

Lamb wave propagation in vibrating structures for effective health monitoring

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ABSTRACT

Lamb wave based Structural Health Monitoring (SHM) has received much attention during the past decades for its broad coverage and high sensitivity to damage. Lamb waves can be used to locate and quantify damage in static structures successfully. Nonetheless, structures are usually subjected to various external vibrations or oscillations. Not many studies are reported in the literature concerning the damage detecting ability of Lamb wave in oscillating structures which turns out to be a pivotal issue in the practical application of the SHM technique. For this reason in this study, the propagating capability of Lamb waves in a vibrating thin aluminum plate is examined experimentally. Two circular shaped piezoelectric wafer active transducers are surface-bonded on the aluminum plate where one acted as an actuator and another as a sensor. An arbitrary waveform generator is connected to the actuator for the generation of a windowed tone burst on the aluminum plate. An oscilloscope is connected to the sensor for receiving the traveled waves. An external shaker is used to generate out-of-plane external vibration on the plate structure. Time of flight (TOF) is a crucial parameter in most Lamb wave based SHM studies, which measures wave traveling time from the actuator to sensor. In the present study the influence of the external vibrations on the TOF is investigated. Experiments are performed under different boundary conditions of the plate, such as free-free and fixed by gluing. The effects of external vibrations in the frequency range between 10 Hz to 1000 Hz are analyzed. Comparisons are carried out between the resulting Lamb wave signals from the vibrating plate for different boundary conditions. Experimental results show that the external vibrations in relatively low frequency range do not change the TOF during the application of Lamb wave based SHM.

Keywords: Structural health monitoring; Lamb wave; external vibration; piezoelectric transducer

1. INTRODUCTION

Lamb waves based structural health monitoring (SHM) is attracting more and more attention. Lamb wave has an advantage over longitudinal wave as it can propagate a significant long distance along the structure. A lot of work can be found on SHM and damage detection techniques which successfully make use of Lamb waves. Rose and Su summarized some recent developments in Lamb wave SHM and integrity inspections^{1, 2}. Giurgiutiu³ explored the capability of piezoelectric wafer active sensors for tuned Lamb waves generating and damage detection. Maslov and Kundu⁴ studied the prediction of the most efficient leaky Lamb mode and frequency to detect internal defects in composite plates. Wilcox et al.⁵ proposed a procedure for selecting suitable Lamb wave modes and frequencies for a particular inspection task. Wang⁶ developed a digital imaging method for locating and sizing structural damage based on the principle of time-reversal, through which temporal and spatial focusing can be achieved.

The time of flight (TOF) is an important parameter in Lamb wave tomography even though there may be alternative methods^{7, 8} such as amplitude/magnitude of signals to locate the damage. Tua et al⁹ proposed a methodology to locate and determine the extent of linear cracks in plates utilizing TOF analysis of Lamb waves. Cui et al.¹⁰ proposed a technique to estimate the axial crack location in hollow section cylindrical structures using TOF. Lemistre and Balageas¹¹ Presented a multi resolution processing to measure the time delay between the arrivals of the actuation burst and of a specific converted Lamb mode for crack localization and estimation of its extend. Yan¹² proposed a particle filter approach to localizing damage in plate-like structures using TOF of Lamb waves.

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Though successful applications of Lamb waves in SHM are abundant, it has to be pointed that the above-mentioned work is based on the premise that the structure is not in motion. Structures are generally and inevitably subjected to vibrations from varied sources during their service lives. Extra stress induced by vibrations could possibly lead to the distortion of the Lamb wave signals and hence the misinterpretation of the structural behavior. The aftermath of using such SHM system could be underestimating the damage severity. These factors will probably hinder the effective monitoring of the structures. Thus, in this study the influence of the external vibrations on the TOF was investigated¹³. Piezoelectric lead zirconate titanate (PZT) elements were utilized to excite and receive Lamb waves in plate-like structures as they show outstanding performance in generation and acquisition of waves. Moreover they are easy to be integrated into existing structures². PZT had found its applications in several practical applications of civil, mechanical and aerospace structures^{14, 15}. Thus this paper is expected to be very useful for practical structures influenced by external vibrations such as trains and buses.

