

Coherent perfect absorption and reflection in slow-light waveguides

Nadav Gutman,^{1,*} Andrey A. Sukhorukov,² Y. D. Chong,³ and C. Martijn de Sterke¹

¹*IPOS and CUDOS, School of Physics, University of Sydney, NSW 2006, Australia*

²*Nonlinear Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, ACT 0200, Australia*

³*Division of Physics and Applied Physics and Centre for Disruptive Photonic Technologies, Nanyang Technological University, Singapore 637371*

*Corresponding author: nadav.gutman@gmail.com

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We identify a family of unusual slow-light modes occurring in lossy multimode grating waveguides, for which either the forward or backward mode components, or both, are degenerate. In the fully degenerate case, the response can be modulated between coherent perfect absorption (zero reflection) and perfect reflection by varying the wave amplitudes in a uniform input waveguide. The perfectly absorbed wave has anomalously short absorption length, scaling as the inverse one-third power of the absorptivity. © 2013 Optical Society of America

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In periodically structured optical media such as Bragg gratings and photonic crystals, the propagation of light is strongly modified by coherent self-interference. This is the basis for numerous photonic devices, such as slow-light waveguides [1]. Slow-light waveguides are of considerable scientific and technological interest because light-matter interactions are enhanced in the slow-light regime; for applications involving light nonlinearity, absorption, and amplification, a short slow-light waveguide section integrated on a photonic chip can achieve the same functionality as a conventional bulk device [2,3]. Slow-light waveguides containing loss and gain also can exhibit unidirectional and nonreciprocal light propagation [4,5], with applications to photonic logic [6]. A key challenge in utilizing these waveguides is that the input coupling from conventional (i.e., nonslow light) waveguides tends to be inefficient. This may be addressed by an impedance-matching transition region [7] or by designing the slow-light waveguide to support evanescent modes, which aid in impedance matching [8]. However, such techniques are typically formulated for the ideal case where the slow-light modes have real dispersion curves, without accounting for losses. Losses, however, can be considerable for slow-light modes; they may even be part of the intended functionality, as in the case of effective absorption arising from light-matter interaction.

In this Letter, we show that the problem of input coupling into slow-light waveguides can be substantially modified by absorption. We identify a family of complex slow-light modes in lossy multimode grating waveguides, which can exhibit either coherent perfect absorption or, its opposite effect, perfect reflection. The concept of coherent perfect absorption was developed recently in a general optical scattering context [9–11], as a generalization of critical coupling; interestingly, it has been previously discussed, in the context of lossy gratings, by Poladian [12]. It has also been studied and demonstrated in thin metamaterial slabs by Zhang *et al.* [13], who emphasized that the absorption can be modulated from

zero to unity by tuning the incident wave amplitudes, with a range of practical applications.

Unlike these previous works, we consider a semi-infinite lossy waveguide, rather than a “cavity” like a finite waveguide segment. Light is coupled into one end of the waveguide, and the interference of *copropagating* slow-light modes gives rise to perfect absorption or reflection. Furthermore, coherent perfect absorption is associated with a degeneracy of the lossy waveguide modes. In an appropriately tuned waveguide this leads to anomalously short absorption lengths, which scale as $\alpha^{-1/3}$, where α is the absorptivity of the waveguide medium, rather than α^{-1} as for ordinary lossy modes. In this way, a normally weakly absorbing waveguide can absorb a correctly chosen coherent input both perfectly (i.e., with zero reflection) and efficiently (i.e., with a short absorption length).

