.

4.2 Model

Products are sold through two channels. The unit price of the product in channel i is $p_i, i \in \{1, 2\}$. The demand in channel i is a function of $p_i, p_1 - p_2$ and substitutability between the products β as follows,

$$\begin{aligned} q_1 &= f_1(p_1) - h(p_1 - p_2, \beta), \\ q_2 &= f_2(p_2) + h(p_1 - p_2, \beta). \end{aligned} \tag{4.1}$$

The function f_i returns the amount of demand attracted by channel i. Generally, $f_i(p_i)$ is a decreasing function of p_i because a rise in price reduces demand. The degree to which a change in p_i leads to a change in $f_i(p_i)$ is referred to as price sensitivity. When price sensitivity is high, a price rise leads to a sharp decrease in demand. The function h measures the migration effect caused by the price differential and $\beta \geq 0$ denotes the substitutability between the products sold in the two channels. $\beta = 0$ indicates that the products sold are non-substitutable. For example, one channel sells toothpaste and the other channel sells cleanser. In this situation, the substitution effect is minimal and possibly zero. $\beta > 0$ indicates that the products sold are substitutable, e.g., both channels sell toothpaste, possibly different brands. The higher value β has, the more substitutable the products are and the bigger substitution effect there will be. If $p_1 > p_2$, the demand migrates from channel 1 to channel 2 and thus $h > 0$. If $p_1 < p_2$, the demand migrates from channel 2 to channel 1 and thus $h < 0$. If $p_1 = p_2$, there is no migration of demand and thus $h = 0$. The degree to which a change in $|p_1 - p_2|$ leads to a change in $|h(p_1 - p_2)|$ is referred to as cross price sensitivity. When cross price sensitivity is high, an increase in the price differential leads to a large migration of demand. After taking account of the migration effect, $f_1 - h$ and $f_2 + h$ are the amount of demand secured by channel 1 and channel 2 respectively. It is assumed that $p_1, p_2, f_1, f_2, q_1, q_2 \geq 0$. The firm aims to maximize the total profit which is

$$\pi = (p_1 - c_1)q_1 + (p_2 - c_2)q_2, \tag{4.2}$$

where c_i is the cost of selling one unit through channel i.

4.3 Analysis and Discussion

We are to characterize the total amount of demand when the product is priced at the value regarded as optimal. Let Q_c be the total amount of demand when each channel sells at the cost, i.e., $p_i = c_i$. It is the largest amount sold if each channel aims to sell a unit at a profit. Let Q_p denote the total amount of demand when the price maximizes the total profit in

Equation (4.2). Represent the reduction in demand when the price rises from the cost to the optimal value by L_{c2p}, i.e., $L_{c2p} = Q_c - Q_p$.

Theorem 4.3.1. *When f_1, f_2, h_1, h_2 are differentiable functions, the following must be true.*

(1) If f_1 and f_2 are linear, then $Q_p = \frac{1}{2}Q_c = L_{c2p}$;

(2) If f_1 and f_2 are convex, then $Q_p < \frac{1}{2}Q_c < L_{c2p}$;

(3) If f_1 and f_2 are concave, then $Q_p > \frac{1}{2}Q_c > L_{c2p}$.

Proof. By Fermat's theorem, at the optimal price,

$$\frac{\partial \pi}{\partial p_1} = 0 \tag{4.3}$$

$$\frac{\partial \pi}{\partial p_2} = 0. \tag{4.4}$$

Adding up Equations (4.3) and (4.4) leads to

$$f_1(p_1) + f_2(p_2) + (p_1 - c_1)f_1'(p_1) + (p_2 - c_2)f_2'(p_2) = 0. \tag{4.5}$$

Note that $f_1(p_1) + f_2(p_2) = Q_p$.

(1) If f_1 and f_2 are linear, then $-(p_1 - c_1)f_1'(p_1) - (p_2 - c_2)f_2'(p_2) = L_{c2p}$;

(2) If f_1 and f_2 are convex, then $-(p_1 - c_1)f_1'(p_1) - (p_2 - c_2)f_2'(p_2) < L_{c2p}$;

(3) If f_1 and f_2 are concave, then $-(p_1 - c_1)f_1'(p_1) - (p_2 - c_2)f_2'(p_2) > L_{c2p}$.

□

Theorem 4.3.1 shows that Q_p depends on the shape of f_i. If f_i is a linear function of price, it means that price sensitivity remains the same as the price goes up. In this situation, Q_p is half of Q_c. In the case of convex f_i, price sensitivity is high when the price is low. In other words, a change in the price leads to a big change in the amount sold when the price is low. While selling at the optimal price, demand falls by more than half from Q_c. By contrast, a situation of concave f_i occurs when low price sensitivity goes with low price. When selling at the optimal price, demand falls by less than half from Q_c.

