



**NANYANG
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UNIVERSITY**

**MODELING USING MAGNETIC DIPOLE MOMENT
PRINCIPLE & ORIENTATION SENSING OF AN
ELECTROMAGNETIC SPHERICAL ACTUATOR**

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SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING

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Modeling Using Magnetic Dipole Moment Principle & Orientation Sensing of an Electromagnetic Spherical Actuator

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ABSTRACT

Multi-Degrees-Of-Freedom (MDOF) systems have always been a deep-rooted interest in the field of robotics due to their great potential in enhancing the number of Degrees-Of-Freedom (DOF) and the rigidity of the system. In this work, an electromagnetic MDOF spherical actuator is proposed. The underlying principle in creating MDOF motion is by electromechanical torque generated between the rotor and stator poles. This dexterous drive can achieve 3-DOF motion within a compact rigid joint. The design is configurable and extremely simple to assemble. Three-DOF motion can be created by a basic module consisting of four stator poles and one rotor pole. Moreover, the absence of high permeable elements in the magnetic system translates to a linear relationship between the input and output parameters. The direct advantage of such a system will be fully appreciated during control implementation. A torque model based on the principle of magnetic dipole moment was formulated for this class of spherical actuators. This novel approach not only allows for direct and inverse correlation with the input current and output torque in three-dimensional space, but also provides an insight into the directional determination of the motion generating torque. A prerequisite of this torque model requires the knowledge of the magnetic field distribution within the region of interest. Therefore, an analytical magnetic field model was also formulated using elliptical integrals for the computation of the magnetic field. This analytical model provides an efficient and accurate description of the magnetic field as compared to the conventional numerical method. Coupled with the torque model, the resultant torque acting on the spherical rotor can be determined. A novel orientation sensing methodology for spherical actuator is also proposed in this thesis. By exploiting axial-symmetric magnetic field incorporated within the prototype, the rotor orientation can be ascertained by the use of Hall sensors. A three-dimensional numerical model was developed using ANSYS for the verification of the analytical magnetic field and torque models. Comparison between both analytical and numerical results reveals good fit and hence validates the proposed theory. The numerical model can also be employed for future parametric evaluation and optimization of the spherical actuator.

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1. C. K. Lim, L. Yan, I. M. Chen, G. L. Yang and W. Lin. Mechanical design & numerical electromagnetic analysis of a DC spherical actuator. In *Proceedings of IEEE International Conference on Robotics, Automation and Mechatronics*, volume 1, pages 536-541, December 2004.
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4. L. Yan, I. M. Chen, C. K. Lim, G. L. Yang, W. Lin and K. M. Lee. Torque modeling of a spherical actuator based on lorentz force law. In *Proceedings of IEEE International Conference on Robotics and Automation*, pages 3646-3651, April 2005.
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14. C. K. Lim, L. Yan, I. M. Chen, G. L. Yang and W. Lin. Torque modeling of a permanent magnet spherical actuator based on magnetic dipole moment principle. Submitted for *Proceedings of IEEE Conference on Advanced Intelligent Mechatronics*, 2008. (In Review)

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NOMENCLATURE

Bold	Vector quantity
<i>Italic</i>	Scalar quantity
{ }	Coordinate frame
{UPPER CASE}	Global coordinate frame
{lower case}	Local coordinate frame
'	Primes (') added to the subscripts indicate that rotation is described by Euler angles. Each rotation is complemented by an additional prime.
Π	Vector plane
$\{\text{coordinate frame}\} \mathbf{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	Position vector with reference to the stated coordinate frame {coordinate frame} with default $\mathbf{P}$ referenced to the global coordinate frame.
$\{\text{coordinate frame}\} \hat{\mathbf{P}} = \frac{1}{ \mathbf{P} } \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	Direction vector with reference to the stated coordinate frame {coordinate frame} with default $\hat{\mathbf{P}}$ referenced to the global coordinate frame.
$\begin{matrix} \{\text{coordinate frame 1}\} \\ \{\text{coordinate frame 2}\} \end{matrix} \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$	Rotation mapping matrix mapping vector $\{\text{coordinate frame 2}\} \mathbf{P}$ to $\{\text{coordinate frame 1}\} \mathbf{P}$
$\mathbf{R}_{\text{orientation convention}} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$	Rotation operator matrix rotating vector $\{\text{coordinate frame 1}\} \mathbf{P}_1$ to create $\{\text{coordinate frame 1}\} \mathbf{P}_2$ by the indicated orientation convention

CHAPTER 1

INTRODUCTION

Electrical motors were invented as early as the 1830's as soon as the magnetic effects of electrical currents and the magnetic properties of iron became known. Typical design of these motors utilizes the repulsion and attraction between electromagnetic poles switched by a commutator. Attempts in the 1840's to enhance these motors in terms of power and size failed partly due to the fact that magnetic forces do not scale proportionally to distance and the significance of counter-EMF was also not known. More success was encountered in making generators. This is usually done by moving permanent magnets (thereby creating a moving magnetic field) with respect to coils of wire wound around iron cores to generate alternating currents for supplying arc lights and direct currents for electrolysis tanks (transformer action). Electrical power industries evolved in the later years. To date, industrial motors come in a variety of shape and sizes. The basic working principle of electrical motors involves rotating coils of wire which are driven by the magnetic forces exerted from the magnetic field on the current. The output motion of such motors is primarily limited to single DOF (Degrees-Of-Freedom).

The proliferation of technology has opened the door to an ever-increasing spectrum of applications in all market sectors. The advancement in robotics has motivated the development of multi-DOF motors and actuators. Present industrial robots exploit several single DOF motors in series or parallel to accomplish multi DOF motion as shown in Figure 1.1. However, singularities usually exist within the workspace that restricts the usable workspace. Moreover, inevitable utilization of reduction gear

trains along with backlashes from member motors and joints compromises the overall positional resolution and accuracy. Lower structural rigidity and slower dynamic response are also some of the inherent trade-offs of these systems. Therefore, one would concur that a system capable of providing multi-DOF of controlled motion within a compact structure is extremely desirable. In recent years, there is an increasing demand for programmable servo-controlled actuation in multiple axes for applications as diverse as active vision systems, force-feedback joysticks and automated manufacturing. The spherical actuator, as the name implies, is capable of producing 3-DOF motions with a compact structure. It is a direct drive device that possesses no singularity within workspace except the boundary and has excellent structural rigidity.

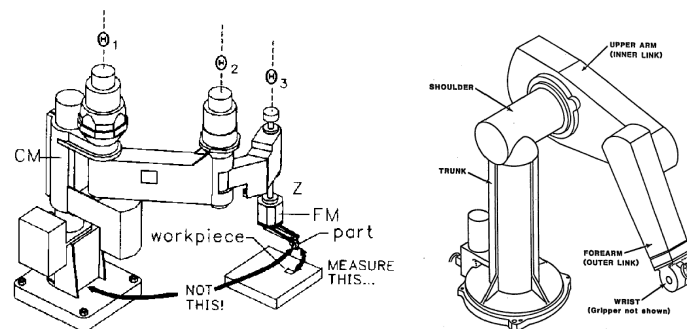


Figure 1.1. Industrial robots with multi-DOF.

In addition, high precision positioning and fast dynamic responses can be also achieved in this class of actuators. Due to the structural design, orientation of such an actuator can be accomplished rapidly and continuously with isotropic resolution in all directions. Direct application of spherical actuators will be in the wrist joints of the manipulator or robotics eyes. Henceforth, by encompassing the potential and benefits of spherical actuators, it will complement and greatly enhance the performance of existing industrial robots and other positioning devices.

1.1 MOTIVATION OF STUDY

Advances in machine and motion control technologies make it possible to realize intelligent and complex motion in robotics systems. Industrial robots are widely found in such applications. In general, the entire robotic system is a linked structure with several single DOF actuators connected together. This structure is simple and yet

powerful. Unfortunately, it possesses singularities within the potential workspace which is inherent in the mechanical design. Furthermore, with the array of actuators and transmission system, it will inevitably compromise the dynamic performance and servo tracking accuracy due to the combined effect of backlash, non-linear friction, inertia and elastic deformation of mechanical parts.

To obtain a Multiple-DOF (MDOF) motion efficiently, it would be ideal if the actuator is capable of generating MDOF motion. From this perspective, the spherical actuator provides the solution to the issues encountered by existing MDOF systems. The spherical actuator not only makes it possible to achieve MDOF motion within one compact structure, it is also expected to consume lesser power as compared to existing MDOF systems. The power efficiency of an MDOF system will contribute significantly in the conservation of energy. Indeed, the spherical actuator has boundless possibilities to not only enhance but also to realize sophisticated functions on existing robotic systems. Some immediate applications of spherical actuators include high-speed laser cutting and micro-machining, opto-electronic components positioning and alignment, MEMS components handling and assembly, micro-surgery and bio-cell manipulation, spacecraft attitude control and actuation of haptic interface.

The potential of MDOF motion which the spherical motor or the actuator can deliver is indeed enormous. A review shows that substantial work and research were conducted in this field. From the 1950's to date, the goal was to design a spherical actuator with superior performance in speed, accuracy, dexterity and volumetric efficiency. Regrettably, most conceptual designs encounter problems in some way or another that hinder further extension work on the proposed prototype. Table 1.1 provides a summary of the characteristics of some prominent spherical actuators proposed to date. We observed that prototypes proposed by Williams *et al.*, Vachtsevanos *et al.*, Baek *et al.* and Kaneko *et al.* are structurally complex in nature due to the technique employed for MDOF generation within the system. In addition, the actuating elements, that is the stator coils are required to be wound in intricate configurations translating into difficulties during the fabrication and assembly process. Another shortfall of these spherical prototypes is in its limited workspace and in the case of Williams *et al.* design, the actual achievable DOF was only two.

Conceptual design by Anders *et al.*, Vachtsevanos *et al.* and Foggia *et al.* utilizes ferromagnetic materials in order to increase the output torque. The accompanying drawbacks of this type of configuration result in non-linear control and the problem of magnetic saturation in the magnetic system. Employing ferromagnetic material increases the output torque but concurrently introduces detent torque due to the magnetization of the ferromagnetic element. In terms of control, the detent forces along with the saturation characteristics of ferromagnetic component poses great challenges. Nevertheless, other than the higher output torque, analysis and modeling of the electromechanical system can be easily conducted using the lump parameter approach in the magnetic circuit. Using a laminated core is one possible solution to circumvent the above mentioned disadvantages.

Chirikjian *et al.* proposed a stepper-like spherical actuator. The downside of such a design lies with the limited step resolution and directional control of the rotor. Since the primary working principle is based on the simple strategy by attracting adjacent poles, the resolution of each step is dependent on the physical location of the poles. Due to this fact, fine motion is not realizable in this manner of configuration. The advantage of Chirikjian *et al.* prototype is that there is no singularity within the targeted workspace. The rotor is free to rotate about any axis in any continuous motion. The challenge for such a design will be the control methodology in order to orientate the rotor.

Lee *et al.* came up with a spherical wrist motor based on the principle of a variable reluctance stepper motor. The challenge for this design revolved around the formulation of a feasible torque model for electromechanical analysis. As air-core coils were exploited for linear control, the determination of magnetic reluctance within the circuit was an uphill task especially in three-dimensional space. Moreover, the lack of an analytical magnetic field model hinders accurate modeling of the proposed system. Recent development by Wang *et al.* offered a spherical actuator capable of generating 3-DOF motion. Based on the Lorentz force law, modeling of the electromagnetic system was conducted. This approach is viable with respect to the proposed design as it computes the force generated along the current carrying wire of the stator poles. The downside in this analysis will be the necessary condition to acquire full magnetic field description of the workspace. With this requirement,

computational time will be exhaustive for real-time control implementation. Besides, customization in terms of rotor magnetization and coil windings are prerequisites increasing the complexity and cost for manufacturing.

The review on existing 3-DOF actuators designs attests to a fact that an effective orientation sensing system has yet been developed. Fundamentally, this is a crucial block within the system design that directly relates to closed-loop control. The major challenge encountered by researchers is the requirement to sense the rotor's spherical surface. Using a rotary encoder has always been an option in addressing this issue [12,15,18]. However, connecting linkages required for this type of encoders compromise and introduce undesirable external forces and inertia. To eliminate this inherent property of a contact-based sensing system, contactless sensing was sought after. Vision and optical-based methods were investigated by Lee *et al.* & Hollis *et al.* [10,53,54]. Pertaining to the vision-based system, factors that deter cost effective implementation of this method originate from its sophisticated image processing algorithm, processing of grid/recognition pattern and the lack of a suitable vision system. Real-time application of such a system is also a potential challenge. A substitute for the vision-based system is by means of utilizing optical sensors. These sensors generate pulse signals which are proportional to the relative motion of the surface being sensed. This approach not only eliminates the need for grid patterns on the surface to be sensed, but also provides concurrent measurement of angular displacements [55-56]. As the primary working principle behind optical sensors depends on the macroscopic changes from the images it acquires, processing of the sensing surface is still a pre-requisite to a certain extent. From the experimental verifications conducted to date, resolution and repeatability of these sensors on spherical surface is still a knotty issue. Other limitations include the imaging frame rate and system stability of these sensors. Hence, one would agree that an actuating system is handicapped if there are no orientation and displacement sensing capabilities. An open-loop control is of little advantage where positioning and directional type of applications are involved. Moreover, for 3-DOF actuators whereby the sensing surface is spherical in shape, it proves to be an uphill task in the implementation of an effective orientation sensing system.

From the above discussion and review on various designs, we agree that for the design of a spherical actuator, the fundamental block lies with how actuating forces are generated to create motion. Complexity in the mechanical design translates to bulk and complicated fabrication processes equating to higher cost of manufacture. The ensuing key factor will be the electromechanical analysis of the system that correlates the input and output parameters of the magnetic system. Appropriate modeling of the system will not only pave the way for design optimization, but also furnish the necessary information for control. The description and computation of magnetic field distribution is of paramount importance during this process. Subsequent vital and final components lie in the feedback loop to complete the control algorithm. For that, an orientation sensing system to sense the instantaneous position of the rotor is crucial for trajectory planning. In summary, there are fundamentally three main challenges that researchers encounter in their work on spherical actuators, namely

1. Complexity in structural design and limited workspace
2. Modeling of the electromechanical system
3. Orientation sensing solution

Henceforth, the motivation behind our research work is to fill the three missing pieces as highlighted above for a successful design in electromagnetic spherical actuators. We intend to design a new electromagnetic spherical actuator which is simple in structure, to formulate a viable analytical torque model for electromechanical analysis and to propose a feasible orientation sensing strategy for feedback control. Figure 1.2 presents our objectives schematically. The significance of this research work with the realization of our objectives will be of great importance and impact in the field of electromagnetic spherical actuator design.

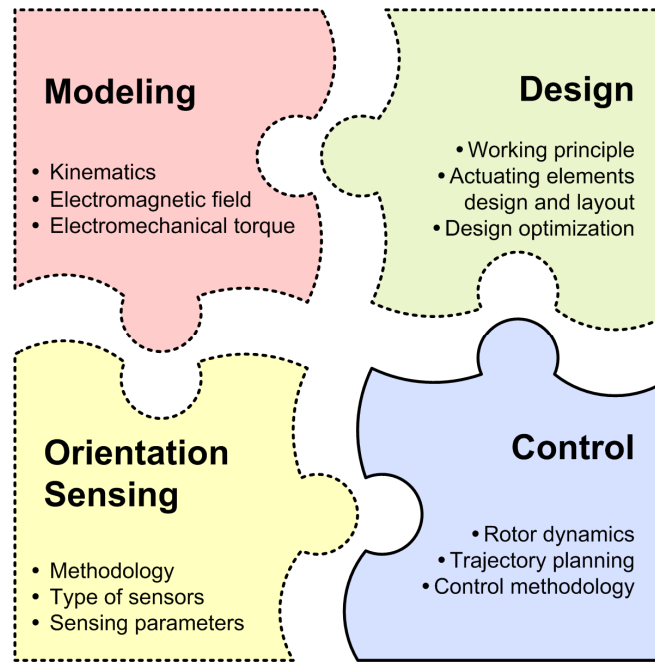


Figure 1.2. Objectives of current work.

1.2 REVIEW ON SPHERICAL MOTOR

The history of spherical motors dates back to the mid 1950's. Before we embark on the journey to review the evolution of this interesting apparatus, the author would like to draw attention to the terminology used in this thesis. The term “motors” will be employed to describe devices designed for continuously operating motion. “Actuators”, on the other hand, would suggest devices that are actuated to achieve specific orientation and region as in positioning applications.

The first spherical induction motor was designed and constructed by Williams and Laithwaite *et al.* [1,2,3] from the University of Manchester. The basic working principle was generating motion by placing conducting material at an angle under a moving magnetic field as illustrated in Figure 1.3. With the rotor spinning at one axis and the placement angle of the stator block that is not accomplished by the wire windings, this design, stringently speaking, has only two-DOF.

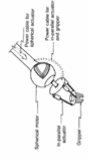
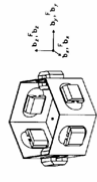
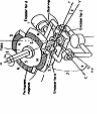
	William <i>et al</i> spherical induction motor 	Vachtsevanos <i>et al</i> spherical motor 	Hollis <i>et al</i> "Magic Wrist" 	Foggia <i>et al</i> electromagnetic actuator 	Kaneko <i>et al</i> spherical DC motor 
Working principle & Modeling method	Generalised squirrel-cage induction motor with rotating field twisted at an angle θ	Composite field wave produced by controlling the phase and excitation level of the windings to impart net rotational motion to the rotor	Lorentz forces generated from the interaction between the magnetic fields vectors and moving charges	Linear induction motor	DC induction motor
Rotor configuration	Ferromagnetic rotor with a spherical barrel shaped surface with inlaid of copper mesh	Conducting shell filled with ferromagnetic lamination	Flat wound coil	Hollow sphere made of solid steel	Four permanent magnets mounted radially to a shaft in an alternating fashion
Stator configuration	Two multiphase winding blocks	Spherical socket with an inlaid of three transversing polyphase windings	2 U-shaped permanent magnet	Three inductors on the inner surface of the rotor along three different directions	Three windings wound to the spherical surface
Maximum inclination	θ	No Limits	$\pm 5\text{mm}$ translational $\pm 4^\circ$ rotational	$\pm 30^\circ$	$\pm 15^\circ$
Pros	First spherical induction motor	Large inclination made possible	i) 6 DOF ii) High precision	Simplicity in the modeling and control of actuator	Rapid rotation is possible
Cons	Only 2 DOF motion	Constructional difficulties in windings	Limited rotation range	Constructional difficulties in windings	i) Limited workspace ii) Constructional difficulties due to coiling

Table 1.1. Characteristics of existing design of spherical motors/actuators.

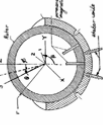
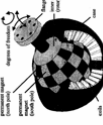
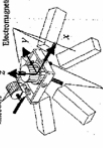
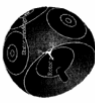
	Lee <i>et al</i> VR spherical motor 	Anders <i>et al</i> "Kugelmotor" 	Baek <i>et al</i> 3 DOF motor 	Chirikjian <i>et al</i> spherical stepper motor 	Wang <i>et al</i> spherical motor 
Working principle & Modeling method	Permanence model in variable reluctance motor	Similar to VR spherical motor	Coulomb Law with respect to magnetism	Standard stepper motor working principle where the rotor will "step" between stable configurations of a rotor/stator pair	Lorentz forces generated from the interaction between the magnetic fields vectors and current density
Rotor configuration	Ball-joint-like body that encloses a number of permanent magnets radially attached to the rotor surface	permanent magnet shells embedded in the surface of the mobile spherical rotor	Alternate arrangement of permanent magnets relating to the positions of stator poles	Plastic sphere with permanent magnets attached in the inner surface	Diametrically magnetized spherical rare earth magnets
Stator configuration	Hemispheric stator houses the stator coils at the vertices of icosahedron	96 stator coils	Electromagnets at six vertices of the lower half of icosahedron	Tapered pedestal houses stator that consists of 16 electromagnets	Windings disposition depends on desired output DOF
Maximum inclination	$\pm 45^\circ$	No limits	$\pm 30^\circ$	No limits	$\pm 45^\circ$
Pros	i) Large workspace ii) Robust and compact design	i) High output torque - 40Nm	i) Relative large workspace ii) Rapid rotation is possible iii) Control for arbitrary position is possible	Much wider range of unhindered motion compared to other spherical stepper motor	i) High specific torque ii) Robust mechanical structure iii) Simple position sensing system
Cons	i) Effect of magnetic saturation ii) Complex control strategy iii) Manufacturing difficulties	i) Effect of magnetic saturation ii) Complex control strategy iii) Manufacturing difficulties	Large energy loss	i) Smooth transitional motion may not be possible ii) Commutation and motion planning is difficult with smaller rotor	i) Constructional difficulties due in windings ii) Potential large detent torque due to iron core iii) Induced eddy current

Table 1.1. Characteristics of existing design of spherical motors/actuators.

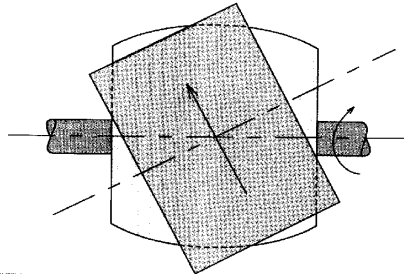


Figure 1.3. Working principle of Williams and Laithwaite *et al.* spherical motor.

Little was done in the area of spherical motors or actuators in the 1960's with sporadic research generated by the interest in space technology [4], whereas in the 1970's, the spherical concept was creatively applied to pumps where the impeller was spherical in shape eliminating the need for shaft seals [5]. Figure 1.4 depicts the invention of a pump with a spherical motor.

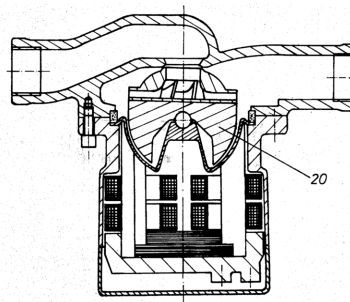


Figure 1.4. Laing *et al.* rotordynamic pump with spherical motor.

Into the 1980's, studies of this innovative device intensify with robotic applications in mind. Much research was done in bid to extend robotic manipulators in several dimensions by separating the manipulation into coarse motion of a general robot arm and fine motion of a dexterous end effector. Precision manipulation task requires small, high bandwidth motions where small weight-to-force ratios and innovative end effectors offers. In view of this, Vachtsevanos *et al.* invented a new robotic manipulator incorporating a novel spherical motor in a single joint shown in Figure 1.5a [6].

The end-effector is based on the principle of in-parallel mechanisms. The inner surface of the stator is provided with three transversing polyphase windings. These windings partitioned the stator surface into a pattern of cells as illustrated in Figure

1.5b [7]. Composite field wave produced by controlling the phase and excitation level of the windings is directed as desired to impart net rotational motion to the rotor. The rotor is in the form of spherical conducting shell containing ferromagnetic material. The drawback of this particular design is the daunting task of inlaying the transversing windings on the inner spherical surface of the stator.