Our structure consists of two joined semi-infinite waveguides, shown schematically in Fig. 1(a). On the left is a lossless uniform waveguide, which supports two

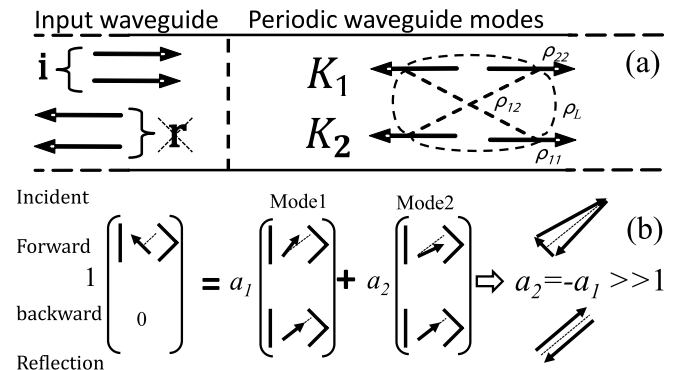


Fig. 1. (a) Schematic of a uniform waveguide coupled to a lossy periodic waveguide. (b) Schematic of the perfect absorption mechanism. The arrows represent the mode distribution of the forward and backward components on each side of the interface. The input is normalized to 1, and the periodic waveguide has two excited modes with amplitudes, a_1 and a_2 .

forward (input) modes and two backward (reflected) modes; these can be either spatial or polarization modes. On the right is a lossy grating waveguide, which likewise supports two forward and two backward modes. The forward grating modes are excited with amplitudes a_1 and a_2 , respectively. The backward grating modes have zero amplitude; in physical terms, this means that the grating waveguide's actual length greatly exceeds the lossy slow-light modes' absorption lengths, so that back-reflection from the far end is negligible. The grating modes are combinations of the uniform waveguide's modes, which we refer to as "basis modes." Each grating mode consists of both forward *and* backward basis modes. Figure 1(b) shows the principle by which reflection can be eliminated at the interface, resulting in coherent perfect absorption. The modes on each side are represented by vectors whose components correspond to forward- and backward-propagating basis modes. As indicated by the arrows, each forward or backward component is itself a vector in a 2D modal subspace (since there are two basis modes for each direction of propagation). Now, suppose the forward grating modes have degenerate (linearly dependent) backward components. We refer to this as *partial mode degeneracy*. If the mode amplitudes satisfy $a_1 = -a_2$, the backward components destructively interfere; then the reflection vanishes, and the incident light is perfectly coupled into the grating waveguide and absorbed. Conversely, the forward components can be made degenerate, resulting in perfect reflection. *Full mode degeneracy*, i.e., degeneracy of both forward and backward components, can also be achieved. In this regime the amplitudes $a_{1,2}$ are much larger than the input amplitude, so the field intensity is strongly enhanced in the region of the lossy waveguide near the interface. We find that this occurs when the waveguide supports *complex degenerate bands* (CDB) satisfying

$$(\omega - \omega_D) = (k - k_D)^2 / \xi, \quad (1)$$

where ω_D is a real frequency, and k_D and ξ are imaginary. Near the degeneracy point, the group velocity $v_g \equiv \partial\omega/\partial\text{Re}(k) \ll c$, so these are indeed slow light modes. When $\omega = \omega_D$, the group velocity vanishes, and we will see that these "frozen light" [14] modes exhibit strongly enhanced absorption.

The above argument can be developed using the coupled-mode theory of grating modes [15,16]. In the uniform waveguide, the modes have propagation constants $\tilde{K}_{1,2}$; there are forward (+) as well as backward (-) modes, and the total field is characterized by four mode amplitudes $E_{1,2}^\pm$. For convenience, we normalize the group velocities of these basis modes to $v = 1$, though this is not essential [15]. In the grating waveguide, losses induce $\exp(\pm\alpha_{1,2}z)$ decays for the basis modes; for convenience, we take $\alpha_1 = \alpha_2 \equiv \alpha$. The grating also couples the basis modes, leading to amplitude variations, which can be described by the envelopes $E_{1,2}^\pm(z, t)$. Assuming solutions of the form $\mathbf{E} \exp(ikz - i\omega t)$, where ω is the operating frequency, the wavenumbers k are the eigenvalues of the wavenumber matrix [15,16]

$$\Theta = \begin{pmatrix} \omega + i\alpha & 0 & \rho_{11} & \rho_{12} \\ 0 & \omega - \delta + i\alpha & \rho_{12} & \rho_{22} \\ -\rho_{11} & -\rho_{12} & -\omega - i\alpha & 0 \\ -\rho_{12} & -\rho_{22} & 0 & -\omega + \delta - i\alpha \end{pmatrix}. \quad (2)$$