Rearrange the terms in Equation 4.2 and the total profit can be also written as

$$\pi = (p_1 - c_1)f_1(p_1) + (p_2 - c_2)f_2(p_2) + [(p_2 - c_2) - (p_1 - c_1)]h(p_1 - p_2, \beta). \quad (4.6)$$

If $p_1 > p_2$, some demand migrates to channel 2. When the profit margin in channel 2 is smaller than that in channel 1, i.e., $p_2 - c_2 < p_1 - c_1$, the migration hurts the total profit. In this situation, the size of the migration should be as small as possible. Hence, the two channels may consider selling non-substitutable products. When the profit margin in channel 2 is larger than that in channel 1, i.e., $p_2 - c_2 > p_1 - c_1$, then the migration boosts profits. In this situation, the two channels should facilitate the migration of demand by offering substitutable products. The situation of $p_1 < p_2$ leads to the opposite conclusion. Findings are represented graphically on Figure 4.1.

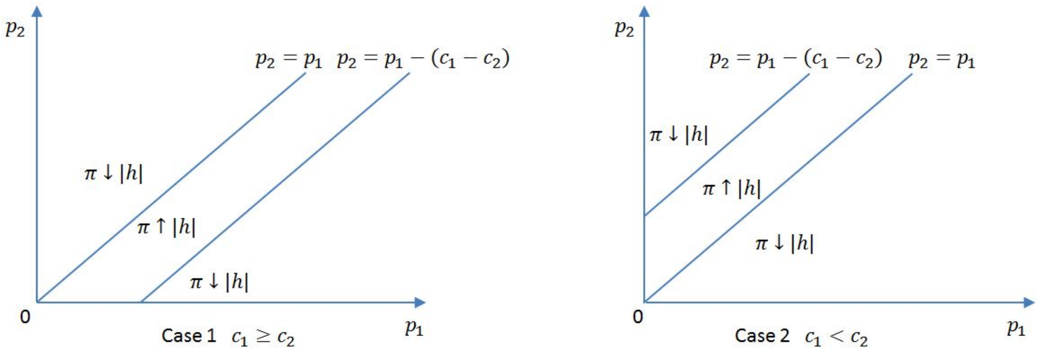

Figure 4.1: The relationship between π and h

4.3.1 An Example

Consider the following form of demand,

$$\begin{aligned} q_1 &= \frac{1}{1+m}[m - \alpha p_1 - \beta(p_1 - p_2)], \\ q_2 &= \frac{1}{1+m}[1 - \alpha p_2 + \beta(p_1 - p_2)], \end{aligned} \tag{4.7}$$

where $m > 0, \alpha > 0$ and $\beta \geq 0$. The form of demand develops from the demand structure in Raju et al. (1995) who assume that customers do not have preference for a channel, i.e., $m = 1$. We note that when $p_1 = p_2 = 0, q_1 = \frac{m}{1+m}, q_2 = \frac{1}{1+m}$ and total demand is 1. The reason we normalize demand is to save the parameter representing the level of base demand. $\frac{1}{1+m}\beta|p_1 - p_2|$ is the size of the migration. It is assumed that $p_1, p_2, m - \alpha p_1, 1 - \alpha p_2, q_1, q_2 \geq 0$.

The firm aims to maximize the total profit in Equation (4.2) by determining the optimal (p_1, p_2). The latter is solved to be

$$(p_1^*, p_2^*) = \left(\frac{\alpha m + \beta + \beta m}{2\alpha(\alpha + 2\beta)} + \frac{c_1}{2}, \frac{\alpha + \beta + \beta m}{2\alpha(\alpha + 2\beta)} + \frac{c_2}{2} \right). \tag{4.8}$$

By substituting (p_1^*, p_2^*) for (p_1, p_2) in Equation (4.7), the optimal amount of demand is obtained as $(q_1^*, q_2^*) = \left(\frac{1}{2(1+m)}[m - \alpha c_1 - \beta(c_1 - c_2)], \frac{1}{2(1+m)}[1 - \alpha c_2 + \beta(c_1 - c_2)]\right)$. In other words, when prices are optimal, demand in each channel is half of that when prices equal costs. This result is consistent with the findings in Theorem 4.3.1. Substitute (p_1^*, p_2^*) for (p_1, p_2) and (q_1^*, q_2^*) for (q_1, q_2) in Equation (4.2), and the maximum total profit π^* can be worked out.

