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RESEARCH ARTICLE

Implementing GeoGebra 3D Calculator With Augmented Reality in Multivariable Calculus Education

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ABSTRACT This paper studies how the GeoGebra 3D Calculator with augmented reality (AR) serves as a tool for visualizing 3D graphs from functions of two variables in a mid-sized multivariable calculus class. We assessed this AR tool's usability as a content delivery system. In our study, 39 students in the AR group received instructions on using the GeoGebra 3D Calculator with AR tool, while another 28 students in the control group worked exclusively with PowerPoint slides. Statistical testing reveals that the GeoGebra 3D Calculator with AR proves more effective than PowerPoint slides in terms of usability for teaching and learning multivariable calculus. However, the students' attitudes and engagement do not show significant differences between the AR and control groups. This preliminary study highlights the crucial role of innovative educational technologies like the GeoGebra 3D Calculator with AR in enhancing the visualization of 3D graphs and other multivariable calculus topics. It suggests an advancement in the evolution of digital tools in education. The increased usability observed in the AR group indicates a promising direction for future educational strategies, especially in fields that require strong spatial visualization skills.

INDEX TERMS Augmented reality, multivariable calculus, visualization, GeoGebra 3D calculator.

I. INTRODUCTION

Calculus plays a crucial role in the educational journey of students, serving as foundational groundwork that equips them with essential skills for a wide range of careers in science, technology, engineering, and mathematics (STEM). It not only lays a strong foundation for understanding advanced concepts in STEM fields but also enhances critical thinking and problem-solving abilities, essential for addressing real-world challenges across various domains. Considering that a substantial number of students worldwide enroll in calculus courses, whether at the secondary or tertiary level, the body of research on calculus education consistently contributes to the enhancement of the educational experience for millions of students annually. Nonetheless, students consistently encounter challenges in visualizing concepts within multivariable calculus. These difficulties can also be attributed to issues with algebraic manipulation and insufficient prior knowledge in certain fundamental calculus topics. The most commonly used method involves utilizing PowerPoint to aid students in visualizing graph functions. Additionally, the model of blended learning in mathematics incorporates elements such as classroom tasks, assessments, computer and web support, and strategies that serve as a framework for classroom instruction. This approach provides students with the chance to benefit from both in-person and

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online instruction. However, methods like PowerPoint have limitations as they do not allow for a 360-degree view, which can restrict the depth of visualization and understanding.

While MATLAB and Python are employed in teaching calculus, they have a steeper learning curve for beginners, requiring programming skills for customization. Therefore, GeoGebra was selected as the study tool for this study. It allows for easy creation of graph functions using various tools to support the purpose of each graph function, and allows users to zoom and rotate. Unlike Desmos 3D Calculator and Wolfram Alpha, GeoGebra offers more interactivity and flexibility. GeoGebra's user-friendly interface and interactive capabilities are expected to enhance comprehension in multivariable calculus and facilitate a more profound grasp of concepts, overcoming the challenges of limited visualization and the steep learning curve associated with other tools. The effectiveness of this approach will be evaluated in the blended learning multivariable calculus course.

Extended Reality (XR) tools, which encompass Virtual Reality (VR) and Augmented Reality (AR), offer a unique opportunity for students to engage with mathematical concepts in an immersive virtual environment or augmented environment. XR-based graphs leverage the power of extended reality technologies to enable students to immerse themselves in three-dimensional visualizations of complex functions, offering a depth of perception that enhances their understanding of the materials [1]. Students can manipulate the graph by viewing it from different angles, or by zooming in and out as needed, allowing them to gain a deeper appreciation of the relationships between different variables. Furthermore, XR tools can offer a more engaging and interactive learning experience by enabling students to interact with text, images, and other media within a fully immersive virtual environment. These tools have been used effectively in physics education [2], enhancing visualization in multivariable calculus topics [3], [4], and in industrial settings [5], [6] to provide real-time information about complex machinery, improving decision-making and productivity [7], [8]. With the increasing computing power and decreasing cost of XR tools, the adoption of blended learning approaches incorporating XR-based tools has become more feasible and practical [9], [10], [11], [12], [13]. XR tools hold the potential to revolutionize mathematics education by making it more engaging, interactive, and accessible to a broader range of learners.

Recent studies highlight the significant impact of metaverse technology on student engagement, academic achievement, and motivation in education, due to its dynamic and curiosity-stimulating learning experiences [14]. The use of an AR sandbox in introductory labs has been shown to enhance engagement and potentially improve learning outcomes, with variations in engagement levels observed based on group interaction dynamics [15]. Additionally, augmented reality markers, implemented through the RA Vufurio platform and Unity3D, have been utilized in developing a mobile application that offers a real-time, three-dimensional graphical representation of geometric surfaces, dynamically adjusting to perspective changes. This technology has been effective in promoting collaborative work among students [16]. Developing graphical intuition is crucial for understanding concepts in mathematics, particularly in multivariable calculus involving two-variable functions. Functions of two variables require a third dimension for depth representation, a feature often inadequately conveyed by traditional methods like PowerPoint slides or still illustrations [17]. AR-based visualization tools significantly enhance students' learning by providing interactive 3D models for a more immersive engagement with complex functions. Research underscores AR technology's effectiveness in mathematics education, enabling students to interact with mathematical objects and concepts for a deeper understanding of multivariable calculus [18].

AR offers advantages over VR in accessibility, functioning in various settings with just a smartphone or tablet [19]. It boosts motivation, attention, and exam performance in education [20]. AR aids in visualizing 3D geometry, improving understanding of spatial and geometric concepts [21]. Notably, Osypova and Tatochenko's study [22] shows that GeoGebra [23], [24], an AR 3D calculator, enhances math learning efficiency, fosters logical thinking, and increases student motivation.