The ρ_{ij} s are overlap integrals of the transverse mode profiles and describe the coupling between basis mode i and the counterpropagating mode j [15]. Such couplings can be achieved either by a deep periodic modulation in a high-index material or by multiple shallow Bragg gratings with different periods; in the latter case, each coefficient is defined by a grating with period $2\pi/\text{Re}(\tilde{K}_i + \tilde{K}_j)$. By appropriate choice of grating structure, we take each ρ_{ij} to be real-valued [15]. We have also included a small relative detuning δ from the Bragg resonance. The eigenvectors of Θ are written as $\mathbf{E}_n = [E_1^+, E_2^+, E_1^-, E_2^-]^T_n$, where the \pm superscripts denote forward and backward components. The eigenvalues $k_n(\omega)$ yield the waveguide's complex band structure.

We now demonstrate how coherent perfect absorption arises in this system. In the input waveguide, the amplitudes are $\mathbf{i} = [i_1, i_2]^T$ for the forward modes, and $\mathbf{r} = [r_1, r_2]^T$ for the backward modes [see Fig. 1(a)]. In the grating waveguide, the mode amplitudes are $a_{1,2}$ for the two forward grating modes, which have $\text{Im}(k) > 0$. Matching these amplitudes gives

$$\mathbf{E}_0 = \begin{pmatrix} \mathbf{i} \\ \mathbf{r} \end{pmatrix} = [i_1, i_2, r_1, r_2]^T = \mathcal{M}[a_1, a_2, 0, 0]^T, \quad (3)$$

where the columns of the matrix \mathcal{M} consist of the eigenvectors of Θ . From \mathcal{M} , we can define a reflection matrix \mathcal{R} , which connects \mathbf{i} and \mathbf{r} by $\mathbf{r} = \mathcal{R}\mathbf{i}$:

$$\mathcal{R} = \begin{pmatrix} \mathcal{M}_{31} & \mathcal{M}_{32} \\ \mathcal{M}_{41} & \mathcal{M}_{42} \end{pmatrix} \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}^{-1} = \mathcal{M}^- / \mathcal{M}^+. \quad (4)$$

Here, \mathcal{M}^\pm are 2×2 matrices. Zero reflection occurs when an eigenvalue of \mathcal{R} is zero, i.e.,

$$\det(\mathcal{R}) = \det(\mathcal{M}^-) / \det(\mathcal{M}^+) = 0. \quad (5)$$

The corresponding eigenvector of \mathcal{R} yields the inputs $i_{1,2}$, which are perfectly absorbed. Equivalently, $\det(\mathcal{M}^-) = 0$, which occurs when the two columns of \mathcal{M}^- are linearly dependent, i.e., Θ has two eigenvectors with equal backward components.

Figures 2(a) and 2(b) show a complex band structure tuned to partial mode degeneracy at frequency ω_D . In this degeneracy case, it can be shown that the two wavenumbers, k_1 and k_2 , must satisfy $\text{Re}(k_1) = -\text{Re}(k_2)$ and $\text{Im}(k_1) + \text{Im}(k_2) = |\rho_{12}|\delta/\sqrt{\det(\rho)}$. One wavenumber has significantly smaller $\text{Im}(k)$ than the other. At $\omega = \omega_D$, the two modes' values of $\text{Im}(k)$ approach one other, but do not meet. The values of $\text{Im}(k)$ at the partial degeneracy point are plotted against the absorptivity α in Fig. 3(a). At the interface $x = 0$, the two modes must have the same amplitude in order for the reflection to cancel exactly; hence, the intensity decay along the lossy