To examine the effect of β on π^*, the first order derivative of π^* with respect to β at $\beta = 0$ is calculated and

$$\frac{\partial \pi^*}{\partial \beta}(0) = \frac{1}{4(1+m)}\left[(c_1 - c_2)^2 - \frac{(m-1)^2}{\alpha^2}\right]. \tag{4.9}$$

Equation (4.9) provides insights into management of products in two channels. (1) The firm has much incentive to offer substitutable products in the two channels when costs in the two channels are far apart ($(c_1 - c_2)^2$ large), there is not a strong preference for a channel ($(m-1)^2$ small) and the price sensitivity is high (α large). It is because the profit margin in the lowly priced channel turns out to be higher than that in the highly priced channel in this situation. Thus, products in the two channels should be substitutable so as to facilitate the migration of demand. (2) By contrast, a firm should offer non-substitutable products when costs in the two channels are close to each other ($(c_1 - c_2)^2$ small), customers express a strong preference to use a certain channel ($(m-1)^2$ large) and the price sensitivity is low (α small). It is because the profit margin in the lowly priced channel turns out to be lower than that in the highly priced channel in this situation. Hence, products should be non-substitutable so as to retain low-price seekers.

4.3.2 Multi-period Demand

The discussion above is on demand management when the planning horizon is one period within which no changes can be made. In this case, the two channels only make a decision prior to the introduction of products. By contrast, if the planning horizon is a number of periods, the two channels may choose to sell substitutable products during some periods and non-substitutable products during the remaining planning horizon. This section discusses demand management in T periods. In period $t, 1 \leq t \leq T$, the two channels determine whether to sell non-substitutable products ($\beta^t = 0$) or substitutable products ($\beta^t = \beta > 0$). Further, they decide the prices of the products. Prices during one period may have an impact on demand in subsequent periods. If substitutable products are sold and the price differential is larger than a tolerance, part of the customer base of the highly priced channel transfer to the customer base of the lowly priced channel in the next period. Otherwise, the customer base remains the same.

Adapt Equation (4.1) for the above-mentioned multi-period setting and the demand in channel i in period t is as follows,

$$\begin{aligned} q_1^t &= f_1^t(p_1^t) - h^t(p_1^t - p_2^t, \beta^t), \\ q_2^t &= f_2^t(p_2^t) + h^t(p_1^t - p_2^t, \beta^t). \end{aligned} \tag{4.10}$$

Furthermore, if $\beta^t = \beta$ and $p_2^t - p_1^t > \tau$,

$$\begin{aligned} f_1^{t+1}(p_1) &= f_1^t(p_1) + \delta, \\ f_2^{t+1}(p_2) &= f_1^t(p_2) - \delta. \end{aligned} \tag{4.11}$$

If $\beta^t = \beta$ and $p_1^t - p_2^t > \tau$,

$$\begin{aligned} f_1^{t+1}(p_1) &= f_1^t(p_1) - \delta, \\ f_2^{t+1}(p_2) &= f_1^t(p_2) + \delta. \end{aligned} \tag{4.12}$$

Otherwise,

$$\begin{aligned} f_1^{t+1}(p_1) &= f_1^t(p_1), \\ f_2^{t+1}(p_2) &= f_1^t(p_2). \end{aligned} \tag{4.13}$$

In the above expressions, $\delta > 0, \tau \geq 0$ are constants. When the two channels sell substitutable products, τ is tolerance towards the price differential. Once the price differential exceeds τ, there are δ units of the customer base (the amount of demand when price is zero) shifting from the highly priced channel to the lowly priced one in the next period. If the price differential is at most τ or the two channels sell non-substitutable products, the customer base in each channel does not change.

The cost of selling one unit in channel i is c_i. The two channels aim to determine the optimal policy that maximizes the total profit over T periods, i.e., $\sum_{t=1}^{T}(p_1^t - c_1)q_1^t + (p_2^t - c_2)q_2^t$. A policy consists of a series of actions, one for each period. An action can be one of the following,

D: to sell non-substitutable products;

$\bar{S}$: to sell substitutable products and the price differential does not exceed τ;

$\grave{S}$: to sell substitutable products and $p_1(t) - p_2(t) > \tau$;

$\acute{S}$: to sell substitutable products and $p_2(t) - p_1(t) > \tau$.

Theorem 4.3.2. *When the optimal policy is unique,*

(1) actions of D should be joined together;

(2) actions of $\bar{S}$ should be joined together.