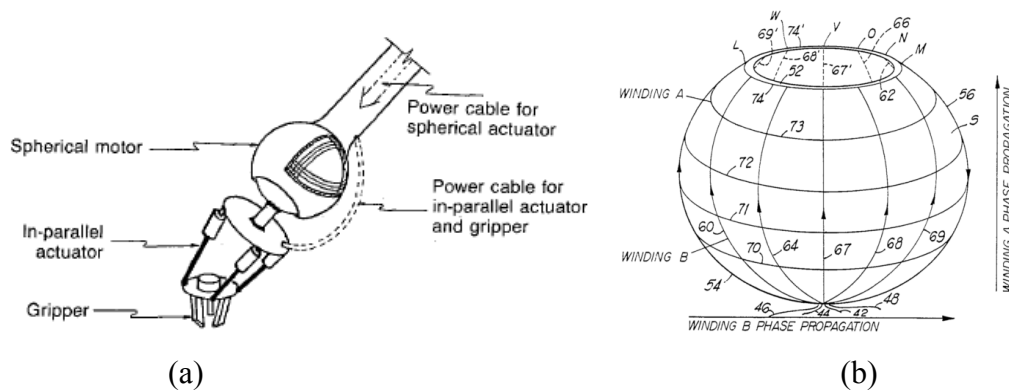


Figure 1.5. Vachtsevanos *et al.* spherical motor. (a) proposed actuator: (b) stator winding means.

Hollis *et al.* carried out concurrent work with the same objective of improving existing robotics systems. The proposed design was a direct current device with 6-DOF named the “Magic Wrist” [8,9]. It consists of a “flotor” and a “stator” as illustrated in Figure 1.6b. The “flotor” is a hollow hexagonal aluminum structure with flat coils imbedded in each of its faces. The sidewalls of the wrist fit between magnets attached to the stator in a hexagonal pattern. The basic actuator composes of two U-shaped magnets and a flat-wound rectangular coil as in Figure 1.6a. When the coils are energized, “current sheets” are created in the direction $\mathbf{I}_k$, $k=1,\dots,6$, along the faces of the hexagonal structure. Lorentz forces are generated from the interaction between the magnetic fields vectors $\mathbf{B}_k$, $k=1,\dots,6$, from the magnets. The longitudinal and latitudinal placed coils create the motion of the cylinder with respect to the magnets achieving the 6-DOF movement (Figure 1.6c). Position and orientation are obtained from an optical sensing system shown in Figure 1.7. The projections of coplanar beams at angles 120° apart from the three LEDs are detected by two-dimensional lateral effect photodiodes. An improved version using a spherical barrel instead of the hexagonal cylinder was also proposed to enhance the workspace,

positioning precision and dynamic response [10]. However, the range of rotation is limited to $\pm 4^\circ$, restricting wide application of this device.

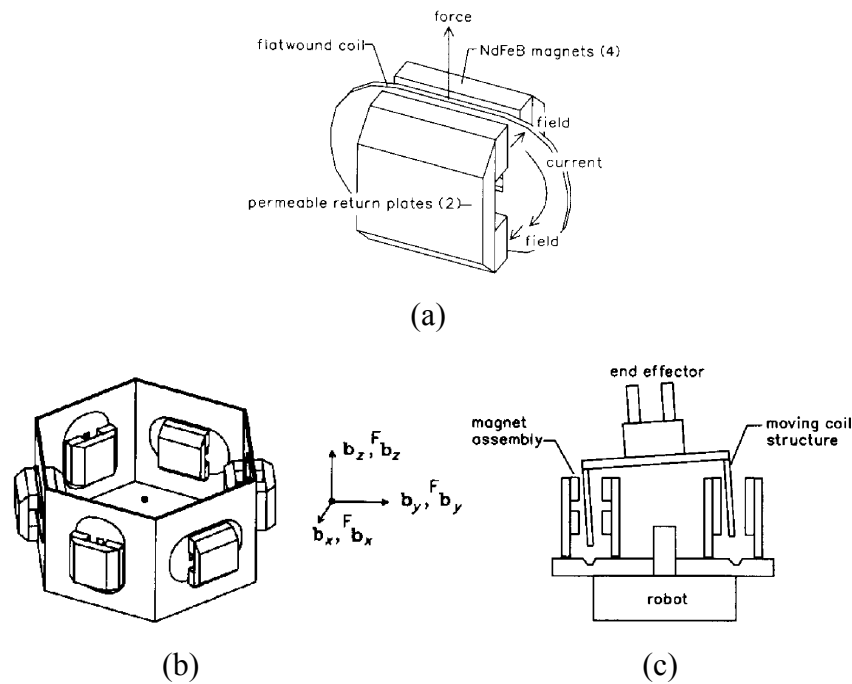


Figure 1.6. Hollis *et al.* “Magic Wrist”. (a) Single actuator: (b) manipulator configuration: (c) sideview of flotor being displaced.

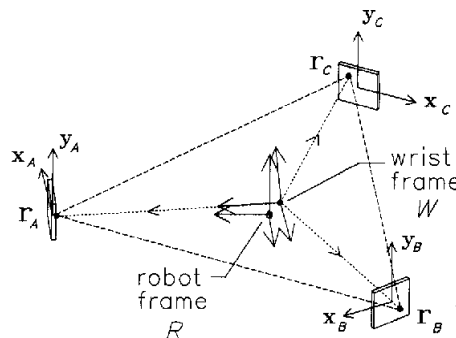


Figure 1.7. Position and orientation sensing scheme by Hollis *et al.*

From France, Foggia *et al.* presented another design of an electromagnetic actuator rotating around three independent axes [11]. The proposed prototype consists of a spherical armature containing three fixed inductors as depicted in Figure 1.8 below. When activated, the three phase windings of each inductor develop moving magnetic field acting along its own plane of symmetry and creating the resulting motion of the armature rotating with a precession. The moving armature is composed of a hollow

sphere and is made of solid steel where the maximum deviation angle is $\pm 30^\circ$. The principle of operation is identical to that of induction motors. Under a 50Hz network, with the input current at 3A per phase, one inductor is capable of generating a maximum starting torque of $\sim 0.8\text{Nm}$. In addition, an electromagnetic sensor was developed to detect the position of the moving armature.

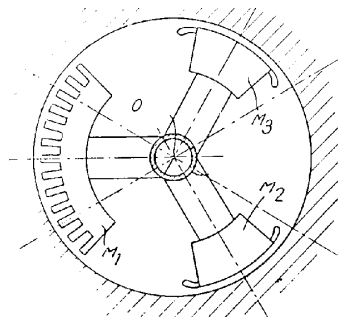


Figure 1.8. Sectional view of Foggia *et al.* actuator.

Kaneko *et al.* developed a spherical DC motor with 3-DOF [12]. The design of the rotor encompasses four permanent magnets mounted radially to a shaft in an alternating fashion shown in Figure 1.9. Closed-loop magnetic fluxes are formed with the adjacent magnets. The shaft is mounted to a gimbal structure to realize three-dimensional rotation. The stator, on the other hand, consists of three windings wound to the spherical surface. Strategic placement of these windings formed eight longitudinal sections in four groups with each group having two sections. Opposite current input to the sections induces three-dimensional rotations. The rated specifications of this motor are reported to have a 0.3 torque constant of spinning, 0.06 torque constant of inclination and a range of inclination of $\pm 15^\circ$.

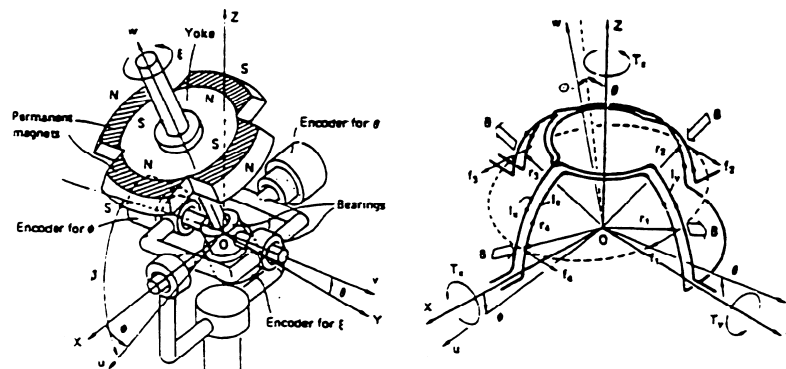


Figure 1.9. Kaneko *et al.* spherical DC servo motor.

In the U.S., Lee *et al.* was involved in the research work of spherical in-parallel actuators [13,14,15]. In the midst of evaluating existing designs, Lee realized that spherical actuators or motors being developed so far encounter the issue of constructional difficulty. Laminations are required to prevent movement of unwanted eddy currents. Complicated phase windings must be embedded in recessed grooves so as to provide rotating magnetic field. Thus, the motivation behind Lee's effort was to overcome these challenges to achieve a true 3-DOF actuator with an alternate arrangement [16]. The concept of a spherical wrist motor based on the principle of variable reluctance (VR) was conceived [17]. This spherical stepper design combines pitch, roll and yaw in a single joint which has significant potential in applications where demand on workspace is low but high speed isotropic manipulation is required. A conceptual schematic of this motor is shown in Figure 1.10.

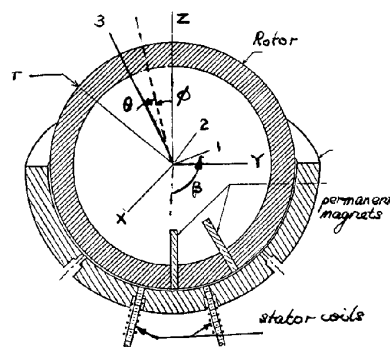


Figure 1.10. Conceptual schematic of spherical wrist motor.

The motor was a ball-joint-like device that encloses a number of permanent magnets radially attached to the rotor surface. The hemispheric stator houses the stator coils with the rotor being supported freely by means of gimbals. The basic working principle was based on the VR motor. As a pair of stator coils adjacent to the magnet is energized, attractive or repulsive magnetic force are produced as a result of the field interaction and hence the resultant torque. Desired movement of the rotor is accomplished by appropriate sequencing of high current pulses to the stator coils. Undesired torque could also be produced to oppose the intended rotation by reversing the current in the coils. The maximum torque and resolution depend primarily on the geometry, arrangement of the coils, number of magnets and the magneto-motive-force (mmf) through the coils. Due to the versatility and potential of this design, a flurry of research effort was underway subsequently in a bid to further establish the

design methodology and theoretical modeling [18,19,20], optimization, characterization and control of this new type of spherical motor [21,22,23,24]. Patents were also filed for the design and methods with respect to this device [25,26,27,28].

With micromotors emerging in the 90s, the demand for high torque to weight ratio, dexterous actuators elevated. Well known for their contribution in the field of ultrasonic motors, the Japanese were also actively involved in the research work of spherical actuators. Toyama *et al.* from Tokyo University of Agriculture and Technology proposed the use of ultrasonic motors (USM) to create multi-DOF actuator [29]. The structure was simple, consisting of a hollow spherical rotor and three set of ring stators. Two stator ring sets are assembled firmly to the rotor at both side and the third set is located in the rotor that drives and output shaft as shown in Figure 1.11a. The driving frictional force is created from the mechanical vibration of the piezoceramic elements that transmits from the elastic body to the rotor as illustrated in Figure 1.11b. Though the advantages of using USMs are evident, due to the nature of frictional forces at work, this type of devices encounter the problem of wear that inherently influences the efficiency and maximum output of the motor. Nevertheless, continued efforts went on with application in a fine orientation stage [30,31].

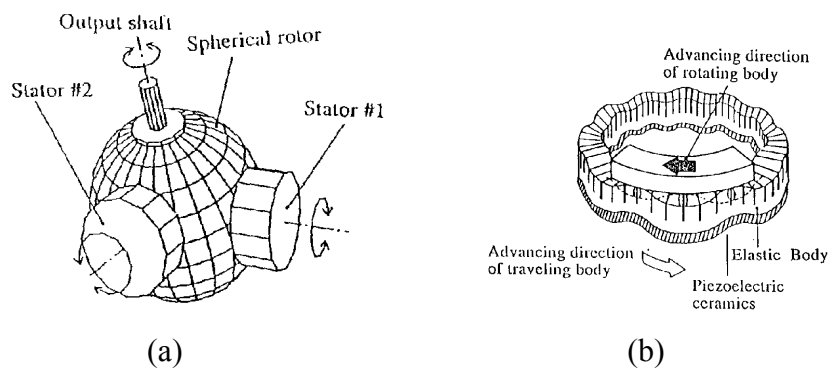


Figure 1.11. Toyama *et al.* spherical ultrasonic motor.

Bederson *et al.* built an interesting miniature spherical pointing motor (SPM) for orientation purposes in small camera sensors [32,33]. The stimulus for this device was in computer active vision where the pan-tilt mechanism should be accurate, fast, small, inexpensive and have low power requirement. The basic operating principle is

to orient a permanent magnet to the magnetic field induced by three orthogonal windings by applying appropriate ratio of currents as depicted in Figure 1.12 below. Due to the constructional simplicity where it consist of wire wound round a metal form, a simple gimbal and a small permanent magnet, SPM was touted to be the smallest and least expensive device capable of pointing a camera accurately at high speed.

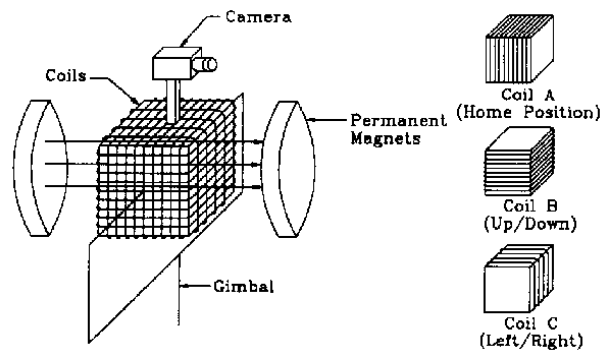


Figure 1.12. Illustration of the internal structure of SPM.

Neto *et al.* from Brazil initiated preliminary investigation into the field of spherical motors [34]. In Germany, a novel spherical linear permanent magnet motor was developed that allows telescope to be controlled in three-dimensions simultaneously by Anders *et al.* [35]. Another research project "Kugelmotor" was financially supported by the Volkswagenstiftung and undertaken by a joint effort of three institutes of Aachen University of Technology, Germany. It investigates the concept implementing multi-coordinate movements with only one motor which is similar to the idea proposed by Lee. The spherical motor is based on a permanent magnet machine design that enables a high-force density but requires a complex control strategy. The special feature of this motor is the sphere that serves as a support and as a drive. Hydrostatic support is utilized and the oil from the support system is also used for cooling the coils. Torque of the motor is generated by electrically excited coils, which are arranged spherically in the stator, and by permanent magnet shells, which are embedded in the surface of the mobile spherical rotor as shown in Figure 1.13 below [36]. This motor has no limitations in motion and produces a high torque (40 Nm) in every position. The motor uses permanent magnets (NeFeB) mounted on the rotor and 96 stator coils to generate the electro-magnetic forces for the motion.

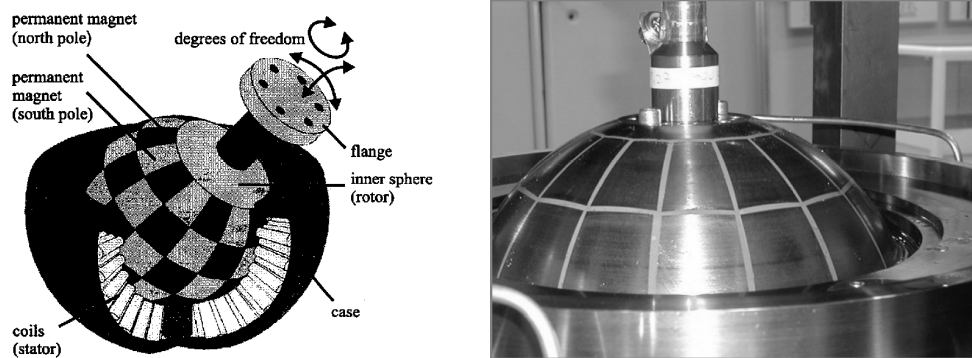


Figure 1.13. Kugelmotor.

Advancing into the 21st century, spherical USMs takes the center stage. Efforts by Hu *et al.* from Nanyang Technological University, Singapore, developed a unique ultrasonic spherical motor [37,38]. The prototype composes of a finger-like rotor, a semi-spherical stator and a supporting machine frame with a spherical bearing as illustrated in Figure 1.14.

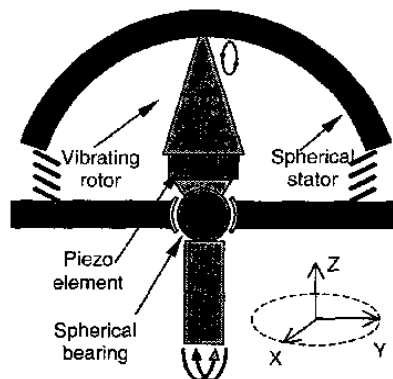


Figure 1.14. Conceptual schematic of Hu *et al.* spherical USM.

From Yonsei University, Korea, a team led by Baek proposes yet another class of 3-DOF motors [39,40]. The principle of motion was based on Coulomb's Law [41] with respect to magnetism. Figure 1.15 shows the interior of the actuator depicting five electromagnets attached to the stator and two permanent magnets attached to the rotor. The electromagnets are located at the five vertices of lower of icosahedron.

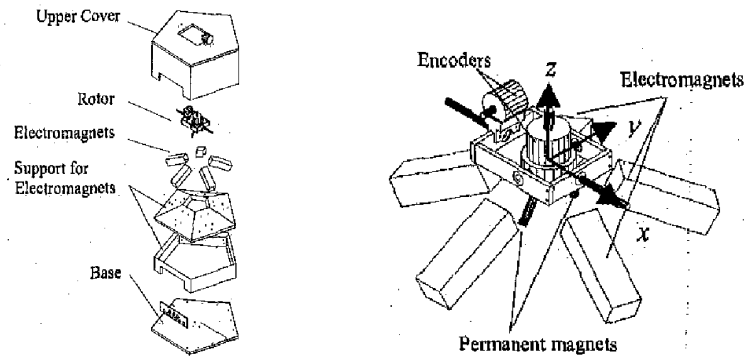


Figure 1.15. Baek *et al.* 3-DOF actuator.

A bar-shaped ultrasonic motor capable of generating multi-DOF was developed by Takemura *et al.* [42,43]. The prototype consists a spherical rotor and a bar-shaped stator as revealed in Figure 1.16a. Motion is produced by utilizing three natural vibration modes of the stator through oval trajectories of contact points. Due to the operating principle, controlling rotor motion precisely under arbitrary combination of vibration mode can be rather challenging. This prompted Takemura and team to conduct further analysis and improvement in their design [44,45]. In view of the structural optimization, Takemura *et al.* developed another new plate type USM with weight and volume advantage over the previous design as illustrated in Figure 1.16b [46]. This was similar to a prototype investigated by Chen *et al.* from Nanyang Technological University [47].

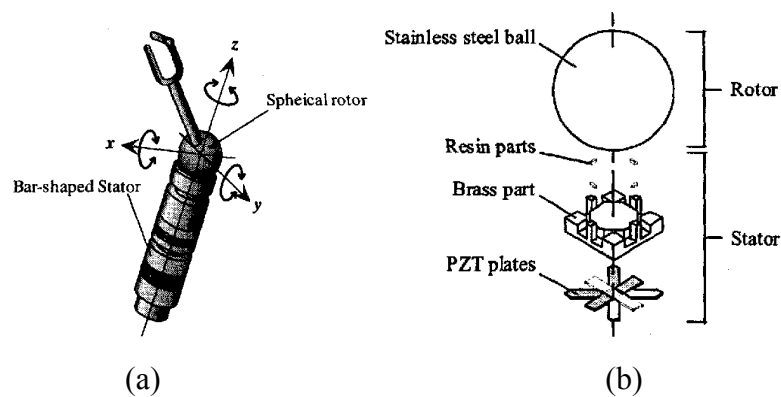


Figure 1.16. Takemura *et al.* multi-DOF USMs. (a) bar-shaped: (b) plate type.

Simplicity in design, structure and control was the goal for the spherical motor fabricated by Nagasawa *et al.* [48] as illustrated in Figure 1.17. The innovative part of

their work was to manipulate the motion of the rotor by four wires which are controlled by a simple algorithm in which the lengths of the wires are regulated.



図1. 試作したワイヤーけん引式球面モータ

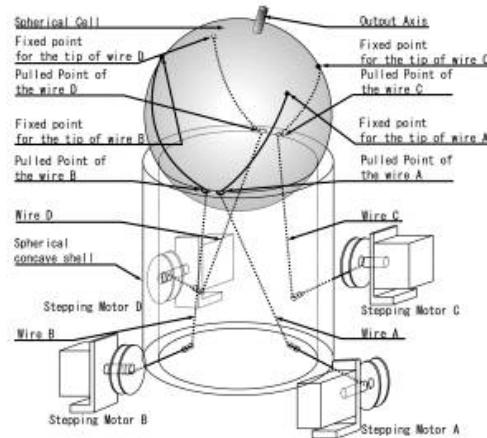


図2. ワイヤーけん引式球面モータの構造

Figure 1.17. Structure of spherical motor manipulated by four wires.

A spherical stepper motor was presented by Chirikjian *et al.* from Johns Hopkins University. The prototype as described was able to achieve a much wider range from the mechanical design of stator by not enveloping the rotor. A 12-inch diameter plastic sphere with permanent magnets attached in the inner surface was used as the rotor. A tapered pedestal acts as the base that houses the stator and supports the rotor that consists of 16 electromagnets. The rotor uses a semi-regular packing based on octahedral symmetry while the stator uses a semi-regular packing based on icosahedral symmetry as depicted in Figure 1.18. The basic working principle is like a standard stepper motor where the rotor will “step” between stable configurations of a rotor/stator pair. Thus, a commutation model was developed for the control of the prototype [49]. Issues surfaced when the rotation point switches during each commutation step. This was credited to the characteristics of the stator electromagnets as they possessed relatively broad band well instead of a desirable sharp minimum.

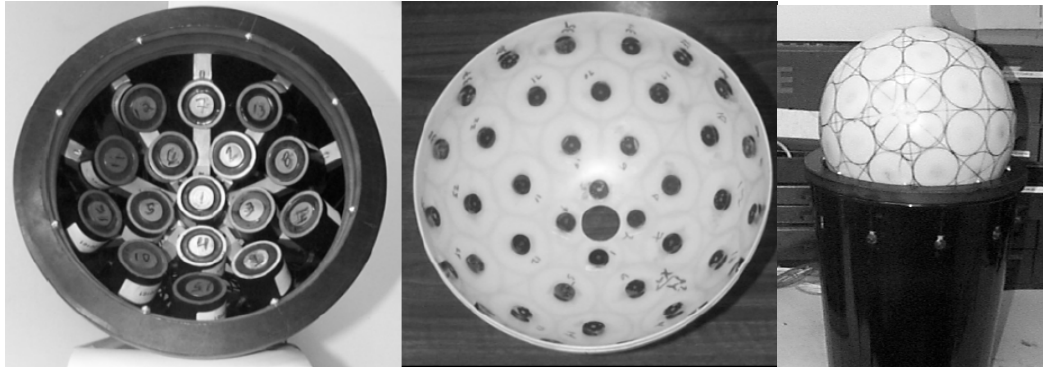


Figure 1.18. Chirikjian *et al.* spherical stepper motor stator and rotor packing.