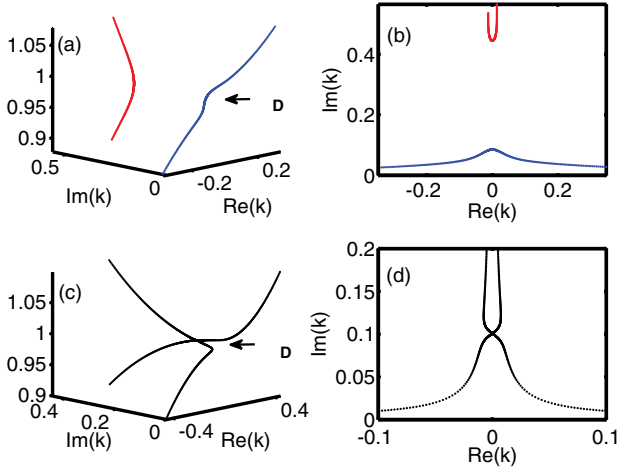


Fig. 2. Complex band structures of absorbing grating waveguides exhibiting coherent perfect absorption. (a) and (b) Partial mode degeneracy, with $\rho_{11} = 1$, $\rho_{22} = 0.9$, $\rho_{12} = 0.2155$, $\delta = 1.8937$, and $\alpha = 10^{-2}$. (c) and (d) Full degeneracy (CDB), with $\rho_{11} = \rho_{22} = 1$, $\rho_{12} = 0.10016$, $\delta = 1.99990$, and $\alpha = 10^{-3}$.

waveguide is dominated by the less strongly damped mode, whose value of $\text{Im}(k)$ scales linearly with α . (For the other mode, $\text{Im}(k)$ is larger and α independent.)

When the forward and backward components are degenerate, which we refer to as *full* mode degeneracy, the absorption characteristics change qualitatively. In this case, the wavenumber matrix Θ is defective; it cannot be diagonalized, only brought to Jordan normal form

$$\mathcal{J} = \begin{pmatrix} k_D & 1 & & \\ & k_D & & \\ & & -k_D & 1 \\ & & & -k_D \end{pmatrix}, \quad (6)$$

where $\mathcal{J} = S^{-1}\Theta S$ and k_D is the complex wavenumber at the degeneracy. Using the characteristic polynomial p , the condition for the Jordan normal form in Eq. (6) is for its first two terms to vanish: $p(k_D + \Delta k) = \sum_{j=0}^4 C_j \Delta k^j$ ($C_0 = C_1 = 0$), where $\Delta k = k - k_D$. We can use this to find the parameters for full mode degeneracy. It has previously been shown that mode degeneracies in lossless waveguides are associated with stationary points in real dispersion relations, of the form $(\omega - \omega_D) \propto (k - k_D)^m$ where m is a positive integer [14]. We find here that full mode degeneracy in a lossy structure yields the dispersion curves of Eq. (1), featuring a *complex* quadratic stationary point. One such band structure is shown in Figs. 2(c) and 2(d).

According to Eq. (4), fully degenerate grating modes can be perfectly absorbed if $\det(\mathcal{M}^-)$ vanishes faster than $\det(\mathcal{M}^+)$ as we approach the degeneracy point. Let us write the forward and backward components of the degenerate modes as $V_k^\pm = [E_1^\pm, E_2^\pm]^T$. Expanding up to third order around the degeneracy, we obtain, from the perturbation theory of Jordan normal forms,

$$\mathbf{V}_{k_{1,2}}^+ = \mathbf{V}_0^+ \pm \Delta k \mathbf{V}_1^+ + \Delta k^2 \mathbf{V}_2^+ \pm \Delta k^3 \mathbf{V}_3^+, \quad (7)$$