The GeoGebra 3D Calculator, equipped with AR, is a simple yet highly useful AR app that allows users to view graph functions on their mobile devices. Markus Hohenwarter initiated GeoGebra in 2001, integrating dynamic geometry and computer algebra in a dynamic teaching tool [23], [24], [25]. As an open-source, user-friendly software, GeoGebra is accessible via its official website.[1] The 3D Calculator enhances self-learning, confidence, motivation, and offers a positive alternative approach in education [26]. Chosen for its efficacy, simplicity, and cost-effectiveness, the 3D Calculator with AR enables accessible educational experiences on diverse devices, leveraging the ubiquity of smartphones [27]. Available for free on Android and iOS, it offers immersive classroom experiences, allowing students to interactively visualize complex calculus concepts and understand challenging mathematical ideas through AR-generated 3D models overlaid on the physical world. Building on the established benefits of the GeoGebra 3D Calculator with AR, this study aims to delve deeper into its practical application in educational settings. We will specifically explore the effectiveness of GeoGebra 3D Calculator with AR as an educational tool in the teaching and learning of multivariable calculus topics, assessing the usability of GeoGebra 3D Calculator with AR as a content delivery system, and comparing it to the business-as-usual method of instruction through the use of PowerPoint slides. In addition, this study will examine the impact of GeoGebra 3D Calculator with AR on student engagement and motivation as compared to

[1] https://www.geogebra.org/

the control group. By measuring these factors, the study aims to determine whether GeoGebra 3D Calculator with AR can enhance the learning experience and usability of AR in classrooms. Our survey respondents comprise 67 students from two classes of the same multivariable calculus course. The two classes were taught by the same instructor. One class was assigned to use the GeoGebra 3D Calculator with AR, complementing the course materials (on PowerPoint slides), and the other group received classroom instruction via PowerPoint slides only. Both groups received the same course content, but the content delivery method differed between the two groups. In summary, this study has two aims:

1) To assess GeoGebra 3D Calculator with AR's usability as a content delivery system in a multivariable calculus classroom.
2) To assess if GeoGebra 3D Calculator with AR (complementing PowerPoint slides) is better than PowerPoint slides alone, shown on flat screen computers, in terms of students' engagement and attitude towards the teaching and learning of multivariable calculus.

The alternative hypotheses are that 1) The use of GeoGebra 3D Calculator as a content delivery system is effective in facilitating the teaching and learning of multivariable calculus and that 2) GeoGebra 3D Calculator is more effective than PowerPoint slides in terms of supporting students' attitudes and engagement towards the learning of multivariable calculus. The corresponding null hypotheses are that 1') The use of GeoGebra 3D Calculator as a content delivery system is not effective in facilitating the teaching and learning of multivariable calculus and that 2') GeoGebra 3D Calculator is no better than PowerPoint slides in terms of supporting students' attitudes and engagement towards the learning of multivariable calculus. The subsequent sections of this paper are outlined as follows: Section II provides an overview of the resources, while Section III introduces the experimental setup. Section IV discusses our results, followed by a discussion in Section V. Finally, Section VI offers a conclusion, summarizing our primary findings.

II. RESOURCES

A. MOBILE APPLICATION: GEOGEBRA 3D CALCULATOR APP

GeoGebra 3D Calculator was specifically chosen as the platform for developing multivariable calculus graph functions in AR due to its user-friendly interface and extensive command library. This software offers a wide range of commands and functions that greatly streamline the development process for various graph functions relevant to multivariable calculus class materials. By leveraging GeoGebra 3D Calculator, the time required for creating these graph functions is significantly reduced, allowing instructors to focus more on teaching and student engagement. The GeoGebra 3D Calculator is readily available for download (free-of-charge) from the Google Play Store or the Apple App Store, enabling users to easily access and view the graph functions in AR on their preferred devices. This accessibility ensures that students can explore and interact with the graph functions anytime and anywhere, thereby enhancing their learning experience. GeoGebra was used to create graph functions for the course materials that were difficult to visualize for this study. Additional graph functions, intersection planes, and curve functions, as well as tangent planes, were included as needed based on the questions.

Users have the flexibility to customize various settings based on their requirements. They can modify properties like color, range, line style, and other visual attributes to enhance the clarity and aesthetics of the graph functions. Additionally, users can easily manipulate the graph functions by holding the 'Shift' button and using the left mouse button to move and drag them within the three-dimensional space. Another useful feature is the ability to change the scale on the axis. By placing the cursor on the axis, pressing the ''Shift'' key, and dragging, users can dynamically adjust the scale to focus on specific regions of interest, allowing for a more detailed analysis of the graph functions. Some of these aspects, including the coding and automation processes, can be pre-configured. This advanced preparation allows students to concentrate solely on the visualization task, eliminating potential distractions and complexities. By streamlining the technical aspects in advance, we ensure a more intuitive and user-friendly experience for the students, enabling them to fully engage with the visual learning process.

Once all the desired settings are in place, users can save the graph functions from the menu, ensuring that all changes and configurations are conveniently stored in their GeoGebra profile. This enables users to easily retrieve and revisit their graph functions for future reference or sharing with others.

Figure 1 illustrates an example of a graph function created using GeoGebra 3D Calculator on the developer's website. It showcases the visualization capabilities of the software in representing complex mathematical concepts in an immersive and interactive manner. Instructors can create graph functions to explain concepts involving tangent plane, lagrange multipliers, and line integral. These visual representations in AR offer students a deeper appreciation of the underlying principles and encourage their active engagement in the learning process. Instructors can also create QR codes for students' easy retrieval of these pre-coded functions for use in the classroom directly.

The first command is typically used to input the graph function itself. After entering the function, the graph function will be displayed on the right side of the screen. The color of the graph function corresponds to the color shown on the small circle on the left side of the graph function which is in red color. The second command is used to create a blue tangent plane function at (0.2, 0.2, 0.08), while the third command is used to display the intersection point in purple color between the graph function and the tangent plane. The last command is used to create a range on both the x-axis and y-axis for the graph function, which is displayed in yellow color. It is important to note that there are additional