2. LAMB WAVE FUNDAMENTAL THEORY AND MONITORING USING TIME OF FLIGHT

2.1 Fundamental theories of Lamb waves

Lamb waves refer to guided waves in a free thin plate. According to the theory of elasticity, the governing equation describing the wave motion of an isotropic plate (without body force) will be in the form of

$$(\lambda + \mu)\nabla\nabla \cdot \mathbf{u} + \mu\nabla^2\mathbf{u} = \rho\ddot{\mathbf{u}} \quad (1)$$

where \mathbf{u} is the displacement vector, ρ is the density, μ is the shear modulus and λ is the Lamé constant of the material. The plane strain assumption is considered for simplicity in the x-z plane comprised of the wave propagation along x and the plate thickness along z.

The displacement can be decomposed according to Helmholtz decomposition and equation 1 can then be solved by the method of potentials. By assuming certain forms of the potentials satisfying equation 1, the displacements, strains and stresses can be represented by the potentials with finite number of coefficients to be determined^{16, 17}.

Since we are interested in plate structures, traction-free boundary conditions on the upper and lower surfaces are applied. A system of homogeneous equations by letting the determinant of the above derived set of equations equal to zero, the characteristic equations of the system are thus acquired. Since Lamb wave solutions can be split into symmetric and antisymmetric modes¹⁶ depending on the particle motions of the plate with respect to the mid plane along the thickness (z), the characteristic equations of these two modes are considered separately as¹⁷

$$\frac{\tan(k_{S_y}d)}{\tan(k_{P_y}d)} = -\frac{4k_x^2k_{P_y}k_{S_y}}{(k_x^2 - k_{S_y}^2)^2} \quad (2)$$

$$\frac{\tan(k_{S_y}d)}{\tan(k_{P_y}d)} = -\frac{(k_x^2 - k_{S_y}^2)^2}{4k_x^2k_{P_y}k_{S_y}} \quad (3)$$

where $k_{P_y}^2 = \frac{\omega^2}{c_P^2} - k_x^2$, $k_{S_y}^2 = \frac{\omega^2}{c_S^2} - k_x^2$, $c_P^2 = \frac{\lambda + 2\mu}{\rho}$, $c_S^2 = \frac{\mu}{\rho}$ and k_x is the wave number. Equation 2 corresponds

to the symmetric mode while equation 3 corresponds to the anti symmetric mode. The real solutions of the equations are the phase velocity or wave number frequency relations for different modes. These transcendental equations can only be solved by numerical methods. The plotted numerical solutions for the symmetric case are the widely used dispersion curves as shown in Figure 1.