From the United Kingdom, Wang *et al.* developed another novel spherical permanent magnet motor as shown in Figure 1.19 [50,51,52]. It builds on the foundation of the advantages from existing spherical motor designs and proposed a creative way in the structure of the rotor. The rotor is a four-pole spherical permanent magnet that is made up of two pairs of parallel-magnetized quarter-spheres. A low-friction coating was applied on the rotor surface that serves the function of bearing support. The stator composed of four sets of windings that are arranged in way that three independently controllable torque components can be developed by energizing the appropriate windings. Experimental tests revealed that this prototype could achieve a torque of 0.25Nm with a pan and tilt excursion of $\pm 40^\circ$. The drawback in this design lies in the support system. The direct contact between stator housing and rotor with the low-friction coating in between requires manufacturing tolerances must be kept at the minimum. A result of non-concentricity will be non-uniform distributed stick-slip friction that hinders control and operation of the motor.



Figure 1.19. Wang *et al.* spherical permanent magnet motor.

For the study of magnetic systems that comprises of ferromagnetic components, it is convenient to resort to the magnetic-circuit concept which is analogous to the electric circuit. The key underlying principle behind this method owes to the presence of ferromagnetic material that confines the magnetic flux of the system entirely that permits the use of magnetic circuit concept.

1.3 SCOPE OF STUDY

In accordance with the objectives listed in Section 1.2, we embark on a study in the theoretical modeling of spherical actuators. The aim is to formulate a versatile and accurate analytical torque model for subsequent analysis and design of the prototype. This model will establish and correlate the input and output parameters of the system. With a feasible torque model, we will be able to evaluate the output torque from the input parameter that the actuator is capable of generating. Conversely, from the formulation, we hope to acquire the inverse of the relation whereby we ascertain the input from the output parameters. This will put in place the groundwork for subsequent analysis and design of the spherical actuator. As magnetic elements are utilized in the prototype design, magnetic field investigation will be carried out. This is crucial for the formulation of the torque model since forces are generated between the electromagnetic elements. Without prior knowledge of the magnetic field distribution within the system, it would be impossible to conduct electromechanical analysis. In extension to this research work, we target to formulate an analytical magnetic field model that is able to describe magnetic field in 3-D space. The most common approach for the determination of magnetic field is by numerical methods like Finite Element Method and Finite Difference Method. J. T Oden made a noteworthy remark in 1991 on the power and versatility of the finite element method [64]. The computation of stationary and static electromagnetic fields by means of finite element methods has always been a topic of interest addressed in the literature [62,63,65]. Most of the computation utilizes the vector or scalar potentials approach [67-71]. This is related to the simplification of the formulation that this approach promises as it allows the use of very convenient nodal elements. However, it was observed that it is not the potentials but the field strength that has physical significance. When one is able to compute the required field strength directly, the

resulting accuracy would be one order higher than the potential based formulations [66]. Moreover, the computation of the magnetic field by numerical methods are traditionally time consuming and computationally exhaustive which makes it unsuitable to be implemented during real-time control. Hence, the scope of our research effort will also focus on how to resolve this existing problem by proposing an analytical magnetic field that is capable of calculating the field strength directly.

In view of the advantages and disadvantages of existing spherical actuator design, the objective of this research is to build on the foundation and develop a robust and dexterous spherical actuator for high precision manipulation application. A new electromagnetic spherical actuator will be designed and fabricated in this project. In general, the design of the spherical actuator can be categorized into the following components shown in Figure 1.20. The primary objective is to have a MDOF actuator with a simple structure. Simplicity in the mechanical design not only offers flexibility during operation but also reduces manufacturing cost. As in past research work, the foreseeable challenges are the design of a feasible support mechanism and the actuating element design. Since the workspace of a spherical actuator spans across three-dimensional space, the accompanying support system needs to adhere to this requirement. Existing spherical actuator employs a variety of support mechanism like air bearings, transfer bearings, friction reducing coating and magnetic levitation [8,49,53-59]. Each different scheme has its pros and cons. The complexity involved in implementing the suggested support mechanism plays an integral part in the performance of the actuator. Factors to be considered will be the frictional and inertial forces that the support mechanism introduces to the system. Stability and the effect of inherent magnetic field within the system are aspects that influence the suitability of any potential system. Henceforth, the ideal support mechanism will be one that is structurally rigid to bear both actuator and external loading and at the same time keeping frictional and inertial forces to the minimum. Assessment of past spherical designs also reveals that some designs require complex coiling methods for the stator component. The motive can be traced back to the actuating method that the particular actuator was designed for. Inevitable consequences of complex coiling structure not only impede the design and assembly process, but also complicate the electromechanical analysis of the system.

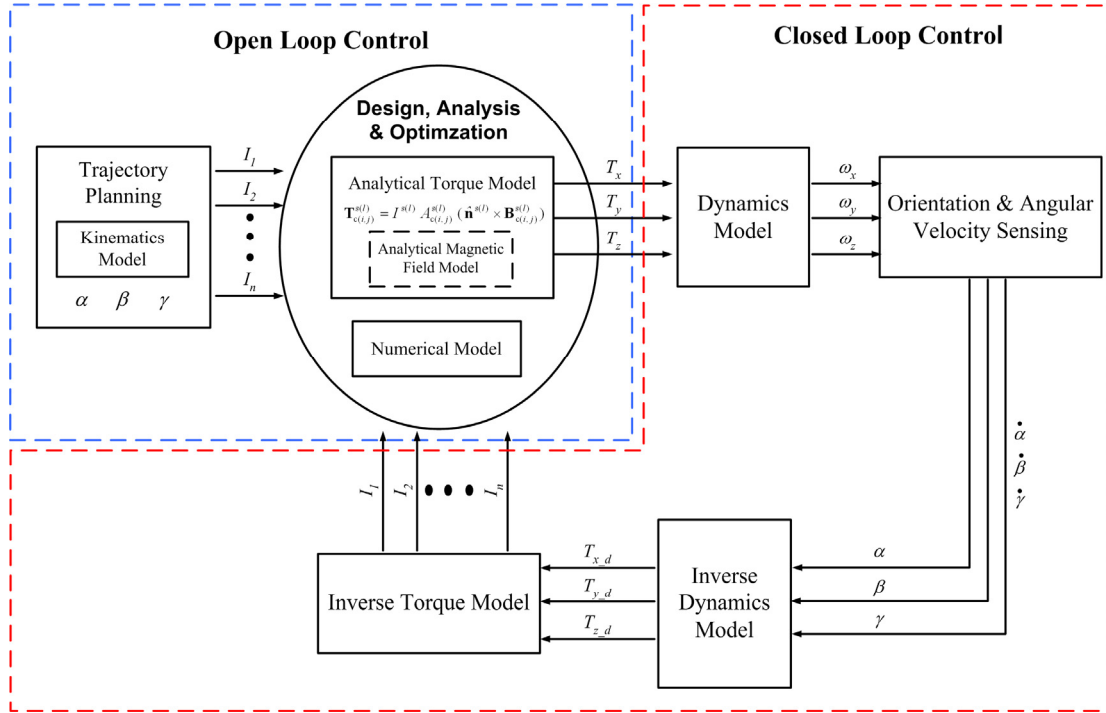


Figure 1.20. Design components of a spherical actuator.

Therefore, considering the shortfalls mentioned above, a study will be carried out to research and identify a way to circumvent the issues encountered. The new design will be a structurally simple electromagnetic spherical actuator that is able to achieve 3-DOF motion within a compact joint. As the objective of this project is to propose a new spherical actuator for positioning application, motion planning and control is an essential and integrated component of the complete system. In order for trajectory planning, the basic requirement will be the availability of orientation information of the part in motion. This translates to the need for an orientation sensing system for the spherical actuator. As highlighted previously, a number of works have been carried out in this aspect. Contactless sensing will be the preferred option as it does not contribute further loading to the existing system. Vision-based orientation sensing is stable with drawbacks in terms of the need for surface processing. On a spherical surface, at higher resolution, the surface preparation degenerates into an arduous task. The optical sensing approach on the other hand has shortfalls of being erratic and unstable during varying speed condition. If we are to take a step back and evaluate the issue of sensing a spherical surface, we will realize that it is the measurand that is the contributory factor for a feasible solution. The preparation and inconsistency of the measurand give rise to the complexities involved. Henceforth, we will embark on an

investigation into how we can systematically acquire the orientation of the spherical actuator from the inherent measurands available within the system.

In summary, for this research effort, emphasis will be on the design and theoretical modeling of a new spherical actuator. The scope includes the following areas:

- i) Formulation of an analytical magnetic model.
- ii) Formulation of an analytical torque model.
- iii) Mechanical design, fabrication and assembly of a new electromagnetic spherical actuator.
- iv) Numerical modeling of prototype.
- v) Orientation sensing methodology

1.4 ORGANIZATION OF THESIS

Chapter 1 provides a summary of the literature review in the field of spherical actuators and motors. From the assessment, potential areas of future research are identified. In this chapter, the motivations are clearly stated with the objectives systematically identified. The scope of study lists the extent and areas that the current research work will encompass. Chapter 2 presents the magnetic field model derived and employed for magnetic field description. It details the derivation of the field model based on the Biot-Savart Law with the analytical solution furnished by elliptical integrals. The advantages and flexibility of this field model is demonstrated from the ability to model various electromagnetic regions. The current model and the method of image are also introduced in this chapter. The ensuing Chapter 3 embarks on the description of the theoretical torque model established from the magnetic dipole moment principle. The details of this novel approach and methodology are discussed within this chapter. Chapter 4 gives an overview into the design of the electromagnetic spherical actuator. Key anatomies of the prototype are illustrated with information on the dimensional and material specifications. This chapter also incorporates the motion planning methodology of the spherical actuator.

Chapter 5 reveals a new scheme in the orientation sensing of spherical actuators. The chapter details the steps and process of this novel approach. Validation of the proposed method was carried through simulation and experimental setup. Verification of the magnetic field and torque models are conducted with reference to numerical results. Following that, Chapter 6 presents the numerical models constructed for this authentication process. The end results, observations and discussions are summarized and concluded in this chapter. Finally, Chapter 7 recapitulates the areas of work conducted and the contributions of this research effort. The chapter concludes with the recommendations of potential forthcoming work.

CHAPTER 2

MAGNETIC FIELD MODELING

This chapter details the derivations and approach employed in the computation of magnetic field pertaining to the elements of the spherical actuator. Primarily, the components of interest will be the permanent magnet rotor pole region, the electromagnetic stator pole region and the region of high permeability.

2.1 MAXWELL'S EQUATIONS

The governing laws between the interaction of charged matter and electromagnetic fields are well established under Maxwell's equations. These equations provide a fundamental understanding of a wide range of electromagnetic phenomena. In differential form, Maxwell's equations are as follow

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (2.1)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.3)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (2.4)$$

where

<i>Field</i>	<i>Description</i>
E	Electric field intensity (V/m)
D	Electric flux density (C/m ²)
H	Magnetic field intensity (A/m)
B	Magnetic flux density (T)
<i>Source terms</i>	<i>Description</i>
J	Free current density (A/m ²)
ρ	Free charge density (C/m ³)

The source terms in the field equations are the free current density **J** and the free charge density, ρ . A charge that has been separated from its atomic or molecular source and is free to move through a material is defined as free charge. The free current density, **J**, is then due to the motion of free charge. In addition to free charge, there is also bound charge where the charge is displaced from its atomic or molecular source but does not separate from it. This usually occurs during the polarization process of dielectric material.

The fields in Equations (2.1)-(2.4) are vector-valued functions of space and time,

$$\mathbf{E} = E_x(x, y, z, t)\hat{x} + E_y(x, y, z, t)\hat{y} + E_z(x, y, z, t)\hat{z}. \quad (2.5)$$

As each of the four fields has three components, the full field equations represent a system of 12 unknowns. By imposing the continuity of charge equation

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0, \quad (2.6)$$

one can obtain the two divergence equations from the two curl equations. In order to complete the field theory, the field equations must be augmented by additional independent equations taking the form of constitutive relations as follow,

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \quad (2.7)$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (2.8)$$

where

<i>Field</i>	<i>Description</i>
M	Magnetization (A/m)
P	Polarization (C/m ²)
<i>Constants</i>	<i>Description</i>
μ_0	Permeability of free space ($4\pi \times 10^{-7}$ Tm/A)
ϵ_0	Permittivity of free space (8.854×10^{-12} F/m)

Maxwell's equations can be written in the integral form by applying integral theorems to the equations. Taking surface integral of Equations (2.1) and (2.3) over an open surface S bounded by a contour C and applying Stokes theorem to the left hand side, we obtain,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) \cdot d\mathbf{s}, \quad (2.9)$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = - \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \quad (2.10)$$

respectively.

In the same way, by integrating the divergence equations, Equations (2.2) and (2.4) over a volume V with a closed surface S and applying divergence theorem to the left-hand side, we have,

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0, \quad (2.11)$$

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \rho \, dv \quad (2.12)$$

respectively.

Equations (2.9)-(2.12) constitutes the integral forms of the field equations that establish the following laws,

Ampere's circuital law

Equation (2.9) states that the circulation of the magnetic field intensity around any closed path is equal to the free current flowing through the surface bounded by the path. This is a generalization of Ampere's circuital law.

Faraday's and Lenz's law of electromagnetic induction

Equation (2.10) embodies Faraday's law where the electromotive force induced in a stationary closed circuit equates to the negative rate of increase of the magnetic flux linking the circuit. The negative sign in Equation (2.10) also reveals Lenz's law which postulate that induced **E**-field will act to oppose the change in magnetic flux.

Gauss' Law

Gauss' law states that the total outward flux of the **D**-field over a closed surface equals the total free charge enclosed by the surface which is shown in Equation (2.12).

No magnetic monopoles

Equation (2.11) implies that the total outward flux of the **B**-field over a closed surface equals to zero indicating that there are no isolated sources or sinks of magnetic field.

2.2 BIOT-SAVART LAW

The fundamental source of magnetic field originates from the spin and orbital motion of atomic electrons. From a macroscopic point of view, steady currents are the basic source of **B**-field found in many electromagnetic devices. Biot-Savart law governs the computation of **B**-field around a current source. Let us consider the field $d\mathbf{B}$ at a point

P due to an infinitesimal current element $I dl$ at point P' as shown in Figure 2.1. For this elemental case, we have

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{Idl \times \hat{\mathbf{r}}}{r^2}. \quad (2.13)$$

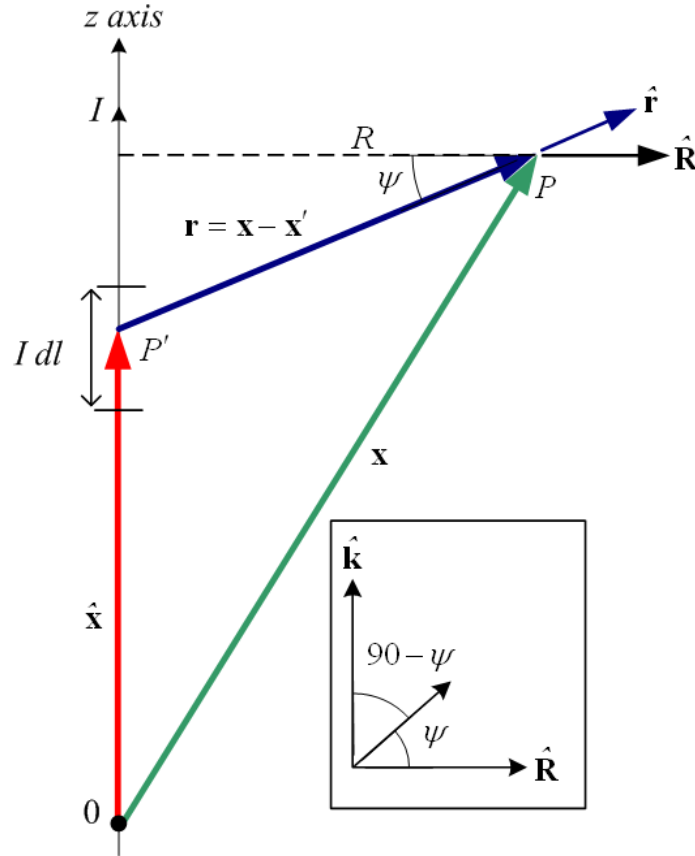


Figure 2.1. A long straight current.

From the Figure 2.1, r is the distance between the source point, P' and the field point P . $\hat{\mathbf{r}}$ is the unit vector in the direction of P' to P and $\mathbf{r} = r\hat{\mathbf{r}} = \mathbf{x} - \mathbf{x}'$. Similar to Coulomb's law, Biot-Savart equation illustrates an inverse square relationship with azimuthal magnetic field direction around the axis of Idl . To compute the field due to a macroscopic wire, we integrate Equation (2.13),

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{Idl \times \hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{Idl \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}. \quad (2.14)$$

Equation (2.14) computes the field produced by a one-dimensional current. Generalizing this equation for the case of an arbitrary volume current density, $\mathbf{J}(\mathbf{x})$,

with an infinitesimal volume d^3x' at $\mathbf{x}'$ and area $d\mathbf{A}$. The current through $d\mathbf{A}$ is $\mathbf{J} \cdot d\mathbf{A} = JdA$ and the contribution to $\mathbf{B}(\mathbf{x})$ from this small current element is

$$d\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{JdA d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}. \quad (2.15)$$

Since $dAd\mathbf{l}$ is just the volume d^3x' and $d\mathbf{l}$ is parallel to $\mathbf{J}(\mathbf{x}')$, hence the complete field at $\mathbf{x}$ will be described as

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3x'. \quad (2.16)$$

This is the generalization of the Biot-Savart law to an arbitrary current distribution throughout a volume of space. From vector calculus, we replace the integral representation of Equation (2.16) with

$$\frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} = -\nabla \frac{1}{|\mathbf{x} - \mathbf{x}'|}. \quad (2.17)$$

Rewriting the integrand noting that $\nabla \times$ is the curl with respect to $\mathbf{x}$ whereas the integral is over $\mathbf{x}'$, we have

$$\mathbf{B}(\mathbf{x}) = \nabla \times \left[\frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') d^3x'}{|\mathbf{x} - \mathbf{x}'|} \right]. \quad (2.18)$$

Equation (2.18) states that $\mathbf{B}(\mathbf{x})$ is equal to the curl of a vector function $\mathbf{B} = \nabla \times \mathbf{A}$ where $\mathbf{A}$ is the vector potential.

2.3 MAGNETIC FIELD MODEL FOR A CURRENT LOOP BY ELLIPTICAL INTEGRALS

From the microscopic view of electromagnetism, starting at the atomic level, we understand that electron magnetic moment and field is generated from its orbital motion and spin. Extending this to a macroscopic perspective, we can envision these orbiting electrons in terms of charges forming a closed loop of current I . It seems natural for us to extend this observation further for this highly symmetric circular loop as the elementary source of a magnetic field. From Chapter 1, we can perceive that the vital information in the analysis of an electromechanical system lies in the computation of the $\mathbf{B}$ -field. In another words, if we are able to acquire the $\mathbf{B}$ -field within the system, we would be able to evaluate the respective mechanical output parameters from the inputs and thus formulating the desired electromechanical theoretical model. As highlighted, a circular current loop is widely used and found in many electromagnetic systems, especially in the form of solenoid. In this section, we will embark on the derivation for the full field description of a circular current loop. Consider a current loop lying in the xy plane as depicted in Figure 2.2 below.

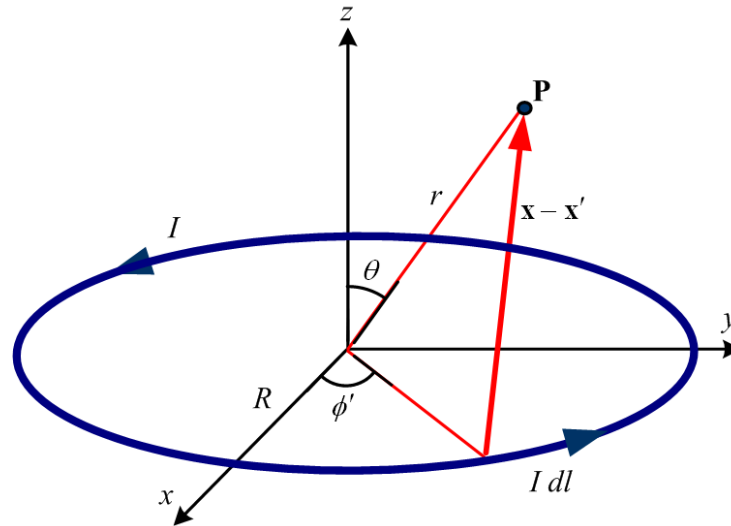


Figure 2.2. Geometry of a current loop.

By symmetry, we comprehend that the magnitude of $\mathbf{B}(\mathbf{x})$ is independent of the azimuthal angle ϕ' . For convenience, we choose a field point P in the yz plane with coordinates $(0, y, z)$. The position vector $\mathbf{P}$ can be described by,

$$\mathbf{x} = y \hat{\mathbf{j}} + z \hat{\mathbf{k}}.$$

Consider an arbitrary source point with coordinates $(x', y', z') = (R \cos \phi', R \sin \phi', 0)$ and position vector,

$$\mathbf{x}' = R \cos \phi' \hat{\mathbf{i}} + R \sin \phi' \hat{\mathbf{j}}.$$

The current element $I d\mathbf{l}$ as shown in Figure 2.1 is tangent to the loop at $\mathbf{x}'$, counter-clockwise in the xy plane, $d\mathbf{l} = R d\phi' (-\sin \phi' \hat{\mathbf{i}} + \cos \phi' \hat{\mathbf{j}})$.