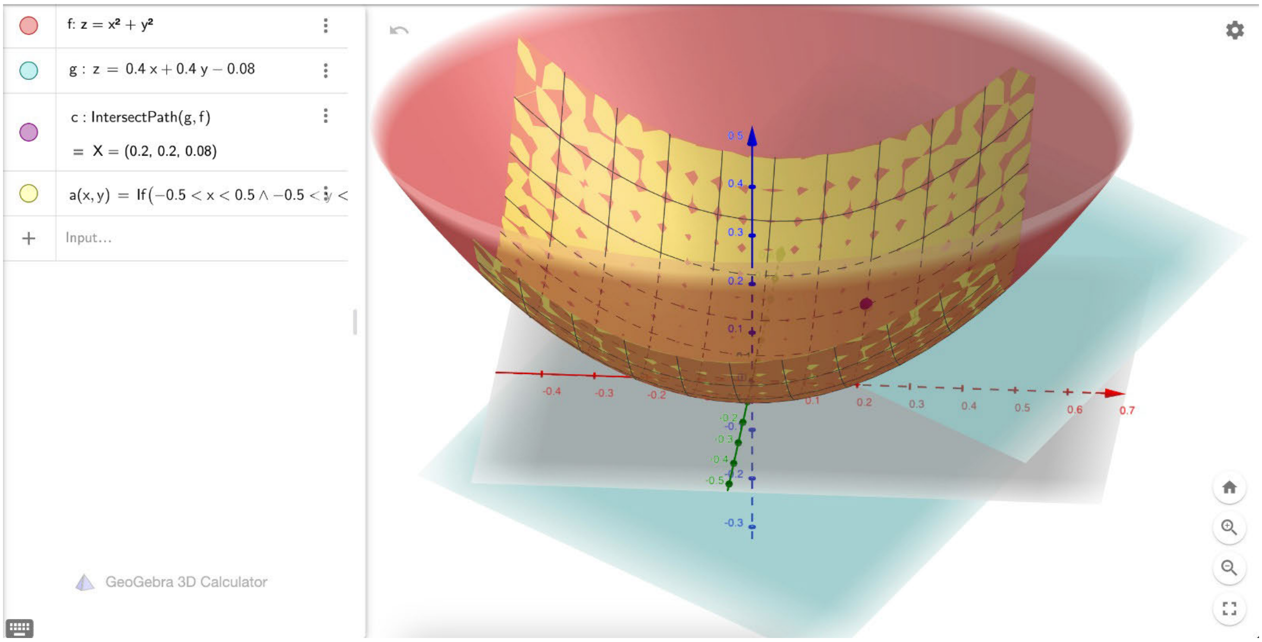

FIGURE 1. Geogebra 3D calculator on developer website which can create and modify graph functions.

commands available on the GeoGebra website. All the steps described above are performed on the GeoGebra website itself to ensure that the user does not accidentally overwrite any changes.

The generated graph functions can be kept private during the development phase before being published on the website. For this study, the graph functions were shared via links, and a QR code was generated for each graph function using a QR code generator. Students can use a QR scanner on their mobile device to easily scan and open the graph function in the GeoGebra 3D Calculator application.

Next, we provide an example of graph function on a mobile application in Figure 2 (not yet in AR mode) which is initiated by scanning the provided QR code. By pressing the AR button located at the bottom right of the graph function, the graph can be viewed in AR mode. It is important to remind students of the need to move slowly in order to allow the surfaces to be detected, as shown in Figure 3. This is an important aspect that users always miss. From our experience, Android devices might require more time to detect surfaces, so moving the phone slowly can help ease surfaces detection. Once the surfaces are detected, tapping the square bracket in the middle allows the graph function to be viewed in AR mode. Note that the graph function in AR might appear smaller at times. This issue can be resolved by restarting the GeoGebra 3D Calculator app. The graph function in AR mode is displayed as shown in Figure 4. Pinching can be used to zoom and rotate the graph function, allowing users to view it from different angles. User can input additional commands below the graph function while keeping the original graph function intact, without overwriting it from the developers' side. This enables students to explore the graph functions on their own. Figure 5 shows an example of graph functions that had been created for this study.

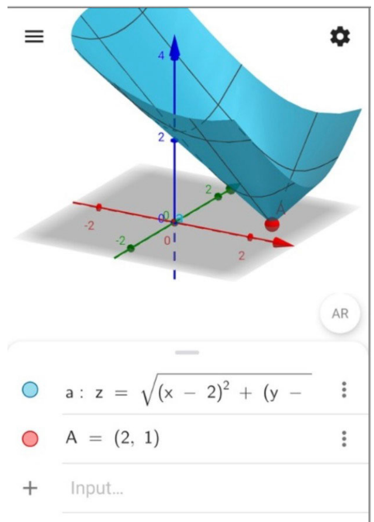

FIGURE 2. Example of graph function on Geogebra 3D Calculator mobile application (Not in AR mode yet).

III. EXPERIMENTAL SETUP

In this study, we leveraged a quasi-experimental design [28] to assess the impact of AR tools on learning outcomes within a first-year multivariable calculus module. Quasi-experimental designs are empirical study methods used to estimate the causal impact of an intervention on target populations without random assignment to treatment or control groups. In our setup, students enrolled in the multi-variable calculus module were initially randomly assigned to one of the nine classes, with two of these classes

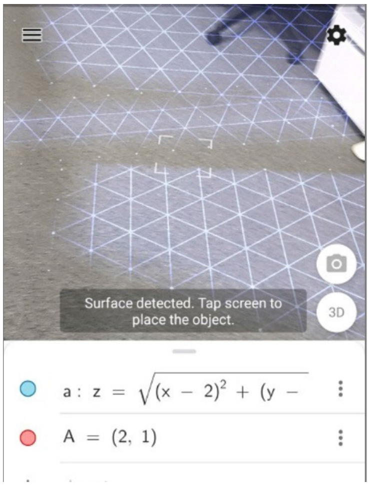

FIGURE 3. Move slowly so that camera detecting surfaces on mobile device.

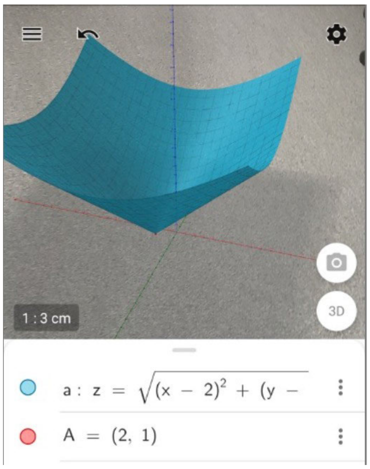

FIGURE 4. Example of graph function displayed in AR mode on mobile device.

being randomly assigned to the same instructor, and each hosting approximately 45 students. Despite the initial random assignment to classes, the specific designation of one class as the AR group (intervention) and the other as the control group introduces the quasi-experimental aspect of our study. This approach allowed us to maintain a level of control over extraneous variables by ensuring both groups were taught by the same instructor, thereby focusing on the effects of the AR intervention.