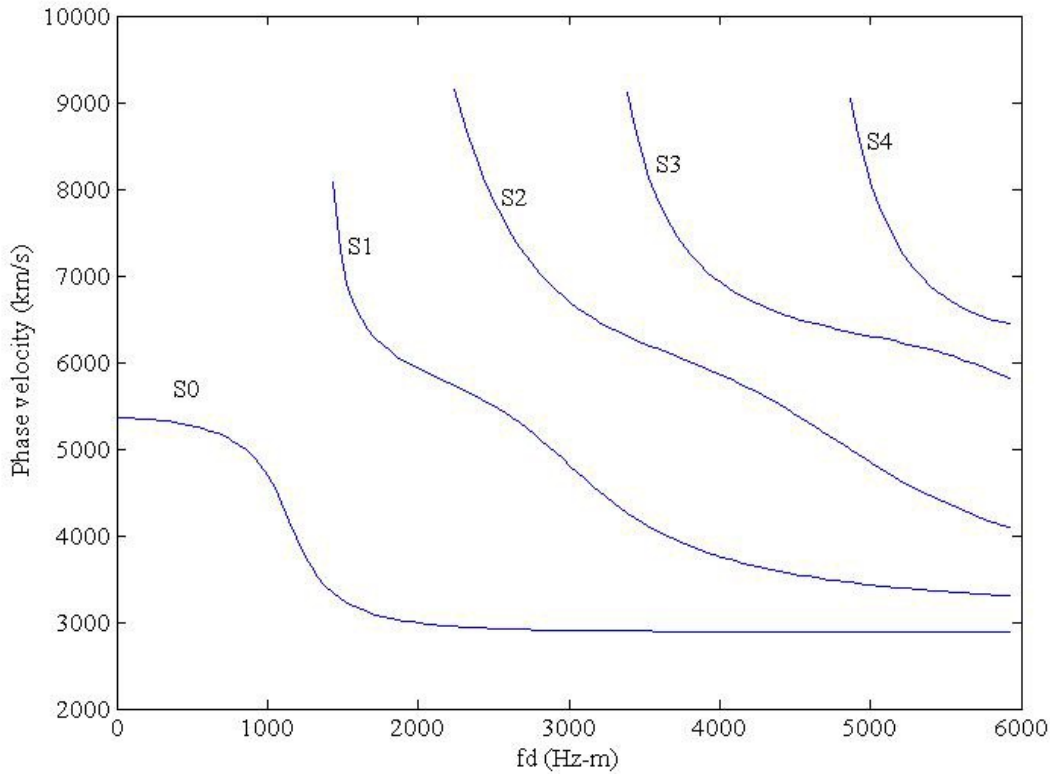


Figure 1. Phase velocity dispersion curves of symmetric modes for an aluminum plate.

2.2 SHM based on TOF

In traditional nondestructive testing (NDT) technologies, two algorithms often used are pitch-catch and pulse-echo, which can also be used in Lamb wave based SHM technique. In both algorithms, usually the TOF would be the key feature that needs to be extracted from the signals. Damage or structural discrepancies can be localized/identified using certain identification algorithms with the TOF measured from the acquired signals of sensors. TOF in Lamb wave is defined as the time for a wave packet to propagate from one transducer to another transducer through a certain path. There could be a structural discrepancy such as a crack or boundary along this path. The TOF can be expressed as

$$T_{A-S} = \frac{D_{A-S}}{V_{S_0}} \quad (4)$$

where V_{S_0} is the group velocity of the wave packet, $D_{A-S} = \sqrt{(x_S - x_A)^2 + (y_S - y_A)^2}$ is the distance between the actuator and the sensor.

The TOF can be experimentally determined from the time domain signals. Its measurement from experimental data can be achieved through envelop construction of wave packets¹⁰. The envelopes of the wave packets are constructed using Hilbert Transfer. The TOF is easily determined by a peak identifying process in a wave packet envelope.

3. EXPERIMENTAL SETUP

The effect of external vibrations on the ultrasonic Lamb wave propagation capability was studied through experiments. Specimens 1 and 2 made of aluminum (T6061) of dimensions 600 x 400.5 x 1.2 mm and 596 x 420 x 1 mm, respectively, were used in the experiment setup as shown in Figure 2. Two circular PZT wafers (27 mm in diameter) were surface-mounted where one was used as an actuator and the other was used as a sensor. The distance between the two PZTs was 400 mm for specimen 1 and 396 mm for specimen 2. A Tabor WW1071 arbitrary waveform generator was used to generate the tone-burst signal for actuation. A Yokogawa DL 1620 oscilloscope was used to capture the signal from the sensor. An external shaker was used to generate sinusoidal vibrations (acceleration of 0.1g peak to peak was adopted) on the specimen along the thickness direction. Specimen 1 was placed freely on top of the shaker in the first case while specimen 2 was fixed on the shaker using an extra platform bonded to the bottom of the plate for the second case. However specimen 2 was used for the study under fixed boundary condition whereas specimen 1 was used in free-free condition.