Proof. (1) We prove by contradiction and assume that the optimal policy has a chain of $(D, \vec{A}, D)$ where the former D is in period t, and $\vec{A}$ is made up of $a \geq 1$ actions and does not include D. Let f_i^s be the demand attracted by channel i in period t. The demand attracted by each channel in period $t+1$ remains the same since the action in period t is D. Let f_i^e be the demand attracted by channel i in period $t+a$. Let $\Pi(f_1, f_2, A)$ denote the total profit in a period when the demand attracted by channel i is f_i and the action is A. By comparing $(D, \vec{A}, D)$ with $(D, D, \vec{A})$, we get $\Pi(f_1^e, f_2^e, D) > \Pi(f_1^s, f_2^s, D)$. By comparing $(D, \vec{A}, D)$ with $(\vec{A}, D, D)$, we get $\Pi(f_1^s, f_2^s, D) > \Pi(f_1^e, f_2^e, D)$ and come to a contradiction. Therefore, Ds should be joined together.

(2) Similar to (1). □

Theorem 4.3.2 states that given the uniqueness of the optimal policy, if the two channels decide to sell substitutable products at similar prices or sell non-substitutable products in a certain number of periods, then those periods should be grouped together. This result helps facilitate the search for the optimal policy on product management and pricing. When the planning horizon is T periods, one needs to consider where to position m consecutive Ds and n consecutive $\bar{S}$ where m and n are non-negative integers and $m+n \leq T$. The policy that produces the highest total profit is the optimal one.

4.4 Conclusions

This chapter considers centralized demand management in two channels where the demand takes a general form of prices. It characterizes the total amount of demand in the two channels when price is optimal. Further, if there exists a price differential of the products in the two channels, demand migrates from the highly priced channel to the relatively lowly priced one. The study demonstrates that the effect of the migration of demand on the total profit is mediated by the profit margins. Finally, it is shown that given the uniqueness of the optimal policy, if the firm decides to sell substitutable products at similar prices or sell non-substitutable products in a certain number of periods, then those periods should be grouped together. Future research may extend to demand management of multiple products in more than two channels in the face of competition.

Chapter 5

Summary and Conclusions

This thesis studies inventory management in firms that market products in multiple channels. Chapter 1 summaries theoretical findings and industrial practices in this field. Chapter 2 compares continuous and end-of-period accounting of inventory in serial systems implementing echelon (r,nQ,T) policies. It is proved that continuous accounting leads to the otherwise overshot reorder points. Chapter 3 considers one-warehouse multi-retailer systems where inventory is redistributed periodically. In this situation, it develops an upper bound of the optimal reorder interval. Chapter 4 discusses the demand management in two channels for which the total profit is a major concern.

As shown in Chapters 2 and 3, if demand demand migrates from one channel to another, it could help reduce inventory-related costs because inventory can be pooled. On the other hand, Chapter 4 shows that migration of demand could have a negative effect on the total profit. Future research may build an integrated model that combines inventory management and revenue management so as to get a bird's-eye view of the role of migration.

Bibliography

Abouee-Mehrizi, H., O. Berman, and S. Sharma

2015. Optimal joint replenishment and transshipment policies in a multi-period inventory system with lost sales. *Operations Research*, 63(2):342–350.

Agrawal, V., X. Chao, and S. Seshadri

2004. Dynamic balancing of inventory in supply chains. *European Journal of Operational Research*, 159(2):296–317.

Amazon

2015. Amazon media room: Press releases, amazon launches first-ever staffed campus pickup and drop-off location, free one-day pickup at purdue university. `http://phx.corporate-ir.net/phoenix.zhtml?c=176060&p=irol-newsArticle&ID=2013031`. (Visited on 04/09/2015).

Ang, M., J.-S. Song, M. Wang, and H. Zhang

2013. On properties of discrete (r, q) and (s, T) inventory systems. *European Journal of Operational Research*, 229(1):95–105.

Ansari, A., C. F. Mela, and S. A. Neslin

2008. Customer channel migration. *Journal of Marketing Research*, 45(1):60–76.

Argos

2014. Home retail group annual report 2014.

Argos

2015. Check & reserve with argos. `http://www.argos.co.uk/static/StaticDisplay/includeName/Reserving.htm`. (Visited on 04/09/2015).

Arnold, J., S. Chapman, and L. Clive

2013. *Introduction to Materials Management*, Always learning. Pearson Education Limited.

Arya, A., B. Mittendorf, and D. E. Sappington

2007. The bright side of supplier encroachment. *Marketing Science*, 26(5):651–659.

Aussadavut, D., F. Ming, J. Apurva, and M. Kamran

2008. A supply chain model with direct and retail channels. *European Journal of Operational Research*, 187(3):691–718.

Avery, J., T. J. Steenburgh, J. Deighton, and M. Caravella

2012. Adding bricks to clicks: Predicting the patterns of cross-channel elasticities over time. *Journal of Marketing*, 76(3):96–111.

Axsäter, S.

2000. Exact analysis of continuous review (R, Q) policies in two-echelon inventory systems with compound poisson demand. *Operations research*, 48(5):686–696.