Quasi-experimental designs, while lacking the random assignment feature of true experiments, offer a practical alternative for educational research, where such randomization may be impractical or impossible. Our design was carefully chosen to approximate the conditions of a randomized controlled trial as closely as possible, within the constraints of a natural classroom setting. For example, in order to enhance the validity of our findings, we utilized a matched-pairs approach by selecting two classes taught by the same instructor. This strategy aimed to control for teaching style and instructor effectiveness, which are critical variables that could influence student outcomes. The initial random assignment of students to the nine classes, including the two involved in our study, further supports the fairness of our design by ensuring that any pre-existing differences among students were evenly distributed, thereby minimizing selection bias.

Both classes were given the same lecture slides in PowerPoint, which included simple 2D visualizations of 3D graphs. The intervention group was further provided with the AR intervention during the class using the Geogebra 3D app. The study lasted for 6 weeks for both classes. The instructor informed the students that participation in the survey was voluntary and anonymous. They could choose not to participate in the survey. The students were also informed that the survey results would be used for educational research. After the study period, a posttest survey was administered, consisting of 75 Likert scale questions and 1 open-ended item for the intervention group, and 68 Likert scale questions with 2 binary questions for the control group. The AR group received additional questions aimed at exploring their learning experiences with AR during the class activities. In our case, the voluntary nature of the survey resulted in differing response rates between the two groups, with 39 students from the AR group and 28 students from the control group participating.

For the AR group, we periodically provided the QR code so that students could access the pre-coded functions easily and view it on their mobile devices/tablet. The students were actively engaged in using the GeoGebra 3D Calculator on their mobile devices. The students actively participated in visualizing the graph functions in AR. An AR engineer was deployed in the classroom to provide assistance to students in case they encountered any difficulties while setting up the GeoGebra 3D Calculator on their mobile devices. As shown on Figure 6 and Figure 7, students were able to use their own mobile devices and tablets to view the graph functions. Based on our observations, students were able to set up the GeoGebra 3D Calculator on their own without much issue. Most of them effortlessly navigated the installation process and configured the necessary settings, showcasing their competence in seamlessly integrating this tool into their learning experience. Furthermore, we observed that students were engaging in discussions with each other more actively compared to a typical class, indicating a higher level of engagement. During our classroom teaching, the instructor adeptly manipulated a tablet device using the GeoGebra with AR, as seen in Figure 8, to illustrate the complex concept of line integrals, a fundamental topic in vector calculus. The tablet's display was simultaneously projected onto several large screens placed at the sides of the classroom, transforming the classroom into an immersive learning environment. This setup provided a comprehensive, large-scale visualization of line integrals, thereby facilitating a deeper understanding of the subject matter among the students.

We incorporated survey questions from [29], [30], [31], and [32], which comprehensively assessed the constructs

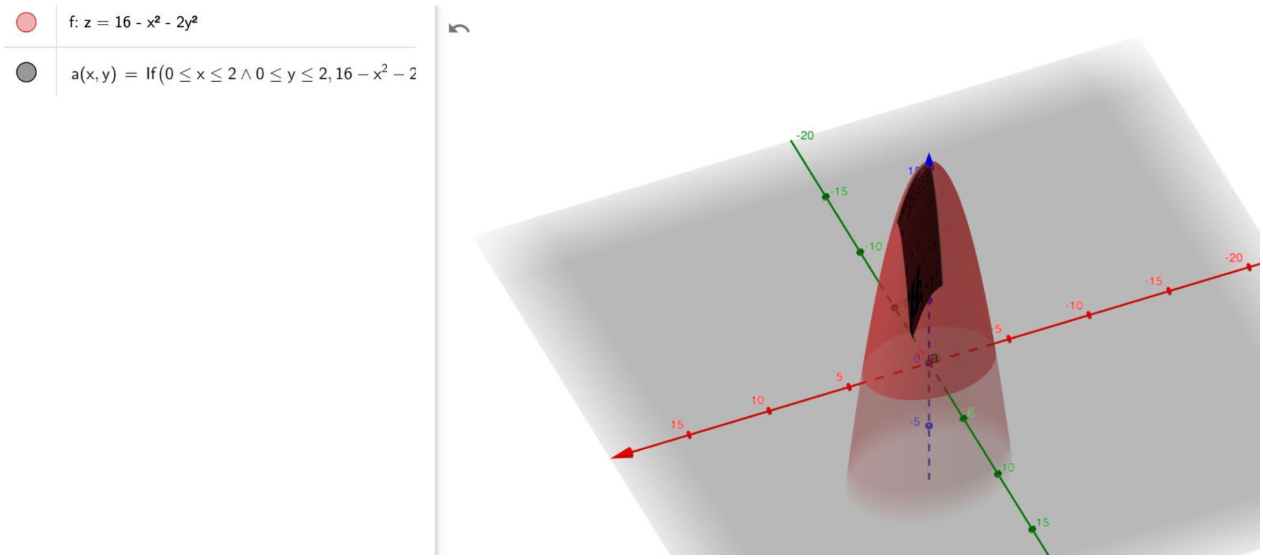

FIGURE 5. Examples of graph functions with shaded region as range created on developer website.

of "students' attitude towards teaching and learning of multivariable calculus," "engagement towards multivariable calculus," and "usability of the mobile application" [4]. The *attitude construct* is composed of 4 sub-constructs: attitude, self-confidence, enjoyment, and value. The *engagement construct* consisted of 4 sub-constructs: emotional engagement, behavioral engagement, social engagement, and cognitive engagement. The *usability construct* was a uni-dimensional construct. The 5-point Likert scale with five response options was used, with 5 indicating "strongly agree", 3 indicating "neutral", and 1 indicating "strongly disagree". The survey items were modified to make them relevant to assessing the usability of both PowerPoint slides and AR on GeoGebra 3D Calculator. The survey was designed to measure different constructs associated with individual surveys [29], [30], [31], [32].

As the sample size of each group was less than 40, we do not expect the sample data to be normally distributed even if the population data are truly normally distributed. Nonetheless, the application of t-test remains valid for comparing groups even when the variable is measured on a Likert scale, and the sample data do not exhibit a normal distribution [33]. Additionally, we conducted both one-sample and independent samples t-tests. The one-sample t-tests compared the mean response of each group to a neutral value, allowing us to assess the deviation from neutrality in participants' responses. The independent samples t-test was employed to compare the means between the two groups directly, providing a comparative analysis of the intervention's effect. This approach enhances the reliability of our findings, as it accounts for both absolute and relative measures of impact. Moreover, we calculated Cohen's d to quantify the effect size, offering a comprehensive understanding of the practical significance of our results alongside the statistical significance reported through t-tests. Such methodological rigor ensures that our findings contribute valuable preliminary insights into the efficacy of AR tools in enhancing mathematical education.