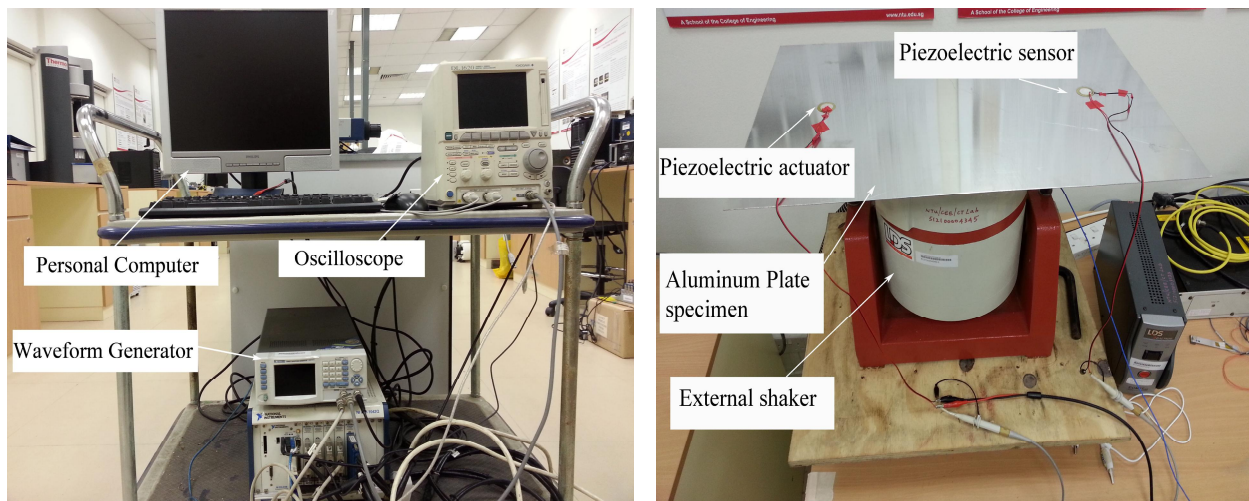


Figure 2. Experimental setup: Waveform generator and oscilloscope (left); Specimen 1 and shaker (right).

4. RESULTS AND DISCUSSION

In this study, a five cycle Hanning windowed tone burst signal with a center frequency of 150 kHz and peak to peak input voltage of 10 volts was sent to the actuator for Wave generation. For the considered frequency only S0 and A0 modes would be actuated in both specimens according to the dispersion curves for aluminum plates.

Figure 3 shows comparison of the wave signals measured by the oscilloscope for static condition and vibration at 10 Hz frequency under free boundary condition. The first wave packet in the sensor signal is due to electromagnetic interference and it was ignored. It can be seen from Figure 3 that the signal obtained at vibration deviates from static condition. The deviation makes it difficult to evaluate the TOF with common data processing techniques such as Hilbert Transform.