Axsäter, S.

2003. Approximate optimization of a two-level distribution inventory system. *International Journal of Production Economics*, 81:545–553.

Axsäter, S.

2007. *Inventory control*, volume 90. Springer Science & Business Media.

Balakrishnan, A., S. Sundaresan, and B. Zhang

2014. Browse-and-switch: Retail-online competition under value uncertainty. *Production and Operations Management*, 23(7):1129–1145.

Bernstein, F., J.-S. Song, and X. Zheng

2009. Free riding in a multi-channel supply chain. *Naval Research Logistics (NRL)*, 56(8):745–765.

BestBuy

2015a. About best buy. `https://corporate.bestbuy.com/about-best-buy/`. (Visited on 04/09/2015).

BestBuy

2015b. Price match guarantee - best buy. `http://www.bestbuy.com/site/Global/%20Price-Match-Guarantee/pcmcat290300050002.c?id=pcmcat290300050002`. (Visited on 04/09/2015).

Biyalogorsky, E. and P. Naik

2003. Clicks and mortar: the effect of on-line activities on off-line sales. *Marketing Letters*, 14(1):21–32.

Boyaci, T.

2005. Competitive stocking and coordination in a multiple-channel distribution system. *IIE transactions*, 37(5):407–427.

Cai, G. G.

2010. Channel selection and coordination in dual-channel supply chains. *Journal of Retailing*, 86(1):22–36.

Cai, G. G., Z. G. Zhang, and M. Zhang

2009. Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes. *International Journal of Production Economics*, 117(1):80–96.

Carrillo, J. E., A. J. Vakharia, and R. Wang

2014. Environmental implications for online retailing. *European Journal of Operational Research*, 239(3):744–755.

Cattani, K., W. Gilland, H. S. Heese, and J. Swaminathan

2006. Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. *Production and Operations Management*, 15(1):40.

Chao, X. and S. X. Zhou

2009. Optimal policy for a multiechelon inventory system with batch ordering and fixed replenishment intervals. *Operations Research*, 57(2):377–390.

Charlton, G.

2013. Evans cycles on the key drivers for multichannel shopping — econsultancy. https://econsultancy.com/blog/63506-evans-cycles-on-the-key-drivers-for-multichannel-shopping. (Visited on 04/08/2015).

Chen, F. and Y.-S. Zheng

1994. Evaluating echelon stock (R, nQ) policies in serial production/inventory systems with stochastic demand. *Management Science*, 40(10):1262–1275.

Chen, F. and Y.-S. Zheng

1997. One-warehouse multiretailer systems with centralized stock information. *Operations Research*, 45(2):275–287.

Chen, K.-Y., M. Kaya, and Ö. Özer

2008. Dual sales channel management with service competition. *Manufacturing & Service Operations Management*, 10(4):654–675.

Chiang, W.-y. K.

2010. Product availability in competitive and cooperative dual-channel distribution with stock-out based substitution. *European Journal of Operational Research*, 200(1):111–126.

Chiang, W.-y. K., D. Chhajed, and J. D. Hess

2003. Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science*, 49(1):1–20.

Chiang, W.-y. K. and G. E. Monahan

2005. Managing inventories in a two-echelon dual-channel supply chain. *European Journal of Operational Research*, 162(2):325–341.

Chiu, H.-C., Y.-C. Hsieh, J. Roan, K.-J. Tseng, and J.-K. Hsieh

2011. The challenge for multichannel services: Cross-channel free-riding behavior. *Electronic Commerce Research and Applications*, 10(2):268–277.

Clarins

2015. Club clarins: Be rewarded. `http://club.clarinsusa.com/home.action`. (Visited on 04/09/2015).

Clark, A. J. and H. Scarf

1960. Optimal policies for a multi-echelon inventory problem. *Management Science*, 6(4):475–490.

Das, C.

1975. Supply and redistribution rules for two-location inventory systems: One-period analysis. *Management Science*, 21(7):765–776.

David, A. and E. Adida

2014. Competition and coordination in a two-channel supply chain. *Production and Operations Management*.

DBS

2014. Dbs paylah! - promotions. `http://www.dbs.com.sg/personal/landing/paylah/promotions.html`. (Visited on 04/09/2015).

Dinner, I. M., H. J. Van Heerde, and S. A. Neslin

2014. Driving online and offline sales: The cross-channel effects of traditional, online display, and paid search advertising. *Journal of Marketing Research*, 51(5):527–545.

DMA

2014. *DMA Statistical Fact Book 2014: The Definitive Source for Direct Marketing Benchmarks*. Direct Marketing Assn.