$$\mathbf{V}_{k_{1,2}}^- = \mathbf{V}_0^- \pm \Delta k \mathbf{V}_1^- + \Delta k^2 \mathbf{V}_2^- \pm \Delta k^3 \mathbf{V}_3^-, \quad (8)$$

which represent the energy carried forward and backward by the modes' components, respectively. The vectors are degenerate for $\Delta k = 0$, and only the odd terms contribute to lifting the degeneracy. Perfect absorption requires $\mathbf{V}_1^- = 0$. Hence, $\det(\mathcal{M}^-) \propto \Delta k^3$, since up to third order the vectors are identical. In contrast, for $\mathbf{V}_1^+ \neq 0$ we find $\det(\mathcal{M}^+) \propto \Delta k$. Thus, $|\det(\mathcal{M}^+)|^2 \propto \Delta k^2 \propto \Delta \omega$ and $|\det(\mathcal{M}^-)|^2 \propto \Delta k^6 \propto \Delta \omega^3$ and $|\det(\mathcal{R})|^2 \propto \Delta k^4 \propto \Delta \omega^2$.

Using Eq. (2), we find that full degeneracy is achieved when $\rho_{11} = \rho_{22}$ and $\delta = 2\rho_{11}\rho_{12}(\alpha^2 + \rho_{12}^2)^{-1/2}$. The degeneracy occurs at frequency $\omega_D = \delta/2$, with wavenumber $k_D = [-(\alpha^4 + \rho_{12}^4 + \alpha^2(\rho_{11}^2 + 2\rho_{12}^2))/(\alpha^2 + \rho_{12}^2)]^{1/2}$. In the limit of small absorption, $\alpha \rightarrow 0$, the degeneracy condition reduces to $\alpha \simeq \rho_{12}^3/\rho_{11}^{2/3}$, with wavenumber $k_D \simeq i\rho_{12} \simeq \alpha^{1/3}\rho_{11}^{2/3}$. Hence, the absorption increases compared to a homogeneous waveguide. We have confirmed this scaling numerically [see Fig. 3(b)] by fulfilling conditions $C_0 = C_1 = 0$, $V_1^- = 0$ for different α 's.

When the grating waveguide supports fully degenerate modes, we can modulate between perfect absorption and perfect reflection using the input wave amplitudes. Let us parameterize the input amplitudes as

$$i_1 = \cos(\beta_1)e^{i\beta_2}, \quad i_2 = \sin(\beta_1). \quad (9)$$

Figure 3(c) shows the variation of the reflectance with $\beta_{1,2}$ at the degeneracy point.

Insight into the perfect absorption/reflection phenomenon can be obtained by expressing the incident fields \mathbf{E}_0 as a linear combination of the modes' Taylor expansion: $\mathbf{i}_0 = b_0 \mathbf{V}_0^+ + b_1 \mathbf{V}_1^+$. We find the amplitudes of the two degenerate modes $a_1, a_2 \propto b_1 \Delta k^{-1}$. When $\Delta k \rightarrow 0$, the field in the lossy waveguide is

$$\mathbf{E} = b_1 \begin{pmatrix} \mathbf{V}_1^+ + i\mathbf{V}_0^+ z \\ i\mathbf{V}_0^- z \end{pmatrix} e^{ik_D z}. \quad (10)$$