By Biot-Savart law,

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I d\mathbf{l} \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \quad (2.19)$$

with relative vectors and magnitude as

$$\mathbf{x} - \mathbf{x}' = -R \cos \phi' \hat{\mathbf{i}} + (y - R \sin \phi') \hat{\mathbf{j}} + z \hat{\mathbf{k}},$$

$$|\mathbf{x} - \mathbf{x}'| = \sqrt{R^2 + y^2 + z^2 - 2 y R \sin \phi'}.$$

Taking cross product of the numerator of Equation (2.19), we have

$$\mathbf{B}(0, y, z) = \frac{\mu_0 IR}{4\pi} \int_0^{2\pi} \frac{z \cos \phi' \hat{\mathbf{i}} + z \sin \phi' \hat{\mathbf{j}} + (R - y \sin \phi') \hat{\mathbf{k}}}{(R^2 + y^2 + z^2 - 2 y^2 R \sin \phi')^{\frac{3}{2}}}. \quad (2.20)$$

From the symmetry of this problem, we can infer that B_x is 0. Since our physical system is in spherical coordinates, it will be logical to express the field in terms of spherical components B_r and B_θ as well. Similarly from symmetry in the yz plane, we have $B_\phi = 0$ since the ϕ' -component of $\mathbf{B}$ is parallel to the x axis. Thus we have $\mathbf{B}$ in the form of $\hat{\mathbf{r}} B_r + \hat{\boldsymbol{\theta}} B_\theta$ where the components are independent of ϕ' with the connections between the Cartesian and spherical components at P as,

$$B_r = B_y \sin \theta + B_z \cos \theta,$$

$$B_\theta = B_y \cos \theta - B_z \sin \theta.$$

Substituting $y = r \sin \theta$ and $z = r \cos \theta$ along with the above relationships into Equation (2.20), we acquire $B_r(r, \theta)$ and $B_\theta(r, \theta)$ as,

$$B_r(r, \theta) = \frac{\mu_0 IR^2 \cos \theta}{4\pi} \int_0^{2\pi} \frac{d\phi'}{(R^2 + r^2 - 2rR \sin \theta \sin \phi')^{\frac{3}{2}}}, \quad (2.21)$$

$$B_\theta(r, \theta) = \frac{\mu_0 IR}{4\pi} \int_0^{2\pi} \frac{(r \sin \phi' - R \sin \theta) d\phi'}{(R^2 + r^2 - 2rR \sin \theta \sin \phi')^{\frac{3}{2}}}. \quad (2.22)$$

The above equations do not appear to be a particularly difficult integral, but all attempts to express it in terms of elementary functions (rational, exponential, logarithmic, trigonometric) fail. Conventional approach is to analyze the full field using the vector potential, $\mathbf{A}(\mathbf{x})$, whereby it reduces to only one finite integral instead of two as listed above. However, evaluation of the integrals must be done numerically with an additional step of acquiring $\mathbf{B}(\mathbf{x})$ from the derivatives of $\mathbf{A}(\mathbf{x})$. Furthermore, to calculate the derivatives numerically, it would be necessary to evaluate $\mathbf{A}(\mathbf{x})$ on a finer mesh of points translating to an increase in computational time and accuracy. The inherent issues with numerical analysis involve the rate of convergence, the accuracy (or even validity) of the answer, and the completeness of the response. Therefore in our approach, we endeavor to solve B_r and B_θ analytically using elliptical integral as compared to the conventional numerical methods. Being able to solve the above equations analytically, precise solution to the integrals can be obtained directly and thus reducing computational time for real time control. Hence, to solve the full field equations analytically, we exploit a new function, the elliptic integral.

2.3.1 Elliptical Integrals

An elliptic integral is an integral of the form $\int R(x, \sqrt{P(x)}) dx$, in which R is a rational function of its arguments and $P(x)$ is a third or fourth-degree polynomial with distinct zeros.

Elliptic integral of the first kind is

$$\int \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \quad [k^2 < 1].$$

Elliptic integral of the second kind is

$$\int \frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}} dx \quad [k^2 < 1].$$

Elliptic integral of the third kind is

$$\int \frac{dx}{(1-nx^2)\sqrt{(1-x^2)(1-k^2x^2)}} \quad [k^2 < 1].$$

These integrals are expressed in Legendre normal form. The number k is called the modulus of the elliptic integral. It is usual to set $k = \sin \alpha$, with α being the modular angle and $m = k^2 = \sin^2 \alpha$ as the parameter. The number n is defined as the characteristic parameter of the elliptic integral of the third kind. When the integration extends over the interval $0 \leq \vartheta \leq \pi/2$, the integrals are considered as complete elliptic integrals. For our case, it is more convenient to work with trigonometric forms of the elliptic integrals. By setting $x = \sin \vartheta$, and regarding the integrals as functions of their upper limits by evaluating them over the interval $[0, \varphi]$, we derive the following set of incomplete elliptic integrals

Incomplete elliptic integral of the first kind is

$$F(\varphi, k) = \int_0^\varphi \frac{d\vartheta}{\sqrt{(1-k^2 \sin^2 \vartheta)}} \quad [k^2 < 1].$$

Incomplete elliptic integral of the second kind is

$$E(\varphi, k) = \int_0^\varphi \sqrt{(1-k^2 \sin^2 \vartheta)} d\vartheta \quad [k^2 < 1].$$

Incomplete elliptic integral of the third kind is

$$\Pi(\varphi, n, k) = d\vartheta \int_0^\varphi \frac{d\vartheta}{(1-n \sin^2 \vartheta) \sqrt{(1-k^2 \sin^2 \vartheta)}} \quad [k^2 < 1].$$

2.3.2 Derivation of Magnetic Field Model for a Current Loop using Elliptical Integrals

As highlighted, in order to solve Equations (2.21) and (2.22) analytically, the strategy is to express the integrals in elliptic functions so as to evaluate the respective components of the **B**-field. Hence, solving for B_r ,

$$B_r(r, \theta) = \frac{\mu_0 I R^2 \cos \theta}{4\pi} \int_0^{2\pi} \frac{d\phi'}{(R^2 + r^2 - 2rR \sin \theta \sin \phi')^{3/2}}. \quad (2.23)$$

Let $a = R^2 + r^2$ and $b = 2rR \sin \theta$,

$$B_r(r, \theta) = \frac{\mu_0 I R^2 \cos \theta}{4\pi} \int_0^{2\pi} \frac{d\phi'}{(a - b \sin \phi')^{3/2}}. \quad (2.24)$$

Resolving $\int \frac{d\phi'}{(a - b \sin \phi')^{3/2}}$ by substituting $\sin \phi' = \frac{a \cos 2x + b}{a + b \cos 2x}$, we have

$$\int \frac{1}{(a - b \sin \phi')^{3/2}} d\phi' = -\frac{2}{a^2 - b^2} \sqrt{a+b} \int \sqrt{1 - \frac{2b}{a+b} \sin^2 x} dx,$$

$$\int \frac{1}{(a-b \sin \phi')^{3/2}} d \phi' = -\frac{2}{(a-b)\sqrt{a+b}} E(x, \sqrt{\frac{2b}{a+b}}) . \quad (2.25)$$

Restoring the above equation back to $B_r(r, \theta)$, we arrived at the final form in terms of elliptic integral with $k = \sqrt{\frac{2b}{a+b}}$,

$$B_r(r, \theta) = \frac{-\mu_0 IR^2 \cos \theta}{\pi} \frac{E(\frac{\pi}{2}, k)}{(a-b)\sqrt{a+b}} . \quad (2.26)$$

Likewise for B_θ , we have

$$B_\theta(r, \theta) = \frac{\mu_0 IR}{4\pi} \int_0^{2\pi} \frac{(r \sin \phi' - R \sin \theta) d \phi'}{(R^2 + r^2 - 2 r R \sin \theta \sin \phi')^{3/2}} , \quad (2.27)$$

$$B_\theta(r, \theta) = \frac{\mu_0 IR}{4\pi} \left\{ \int_0^{2\pi} \frac{r \sin \phi' d \phi'}{(R^2 + r^2 - 2 r R \sin \theta \sin \phi')^{3/2}} - \int_0^{2\pi} \frac{R \sin \theta d \phi'}{(R^2 + r^2 - 2 r R \sin \theta \sin \phi')^{3/2}} \right\} \quad (2.28)$$

From the above, the challenge lies in resolving the respective integrals in elliptical forms. There are standard elliptical functions that can be found from mathematical handbooks [89,90]. Unfortunately, there are no existing integral functions that can be implemented directly to resolve Equation (2.28). Henceforth, we derive the following relationship,

Substituting $\sin x = \frac{a \cos 2\delta + b}{a + b \cos 2\delta}$ into $\int \frac{\sin x dx}{\sqrt{a - b \sin x}^3}$, we arrive at

$$\int \frac{\sin x dx}{\sqrt{a - b \sin x}^3} = \int \left(\frac{-2a \cos 2\delta}{(a^2 - b^2)\sqrt{a + b \cos 2\delta}} - \frac{2b}{(a^2 - b^2)\sqrt{a + b \cos 2\delta}} \right) d\delta .$$

From the table of integrals, we have

$$\int \frac{\cos 2\delta d\delta}{\sqrt{a+b\cos 2\delta}} = \frac{1}{b\sqrt{a+b}} \left\{ (a+b)E\left(\frac{x}{2}, k\right) - aF\left(\frac{x}{2}, k\right) \right\}.$$

Resolving the above equations, we acquire

$$\int \frac{\sin x dx}{\sqrt{a-b\sin x}^3} = -\frac{2}{(a^2-b^2)} \left(\frac{a}{b\sqrt{a+b}} \left\{ (a+b)E\left(\frac{x}{2}, k\right) - aF\left(\frac{x}{2}, k\right) \right\} + \frac{b}{\sqrt{a+b}} F(\delta, k) \right)$$

Thus, we derive the integral function

$$\int \frac{\sin x dx}{\sqrt{a-b\sin x}^3} = \frac{2}{b\sqrt{a+b}} \left(F\left(\frac{\pi}{2}, k\right) - \frac{a}{(a-b)} E\left(\frac{\pi}{2}, k\right) \right). \quad (2.29)$$

Replacing the above formula and Equation (2.25) into the original equation, we redress $B_\theta(r, \theta)$ as,

$$B_\theta(r, \theta) = \frac{\mu_0 IR}{4\pi} \left\{ 2r \left(\frac{2}{b\sqrt{a+b}} \left(F\left(\frac{\pi}{2}, k\right) - \frac{a}{(a-b)} E\left(\frac{\pi}{2}, k\right) \right) \right) - 2R \sin \theta \left(\frac{-2}{(a-b)\sqrt{a+b}} E\left(\frac{\pi}{2}, k\right) \right) \right\}$$

Finally, we derive the final form of $B_\theta(r, \theta)$ in terms of elliptic functions

$$B_\theta(r, \theta) = \frac{\mu_0 IR}{\pi b\sqrt{a+b}} \left\{ rF\left(\frac{\pi}{2}, k\right) - \frac{ra-Rb\sin \theta}{(a-b)} E\left(\frac{\pi}{2}, k\right) \right\}. \quad (2.30)$$

Conclusively, we have solved and derived an elegant analytic model for the full field description of a current loop in terms of the complete elliptic integrals,

$$B_r(r, \theta) = \frac{-\mu_0 IR^2 \cos \theta}{\pi} \frac{E\left(\frac{\pi}{2}, k\right)}{(a-b)\sqrt{a+b}}, \quad (2.31)$$

$$B_{\theta}(r, \theta) = \frac{\mu_0 IR}{\pi b \sqrt{a+b}} \left\{ r F\left(\frac{\pi}{2}, k\right) - \frac{ra - Rb \sin \theta}{(a-b)} E\left(\frac{\pi}{2}, k\right) \right\}. \quad (2.32)$$

where

$$k = \sqrt{\frac{2b}{a+b}},$$

$$F\left(\frac{\pi}{2}, k\right) = \int_0^{\pi/2} \frac{d\vartheta}{\sqrt{(1-k^2 \sin^2 \vartheta)}} \quad [k^2 < 1],$$

Complete elliptic integral of the first kind,

$$E\left(\frac{\pi}{2}, k\right) = \int_0^{\pi/2} \sqrt{(1-k^2 \sin^2 \vartheta)} d\vartheta \quad [k^2 < 1],$$

Complete elliptic integral of the second kind.

2.4 MAGNETIC FIELD MODEL FOR PERMANENT MAGNET

In the modeling of permanent magnets, there are basically two methods, namely, the current model and the charge model. For the current model, a distribution of equivalent current is used to replicate the actual magnet and the field is obtained using standard methods for steady current. As for the charge model approach, the magnet is reduced to a distribution of “magnetic charge”. As the stator and the rotor poles for our spherical actuator shares the same physical shape, it will be suitable to utilize the current model for modeling the permanent magnet rotor poles. Again, derived from Maxwell’s equations using vector potential and the constitutive relations, we have

$$\nabla^2 \mathbf{A} - \nabla(\nabla \cdot \mathbf{A}) = -\mu_0 (\mathbf{J} + \nabla \times \mathbf{M}).$$

Imposing Coulomb gauge condition $\nabla \cdot \mathbf{A} = 0$ and obtain

$$\nabla^2 \mathbf{A} = -\mu_0 (\mathbf{J} + \nabla \times \mathbf{M}). \quad (2.33)$$

The above equation evokes the definition of the equivalent magnetic volume current density $\mathbf{J}_m \equiv \nabla \times \mathbf{M}$. By superposition principle for a linear system, we can acquire $\mathbf{A}$ or $\mathbf{B}$ by summing the solutions for $\mathbf{J}$ and $\mathbf{J}_m$,

$$\nabla^2 \mathbf{A} = -\mu_0 (\mathbf{J} + \mathbf{J}_m).$$

If we are to assume an infinite homogenous material with no free current, $\mathbf{J}=0$, the solution of Equation (2.33) can be written in the integral form using the free-space Green's function $G(\mathbf{x}, \mathbf{x}') = -(1/4\pi)(1/|\mathbf{x} - \mathbf{x}'|)$, hence we have

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}_m(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} dv'. \quad (2.34)$$

Computing $\mathbf{B} = \nabla \times \mathbf{A}$,

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \mathbf{J}_m(\mathbf{x}') \times \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} dv'. \quad (2.35)$$

If in the case of magnetization $\mathbf{M}$ being confined to a volume V of permeability μ_0 and falls abruptly to zero outside of V , Equations (2.34) and (2.35) reduce to

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}_m(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} dv' + \frac{\mu_0}{4\pi} \oint_S \frac{\mathbf{j}_m(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} ds', \quad (2.36)$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_V \mathbf{J}_m(\mathbf{x}') \times \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} dv' + \frac{\mu_0}{4\pi} \int_S \mathbf{j}_m(\mathbf{x}') \times \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} ds'. \quad (2.37)$$

with S representing the magnet surface, $\mathbf{J}_m = \nabla \times \mathbf{M}$ (A/m²) and $\mathbf{j}_m = \mathbf{M} \times \hat{\mathbf{n}}$ (A/m) are the equivalent volume and surface current density respectively. Henceforth, by employing the current model, we can acquire the full $\mathbf{B}$ -field of a cylindrical permanent magnet by substituting the equivalent I into Equation (2.19).

2.5 MAGNETIC FIELD MODEL IN REGION OF HIGH PERMEABILITY

For the analysis of magnetic field distribution near regions containing high permeability materials, the method of images approach is exceptionally useful. In this method, regions of high permeability are replaced by carefully chosen image sources that render the appropriate boundary conditions at the interface. The efficiency of this method is further manifested when the solution region exhibits a high degree of symmetry. The fundamental of this approach is illustrated in Figure 2.3. From the illustration, a single turn of wires with input current i is located at a height, h , above an infinite nonconducting high permeability material with $\mu \approx \infty$. To determine the field distribution in the free space of this system, both the magnitude and position of the image sources need to be determined. Once the image sources are known, a field solution is obtained by taking superposition of the solutions for the individual source terms as if they were in free space. As the media is nonconducting, there is no surface current. In the plate, $\mathbf{H} = 0$, but $\mathbf{B}$ is finite.

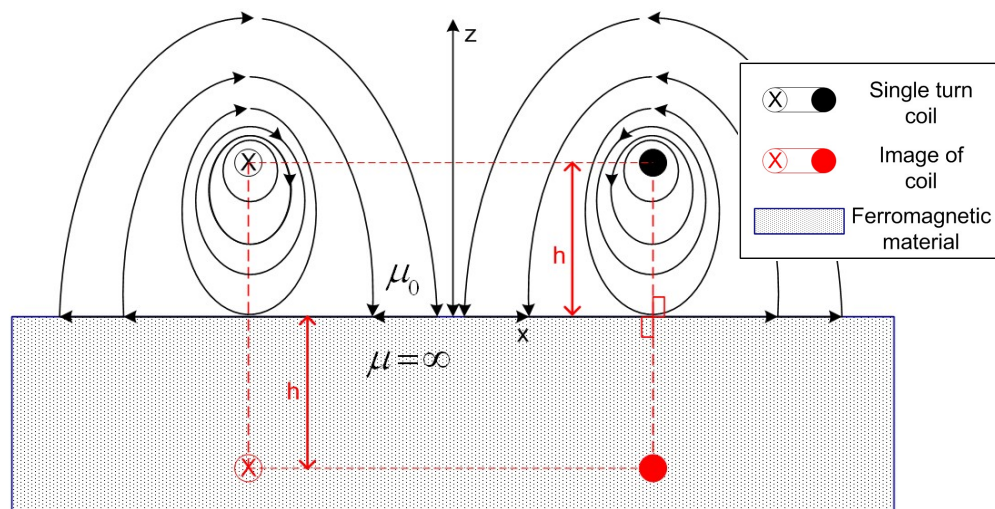


Figure 2.3. Physical system of coil above a material with high permeability.

From Equations (2.38) and (2.39), at the surface of the media, the normal component of $\mathbf{B}$ must be continuous and the tangential component of $\mathbf{H}$ must be zero,

$$B_{1n} = B_{2n}, \quad (2.38)$$

$$H_{1t} = H_{2t}, \quad (2.39)$$

$$B_n^{in} = B_n^{out}, \quad (2.40)$$

$$H_t = 0, \quad (2.41)$$

where B_n^{in} and B_n^{out} are the normal components of $\mathbf{B}$ inside and outside the media respectively. These are the same boundary conditions if the media is being replaced by an image current lying directly beneath the source at distance h into the media as shown in Figure 2.4.

The magnitude and direction of the image coil must be the equivalent of the source in order to ensure that the boundary conditions are satisfied at the surface of the media. The validity of the solution is limited in the region outside the material where the real sources are located and the uniqueness of the field solution.

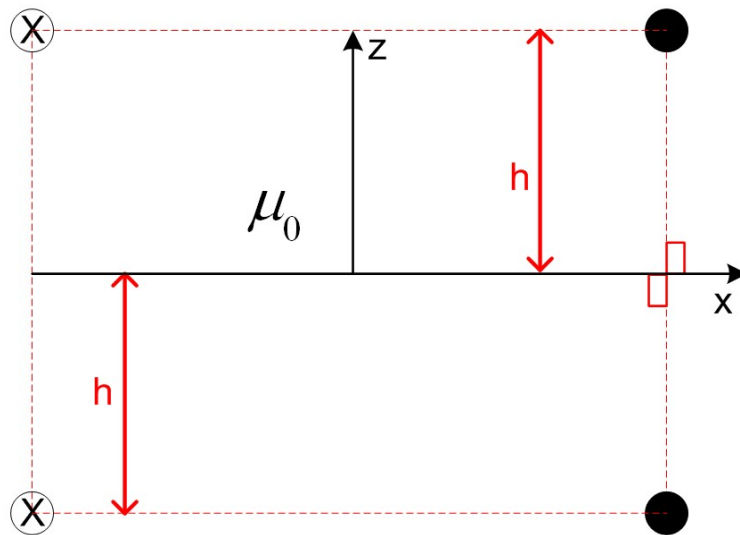


Figure 2.4. Equivalent system of coil above a material with high permeability.

The advantage of this approach lies in the fact that the original boundary-value problem with material interface has been reduced to a free-space problem as shown in Figure 2.4. As described, the original problem is transformed into one in which there

are two coils in free space separated by a distance $2h$. The field solution to this equivalent problem is the superposition of the solutions of the individual coils and is valid above the media. The uniqueness of this method is established as the image sources chosen render the prescribed boundary conditions satisfying the field equations in the solution region. The steps using method of images can be summarized as follows:

- 1) Establish the location and magnitude of image sources rendering the boundary conditions, Equation (2.41) at the media surface.
- 2) Replace media with the equivalent image sources in the physical system.
- 3) Compute field solution as a superposition of the fields generated from both the real and image sources in free space.

Hence, the modeling of the high permeable region for the spherical actuator presented in this dissertation is shown in Figure 2.5. This simulates the cylindrical boundary condition between the high permeable material of the spherical bearing and the permanent magnet rotor pole. From the steps listed above, we are able to model the resultant field by the proposed method.

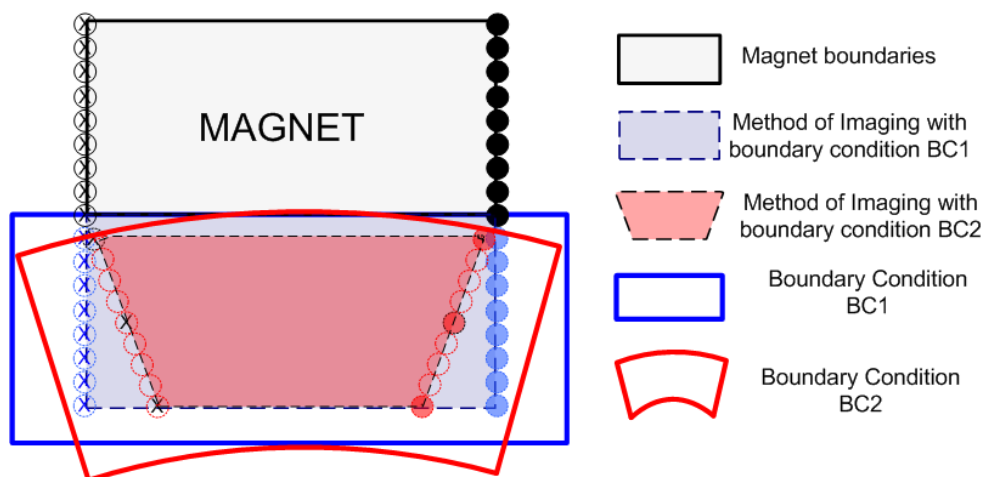


Figure 2.5. Equivalent system of coil of current spherical actuator setup.

2.6 CHAPTER SUMMARY

This chapter presents a generic magnetic field model derived from Biot-Savart law with the analytical field solution obtained by elliptical integrals. This analytical model is capable of describing three-dimensional field of a current loop. Extending this analytical field model, the current model was introduced for the modeling of permanent magnets. This approach replaces physical system consisting of permanent magnet with equivalent current density generating the same magnetic field. High permeable electromagnetic regions containing ferrous material are resolved utilizing the method of imaging principle. The advantage of this method is exemplified as boundary-value problems are reduced into equivalent free-space systems that simplifies the entire electromagnetic analysis. Hence, the analytical field model detailed in this chapter completes the modeling of the three critical regions commonly found in all electromagnetic system.

CHAPTER 3

MAGNETIC TORQUE MODELING

Theoretical modeling in any engineering design is of paramount importance as it provides the interrelationship between variables being analyzed in a given condition. Modeling of such nature helps the researcher to clarify and predict the possible outcome from the proposed hypotheses. With regards to the design of a spherical actuator, torque modeling is crucial as it correlates the input parameters such as current to the output mechanical torque. In this chapter, a new approach in the torque formulation for spherical actuator employing the magnetic dipole moment principle is being discussed. Derivation from first principle and the extension of this novel method in the acquisition of the resultant torque induced on the rotor will be presented.