TABLE 1. Statistics of the one-tailed one-sample t-test, testing against a neutral response of 3.

	Platform	Mean	SD	df	t	p (one-tail)
Attitude	AR	3.36	0.91	38	4.38	<0.001*
	nonAR	3.24	0.92	27	2.17	<0.02*
Engagement	AR	3.60	0.84	38	9.79	<0.001*
	nonAR	3.55	0.82	27	6.66	<0.001*
Usability	AR	4.04	0.80	38	12.41	<0.001*
	nonAR	2.95	0.91	27	0.47	0.32

* p is significant

Three statistical tests were conducted using the data of the survey to test the hypotheses mentioned in Section I. First, the one-tailed one-sample t-tests were conducted to determine if there were significant differences between a neutral response of 3 and the mean of the Likert scale for each group.This is because the t-test typically offers equivalent safeguards against Type II errors and consistently ensures the same level of protection against Type I errors [34]. An independent sample t-test was conducted to compare the two groups. Cohen's d was also calculated to measure the effect size for each construct.

IV. RESULTS

Referring to Table 1, the mean value of the attitude construct for both the intervention and control groups were significantly greater than the neutral response of 3. This indicates that students in both groups had a positive attitude towards the teaching and learning of multivariable calculus. For the engagement construct, both the intervention group and the control group were significantly greater then a neutral response of 3. In terms of the usability construct, the mean value for the intervention group was significantly greater than the neutral response of 3 while mean value for the control group is lower than - albeit not significantly different from,

TABLE 2. Statistics of the independent samples t-test comparing both groups against each other.

	df	t	p	cohen's d	95% confidence interval	
					lower limit	upper limit
Attitude	65	0.61	0.55	0.147	-0.267	0.561
Engagement	65	0.48	0.63	0.117	-0.226	0.460
Usability	65	6.71	$< 0.001^*$	1.628	1.274	1.982

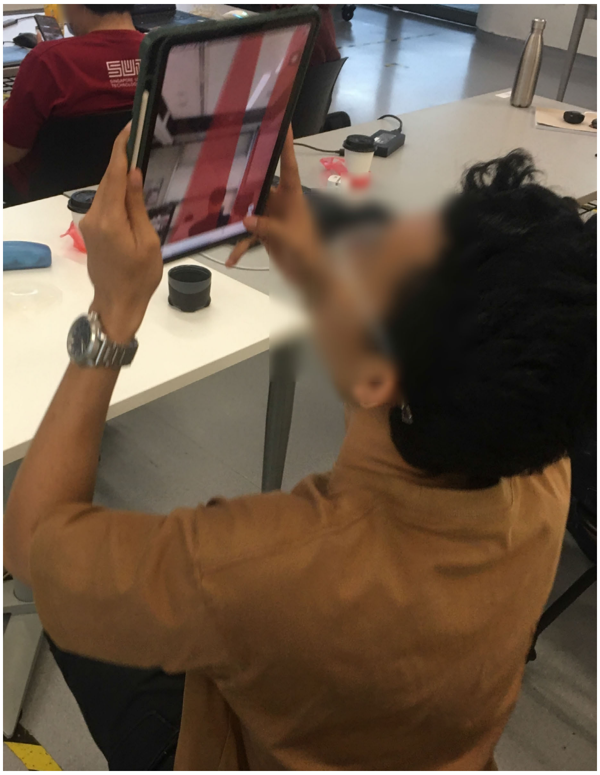

FIGURE 6. Student viewing AR graph function using his iPad.

the neutral response of 3. This indicates that students found the AR intervention with GeoGebra 3D Calculator to be of good usability for learning.

Table 2 shows the statistics of the independent samples t-test comparing between the intervention and control groups for each construct. The mean value of the usability construct was significantly greater in the intervention group compared to the control group. This indicates that students in the intervention group perceived the AR intervention with GeoGebra 3D Calculator to be of high usability, as compared to the control group's use of PowerPoint slides in the learning of multivariable calculus (notably, the Cohen's d here indicates a 'large' effect size).

However, there is no statistical difference between the mean of intervention group and control group for both the attitude and engagement constructs. When it comes to influencing students' attitudes and engagement levels, the AR intervention did not demonstrate a statistically significant difference. In essence, while students found the GeoGebra 3D Calculator with AR to be a more user-friendly tool (than PowerPoint slides) for learning, it did not notably alter their engagement or attitudes as compared to the control group.

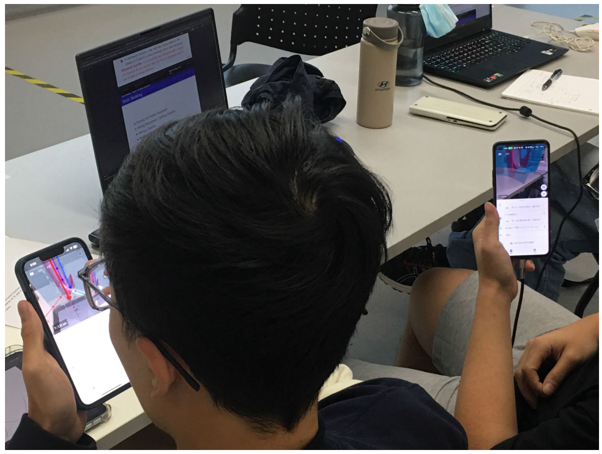

FIGURE 7. Students viewing AR graph function using their own mobile devices.

We used the t-test with a confidence level of $\alpha = 0.05$. The p-value obtained allow us to reject null hypothesis for 'usability' construct but fails to reject the corresponding null hypothesis for the Attitude and Engagement constructs. This leads us to conclude that usability of GeoGebra 3D Calculator as a content delivery system is effective in facilitating the teaching and learning of multivariable calculus. However, GeoGebra 3D Calculator is no better than PowerPoint slides in terms of enhancing students' attitudes and engagement towards the learning of multivariable calculus.