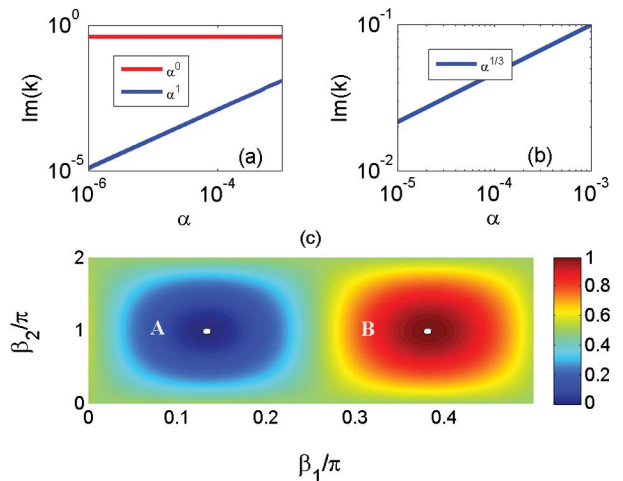


Fig. 3. Decay rate $\text{Im}(k)$ versus absorptivity α for (a) partial mode degeneracy and (b) full degeneracy. For each α , the system is retuned by varying ρ_{12} and δ so as to maintain partial mode degeneracy at some real ω_D . (c) Reflection at full degeneracy versus the relative amplitude (β_1) and phase (β_2) of the inputs, as parameterized in Eq. (9).

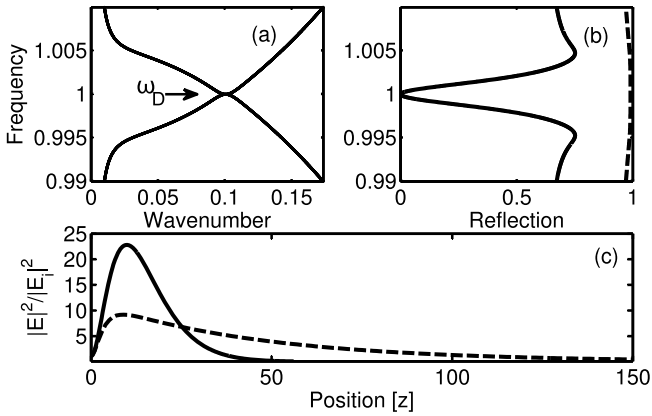


Fig. 4. (a) Real band structure and (b) reflection spectrum near a full mode degeneracy point. In (b), the two curves correspond to the input combinations, which would yield perfect absorption (black) and perfect reflection (dashed) at the degeneracy, corresponding to the A and B points in Fig. 3(c). (c) Field intensity in a grating waveguide with partial mode degeneracy (dashed) and full degeneracy (black), with absorptivity of $\alpha = 10^{-3}$.

This grows polynomially before decaying exponentially with decay length $1/k_D$. At the degeneracy point, only b_1 (which represents the dependence of the input field on \mathbf{V}_1^+) is important. To maximize (minimize) the field, the input \mathbf{i} should be parallel to \mathbf{V}_1^+ (\mathbf{V}_0^+); this corresponds to the points A (zero reflection) and B (total reflection) in Fig. 3(c), respectively. The reflectances for the two input combinations are plotted in Fig. 4(b). Figure 4(a) shows the real part of the band structure, demonstrating that $\partial\omega/\partial k \rightarrow 0$ at the degeneracy point.

Figure 4(d) shows the field intensities inside a grating waveguide with partial and full mode degeneracy. Both the partially and fully degenerate modes produce a slow-light intensity enhancement near the interface. However, in the partially degenerate case, the intensity subsequently decays at the same rate as in the uniform waveguide. In the fully degenerate case, the decay length is significantly shortened. For comparison, in a uniform waveguide with the same absorptivity ($\alpha = 10^{-3}$), the decay length is two orders of magnitude longer.

In conclusion, we have shown that a multimode periodically modulated lossy waveguide can be designed to feature a complex band degeneracy, which allows one to modulate between perfect absorption and perfect reflection by varying the input wave amplitudes. In particular,

the absorption can take place within a much shorter waveguide section than indicated by the underlying absorptivity, due to an anomalous scaling of the absorption length. Although we have analyzed this phenomenon in the context of the coupled mode theory of grating waveguides, the basic physical principle can be applied not only to low-index waveguides [17] but also to high-index [1], such as photonic crystal waveguides, suitable for on-chip slow-light applications.

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