3.1 WORKING PRINCIPLE OF PROTOTYPE

Prior to the introduction of our torque model, let us review the basic working principle of the spherical actuator. A spherical actuator possesses three rotational degrees of freedom. To better understand how three-dimensional motion is generated, we can envision it from schematic depicted in Figure 3.1. From the fundamental universal law of like poles repel and unlike poles attracting each other, rotary and directional control can be achieved by the manipulation of the electromagnets input current.

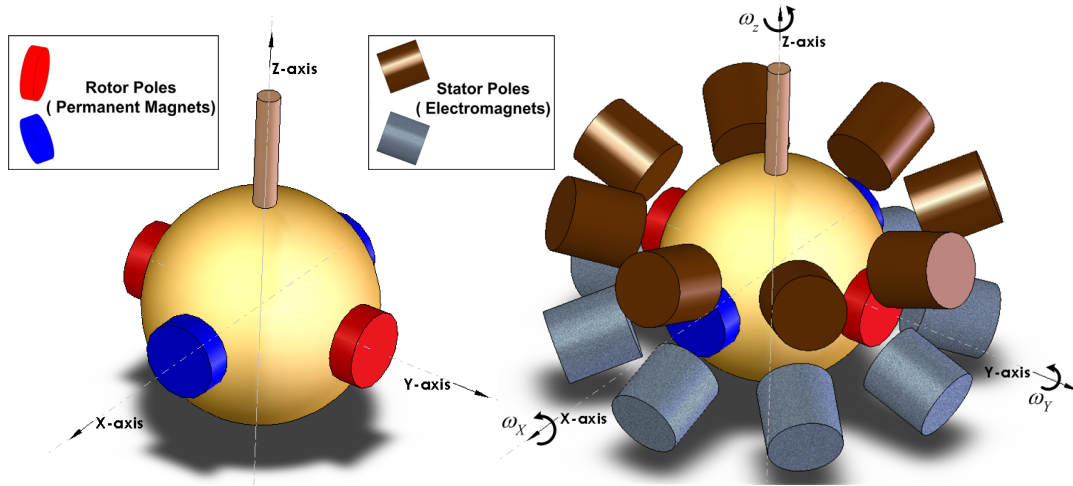


Figure 3.1. Schematic of the working principle of spherical actuator.

Extending the above working principle by arranging permanent magnets around the surface of a sphere and distributing electromagnets surrounding the spherical surface, we are able to realize the three-dimensional motion as desired. Since there is a moving and a stationary component in a spherical actuator, we shall assign the stator coordinate frame, $\{XYZ\}$ as the Global Coordinate System (GCS) and rotor coordinate frame, $\{xyz\}$ as Local Coordinate System (LCS). Effectively, with two pairs of electromagnets interacting with the permanent magnets, we are able to achieve inclination and generate spin about the rotor output shaft as shown. From the reference coordinate frame depicted, we define rotor pan and tilt as the rotational motion, ω_x and ω_y , about the stator X and Y -axes respectively. Correspondingly, the spin, ω_z , is defined as the rotation about the rotor z -axis. In order to attain higher resolution and larger working range, one will only need to integrate more electromagnets and magnets pair into the system. This forms the basis for the fundamental working principle of our prototype.

3.2 TORQUE MODEL

In most electromagnetic setup, the magnetic circuits consist of field sources such as magnets or coils with the utilization of ferromagnetic materials guiding the path of the magnetic flux. For this class of systems, the equivalent lumped-parameter model is widely exploited for the electromagnetic analysis. This approach follows from the integral form of the magnetostatic field equations,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} = I_{\text{total}} \quad \text{and} \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

where I_{total} is the total current passing through the area S bounded by contour C . From the first equation, it is imperative to identify the orientation and magnitude of $\mathbf{H}$ relative to the integration path $d\mathbf{l}$ in order to evaluate the equation. There are basically two fundamental assumptions in this equivalent lumped-parameter analysis. For most practical applications, an important assumption is that $d\mathbf{l}$ is chosen based on circuit geometry and $\mathbf{H}$ is assumed to be parallel to $d\mathbf{l}$. As of this presumption, vector product $\mathbf{H} \cdot d\mathbf{l}$ reduces to a scalar product $H dl$. The second key assumption for this method assumes that the flux generated by the field sources is confined to the circuit with negligible flux leakages. From the above brief introduction of the equivalent lumped-parameter model, we can infer that this approach is not suitable for the analysis of our spherical actuator simply base on the fact that magnetic flux in our system is not confined to any predetermined path. Furthermore, magnetic flux leakages are inherent due to the absence of ferromagnetic material in the system so as to maintain the electromagnetic linearity relationship.

Pertaining to the torque analysis of the spherical actuator in particular, due to the absence of high permeability material, we understand that the conventional magnetic circuit approach is not viable as there are no return flux paths connecting the interacting elements. Furthermore, magnetic energy in the system is not confined within the air gap between the rotor and stator poles. Virtual work approach can be used to analyze the mechanical output by evaluating the magnetic potential energy of the system. Unfortunately, this method is greatly hindered by the complexity involved during the formulation of the energy density function. In addition, fluctuating field variation in space and flux leakages poses even more challenges for this technique. Hence, discerning from the setup of the air-core spherical actuator, it is instinctively clear that magnetic circuit approach cannot be considered. As for the virtual work method, the precondition for analysis in determining the energy density hampers the efficiency of this technique. Thus, it would be ideal if torque analysis can be conducted disregarding the magnetic energy of the system. There are two such strategies namely, Maxwell stress tensor and Lorentz force law. Lorentz law governs the electromechanical interaction between moving charge and fields and Maxwell

stress tensor deals with the force density function. In these two methods, the prerequisite will be the knowledge of magnetic field distribution in the region of interest and hence circumvent the inclusion of magnetic energy function. The downside to these two schemes lies with the need to know the surrounding magnetic field distribution in the region of interest that can be computationally exhaustive. For real-time control implementation, it is still a step from ideal. For our torque analysis, we are proposing an innovative strategy using the magnetic dipole moment method in acquiring the output torque of the spherical actuator. This is the first time the magnetic dipole moment method is exploited for 3-D torque analysis of a spherical actuator. This approach does not require the full $\mathbf{B}$ -field description of the surface of interest and hence significantly reducing computational time. With this new technique coupled with the analytical field model developed previously, a complete torque model can be formulated and computed with high efficiency and accuracy. The ensuing sections will presents the minutiae of this torque model.

3.2.1 Magnetic Dipole

For an arbitrary current distribution, vector potential $\mathbf{A}(\mathbf{x})$ has multipole expansion in order of dipole, quadrupole, octopole terms etc. except the monopole term. For the asymptotic analysis, the dominant term at large distances is the dipole term. The vector potential is given by

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') d^3x'}{|\mathbf{x} - \mathbf{x}'|}. \quad (3.1)$$

Source $\mathbf{J}(\mathbf{x}')$ is assumed to be localized in a limited region of space. Far from the source, for $r \gg r'$ where $r = |\mathbf{x}|$ and $r' = |\mathbf{x}'|$, the following approximation is valid

$$\frac{1}{|\mathbf{x} - \mathbf{x}'|} = \frac{1}{r} + \frac{\mathbf{r} \cdot \mathbf{x}'}{r^2} + O\left(\frac{r'^2}{r^3}\right). \quad (3.2)$$

with notation $O\left(\frac{r'^2}{r^3}\right)$ representing terms proportional to powers of $1/r$ for the higher multipole terms which can be neglected for the asymptotic analysis. As first term of

Equation (3.2) gives no contribution to the vector potential, hence the asymptotic potential is obtained by substituting the second term of Equation (3.2) into (3.1),

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi r^2} \int \mathbf{J}(\mathbf{x}') \hat{\mathbf{r}} \cdot \mathbf{x}' d^3 x' . \quad (3.3)$$

Expressing the integral in Equation (3.3) we have,

$$\int \mathbf{J}(\mathbf{x}') \hat{\mathbf{r}} \cdot \mathbf{x}' d^3 x' = \mathbf{m} \times \hat{\mathbf{r}} . \quad (3.4)$$

Therefore, the magnetic dipole moment of the current distribution can be defined by

$$\mathbf{m} = \frac{1}{2} \int \mathbf{x}' \times \mathbf{J}(\mathbf{x}') d^3 x' . \quad (3.5)$$

The dipole moment $\mathbf{m}$ is an intrinsic property of the current distribution that does not depend on the observation point $\mathbf{x}$. From the above, the dipole vector potential can be expressed as

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{|\mathbf{x}|^3} . \quad (3.6)$$

Extending to the case of a planar current loop C with current I , we recall that $\mathbf{J}(\mathbf{x}') d^3 x'$ is the same as $I d\mathbf{l}$ for current in a wire. The shape of the wire is irrelevant as long as it is planar. We may picture this wire loop carrying the current in Figure 3.2.

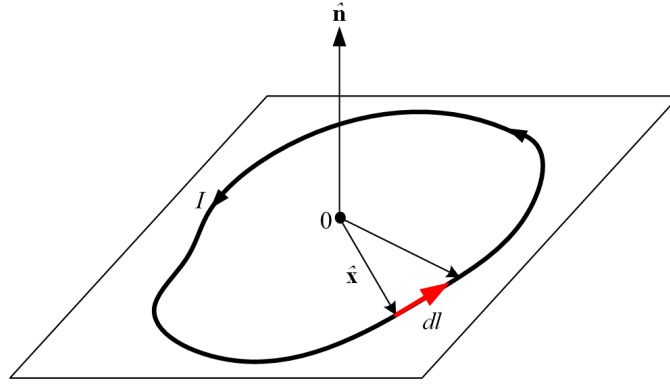


Figure 3.2. A planar current loop.

Therefore, by replacing the parameters, we have the dipole moment of a wire loop as

$$\mathbf{m} = I \oint_C \frac{1}{2} \mathbf{x}' \times d\mathbf{l} . \quad (3.7)$$

The vector $\frac{1}{2} \mathbf{x}' \times d\mathbf{l}$ is perpendicular to the plane of the loop with the magnitude equals to the area of the infinitesimal triangular sector bounded by $\mathbf{x}'$, $\mathbf{x}' + d\mathbf{l}$ and $d\mathbf{l}$. Thus

$$\mathbf{m} = I A \hat{\mathbf{n}} \quad (3.8)$$

where $\hat{\mathbf{n}}$ is the normal unit vector and $A = \pi r^2 \equiv \frac{\pi}{4} \phi_{stator}^2$, is the planar area of the current loop.

3.2.2 Torque and Potential Energy of Magnetic Dipole

In order to formulate a torque model for a spherical actuator based on the magnetic dipole phenomenon, it is essential to consider the dynamics of the dipole itself. The key relationship which we want to establish will be the force or torque effect by an external field $\mathbf{B}$ on the dipole. Considering the case of a planar current loop, the torque on the planar loop due to the magnetic forces on current distribution can be expressed by

$$\mathbf{N} = \oint_C \mathbf{x} \times d\mathbf{F} = \oint_C \mathbf{x} \times (I d\mathbf{x} \times \mathbf{B}) \quad (3.9)$$

where $d\mathbf{x}$ is an infinitesimal segment of the curve at $\mathbf{x}$. If $\mathbf{B}$ -field is assumed to be uniform and constant in the region of the current, by cross product identity,

$$d[\mathbf{x} \times (\mathbf{x} \times \mathbf{B})] = \mathbf{x} \times (d\mathbf{x} \times \mathbf{B}) + d\mathbf{x} \times (\mathbf{x} \times \mathbf{B}).$$

Since for a loop, the start and end point of the integration is the same, we will have the loop integral equals to 0. Inferring from above, the remaining two terms on the right hand side are equal but opposite. Therefore, we may write

$$\mathbf{N} = \frac{I}{2} \oint_C [\mathbf{x} \times (d\mathbf{x} \times \mathbf{B}) - d\mathbf{x} \times (\mathbf{x} \times \mathbf{B})].$$

Given that,

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0,$$

we rewrite $\mathbf{N}$ as,

$$\mathbf{N} = \frac{I}{2} \oint_C [(\mathbf{x} \times d\mathbf{x}) \times \mathbf{B}] = \mathbf{m} \times \mathbf{B} \quad (3.10)$$

with $\mathbf{m}$ defined by Equation (3.8). The result above reveals that the torque on a magnetic dipole is perpendicular to $\mathbf{B}$ in the direction twisting the dipole moment $\mathbf{m}$, towards the alignment with the $\mathbf{B}$ -field. This result is applicable to any point-like magnetic dipole. To gain some intuition, Figure 3.3 illustrates Equation (3.10) pictorially. The orientation potential energy associated with this torque can be derived from a mechanical argument. Assume that the internal current that induced the rotation of $\mathbf{m}$ remains unchanged, we will have magnitude $|\mathbf{m}|$ as constant. The potential energy U is defined as the work done by torque. With reference to Figure 3.4, for an infinitesimal rotation of the dipole, we have

$$-dU = \mathbf{N} \cdot d\boldsymbol{\theta}. \quad (3.11)$$

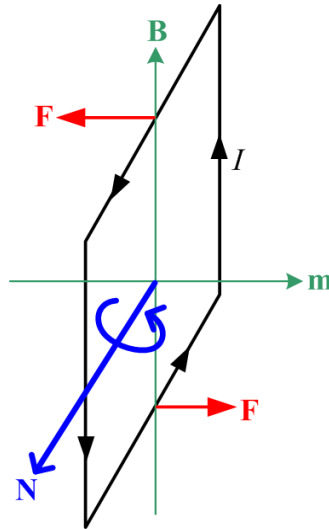


Figure 3.3. Torque on a square loop.

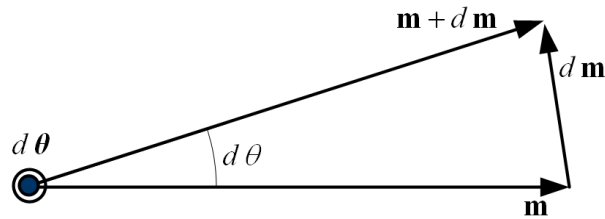


Figure 3.4. Rotation of $\mathbf{m}$.

The change of $\mathbf{m}$ is $d\mathbf{m} = d\boldsymbol{\theta} \times \mathbf{m}$, and along with Equation (3.11), we have

$$dU = -(\mathbf{m} \times \mathbf{B}) \cdot d\boldsymbol{\theta}. \quad (3.12)$$

By vector formula

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}),$$

we derive

$$dU = -(d\boldsymbol{\theta} \times \mathbf{m}) \cdot \mathbf{B} = -d\mathbf{m} \cdot \mathbf{B}.$$

Consequently, we have the orientation potential energy as

$$U = -\mathbf{m} \cdot \mathbf{B}. \quad (3.13)$$

A natural occurrence illustrating the magnetic dipole moment phenomenon can be observed from a magnetized needle put through a cork floating on the surface of water.

Due to the Earth's magnetic field and the above mentioned torque, the needle becomes aligned with the field pointing north. From the above, we have shown that for a current loop under the influence of $\mathbf{B}$ -field, will experience a magnetic moment about its normal as prescribed by Equation (3.10). Hence, by extending this phenomenon to a solenoid consisting of a series of current loops, by superposition principle, we can resolve the resultant torque through the summation of the individual loops as illustrated by Figure 3.5.

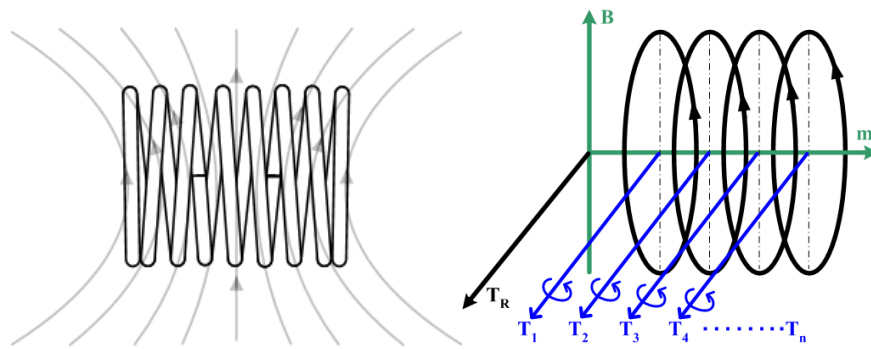


Figure 3.5. Superposition of resultant torque of a solenoid.

3.2.3 Torque Model for Spherical Actuator

Applying the preceding notion to our specific application, the objective is to evaluate the torque acting on the rotor when the stator coils are being energized. With reference to Figure 3.6, to generate 3-DOF motions, two pairs of stator coils are required. From the illustration, stator coils **S1**, **S2**, **S3** and **S4** constitute the actuating elements of the spherical actuator. By the universal attraction law between unlike poles, forces will be generated upon activation of the stator coils. These forces are denoted by **F1**, **F2**, **F3** and **F4** resulting from energizing **S1**, **S2**, **S3** and **S4** respectively. Correspondingly, the figure also depicts the resultant torque about the center of the rotor by **T1**, **T2**, **T3** and **T4** in the same order. It is through the combination of torque **T1** to **T4** that 3-DOF motions are achieved. Hence, in order to control the spherical actuator, analyzing the output torque is of paramount

importance. The resultant output torque on the rotor is in fact the summation of the respective torque induced by the individual stator coil. Therefore, torque formulation for a set of rotor and stator pole will suffice as the remaining sets can be modeled in the similar fashion with positional variation.

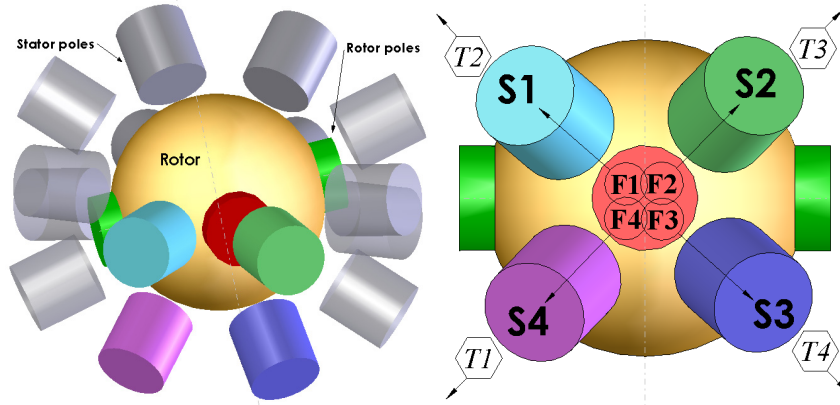


Figure 3.6. Actuation strategy for 3-DOF motions.

An obvious and yet important kinematic constraint of a spherical actuator is that the rotor rotates about a common point. For our prototype, the center of rotation is at the center of the spherical bearing that coincides with the rotor center. The poles of the rotor and the stator are designed to be concentric with their axial axis aligned in the radial direction from the rotor center. This physical constraint simplifies the analysis as the resolving of the individual torque by the coil can be made reference to the rotor center. Figure 3.7 shows the schematic of a set of rotor and stator pole. The strategy behind this torque formulation is to establish the torque induced on the individual energized coils under the influence of $\mathbf{B}$ -field from the rotor pole. Thereafter, by summing the respective torque, we will acquire the resultant torque of the stator coil. Since the stator coils are the stationary part whereas the rotor is the moving part, from the Third law of motion, we are able to derive the resultant torque acting on the rotor. According to the magnetic dipole moment principle, to compute the torque acting on a single current coil, we have

$$\mathbf{T}_1 = \mathbf{m}_1 \times \mathbf{B}_1 \quad (3.14)$$

between the local and global coordinate frames. Without any loss of generality, rotor pole $\mathbf{P}_{r1}$, is positioned along the $\mathbf{X}$ -axis. An arbitrary position of stator coil, $\mathbf{P}_{c1}$, can express by its spherical coordinates parameters, r_{c1} , θ_{c1} and ϕ_{c1} . Assuming a single layer stator coil made up of n number of coils, the individual position vector of each coil can be defined by its respective spherical coordinate parameters. Therefore, we obtain the following position vectors

$$\mathbf{P}_{r1} = r_{r1} \begin{pmatrix} \cos \phi_{r1} \sin \theta_{r1} \\ \sin \phi_{r1} \sin \theta_{r1} \\ \cos \theta_{r1} \end{pmatrix}, \quad (3.15)$$

$$\mathbf{P}_{c1} = r_{c1} \begin{pmatrix} \cos \phi_{c1} \sin \theta_{c1} \\ \sin \phi_{c1} \sin \theta_{c1} \\ \cos \theta_{c1} \end{pmatrix} \dots \dots \dots \mathbf{P}_{cn} = r_{c(n)} \begin{pmatrix} \cos \phi_{c(n)} \sin \theta_{c(n)} \\ \sin \phi_{c(n)} \sin \theta_{c(n)} \\ \cos \theta_{c(n)} \end{pmatrix}. \quad (3.16)$$

When current is passed through the stator coil, the torque induced will attempt to align the coil with the $\mathbf{B}$ -field from the rotor pole. This is analogous to the alignment of a magnetized needle with the Earth's magnetic field mentioned previously. Hence, the trajectory path that the rotor will undertake in order to minimize its potential energy will be the shortest path between the axial axis of the rotor, $\mathbf{P}_{r1}$ and stator pole $\mathbf{P}_{c1}$. We define this shortest path between the two interacting element as $\mathbf{P}_{r1-c1}$ as shown in Figure 3.8 below.

$$\mathbf{P}_{r1-c1} = \mathbf{P}_{c1} - \mathbf{P}_{r1} \quad (3.17)$$

As depicted in the figure, $\mathbf{P}_{r1-c1}$ lies on the local plane, Π_{r1-c1} . This plane is in fact located on the coordinate frame $\{x'_{rotor} \ y'_{rotor} \ z'_{rotor}\}$ characterized by $\mathbf{Z}$ - $\mathbf{Y}$ - $\mathbf{X}$ Euler angles, α_{r1} , β_{r1} and γ_{r1} . Carrying out our objective of acquiring the $\mathbf{B}$ -field at coil C_1 , we have identified the position vector from the rotor pole surface to the center of the stator coil in the global coordinate frame $\{\mathbf{XYZ}\}$. Similarly, we can establish the respective position vectors of the remaining current coils as illustrated in Figure 3.9. We will now employ our analytical field model for the computation of $\mathbf{B}_1$. Utilizing

the formulation presented in Chapter 2, we will be able to obtain $\mathbf{B}_l$ from the position vector $\mathbf{P}_{r1-cl}$ by first mapping it to the local coordinate frame $\{x'_{rotor} y'_{rotor} z'_{rotor}\}$.

$${}^{x'_{rotor} y'_{rotor} z'_{rotor}}_{\mathbf{P}_{r1-cl}} = {}^{x'_{rotor} y'_{rotor} z'_{rotor}}_{XYZ} \mathbf{R} \cdot \mathbf{P}_{r1-cl} \quad (3.18)$$

where ${}^{x'_{rotor} y'_{rotor} z'_{rotor}}_{XYZ} \mathbf{R}$ is the rotation matrix that maps the global to the local coordinate frame with the respective α_{r1}, β_{r1} and γ_{r1} Euler angles.