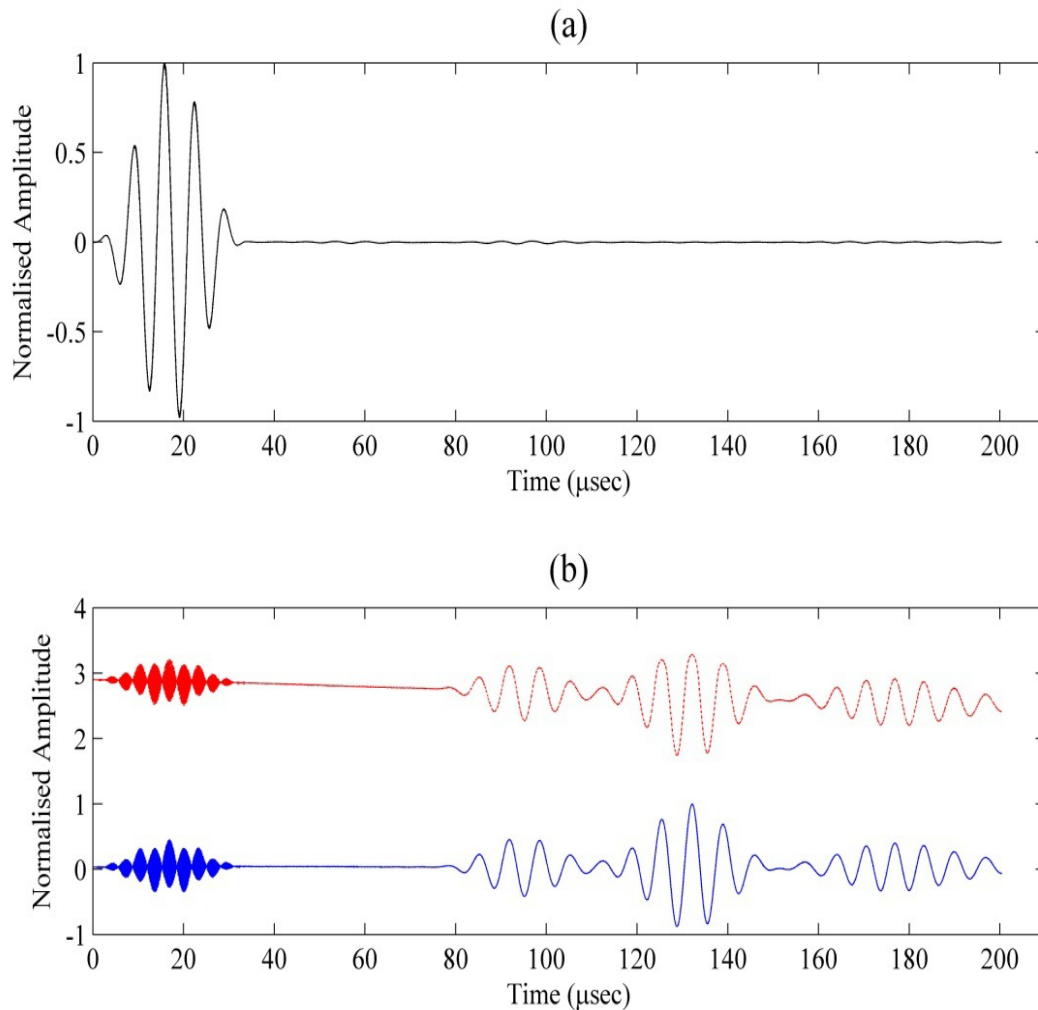


Figure 3. Comparison of sensor signals of specimen 1: (a) Actuation signal; (b) Sensor acquired signal: static (solid blue) and vibrating 10Hz (dash-dotted red).

The external vibrations pose some difficulty in estimating the TOF from the raw signals as the received Lamb wave signals were distorted. Hence in order to estimate the TOF, the vibration induced deviation was first eliminated by a curve fitting based compensation process the result of which is shown in Figure 4 (a). Next, cross correlation between the compensated signal and the actuation signal was calculated as shown in Figure 4 (b). The TOF was then estimated by searching for the local maximum in the envelope of the cross correlation (Figure 4 (c)).

Figure 5 shows the process of extracting the TOF for specimen 1 in static condition. The received signals and extracted TOF for specimen 2 in static and vibrating cases are shown in Figure 6 and Figure 7, respectively.

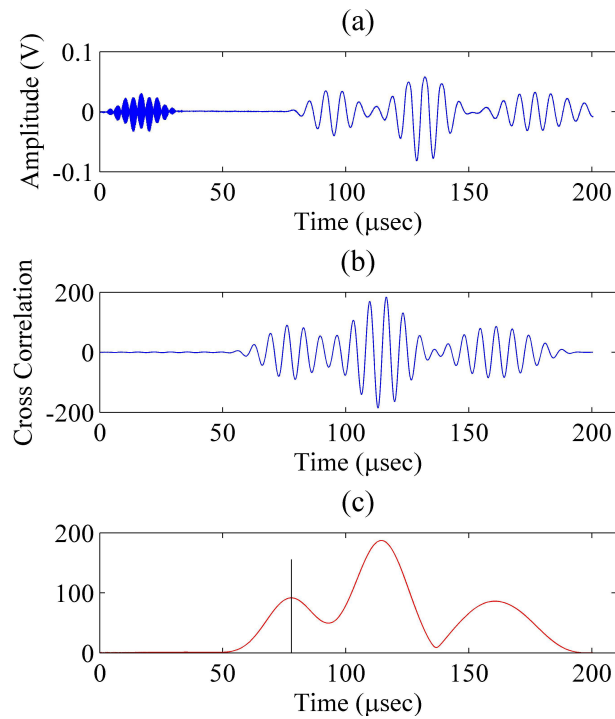


Figure 4. Determination of TOF for specimen 1 in vibration at 10 Hz: (a) modified sensor signal; (b) cross correlation between the modified sensor signal and the actuation signal; (c) envelope of the cross correlation.