Fernández-Sabiote, E. and S. Román

2012. Adding clicks to bricks: A study of the consequences on customer loyalty in a service context. *Electronic Commerce Research and Applications*, 11(1):36–48.

Frambach, R. T., H. C. Roest, and T. V. Krishnan

2007. The impact of consumer internet experience on channel preference and usage intentions across the different stages of the buying process. *Journal of Interactive Marketing*, 21(2):26–41.

Gallego, G., Ö. Özer, and P. Zipkin

2007. Bounds, heuristics, and approximations for distribution systems. *Operations Research*, 55(3):503–517.

Gallego, G. and D. Simchi-Levi

1990. On the effectiveness of direct shipping strategy for the one-warehouse multi-retailer R-systems. *Management Science*, 36(2):240–243.

Gallino, S. and A. Moreno

2014. Integration of online and offline channels in retail: The impact of sharing reliable inventory availability information. *Management Science*, 60(6):1434–1451.

Gao, F. and X. Su

2016. Online and offline information for omnichannel retailing. *Manufacturing & Service Operations Management*.

Gensler, S., P. Leeflang, and B. Skiera

2012a. Impact of online channel use on customer revenues and costs to serve: Considering product portfolios and self-selection. *International Journal of Research in Marketing*, 29(2):192–201.

Gensler, S., P. C. Verhoef, and M. Böhm

2012b. Understanding consumers multichannel choices across the different stages of the buying process. *Marketing Letters*, 23(4):987–1003.

Gilt

2013. Gilt.com unveils gilt insider program. `http://www.gilt.com/company/press/giltcom-unveils-gilt-insider-program`. (Visited on 04/09/2015).

Glazebrook, K., C. Paterson, S. Rauscher, and T. Archibald

2015. Benefits of hybrid lateral transshipments in multi-item inventory systems under periodic replenishment. *Production and Operations Management*, 24(2):311–324.

Hadley, G. and T. M. Whitin

1963. *Analysis of inventory systems*. Prentice Hall.

Homburg, C., J. Vollmayr, and A. Hahn

2014. Firm value creation through major channel expansions: Evidence from an event study in the united states, germany, and china. *Journal of Marketing*, 78(3):38–61.

Hsiao, L. and Y.-J. Chen

2013. The perils of selling online: Manufacturer competition, channel conflict, and consumer preferences. *Marketing Letters*, 24(3):277–292.

Hsiao, L. and Y.-J. Chen

2014. Strategic motive for introducing internet channels in a supply chain. *Production and Operations Management*, 23(1):36–47.

Hua, G., S. Wang, and T. E. Cheng

2010. Price and lead time decisions in dual-channel supply chains. *European journal of operational research*, 205(1):113–126.

Huang, W. and J. M. Swaminathan

2009. Introduction of a second channel: Implications for pricing and profits. *European Journal of Operational Research*, 194(1):258–279.

Inditex

2014. Inditex interim nine months 2014 results. `http://www.inditex.com/documents/10279/145048/INDITEX-Interim+3Q2014+Results.pdf/e8443dcc-28c7-4939-b2d5-fa3d6307fc18`. (Visited on 04/09/2015).

Jönsson, H. and E. A. Silver

1987. Analysis of a two-echelon inventory control system with complete redistribution. *Management Science*, 33(2):215–227.

Keller, K. L.

2010. Brand equity management in a multichannel, multimedia retail environment. *Journal of Interactive Marketing*, 24(2):58–70.

Konuş, U., S. A. Neslin, and P. C. Verhoef

2014. The effect of search channel elimination on purchase incidence, order size and channel choice. *International Journal of Research in Marketing*, 31(1):49–64.

Kushwaha, T. and V. Shankar

2013. Are multichannel customers really more valuable? the moderating role of product category characteristics. *Journal of Marketing*, 77(4):67–85.

Kwon, W.-S. and S. J. Lennon

2009. Reciprocal effects between multichannel retailers' offline and online brand images. *Journal of Retailing*, 85(3):376–390.

Lagodimos, A., I. Christou, and K. Skouri

2012. Computing globally optimal (s, S, T) inventory policies. *Omega*, 40(5):660–671.

Larivière, B., L. Aksoy, B. Cooil, and T. L. Keiningham

2011. Does satisfaction matter more if a multichannel customer is also a multicompany customer? *Journal of Service Management*, 22(1):39–66.

Liao, C.-H., H. Rebecca Yen, and E. Y. Li

2011. The effect of channel quality inconsistency on the association between e-service quality and customer relationships. *Internet Research*, 21(4):458–478.

Lien, R. W., S. M. Iravani, K. Smilowitz, and M. Tzur

2011. An efficient and robust design for transshipment networks. *Production and Operations Management*, 20(5):699–713.

Little, J. D.