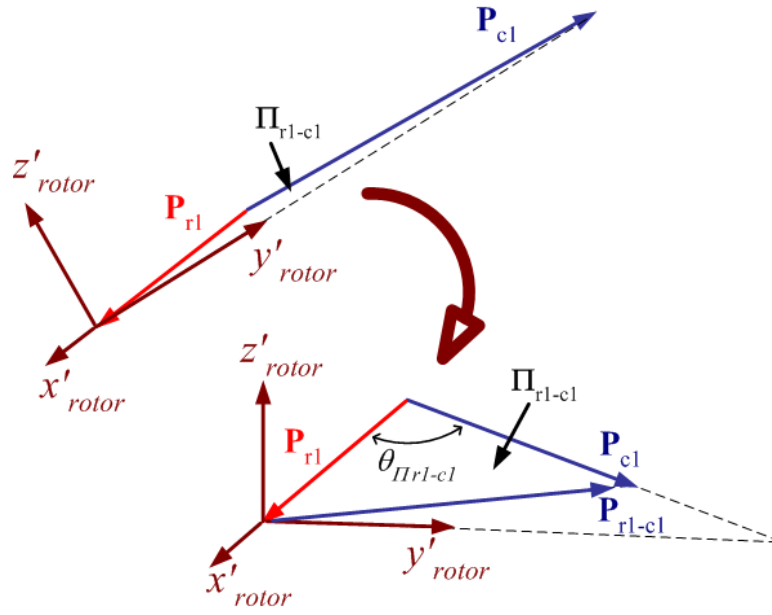


Figure 3.8. Illustration of shortest path between rotor and stator poles.

At this juncture, from our prototype design, we have rotor poles located around the circumference and on the center plane of the rotor. Hence, the local coordinate frame for any rotor pole can be defined by the **Z-Y-X** Euler angles, α_{r1} and γ_{r1} with $\beta_{r1} = 0$. Angle α_{r1} can be easily determined from the position of the rotor pole. As of Figure 3.9, viewing along the line of intersection between plane Π_{r1-cl} and $\Pi_{xrotor-yrotor}$, we can acquire the dihedral angle γ_{r1} . This can be computed by performing the dot product between the two normal unit vectors of the planes. Hence we have,

$$\hat{\mathbf{n}}_{\Pi_{xrotor-yrotor}} = \mathbf{x}_{rotor} \times \mathbf{y}_{rotor} \quad (3.19)$$

$$\hat{\mathbf{n}}_{\Pi_{r1-cl}} = \mathbf{x}_{rotor} \times \mathbf{P}_{r1-cl} \quad (3.20)$$

Consequently we acquire

$$\gamma_{rl} = \cos^{-1} \left(\frac{\hat{\mathbf{n}}_{\Pi_{xrotor-yrotor}} \cdot \hat{\mathbf{n}}_{\Pi_{rl-cl}}}{\|\hat{\mathbf{n}}_{\Pi_{xrotor-yrotor}}\| \|\hat{\mathbf{n}}_{\Pi_{rl-cl}}\|} \right). \quad (3.21)$$

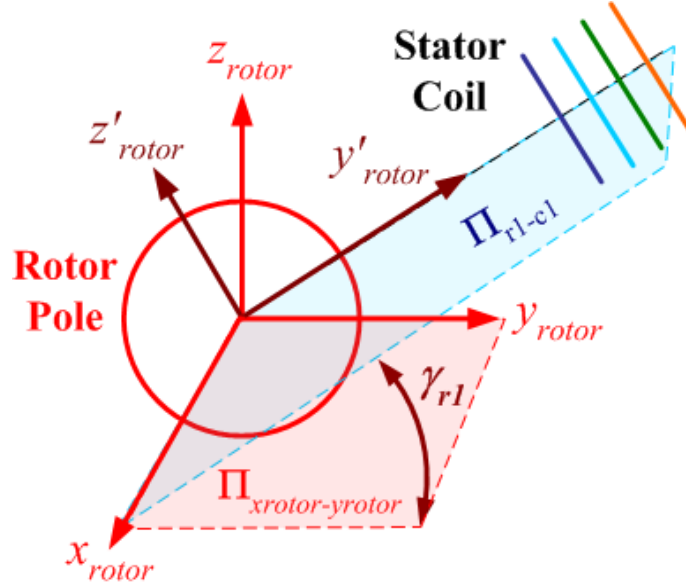


Figure 3.9. Illustration of Euler angle γ_{rl} .

From the inner product operation, we are able to determine the magnitude of the rotation angle γ_{rl} . A limitation of this approach will be the inability to differentiate angle beyond 180° . For that reason, with only the magnitude of the Euler angle γ_{rl} from the inner product operation is not sufficient to fully describe the characteristics of this rotation angle. Adhering to the right hand rotational convention, we would exploit from prior knowledge of the position vector between the rotor and stator pole, $\mathbf{P}_{rl-cl}$ to determine the magnitude of this Euler angle γ_{rl} . In view of that, we establish the following necessary conditions for $\mathbf{P}_{rl-cl}$ as illustrated in Figure 3.10.

$$\left. \begin{array}{l} \text{sgn}(P_{rl-cl})_y = -\text{ve} \\ \text{sgn}(P_{rl-cl})_z = +\text{ve} \end{array} \right\} \text{Region I} \Rightarrow \gamma_{rl} \quad (3.22)$$

$$\left. \begin{array}{l} \text{sgn}(P_{rl-cl})_y = +\text{ve} \\ \text{sgn}(P_{rl-cl})_z = +\text{ve} \end{array} \right\} \text{Region II} \Rightarrow \gamma_{rl} \quad (3.23)$$

$$\left. \begin{array}{l} \text{sgn}(P_{r1-cl})_y = +ve \\ \text{sgn}(P_{r1-cl})_z = -ve \end{array} \right\} \text{Region III} \Rightarrow -\gamma_{rl} \quad (3.24)$$

$$\left. \begin{array}{l} \text{sgn}(P_{r1-cl})_y = -ve \\ \text{sgn}(P_{r1-cl})_z = -ve \end{array} \right\} \text{Region IV} \Rightarrow -\gamma_{rl} \quad (3.25)$$

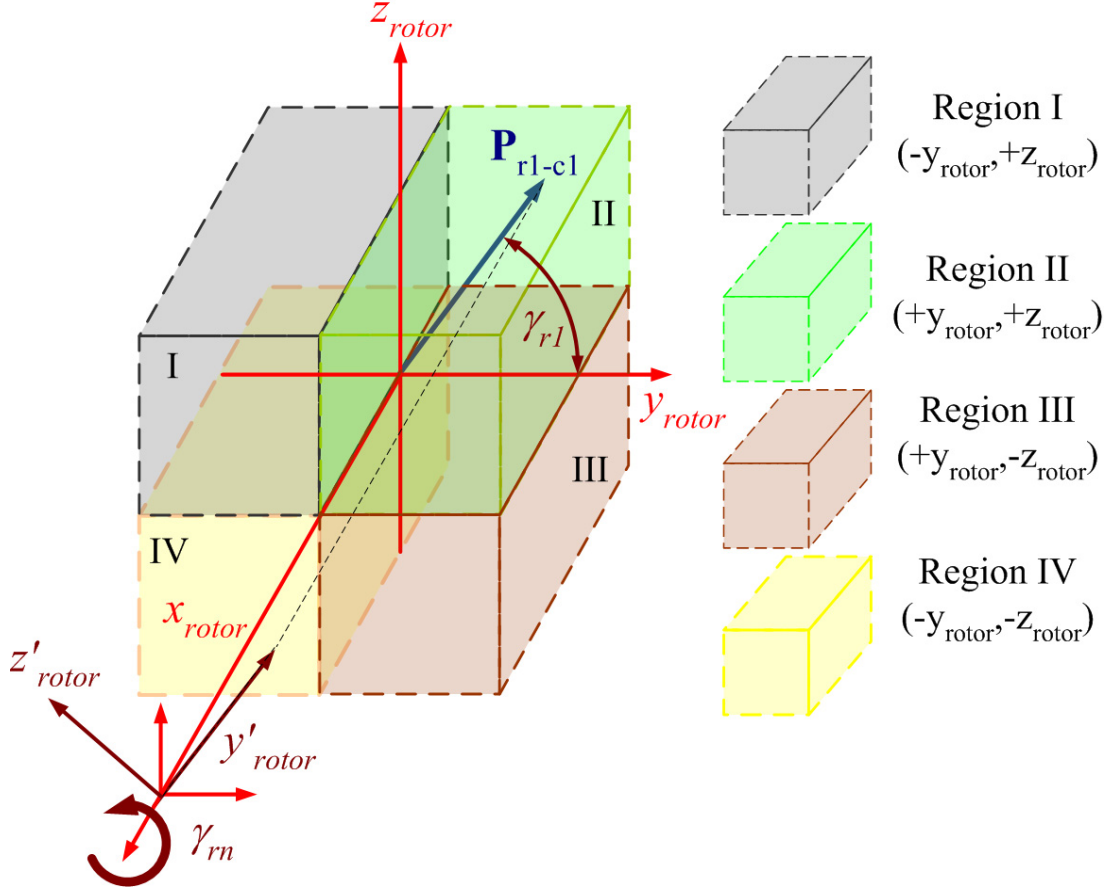


Figure 3.10. Conditions for signed Euler angle γ_{rl} .

With the Euler angle γ_{rl} known, we will be able to proceed with the mapping of $\mathbf{P}_{r1-cl}$ onto the rotated plane, Π_{r1-cl} so as to obtain ${}^{x'_{rotor}, y'_{rotor}, z'_{rotor}}\mathbf{P}_{r1-cl}$. Substituting this position vector into our analytical field model, we will acquire the magnetic flux density $\mathbf{B}_1$ acting on coil C_1 . Knowing $\mathbf{B}_1$, we can hence compute the torque at coil C_1 by the following equation,

$$\mathbf{T}_1 = I_1 A_1 (\hat{\mathbf{n}}_1 \times \mathbf{B}_1) \quad (3.26)$$

where I_1 is the current passing through the stator coil, $\hat{n}_1$ is the unit normal vector along the coil axial axis, A_1 is the surface area of the loop and $\mathbf{B}_1$ is the magnetic flux density at the center of coil C_1 . The accompanying advantage of using the magnetic dipole moment approach can be clearly seen from the fact that there are only two varying parameters, A_1 and $\mathbf{B}_1$ in the torque formulation with the remaining inputs as constants. In addition, with only one parameter being a vector and the other a scalar, inherent benefit will be the reduction of computational steps and time.

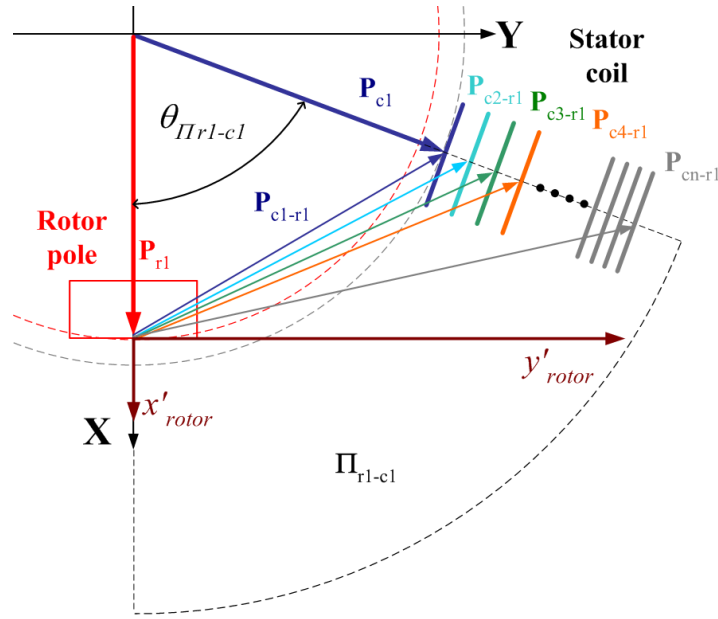


Figure 3.11. Position vectors for $\mathbf{B}$ -field calculation on rotated plane Π_{r1-c1} .

What we have shown so far is the torque derivation for a single coil within a stator pole that comprises of n number of coils. Extending the same approach and summing up all the individual coils within the stator pole, we will be able to determine the resultant torque on the rotor induced by the input current into the coils. This notion is encapsulated in Figure 3.11. Therefore, we have the resultant torque as,

$$\mathbf{T}_R = \mathbf{T}_{c(1,1)} + \mathbf{T}_{c(1,2)} + \dots + \mathbf{T}_{c(i,j)} = \sum_{j=1}^j \sum_{i=1}^i \mathbf{T}_{c(i,j)} \quad (3.27)$$

where i indicates the i^{th} turn and j denotes the j^{th} layer of the stator coil comprising of $n = i \times j$ number of turns along with the following assumptions,

Current loop is assumed to be planar with respect to the coil axial axis. (A.1)

Current loop are adjacent to each other with no overlap. (A.2)

Figure 3.12 illustrates a generic schematic of the cross sectional area of the stator coil reflecting the n number of current loops bounded by i number of turns and j number of layers within the stator structure.

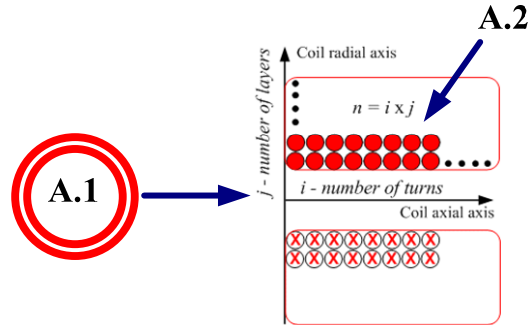


Figure 3.12. Cross section view of stator coil.

Henceforth, we have expounded on the conceptual idea behind our proposed torque model and its derivation process for the spherical actuator. As highlighted, motion generation of this type of spherical actuators can be achieved by a single pair of rotor and stator poles. Whereas to control 3-DOF orientation of the spherical rotor, the minimum number of stator poles required is four. Through the pairing of various stator poles, we will be able to control and position the rotor accordingly as illustrated in Figure 3.13 and 3.14. By grouping a set of rotor poles, we will achieve directional control dictated by the resultant torque, T_D . As for positioning, we will require the holding torque, T_H in order to maintain the rotor orientation at the intended position.

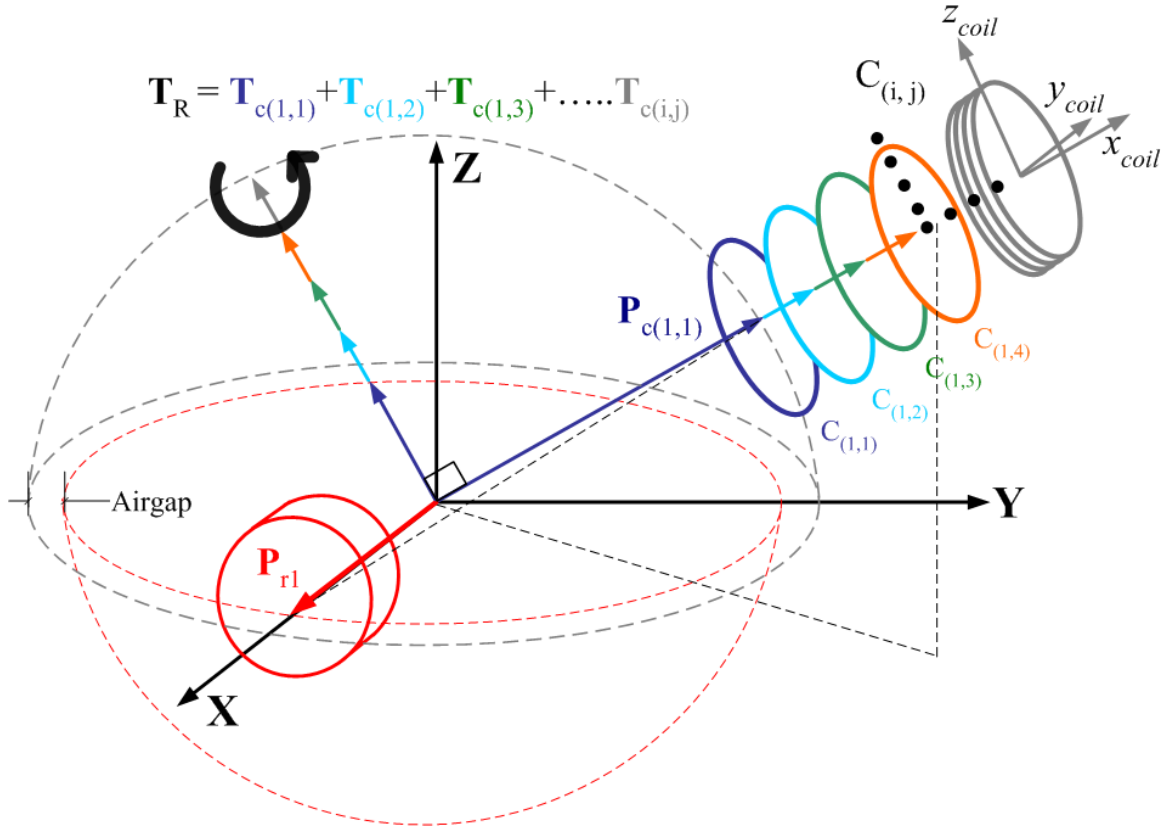


Figure 3.13. Torque model for a set of rotor and stator pole.

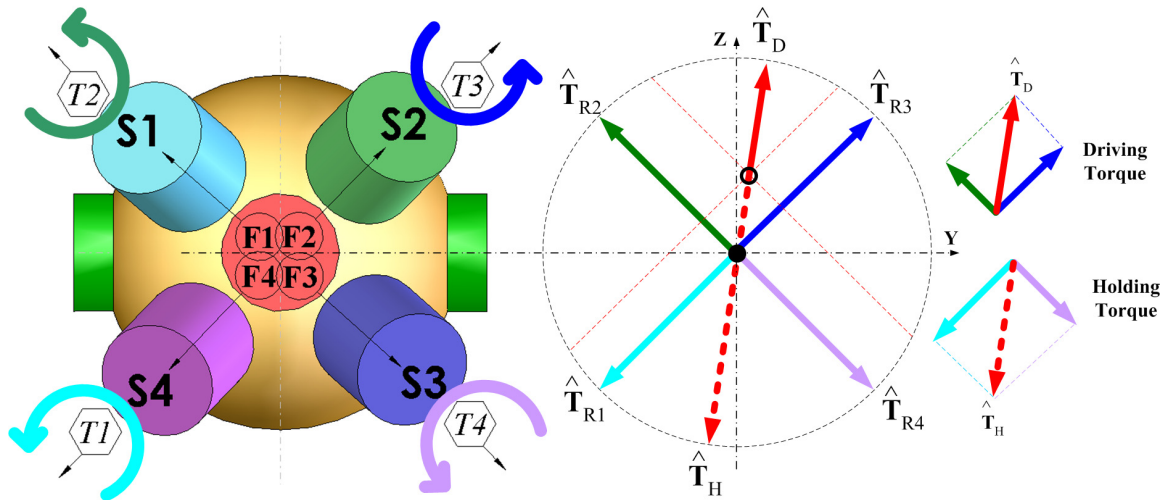


Figure 3.14. Control strategy of spherical actuator.

From the preceding section, we have illustrated the derivation process for a set of rotor and stator poles. As the input parameters are expressed in vectorial form in terms of its position vectors, we can conveniently include additional poles into this existing torque model and acquire its total resultant torque consequently. Since the fundamental configuration for 3-DOF motion generation require a set of four stator

poles and one rotor pole, we can establish the full torque model based on the extension of the prescribed basic arrangement and obtain the resultant as follow,

$$\mathbf{T}_{\text{rotor}} = \mathbf{T}_{R1} + \mathbf{T}_{R2} + \mathbf{T}_{R3} + \mathbf{T}_{R4} . \quad (3.28)$$

Equivalently, without any loss in generality, for l number of stator poles, we have,

$$\mathbf{T}_{\text{rotor}} = \sum_{l=1}^l \mathbf{T}_{R(l)} . \quad (3.29)$$

The above equation conveys the output torque of the spherical actuator at any instantaneous orientation. As this is the underlying module for generating 3-DOF motion, additional sets of similar rotor and stator poles configuration can be incorporated to increase the output torque. From the inclusion of extra set of poles, the only changes will be in the magnitude of the output torque. Graphically, we can perceive this from Figure 3.15 where it presents the schematic of the poles configuration of the spherical actuator. Viewing from the top, during operation, a total of four sets of stator coils will be energized. Each set is denoted by Set 1-4 as depicted in the figure with the remaining coils at its non-energized state. At any instantaneous time, the rotor and stator poles can be fully described by their position vectors and hence, the complete resultant torque can be computed using the proposed magnetic dipole moment approach.

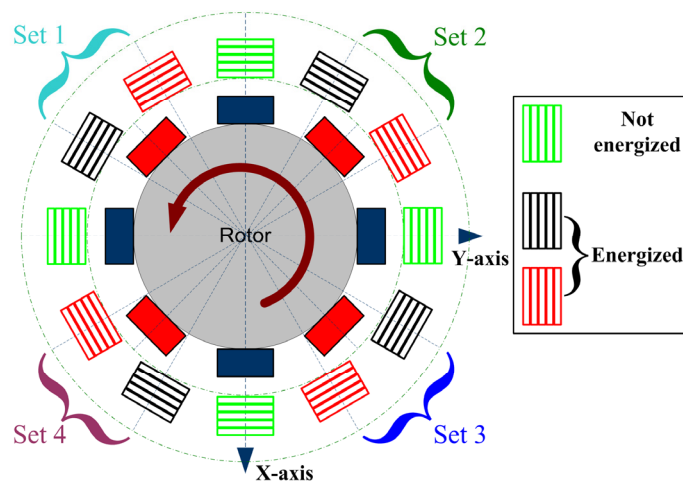


Figure 3.15. Top view of poles configuration of current spherical prototype.

Table 3.1 recapitulates the parameters for the proposed torque model and summarizes the steps undertaken for this analysis of our spherical actuator. The main advantages of this new approach are listed as follow

- Analytical torque model derived from the magnetic dipole moment principle.
- The proposed method circumvents the need to acquire full magnetic field distribution in space for the computation of rotor torque and hence simplifying the entire analysis process.
- The torque model coupled with the new analytical magnetic field model permits direct computation of actual torque induced by stator poles.
- Vectorial presentation of input parameters translates to flexibility in the torque analysis of any rotor/stator poles configuration.
- The proposed torque model correlates rotor resultant torque with the dimension, position, material and input properties of the rotor and stator poles for design optimization.