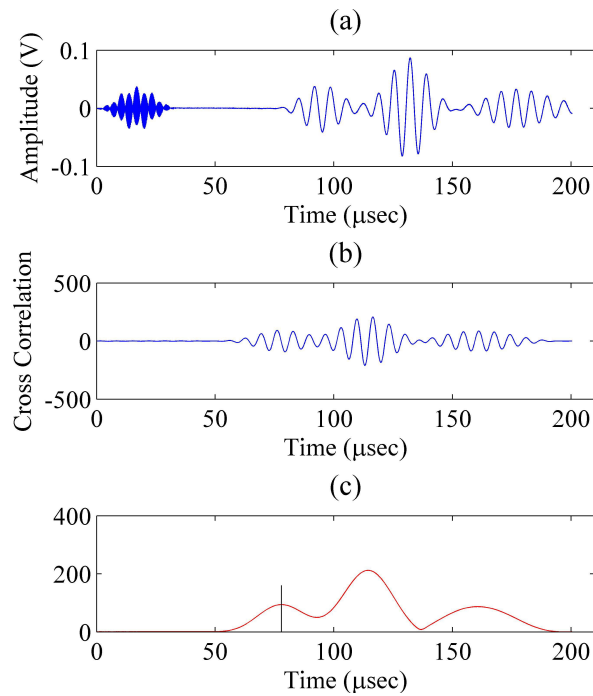


Figure 5. Determination of TOF for specimen 1 in static condition: (a) sensor signal; (b) cross correlation between the sensor signal and the actuation signal; (c) envelope of the cross correlation.

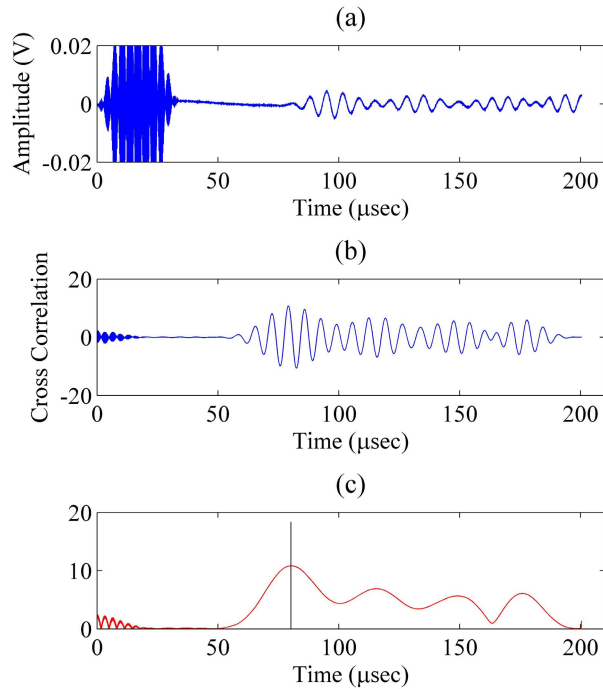


Figure 6. Determination of TOF in specimen 2 in static condition: (a) sensor signal; (b) cross correlation between the sensor signal and actuation signal; (c) envelope of the cross correlation.

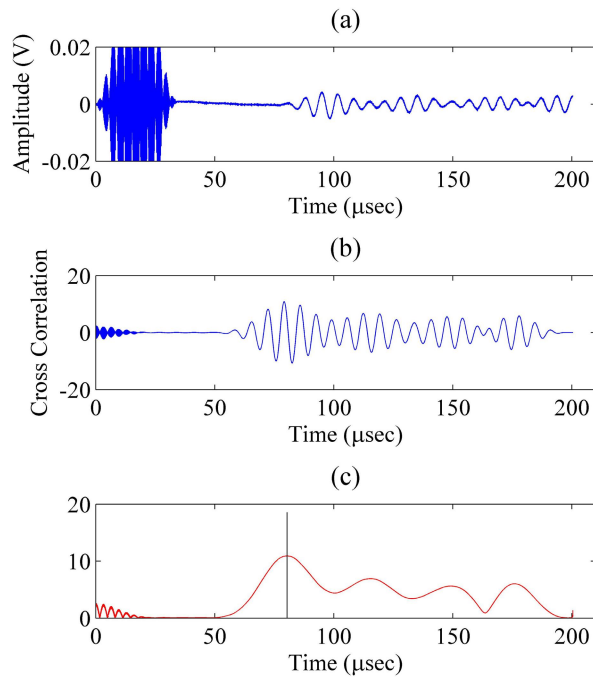


Figure 7. Determination of TOF in specimen 2 in vibration at 10Hz: (a) modified sensor signal; (b) cross correlation between the modified sensor signal and actuation signal; (c) envelope of the cross correlation.

The TOF extracted using the above-mentioned technique at different vibration frequencies for both specimens are shown in Table 1. It can be observed that the TOF for specimen 2 is larger than that for specimen 1. This could be due to the thickness difference between the plates themselves and the additional platform used to fix specimen 2. Nonetheless for both specimens, the TOFs only fluctuate slightly as the vibration frequency varies (shown in Figure 8), which could be attributed to measuring errors. The table presents that the TOFs were not affected by the external vibrations at the frequencies studied.