V. DISCUSSION

Practically, the improved usability offered by GeoGebra 3D Calculator means that students can more effectively engage with the course materials, leading to potentially enhanced learning experiences. This can translate into better comprehension of multivariable calculus concepts, more efficient problem-solving, and, ultimately, improved academic performance. Additionally, the user-friendliness of GeoGebra can reduce frustration and encourage students to explore mathematical concepts in a more interactive and enjoyable manner.

The perceived usability advantage of the GeoGebra 3D Calculator in the context of learning, when compared to PowerPoint slides, could be attributed to its ease of setup and viewing in AR. The intuitive interface of the GeoGebra 3D Calculator, coupled with the immersive and interactive nature of AR, has led to students perceiving it as a more user-friendly tool, hence its advantage in usability. Despite this perceived usability advantage, the GeoGebra 3D Calculator with AR did not significantly influence students' overall engagement

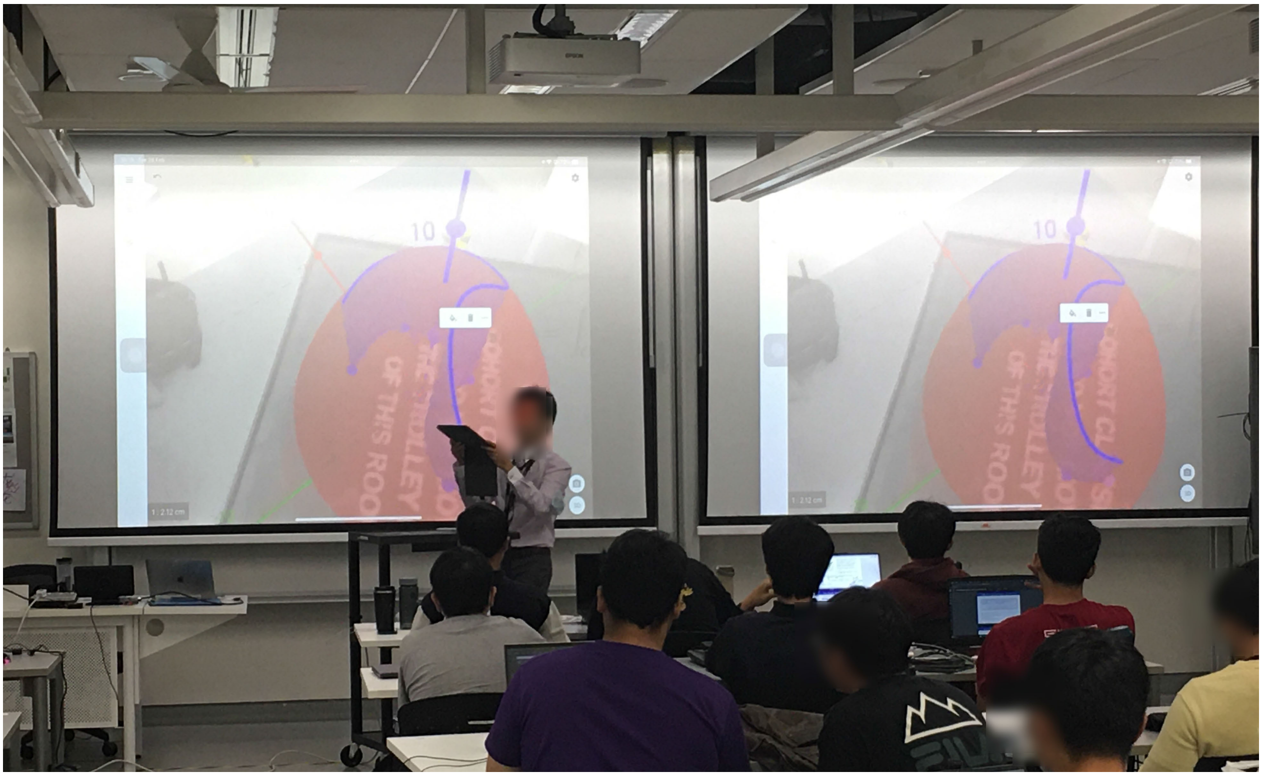

FIGURE 8. An instructor utilizing GeoGebra with AR on tablet for visualization of line integrals, projected on the screen in the front.

or attitudes towards learning of multivariable calculus. These findings, while insightful, are based on a limited sample size. Therefore, further research is needed with a larger sample size to validate these observations and to explore the potential of AR tools like the GeoGebra 3D Calculator in enhancing educational experiences.

The qualitative feedback revealed certain encouraging sentiments that merit further exploration. For instance, students mentioned that '[AR] helps in solving problems in homework as well especially when it is harder to visualise', 'interactive, can see from every angle, easy to use.','I think AR really helps a lot for visualisation, cos when I'm stuck with a question I can use the AR to visualise'. Trying to get used to a new mobile application is one of the difficulties faced by most students. However, we were encouraged by the fact that students started using GeoGebra 3D Calculator more frequently to check their answers and visualize concepts on their own during class activities, as well as after the class. This indicated an increasing comfort level and acceptance of the application over time.

The feedback provided by students in the AR group also highlighted a few issues. These issues range from 'May not work on older devices', 'Controls are a bit iffy, my phone small so hard to see, sitting down hard to move the phone around to see', 'Things that do not work for me are typing the equation in by myself' and 'see points that are far away from origin, difficulty finding the right region'. To address these concerns and improve students' attitude towards GeoGebra 3D Calculator, the instructors may need to provide more guidance in manipulating the graph functions. By ensuring that students have a clear understanding of how to manipulate the graphs and view them from different angles, these issues can be mitigated. Online resources can also be provided to the students in advance, allowing them to explore and create graph functions independently, thereby enhancing their learning experience.