The newly formulated analytical torque model for the spherical actuator can be utilized for design optimization with respect to a given criterion. From the prototype design aspect, the proposed torque model is a function of the following parameters,

$$\mathbf{T}_{\text{rotor}} = f(I_{\text{stator}}, r_{\text{stator}}, t_{\text{stator}}, n_{\text{stator}}, \theta_{\text{stator}}, \phi_{\text{stator}}, r_{\text{rotor}}, \theta_{\text{rotor}}, \phi_{\text{rotor}})$$

with

I_{stator} = Input Current

r_{stator} = Radial distance from rotor center to stator pole surface

t_{stator} = Wire thickness

n_{stator} = Total of number of turns in stator pole

θ_{stator} = Stator pole polar angle from the **Z** - axis

ϕ_{stator} = Stator pole azimuthal angle in the **X** - **Y**plane from the **X** - axis

r_{rotor} = Radial distance from rotor center to rotor pole surface

θ_{rotor} = Rotor pole polar angle from the **Z** - axis

ϕ_{rotor} = Rotor pole azimuthal angle in the **X** - **Y**plane from the **X** - axis

With the design inputs and stated criterion, we can determine the optimized design meeting the specified criterion. From the control aspect, with the position and physical constraint of the stator poles being fixed, the torque model reduces to,

$$\mathbf{T}_{\text{rotor}} = f(I_{\text{stator}}, \alpha_{\text{rotor}}, \beta_{\text{rotor}}, \gamma_{\text{rotor}})$$

with

$$\alpha_{\text{rotor}}, \beta_{\text{rotor}}, \gamma_{\text{rotor}} = \mathbf{Z} - \mathbf{Y} - \mathbf{Z} \text{ Euler angle of rotations}$$

The above relationship determines the maximum achievable torque at any instantaneous orientation of the rotor.

3.2.4 Inverse Torque Model

According to the magnetic dipole moment principle, torque $\mathbf{T}$, is derived from the vector cross product between the magnetic dipole moment, $\mathbf{m}$ and the respective $\mathbf{B}$ -field.

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad (3.30)$$

$$\mathbf{m} = I A (\hat{\mathbf{n}} \times \mathbf{B}) \quad (3.31)$$

Hence, we can infer the direction of the resultant torque by acquiring the directional cosines of the resultant torque vector. This entails that we can perform inverse computation of the torque model by determining the magnitude of the resultant vector. Since current is constant for any stator coil, we formulate the resultant torque, $\mathbf{T}_R$ as,

$$\mathbf{T}_R = I \sum_{j=1}^j \sum_{i=1}^i A_{c(i,j)} (\hat{\mathbf{n}} \times B_{c(i,j)} \hat{\mathbf{B}}_{c(i,j)}) \quad (3.32)$$

Resolving in unit vectors,

$$T_R \hat{\mathbf{T}}_R = I \sum_{j=1}^j \sum_{i=1}^i A_{c(i,j)} B_{c(i,j)} (\hat{\mathbf{n}} \times \hat{\mathbf{B}}_{c(i,j)}) \quad (3.33)$$

Therefore, we establish the inverse relationship as,

$$I = \frac{T_R}{\sum_{j=1}^j \sum_{i=1}^i A_{c(i,j)} B_{c(i,j)}} \quad (3.34)$$

Equation (3.34) allows the determination of the input current of an arbitrary coil in accordance to the required output torque, T_R . This linear correlation is established based on the magnetic dipole principle which is able to furnish the magnitude of the input current with the desired torque output. The above relationship is critical during closed-loop control implementation as it correlates the desired output torque with the input current.

3.3 CHAPTER SUMMARY

In conclusion, a new analytical torque model based on the magnetic dipole moment principle is proposed for torque analysis of the spherical actuator as presented in this chapter. The proposed model is able to provide the rotor resultant torque for any configuration of poles layout. With the establishment of this correlation, key design parameters can be identified in accordance to a given optimization criterion. This theoretical model also lays the foundation for control implementation.

Parameters	Description
$\mathbf{P}_{r(k)}$	Position vector of the k^{th} rotor pole
$\mathbf{P}_{s(l)}$	Position vector of the l^{th} stator pole
$\mathbf{P}_{c(i,j)}^{s(l)}$	Position vector of the $(i,j)^{\text{th}}$ coil of the l^{th} stator pole
$\mathbf{P}_{r(k)-c(i,j)}^{s(l)}$	Position vector from the k^{th} rotor pole to the $(i,j)^{\text{th}}$ coil of the l^{th} stator pole
$\mathbf{B}_{c(i,j)}^{s(l)}$	$\mathbf{B}$ -field acting on $(i,j)^{\text{th}}$ coil of the l^{th} stator pole
$I^{s(l)}$	Input current of the l^{th} stator pole
$A_{c(i,j)}^{s(l)}$	Surface area of $(i,j)^{\text{th}}$ coil of the l^{th} stator pole
$\mathbf{T}_{R(l)}$	Torque acting on rotor by the l^{th} stator pole
$\mathbf{T}_{\text{rotor}}$	Total resultant torque acting on rotor

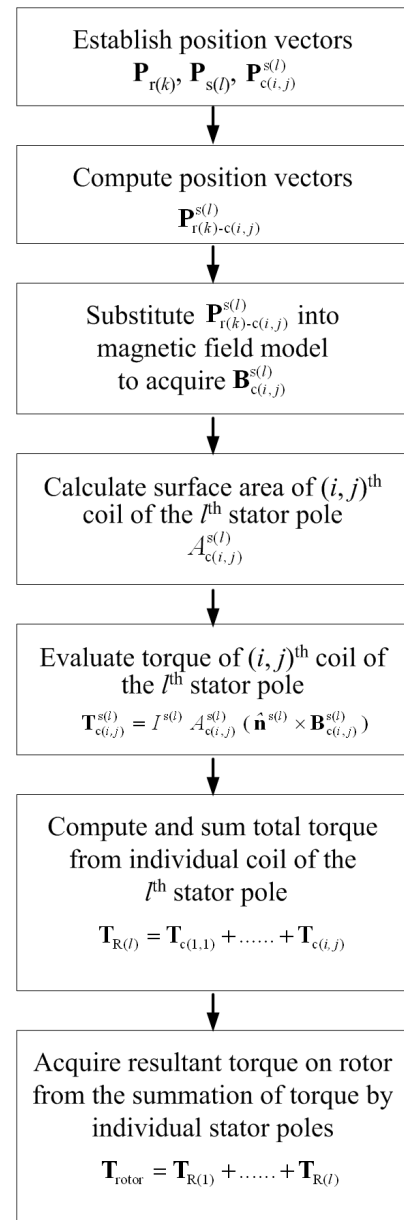


Table 3.1. Torque modeling of spherical actuator using the magnetic dipole moment approach.

CHAPTER 4

DESIGN AND MOTION GENERATION OF SPHERICAL ACTUATOR PROTOTYPE

Much of the different aspects of spherical actuators and the fundamental of electromagnetism had been discussed in the foregoing chapters. In this chapter, the design procedures and assembly of the prototype will be examined and presented. The mechanical design of a spherical actuator can be categorized into the following:

- i) Rotor and rotor pole design
- ii) Stator and stator pole design
- iii) Support mechanism design

Design experience in spherical actuators, unlike that of single DOF electromagnetic motors, has been limited and intricate due to the three dimensional nature of the output motion. This chapter aims to present the design procedure in a concise and orderly manner that may serve as a starting point for design optimization and research. In the following sections, different aspects of the prototype design will be presented. Each subsection will detail the design methodology and specifications of the subsystems. From the review of existing spherical actuators design undertaken in Chapter 1, different aspects in terms of the advantages and disadvantages were considered. With the objective of achieving high precision and linearity for ease

during control, we embarked on the mechanical design of our initial prototype with the following predetermined features:

- Air-core coil as stator pole
- Permanent magnet as rotor pole
- Rotor size $< 100\text{mm}$
- Inclination range $\pm 11^\circ$

With the hardware being established, ensuing sections of this chapter will also expound on the motion planning aspect of the proposed spherical actuator. Based on the fundamental working principle, we will show and establish how MDOF motion can be generated by this new prototype. Accompanying kinematics and dynamics formulations will be included in the concluding segment of this chapter.

4.1 ROTOR POLE DESIGN

Permanent magnets in one form or another can be found in many equipments ranging from consumer products, to industrial machinery and research apparatus. The fundamental advantage is that they can provide strong magnetic field over an extended spatial region for an indefinite period of time with little expenditure of energy. Another advantage of permanent magnets is that they can be fabricated in a variety of shapes and sizes, structural properties and magnetization patterns fit for different applications. An important factor in machine design is the use of new materials, such as rare earth permanent magnets or soft magnetic composite materials to enhance the efficiency of the electrical machine. Replacing the electrical field excitation by permanent magnets, not only increases efficiency, but also reduces electrical time constants and weight/volume ratio, especially for small machines. Therefore, rare earth magnets were considered to serve the function of rotor poles. The properties of primary importance in the selection of magnets are those that define the magnitude and stability of the field that it can provide. These include the coercivity H_c , saturation magnetization M_s , remanence B_r as well as the demagnetization curve. Particularly in our application, it warrants a permanent magnet with high-energy product $(BH)_{max}$ and resistance to elevated temperature as

heat is an issue during the operation of the prototype. Thus Neodymium-Iron-Boron (NdFeB) was chosen, as it possesses higher specific energy density and better temperature stability. In order to maximize magnetic energy within the space constraints of the rotor, stacking of various sizes of NdFeB magnets was resorted. The arrangement of the rotor poles is important with reference to the actuating scheme employed. Having even number of poles distributed uniformly around the spherical surface of the rotor is a method in preventing the inherent self-inducing torque generated from Earth's magnetic field. This phenomenon is analogous to the effect of Earth's magnetic field to the compass needle.

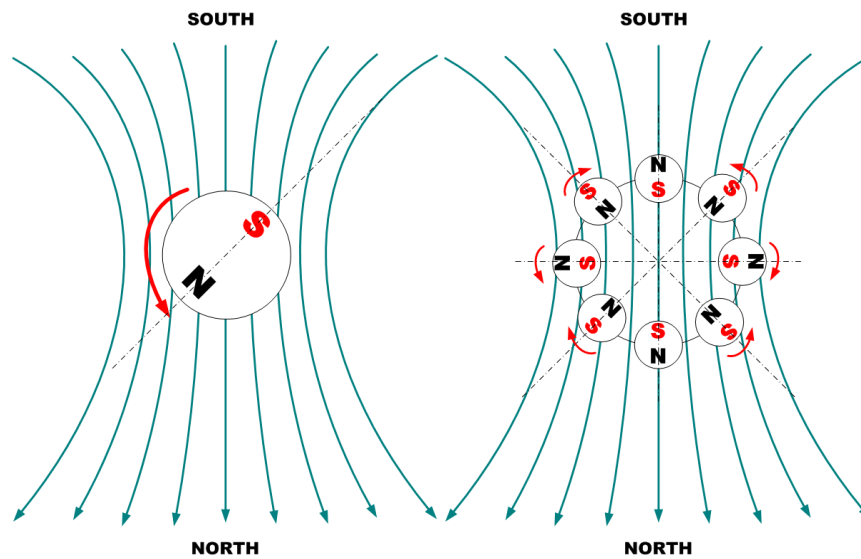


Figure 4.1. Impact of Earth's magnetic field on rotor poles.

Due to the intrinsically high magnetic flux rare earth magnet produces, it tends to align itself to Earth's magnetic field. Therefore, by having evenly distribution array of these magnets, the effect of the torque generated, pivoted at the center of the rotor will be lessen. The eight-rotor poles layout was employed in our design as depicted in Figure 4.1. Figure 4.2 presents an arrangement of a set of rotor poles consisting of stacking various NdFeB magnets employed in our design. Maximization of magnetic flux density of rotor poles can be achieved by means of acquiring the strongest magnets available and optimizing the space constraint within the spherical rotor. The latter can be realized by packing more magnets within the space available. Staggered arrangement of the magnet poles in increasing diameter is determined according to the dihedral-shaped enveloping surface which covers all the magnets as illustrated in

Figure 4.2. This approach maximizes the space limitation by packing cylindrical magnets of various sizes within the rotor structure.

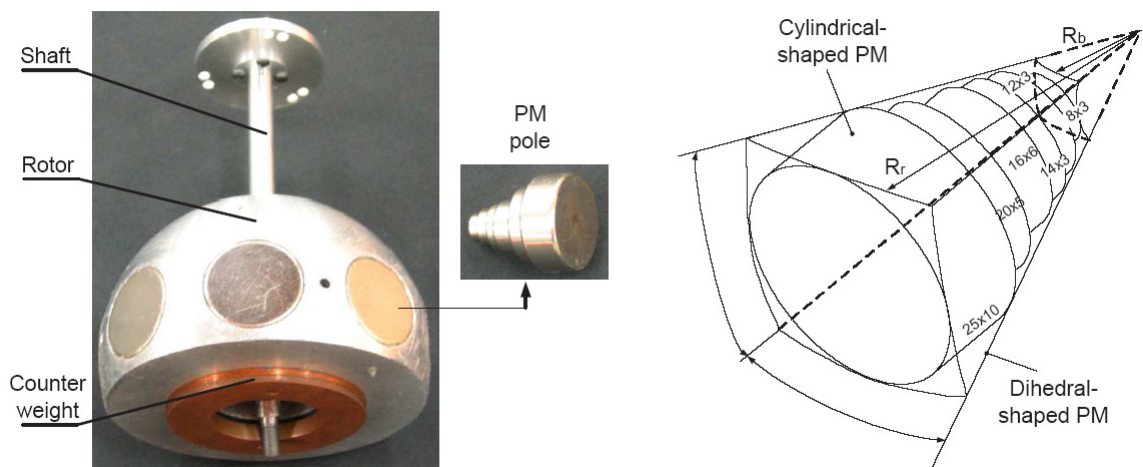


Figure 4.2. Stacking of NdFeB in a set of rotor pole.

4.2 ROTOR DESIGN

The rotor as the name implies, assumes the role of the moving element in the structure of our spherical actuator. Considerations for this component are the material selection and dimensions. Since this is the component that translates forces into displacement, it should contribute minimal inertia during the actuation phase as quick response relates directly to lower moment of inertia. Other than the fact that a lower density material is preferred, additional factors include the thermal and mechanical properties of the material. Thermal effects relates to the heat generated during the operation of the prototype. The material of the rotor should also be magnetically inert so as not to interfere with the magnetic field between the stator and rotor poles. Plastic materials in the likes of Acrylic and Delrin were considered due to their low density and non-magnetic properties. From our evaluation, based on structural integrity under thermal stress and ease of manufacturability, aluminum was selected to be the rotor material. As the rotor houses the permanent magnets, the shape and size must be kept at the minimum so as to lower the overall inertia of the system. The diameter of the rotor was decided at 93mm as this dimension can be found in existing spherical motor design and hence, benchmarking can be conducted in the later stage. The rotor design is shown in Figure 4.3.

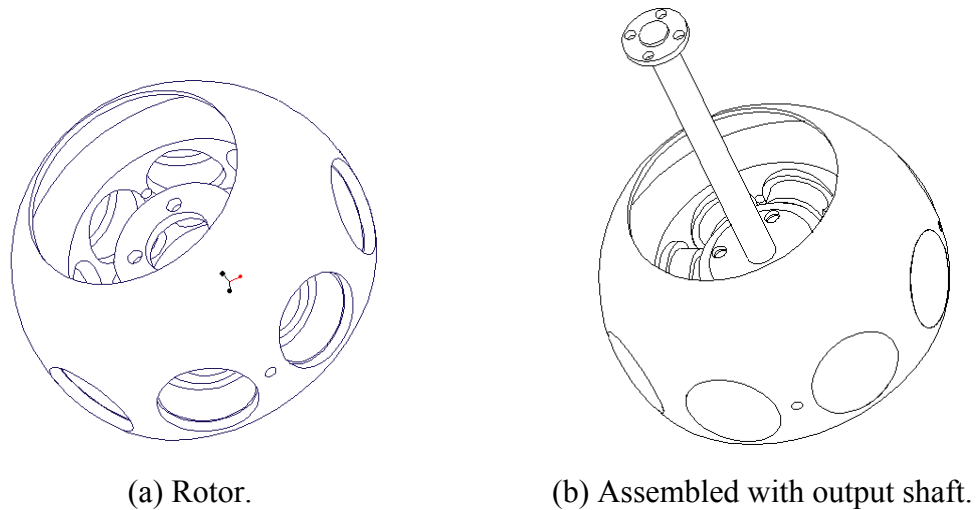


Figure 4.3. Rotor design.

The computed mass of rotor and the full assembly with the rotor pole magnet and balance mass are as follow,

Material	Aluminum
Dimension	Ø93mm

	Mass (Kg)	Inertia (Kgm ² x 10 ⁻⁴)
Rotor only	0.1849	$I = \begin{pmatrix} 1.511 & 0 & 0 \\ 0 & 2.372 & 0 \\ 0 & 0 & 1.511 \end{pmatrix}$
Full assembly of rotor	0.8213	$I = \begin{pmatrix} 5.850 & 0 & 0 \\ 0 & 9.196 & 0 \\ 0 & 0 & 5.851 \end{pmatrix}$

Table 4.1. Rotor properties.

4.3 STATOR DESIGN

The stator as shown in Figure 4.4 not only houses the rotor but also serves the function of a fixture for the positioning of the stator poles. Attraction force between rotor/stator poles forms the basis for motion generation. In order to create three-dimensional motion, we would require at least two pairs of stator poles interacting

with two rotor poles on two orthogonally lying planes. Consequently, if we place another set of rotor/stator pairs in between the orthogonally lying set of rotor/stator pairs, we are able to generate spin at the directed axis and also enhance the resolution of the prototype. Figure 4.5 summarizes the point mentioned.

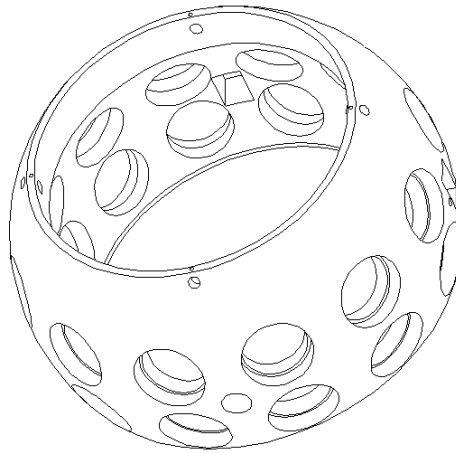


Figure 4.4. Stator.

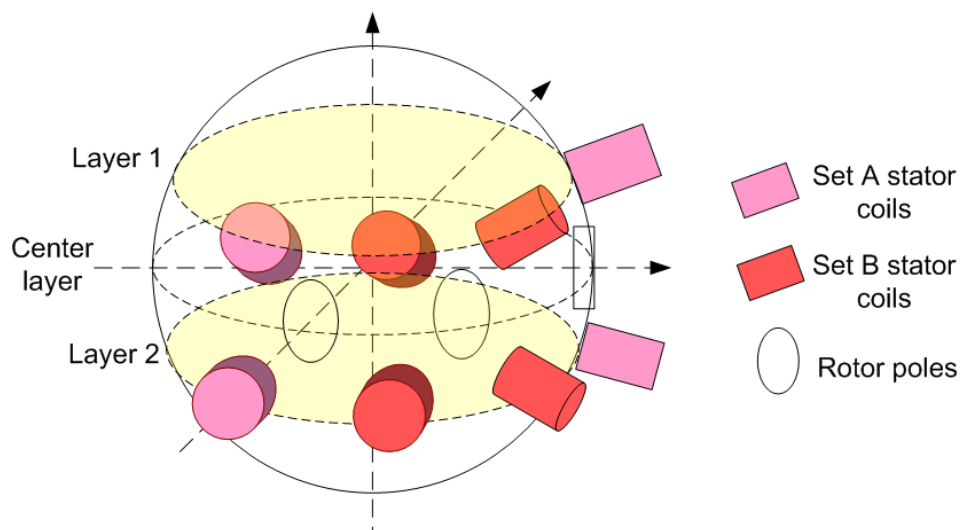


Figure 4.5. Stator pole position.

Shown in Figure 4.6 is a set of rotor/stator poles that occupies a quadrant of a sphere. Therefore, by distributing this basic rotor/stator pair configuration equally around the spherical surface, we would obtain a two-layer design with twelve stator poles each. Stated previously, the predetermined angle of inclination θ was fixed at 11° as illustrated in the figure. Another feature being taken into consideration is the centering of the rotor to the stator origin. A pair of centering pins is designed to be

located orthogonally at the center layer so as to ensure concentricity between rotor and stator.

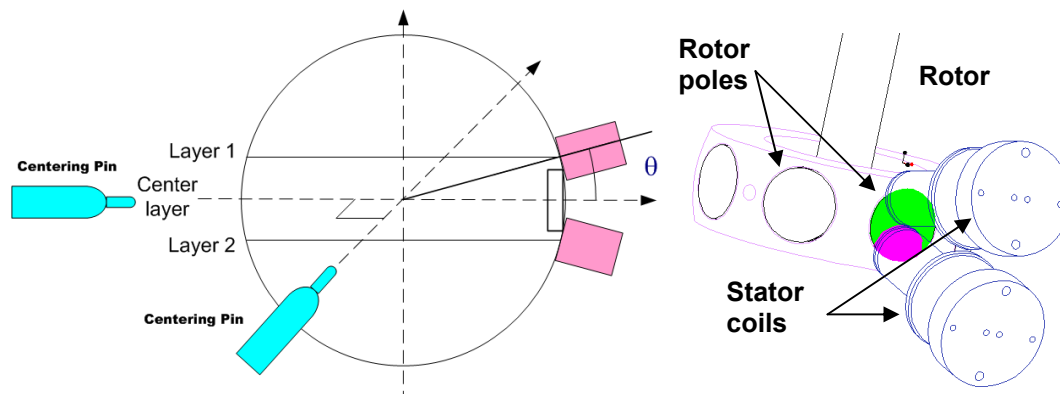


Figure 4.6. Illustrations of two-layer stator pole design.

The material of the stator should not be a ferrous material as that will result in unwanted detent force between the stator and the magnetic rotor poles. Due to ease of manufacturability, aluminum was selected to be the rotor material. The specifications of the stator are summarized in Table 4.2.

Material	Aluminum
Dimension	$S\varnothing_{\text{outer}} = 225\text{mm}$ $\varnothing_{\text{inner}} = 168\text{mm}$
Stator poles	24 poles in total
Angle of inclination	$\pm 11^\circ$

Table 4.2. Stator specifications.

4.4 STATOR POLE DESIGN

The main drawbacks of using ferromagnetic material as core for the stator pole are the potentially large detent force and induced eddy current. These are factors that compromise the achievable dynamics performance of the actuator. In the design of our prototype, the air core coils were employed as the stator pole. Following is a list of the advantages and disadvantages of air core coils,

Advantages

- Air is assumed to be a linear homogenous media with the absence of magnetization leading to the constitutive linear relation, $B = \mu_0 H$. Hence, electrical current through the air core coil creates a magnetic flux proportional to the current.
- Inductance of air core coils are unaffected by the current it carries. This contrasts with the situation with coils using ferromagnetic cores whose inductance tends to reach a peak at moderate field strengths before dropping towards zero as saturation approaches.
- Air coils are also free of the 'iron losses' which affect ferromagnetic cores. This property becomes progressively important as frequency increases as it delivers better Q-factor, greater efficiency, greater power handling, and less distortion.
- Free from non-linear hysteresis, saturation, eddy current effects and the presence of undesirable detent forces.