2011. Little's law as viewed on its 50th anniversary. *Operations Research*, 59(3):536–549.

Liu, F. and J.-S. Song

2012. Good and bad news about the (S, T) policy. *Manufacturing & Service Operations Management*, 14(1):42–49.

Liu, K., Y. Zhou, and Z. Zhang

2010. Capacitated location model with online demand pooling in a multi-channel supply chain. *European Journal of Operational Research*, 207(1):218–231.

Lu, Q. and N. Liu

2015. Effects of e-commerce channel entry in a two-echelon supply chain: A comparative analysis of single-and dual-channel distribution systems. *International Journal of Production Economics*, 165:100–111.

Mahar, S., K. M. Bretthauer, and M. Venkataramanan

2009. The value of virtual pooling in dual sales channel supply chains. *European Journal of Operational Research*, 192(2):561–575.

Mahar, S., P. A. Salzarulo, and P. D. Wright

2012. Using online pickup site inclusion policies to manage demand in retail/e-tail organizations. *Computers & Operations Research*, 39(5):991–999.

Mahar, S. and P. D. Wright

2009. The value of postponing online fulfillment decisions in multi-channel retail/e-tail organizations. *Computers & operations research*, 36(11):3061–3072.

Melewar, T., L. L. Lim, and R. Yan

2010. Product brand differentiation and dual-channel store performances of a multi-channel retailer. *European Journal of Marketing*, 44(5):672–692.

Montoya-Weiss, M. M., G. B. Voss, and D. Grewal

2003. Determinants of online channel use and overall satisfaction with a relational, multichannel service provider. *Journal of the academy of marketing Science*, 31(4):448–458.

M&S

2014. Marks and spencer group plc annual report and financial statements 2014.

M&S

2015. `http://www.marksandspencer.com/`. (Visited on 04/09/2015).

Mukhopadhyay, S. K., X. Zhu, and X. Yue

2008. Optimal contract design for mixed channels under information asymmetry. *Production and Operations Management*, 17(6):641–650.

Neslin, S. A. and V. Shankar

2009. Key issues in multichannel customer management: current knowledge and future directions. *Journal of interactive marketing*, 23(1):70–81.

Nike

2015. Nike store locator. `http://www.nike.com/sg/en_gb/sl/store-locator`. (Visited on 04/09/2015).

Ofek, E., Z. Katona, and M. Sarvary

2011. bricks and clicks: The impact of product returns on the strategies of multichannel retailers. *Marketing Science*, 30(1):42–60.

Oh, L.-B., H.-H. Teo, and V. Sambamurthy

2012. The effects of retail channel integration through the use of information technologies on firm performance. *Journal of Operations Management*, 30(5):368–381.

Özdemir, D., E. Yücesan, and Y. T. Herer

2006. Multi-location transshipment problem with capacitated transportation. *European Journal of Operational Research*, 175(1):602–621.

Paterson, C., G. Kiesmüller, R. Teunter, and K. Glazebrook

2011. Inventory models with lateral transshipments: A review. *European Journal of Operational Research*, 210(2):125–136.

Paterson, C., R. Teunter, and K. Glazebrook

2012. Enhanced lateral transshipments in a multi-location inventory system. *European Journal of Operational Research*, 221(2):317–327.

Perakis, G. and A. Sood

2006. Competitive multi-period pricing for perishable products: A robust optimization approach. *Mathematical Programming*, 107(1-2):295–335.

PUMA

2013. Annual report 2013.

Qiu, X. and G. Q. Huang

2011. On storage capacity pooling through the supply hub in industrial park (ship): The impact of demand uncertainty. In *Industrial Engineering and Engineering Management (IEEM), 2011 IEEE International Conference on*, Pp. 1745–1749. IEEE.

Raju, J. S., R. Sethuraman, and S. K. Dhar

1995. The introduction and performance of store brands. *Management Science*, 41(6):957–978.

Rangaswamy, A. and G. H. Van Bruggen

2005. Opportunities and challenges in multichannel marketing: An introduction to the special issue. *Journal of Interactive Marketing*, 19(2):5–11.

Rao, U. S.

2003. Properties of the periodic review (R, T) inventory control policy for stationary, stochastic demand. *Manufacturing & Service Operations Management*, 5(1):37–53.

Ruddick, G.

2013. 'success online? it's all about shops actually' - telegraph. `http:`

//www.telegraph.co.uk/finance/newsbysector/retailandconsumer/10007746/Success-online-Its-all-about-shops-actually.html. (Visited on 04/08/2015).