In the qualitative survey, students were asked to pinpoint what did not work out for them while using the AR as a visualization tool for multivariable calculus. Some students commented on the practical use of augmented reality, expressing that it might "not be very useful" in their learning. For instance, students suggested that 'sometimes it takes abit of time, maybe can be better integrated into the slides','AR was not a big help to me in general. I hardly used it during my learning' and 'using the AR part (where the function is projected on a real surface) is a bit finnicky, because one has to physically move around to see the function at different angles. I often just use spin the model around in the non-AR visualisation part.' Enhancing the integration of AR tools with learning materials motivates future research. For example, to infuse the QR code within the slides to create a more seamless learning experience. Providing comprehensive training and guidance on the effective use of the GeoGebra 3D Calculator with AR could enhance its utilization in learning. Additionally, familiarizing students with its usage through more examples within teaching

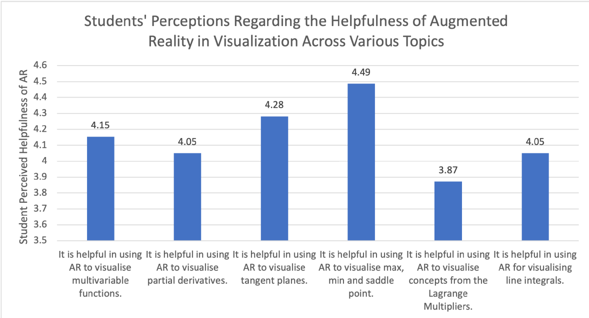

FIGURE 9. **An examination of students' perceptions regarding the helpfulness of augmented reality in visualization across various topics [with 1 - being strongly disagree, 3 - being neutral and 5 - being strongly agree].**

materials could further condition them to its application. Another reason this study shows no significant changes in students' attitude and engagement could be attributed to the large number of survey questions, approximately 70 in total, which might have led to fatigue.

Finally, we examine students' perceived helpfulness of AR in the different topics related to multivariable calculus in Figure 9. The results indicate that the majority of students had positive rating (1 - being strongly disagree, 3 - neutral, 5 - being strongly agree) with regards to the use of GeoGebra 3D Calculator with AR for the various topics. Specifically, most of the topics received a rating higher than 3 (neutral). They found it particularly helpful in visualizing maxima, minima, and saddle points in AR mode. The rating for lagrange multipliers received the lowest rating among all the different topics and perhaps this could be due to the fact that the context was used in proving the method of lagrange multipliers. Thus, GeoGebra has the potential to significantly improve students' visualization skills, as evidenced by their increased usage. It is important to note that this study focused on specific topics within the course content. There are other areas in which Augmented Reality (AR) can be effectively employed, and these avenues are ripe for exploration in future research studies.

VI. CONCLUSION

In conclusion, our study demonstrates the efficacy of GeoGebra augmented with AR technology in the multivariable calculus curriculum. This integration notably enhances students' ability to visualize complex 3D structures and grasp intricate concepts, indicating AR's potential to significantly enrich mathematical education. However, the true scope of AR's benefits across diverse mathematical areas remains to be fully explored. Further research with larger sample sizes and varied educational contexts is essential to validate our findings and expand our understanding of AR's educational impact. Our findings also have practical implications for educators seeking to make math more interactive and comprehensible. The use of AR tools can open new pathways for students to achieve a profound understanding of mathematical principles. We advocate for continued exploration into the application of AR in education, aiming to refine strategies for its integration and to elevate student engagement and learning outcomes in math. Future studies should aim to address the limitations and gaps identified in our research, further contributing to the evolving landscape of digital learning technologies. Ultimately, our work underscores the transformative potential of AR in enhancing the mathematics learning experience, promising a more engaging and effective educational environment.

APPENDIX

The survey questions can be found at: https://osf.io/bq7g8/?view_only=a2fb522a8f5040aa82af76e692475269

CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

[1] J. W. Lai and K. H. Cheong, "Adoption of virtual and augmented reality for mathematics education: A scoping review," *IEEE Access*, vol. 10, pp. 13693–13703, 2022.

[2] J. W. Lai and K. H. Cheong, "Educational opportunities and challenges in augmented reality: Featuring implementations in physics education," *IEEE Access*, vol. 10, pp. 43143–43158, 2022.

[3] K. H. Cheong, J. S. Chen, K. Kang, and D. J. Yeo, "Supporting students' visualization of multivariable calculus partial derivatives via virtual reality," *Mathematics*, vol. 11, no. 4, p. 831, Feb. 2023.

[4] K. H. Cheong, J. W. Lai, J. H. Yap, G. S. W. Cheong, S. V. Budiman, O. Ortiz, A. Mishra, and D. J. Yeo, "Utilizing Google cardboard virtual reality for visualization in multivariable calculus," *IEEE Access*, vol. 11, pp. 75398–75406, 2023.

[5] Y. H. Teo, J. H. Yap, H. An, S. C. M. Yu, L. Zhang, J. Chang, and K. H. Cheong, "Enhancing the MEP coordination process with BIM technology and management strategies," *Sensors*, vol. 22, no. 13, p. 4936, Jun. 2022.

[6] K. H. Cheong, Y. H. Teo, J. M. Koh, U. R. Acharya, and S. C. M. Yu, "A simulation-aided approach in improving thermal-visual comfort and power efficiency in buildings," *J. Building Eng.*, vol. 27, Jan. 2020, Art. no. 100936.

[7] L. Vacchetti, V. Lepetit, M. Ponder, G. Papagiannakis, P. Fua, D. Thalmann, and N. M. Thalmann, "A stable real-time AR framework for training and planning in industrial environments," in *Virtual and Augmented Reality Applications in Manufacturing*, S. K. Ong and A. Y. C. Nee, Eds. London, U.K.: Springer, 2004, pp. 129–145.

[8] X. Li, W. Yi, H.-L. Chi, X. Wang, and A. P. C. Chan, "A critical review of virtual and augmented reality (VR/AR) applications in construction safety," *Autom. Construction*, vol. 86, pp. 150–162, Feb. 2018.

[9] Z. Chen, A. Chia, and X. Bi, "Promoting innovative learning in training and adult education—A Singapore story," *Stud. Continuing Educ.*, vol. 43, no. 2, pp. 196–207, Jun. 2020.

[10] D. Y. Tan and J.-M. Chen, "Bringing physical physics classroom online—Challenges of online teaching in the new normal," *Phys. Teacher*, vol. 59, no. 6, pp. 410–413, Sep. 2021.

[11] D. Y. Tan, W. L. Kwan, L. L. A. Koh, G. M. Pee, K. T. Lur, and Z. Y. Yeo, "Virtual dissection activities as a strategy for blended synchronous learning in the new normal," in *Proc. IEEE Global Eng. Educ. Conf. (EDUCON)*, Mar. 2022, pp. 565–570.

[12] K. H. Cheong, J. M. Koh, D. J. Yeo, Z. X. Tan, B. O. E. Boo, and G. Y. Lee, "Paradoxical simulations to enhance education in mathematics," *IEEE Access*, vol. 7, pp. 17941–17950, 2019.