Table 1. Time of Flight extracted from the received wave signals.

Specimen 1			Specimen 2		
Vibration Frequency (Hz)	Time of Flight (μ s)	Deviation from static (%)	Vibration Frequency (Hz)	Time of Flight (μ s)	Deviation from static (%)
Static	77.90	--	Static	80.26	--
5	77.84	-0.08	5	80.38	0.15
10	77.86	-0.05	10	80.38	0.15
50	77.84	-0.08	50	80.28	0.02
100	77.86	-0.05	100	80.24	-0.02
200	77.88	-0.03	200	80.18	-0.10
500	77.82	-0.10	500	80.28	0.02
1000	77.84	-0.08	1000	80.36	0.12

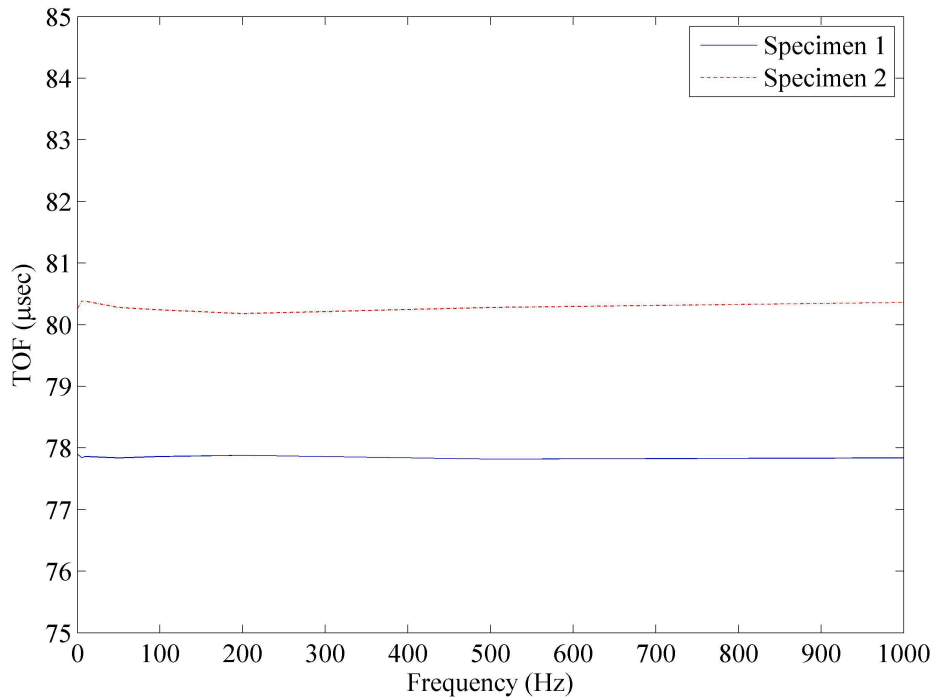


Figure 8. Extracted TOF for different vibration frequencies: specimen 1 (solid line); specimen 2 (dotted dash line).

The experimental results in this study signify that NDT/SHM algorithms developed based on TOF of Lamb waves are still effective even if the structures being monitored are subjected to vibrations. The vibration frequencies studied cover most of the dominant frequency ranges in practical operating conditions for most structures. Thus, the applicability of Lamb wave based SHM technique in structures subjected to external vibrations is verified.

5. CONCLUSIONS

There have been a number of SHM systems making use of Lamb waves. However, most of them did not take into consideration the influence of external vibrations which generally exist in practical structures. In this study the influence of the external vibrations on the TOF, one of the key characteristics of Lamb wave based SHM, was investigated using surface mounted PZT elements as transducers. The conclusions that can be drawn are:

- (1) The received time domain wave signals in vibrating structures will be shifted or rotated. This is due to the external vibrations which generate flexural waves in addition to the Lamb waves actuated by the PZT elements.
- (2) The TOF can be measured from the vibration distorted wave signals utilizing appropriate data processing methodologies.
- (3) The TOF are not affected by the external vibrations within the frequency and acceleration range studied.
- (4) Probably, TOF are also not altered in practical situations in which structures are subject to vibrations with a frequency range much lower than that of the Lamb Waves but further studies are needed to back up this claim.

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