Rudi, N., H. Groenevelt, and T. R. Randall
2009. End-of-period vs. continuous accounting of inventory-related costs. *Operations research*, 57(6):1360–1366.

Seidscher, A. and S. Minner
2013. A semi-markov decision problem for proactive and reactive transshipments between multiple warehouses. *European Journal of Operational Research*, 230(1):42–52.

Shang, K. H., Z. Tao, and S. X. Zhou
2015. Optimizing reorder intervals for two-echelon distribution systems with stochastic demand. *Operations Research*, 63(2):458–475.

Shang, K. H. and S. X. Zhou
2010. Optimal and heuristic echelon (r, nQ, T) policies in serial inventory systems with fixed costs. *Operations Research*, 58(2):414–427.

Sharma, A. and A. Mehrotra
2007. Choosing an optimal channel mix in multichannel environments. *Industrial Marketing Management*, 36(1):21–28.

Tagaras, G. and D. Vlachos
2002. Effectiveness of stock transshipment under various demand distributions and nonnegligible transshipment times. *Production and Operations Management*, 11(2):183–198.

Tiacci, L. and S. Saetta

2011. A heuristic for balancing the inventory level of different locations through lateral shipments. *International Journal of Production Economics*, 131(1):87–95.

Topshop

2015. Topshop personal shopping. `http://www.topshop.com/en/tsuk/category/topshop-personal-shopping-home-2693867/home`. (Visited on 04/09/2015).

Trampe, D., U. Konuş, and P. C. Verhoef

2014. Customer responses to channel migration strategies toward the e-channel. *Journal of Interactive Marketing*, 28(4):257–270.

Tsay, A. A. and N. Agrawal

2004. Channel conflict and coordination in the e-commerce age. *Production and Operations Management*, 13(1):93–110.

Valentini, S., E. Montaguti, and S. A. Neslin

2011. Decision process evolution in customer channel choice. *Journal of Marketing*, 75(6):72–86.

Van Houtum, G.-J., A. Scheller-Wolf, and J. Yi

2007. Optimal control of serial inventory systems with fixed replenishment intervals. *Operations Research*, 55(4):674–687.

Van Nierop, J. E. M., P. S. Leeflang, M. L. Teerling, and K. Huizingh

2011. The impact of the introduction and use of an informational website on offline customer buying behavior. *International Journal of Research in Marketing*, 28(2):155–165.

Vinhas, A. S. and E. Anderson
2005. How potential conflict drives channel structure: Concurrent (direct and indirect) channels. *Journal of Marketing Research*, 42(4):507–515.

Wallace, D. W., J. L. Giese, and J. L. Johnson
2004. Customer retailer loyalty in the context of multiple channel strategies. *Journal of Retailing*, 80(4):249–263.

Wang, Q. and G. Wan
2016. Fixed-interval order-up-to policies for stochastic serial sypply chain systems. Working paper.

Watsons
2015. Watsons singapore. `http://www.watsons.com.sg/`. (Visited on 04/09/2015).

Xia, Y. and G. P. Zhang
2010. The impact of the online channel on retailers' performances: An empirical evaluation. *Decision Sciences*, 41(3):517–546.

Xing, D. and T. Liu
2012. Sales effort free riding and coordination with price match and channel rebate. *European Journal of Operational Research*, 219(2):264–271.

Xu, G., B. Dan, X. Zhang, and C. Liu
2014. Coordinating a dual-channel supply chain with risk-averse under

a two-way revenue sharing contract. *International Journal of Production Economics*, 147:171–179.

Yan, R.
2011. Managing channel coordination in a multi-channel manufacturer–retailer supply chain. *Industrial Marketing Management*, 40(4):636–642.

Yan, R., P. Guo, J. Wang, and N. Amrouche
2011. Product distribution and coordination strategies in a multi-channel context. *Journal of Retailing and Consumer Services*, 18(1):19–26.

Yan, R. and Z. Pei
2011. Information asymmetry, pricing strategy and firm's performance in the retailer-multi-channel manufacturer supply chain. *Journal of Business Research*, 64(4):377–384.

Yoo, W. S. and E. Lee
2011. Internet channel entry: a strategic analysis of mixed channel structures. *Marketing Science*, 30(1):29–41.

Zhang, J., P. W. Farris, J. W. Irvin, T. Kushwaha, T. J. Steenburgh, and B. A. Weitz
2010. Crafting integrated multichannel retailing strategies. *Journal of Interactive Marketing*, 24(2):168–180.

Zhang, J. and M. Wedel
2009. The effectiveness of customized promotions in online and offline stores. *Journal of Marketing Research*, 46(2):190–206.

Zhang, X.

2009. Retailers' multichannel and price advertising strategies. *Marketing Science*, 28(6):1080–1094.