[13] K. H. Cheong and J. M. Koh, "Integrated virtual laboratory in engineering mathematics education: Fourier theory," *IEEE Access*, vol. 6, pp. 58231–58243, 2018.

[14] A. A. Yakin and P. M. I. Seraj, ''Impact of metaverse technology on student engagement and academic performance: The mediating role of learning motivation,'' *Int. J. Computations, Inf. Manuf. (IJCIM)*, vol. 3, no. 1, pp. 10–18, Jun. 2023.
[15] N. A. Soltis, K. S. McNeal, R. M. Atkins, and L. C. Maudlin, ''A novel approach to measuring student engagement while using an augmented reality sandbox,'' *J. Geography Higher Educ.*, vol. 44, no. 4, pp. 512–531, Oct. 2020.
[16] L. M. Herrera, J. C. Pérez, and S. J. Ordóñez, ''Developing spatial mathematical skills through 3D tools: Augmented reality, virtual environments and 3D printing,'' *Int. J. Interact. Design Manuf. (IJIDeM)*, vol. 13, pp. 1385–1399, Jun. 2019.
[17] L. Kurniawati, I. S. Farhana, and R. Miftah, ''Improving students' mathematical intuitive thinking ability using analogy learning model,'' *J. Phys., Conf. Ser.*, vol. 2157, no. 1, Jan. 2022, Art. no. 012042.
[18] M. Cevikbas, N. Bulut, and G. Kaiser, ''Exploring the benefits and drawbacks of AR and VR technologies for learners of mathematics: Recent developments,'' *Systems*, vol. 11, no. 5, p. 244, May 2023.
[19] F. Liarokapis and E. Anderson, ''Using augmented reality as a medium to assist teaching in higher education,'' in *Proc. Eurographics*, May 2010, pp. 9–16.
[20] E. Omurtak and G. Zeybek, ''The effect of augmented reality applications in biology lesson on academic achievement and motivation,'' *J. Educ. Sci., Environ. Health*, vol. 16, no. 3, pp. 40–48, Jan. 2022. [Online]. Available: https://www.learntechlib.org/p/151963
[21] M. Flores-Bascuñana, P. D. Diago, R. Villena-Taranilla, and D. F. Yáñez, ''On augmented reality for the learning of 3D-geometric contents: A preliminary exploratory study with 6-grade primary students,'' *Educ. Sci.*, vol. 10, no. 1, p. 4, 2020. [Online]. Available: https://www.mdpi.com/2227-7102/10/1/4
[22] N. V. Osypova and V. I. Tatochenko, ''Improving the learning environment for future mathematics teachers with the use application of the dynamic mathematics system geogebra AR,'' in *Proc. 4th Int. Workshop Augmented Reality Educ. (AREdu)*, vol. 2898, Kryvyi Rih, Ukraine, S. H. Lytvynova and S. O. Semerikov, Eds., May 11, 2021, pp. 178–196. [Online]. Available: https://ceur-ws.org/Vol-2898/paper10.pdf
[23] M. Hohenwarter, ''GeoGebra: Ein softwaresystem für dynamische geometrie und algebra der Ebene,'' M.S thesis, Paris Lodron Univ., Salzburg, Austria, 2002.
[24] M. Hohenwarter, M. Borcherds, G. Ancsin, B. Bencze, M. Blossier, J. Éliás, K. Frank, L. Gál, A. Hofstätter, F. Jordan, Z. Konečný, Z. Kovács, E. Lettner, S. Lizelfelner, B. Parisse, C. Solyom-Gecse, C. Stadlbauer, and M. Tomaschko. (Oct. 2018). *GeoGebra 5.0.507.0*. [Online]. Available: http://www.geogebra.org
[25] M. Hohenwarter, D. Jarvis, and Z. Lavicza, ''Linking geometry, algebra, and mathematics teachers: Geogebra software and the establishment of the international geogebra institute,'' *Int. J. Technol. Math. Educ.*, vol. 16, no. 2, pp. 83–86, 2009.
[26] S. Yorganci, ''A study on the views of graduate students on the use of GeoGebra in mathematics teaching,'' *Eur. J. Educ. Stud.*, vol. 4, no. 8, pp. 63–78, 2018.
[27] M. Y. Gavali, ''Smartphone, the new learning aid amongst medical students,'' *J. Clin. Diagnostic Res.*, vol. 11, no. 5, pp. JC05–JC08, 2017.
[28] D. T. Campbell and J. C. Stanley, *Experimental and Quasi-Experimental*. Boston, MA, USA: Houghton Mifflin, 1963.
[29] S. Y. Lim and E. Chapman, ''Development of a short form of the attitudes toward mathematics inventory,'' *Educ. Stud. Math.*, vol. 82, no. 1, pp. 145–164, Jun. 2012, doi: 10.1007/s10649-012-9414-x.
[30] Y.-C. Huang and S.-H. Lin, ''Development and validation of an inventory for measuring student attitudes toward calculus,'' *Meas. Eval. Counseling Develop.*, vol. 48, no. 2, pp. 109–123, Apr. 2015.
[31] J. A. Fredricks, M.-T. Wang, J. S. Linn, T. L. Hofkens, H. Sung, A. Parr, and J. Allerton, ''Using qualitative methods to develop a survey measure of math and science engagement,'' in *Learning and Instruction* (Student Engagement and Learning: Theoretical and Methodological Advances), vol. 43. Amsterdam, The Netherlands: Elsevier, 2016, pp. 5–15.
[32] J. A. Frank and V. Kapila, ''Mixed-reality learning environments: Integrating mobile interfaces with laboratory test-beds,'' *Comput. Educ.*, vol. 110, pp. 88–104, Jul. 2017.
[33] P. C. C. Vieira, ''T-test with likert scale variables,'' Apr. 2016, doi: 10.2139/ssrn.2770035.
[34] J. Frost. (2016). *Best Way to Analyze Likert Item Data: Two Sample T-Test Versus Mann–Whitney*. Accessed: Jun. 10, 2017. [Online]. Available: http://blog.minitab.com/blog/adventures-in-statistics-2/best-way-to-analyze-likertitem-data

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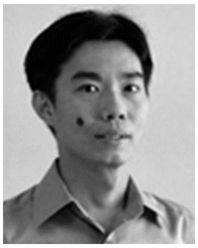

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