Disadvantages

- In the absence of a high permeability core, more and larger turns are required to achieve a given inductance value. This translates to larger coils, lower self-resonance and higher copper loss.
- Without closed magnetic paths as compared to ferromagnetic core coils will results in greater stray field radiation and pickup.
- Lower resultant magnetic forces in the presence of external fields.

In a nutshell, the advantages of using the air coil are that the inductance of the air coil is unaffected by the current it carries, eliminates unwanted detent force and also it is free of the eddy current losses. However, without a high permeability core, one must have more or larger turns to achieve a given inductance value as compared to ferromagnetic core. More turns means larger coils, lower self-resonance and higher

copper loss. AWG 24 magnet wires are used as the winding material of the stator coil. In almost all cases, these wires are single strand 'enamel' insulated. The coating is usually of poly vinyl acetal, polyester or polyurethane. The selection of magnet wires depends on its thermal properties and diameter. This is because the wires will be subjected under thermal stress during operation and the physical size of the wires is directly related to the maximum number of turns achievable with the stator coil frame design. Copper loss or winding loss is the term used to describe the energy dissipated by resistance in the wire used to wind a coil. Naturally, in most cases this wire will be made of copper, whose resistivity at 20°C is about $1.73 \times 10^{-8} \Omega\text{m}$. The resistance of a wire per metre (Ω/m) can be conveniently estimate as,

$$R = \frac{0.022}{D_w^2} \quad (4.1)$$

where D_w is the diameter of the conductor in millimetres. The length of a winding can be calculated from the following,

$$l_w = N\pi D_{av} \quad (4.2)$$

where N is the number of turns and D_{av} is the average diameter of the winding in metres (m). Copper losses (watts) will then be,

$$P_w = 0.022l_w \left(\frac{I}{D_w}\right)^2 \quad (4.3)$$

where I is the RMS current in the winding. Any length of wire has inductance. Inductance is a measure of a coil's ability to store energy in the form of a magnetic field. It is defined as the rate of change of flux with current. In the simple case of a single layer solenoidal coil the inductance may be estimated as follows [85],

$$L = \frac{N^2 r^2}{228r + 254l} \quad (4.4)$$

where L is the inductance in microhenrys, r is the coil radius in millimetres, l is the coil length in millimetres and N is the number of turns as illustrated in Figure 4.7.

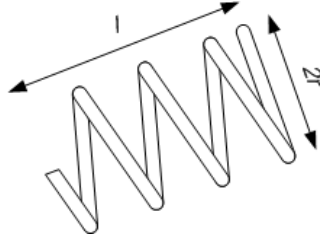


Figure 4.7. A single layer coil.

For a multi-layer air core solenoid, we have the inductance as [86],

$$L = \frac{31.6N^2r_1^2}{6r_1^2 + 9l + 10(r_2 - r_1)} \quad (4.5)$$

with L as the inductance in microhenrys, N represents the total number of turns on coil, r_1 as the radius of the inside of the coil (meters), r_2 relates to the radius of the outside of the coil (meters) and l is the length of the coil (meters). For now, the coil frame to be employed in our prototype has the dimension (in millimeters) presented in Figure 4.8.

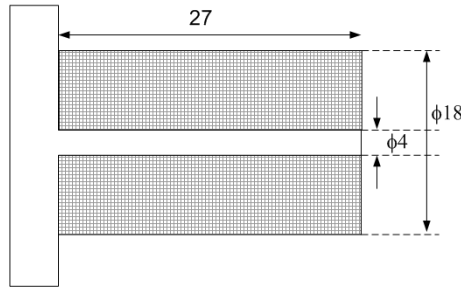


Figure 4.8. Coil frame dimension.

Based on a coil frame of length, $L_{coil}=27\text{mm}$, AWG 24 wire of nominal diameter including insulation, $\phi_{wire}\sim 0.45\text{mm}$, overall external diameter of stator coil $\phi_s=18\text{mm}$, and coil frame core diameter, $\phi_{core}=4\text{mm}$,

$$\text{Number of turns} = \text{Int} \left[\frac{L_{coil}}{\phi_{wire}} \right] \times \text{Int} \left[\frac{\phi_s - \phi_{core}}{2} \right] = 900 \text{ turns}.$$

4.5 SUPPORT MECHANISM

With reference to Chapter 1, from the study of numerous spherical actuators, the support mechanism for the various designs can be summarized in Table 4.3. In our prototype, a spherical bearing as shown in Figure 4.9 is being utilized [87]. The key advantages of this form of bearing over existing support scheme employed can be sum up as follows.

- No contact between bearing and rotor surface.
- No special surface treatment required on rotor surface.
- Easy installation.
- Calibration possible upon installation.
- As bearing is installed at the center of rotor (shown in Figure 3.16b), the decrease in moment arm will result in lower inertia and friction contributed by the bearing.

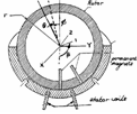
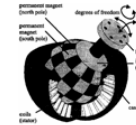
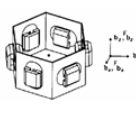
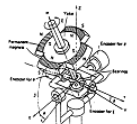
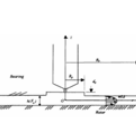
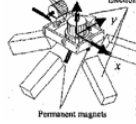
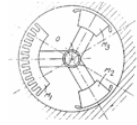
	Mechanical Bearings	Aerostatic/Hydros tatic Bearings	Magnetic levitation	Direct support using frictionless material
Pros	i) Simple to install ii) Mechanically stable	i) Friction-free movement	i) Friction-free movement	i) Simple to install ii) Mechanically stable
Cons	i) Friction at contact surface ii) Fatigue wear effects	i) Complex setup and control system ii) Instability iii) Cost	i) Complex setup and control system ii) Instability iii) Cost	i) Friction at contact surface ii) Fatigue wear effects
	Lee et al VR spherical motor 	Anders et al "Kugelmotor" 	Hollis et al "Magic Wrist" 	Wang et al spherical motor 
	Kaneko et al spherical DC spherical motor 	Lee et al VR spherical motor 	Baek et al 3 DOF motor 	Chirikjian et al spherical stepper motor 
	Foggia et al electromagnetic actuator 			

Table 4.3. Support scheme of existing spherical actuator design.

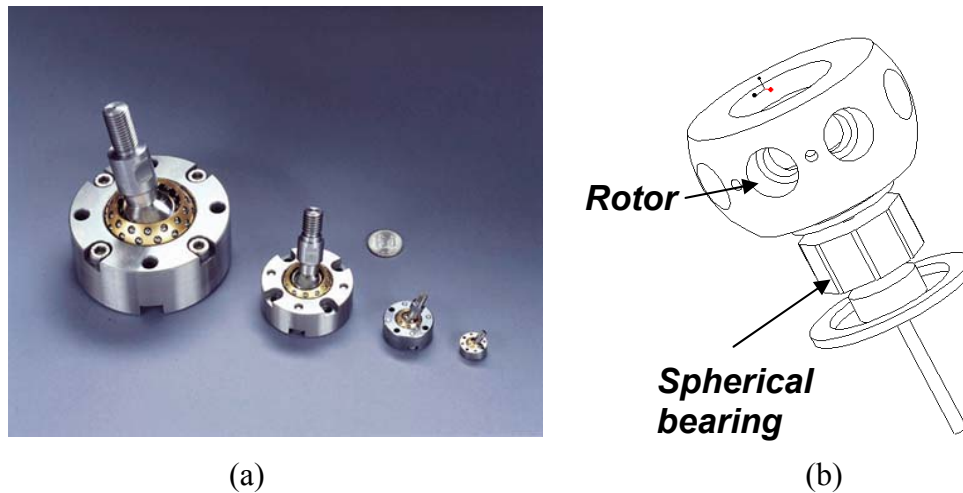


Figure 4.9. Spherical bearing.

4.6 SPHERICAL PROTOTYPE

This section summarizes the specifications of our prototype being designed and fabricated. Table 4.4 presents the specifications and Figure 4.10 (a) and (b) illustrates the exploded view and assembled view of the prototype respectively.

Stator	
<i>Dimension</i>	$S\varnothing_{\text{outer}} = 225\text{mm}$ $\varnothing_{\text{inner}} = 168\text{mm}$
<i>Material</i>	Aluminum
<i>Pole material</i>	Aluminum
<i>Poles layout</i>	2 layer with 12 poles each distributed around $S\varnothing_{\text{outer}} = 225\text{mm}$ at $\pm 11^\circ$ apart from the center plane
Rotor	
<i>Dimension / Material</i>	$\varnothing_{\text{outer}} = 93\text{mm}$ / Aluminum
<i>Pole material</i>	NdFeB
<i>Poles layout</i>	8 poles distributed around $S\varnothing 93\text{mm}$
<i>Windings</i>	Number of turns, $N_{\text{turns}} \sim 900$, AWG 24 / Grade H
Range of inclination	$\pm 11^\circ$

Table 4.4. Specifications of proposed spherical actuator.

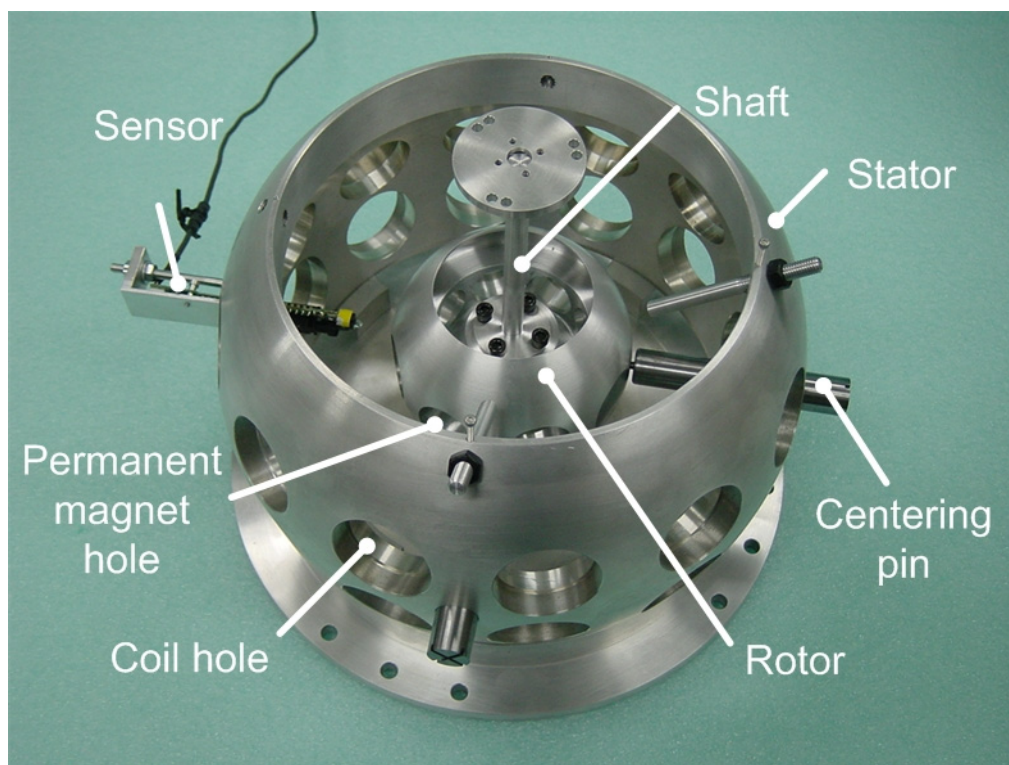
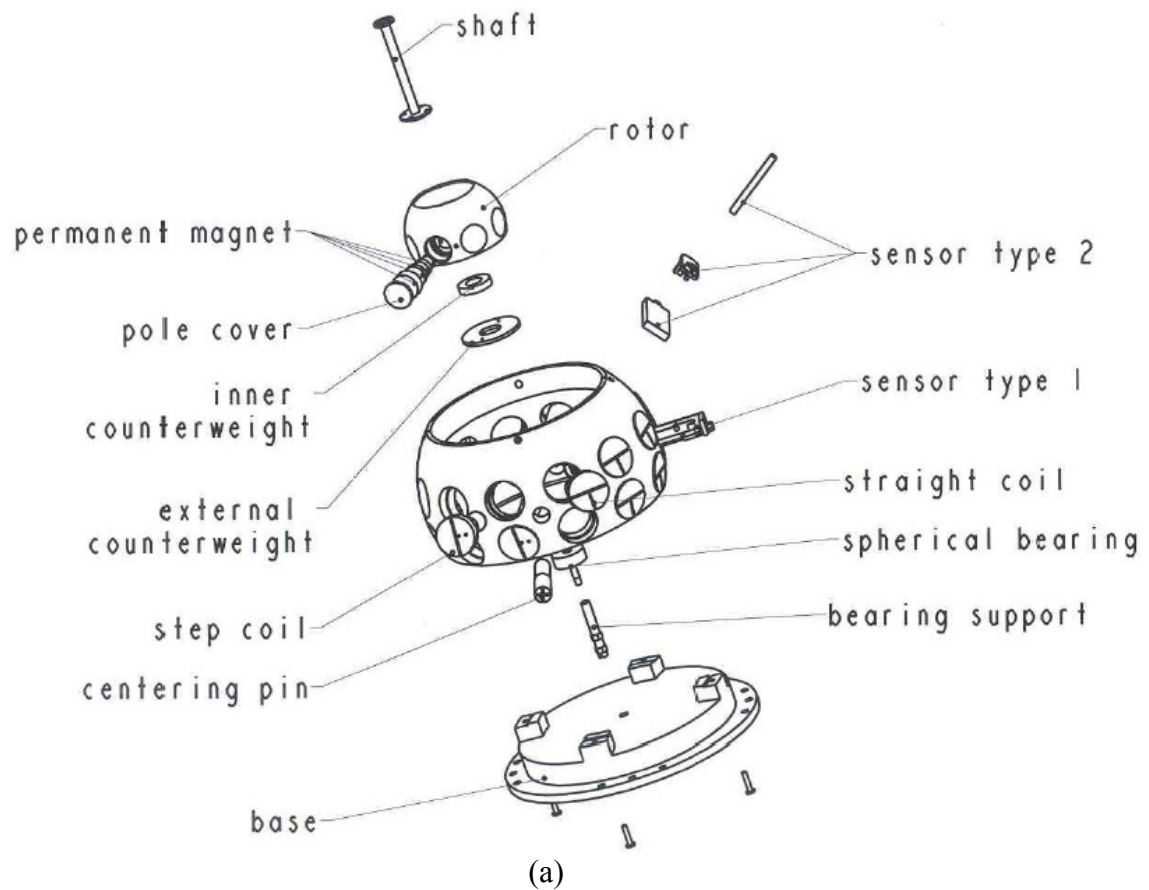


Figure 4.10. Proposed spherical actuator.

The preceding sections present the details of the spherical actuator prototype designed and assembled in this work. This prototype will be able to demonstrate the working principle as proposed in the design and provides the platform for future experimental investigations.

4.7 MOTION GENERATION

There are two essential characteristics of the spherical actuator that are worth mentioning again. First, 3-DOF rotation is about a single point, pivoted at the center of the spherical bearing. Second, actuating forces translating to rotational torques about the rotor center is primarily magnetic forces between the rotor and stator poles. The resultant torque relating each interacting pairs of poles is in the direction along the shortest trajectory between the respective poles. This universal phenomenon conforms to the fact that interacting magnetic dipoles will attempt to align themselves to minimize the resultant magnetic potential energy. This observation implies that directional control of the spherical actuator is dictated and dependent on the rotor pole orientation with respect to the interacting stator pole. Hence, we recognize that in order to control the orientation of this 3-DOF spherical actuator, we would require a minimum set of four stator poles and one rotor pole as shown in Figure 4.11 below.

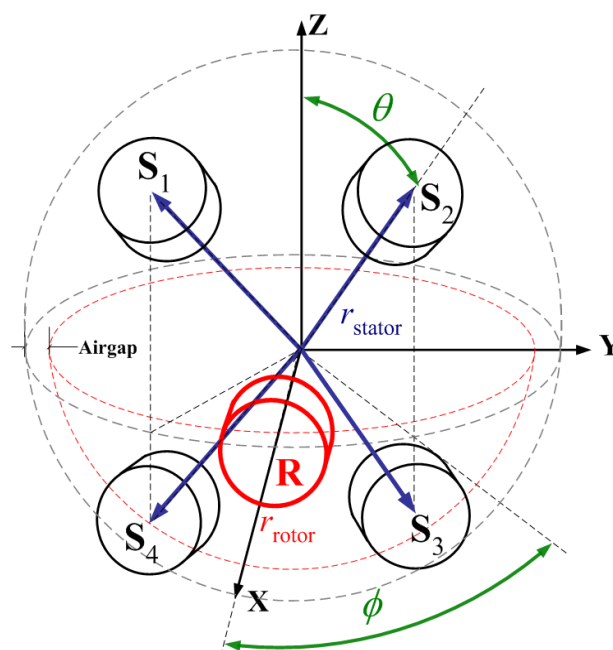


Figure 4.11. Basic configuration for 3-DOF motion control.

The fundamental working principle lies with the attraction forces generated between the rotor and stator poles. By energizing the electromagnetic stator poles, the induced forces translate to rotational torques about the rotor center. Figure 4.12 illustrates the above mentioned description schematically. Torque vectors, T_1 , T_2 , T_3 and T_4 represent the resultant torque induced onto the rotor pole highlighted in red by the individual stator poles. T_D denotes the driving torque, define as the vector sum of T_1 , T_2 , T_3 and T_4 and T_H is termed as the holding torque, define as the counter torque required to hold the rotor pole at the desired instantaneous position.

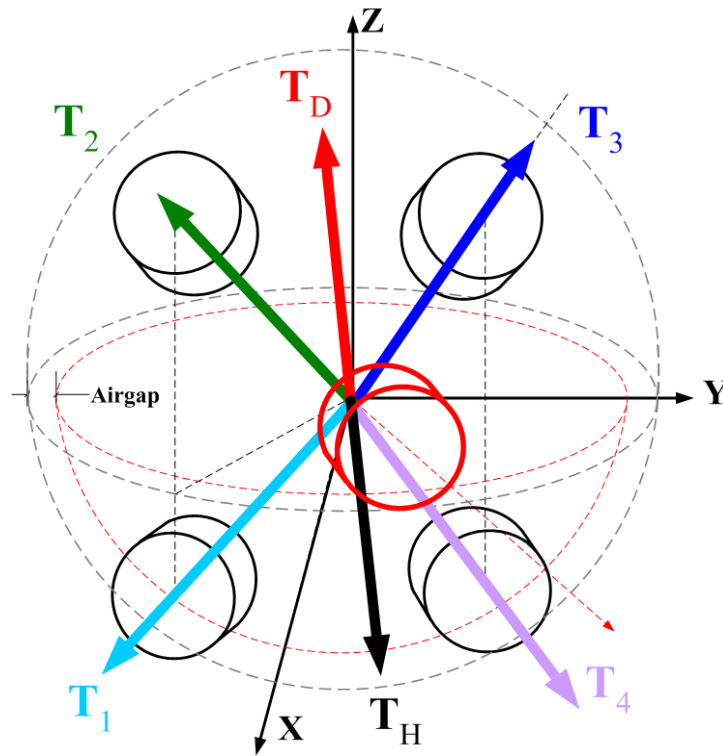


Figure 4.12. Schematic of the resultant torque vectors.

At this juncture, one would concur that the orientation control for this class of actuator lies in the manipulation of resultant torques generated by the respective pairs of rotor and stator poles. Therefore, the vital element for orientation control lies in the resultant torque formulation. In the preceding paragraph, it was highlighted that the physical constraint of the rotor is pivoted about its center. Due to this constraint, we will show that the individual torque by the respective stator poles resides on the same plane. Since the position of stator poles of our prototype shares the same azimuthal (ϕ) and polar angles (θ), the following position vectors denote the rotor pole, $\mathbf{R}$ and the specific stator poles,

$$\mathbf{R} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \mathbf{S}_1 = \begin{bmatrix} x \\ -y \\ z \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{S}_3 = \begin{bmatrix} x \\ y \\ -z \end{bmatrix}, \quad \mathbf{S}_4 = \begin{bmatrix} x \\ -y \\ -z \end{bmatrix}$$

where a, b, c, x, y, z are the Cartesian coordinates of the rotor and stator poles position vectors define by the spherical coordinate parameters, r, ϕ , and θ . From the law of minimization of magnetic potential energy, we understand that the attraction forces induced will attempt to move the rotor pole towards the stator pole by the shortest possible path. Henceforth, the direction of the torque generated by the individual stator poles with the rotor pole can be in fact acquired through the cross product of $\mathbf{R}$ and $\mathbf{S}$. Thus, we have,

$$\begin{aligned} \mathbf{T}_1 = \mathbf{R} \times \mathbf{S}_1 &= \begin{bmatrix} -c \cdot y - b \cdot z \\ a \cdot z - c \cdot x \\ b \cdot x + a \cdot y \end{bmatrix}, & \mathbf{T}_2 = \mathbf{R} \times \mathbf{S}_2 &= \begin{bmatrix} c \cdot y - b \cdot z \\ a \cdot z - c \cdot x \\ b \cdot x - a \cdot y \end{bmatrix}, \\ \mathbf{T}_3 = \mathbf{R} \times \mathbf{S}_3 &= \begin{bmatrix} c \cdot y + b \cdot z \\ -a \cdot z - c \cdot x \\ b \cdot x - a \cdot y \end{bmatrix}, & \mathbf{T}_4 = \mathbf{R} \times \mathbf{S}_4 &= \begin{bmatrix} b \cdot z - c \cdot y \\ -a \cdot z - c \cdot x \\ b \cdot x + a \cdot y \end{bmatrix}. \end{aligned}$$

In order to prove that the resultant torque, $\mathbf{T}_1$ - $\mathbf{T}_4$ lies on the same plane, we compute normal vector of these resultant torques and show that they resides on the same plane.

$$\begin{aligned} \mathbf{T}_{12} = \mathbf{T}_1 \times \mathbf{T}_2 &= \begin{bmatrix} (a \cdot z - c \cdot x) \cdot (b \cdot x - a \cdot y) - (b \cdot x + a \cdot y) \cdot (a \cdot z - c \cdot x) \\ (b \cdot x - a \cdot y) \cdot (c \cdot y - b \cdot z) - (-c \cdot y - b \cdot z) \cdot (b \cdot x - a \cdot y) \\ (-c \cdot y - b \cdot z) \cdot (a \cdot z - c \cdot x) - (a \cdot z - c \cdot x) \cdot (c \cdot y - b \cdot z) \end{bmatrix}. \\ \mathbf{T}_{34} = \mathbf{T}_3 \times \mathbf{T}_4 &= \begin{bmatrix} (-a \cdot z - c \cdot x) \cdot (b \cdot x - a \cdot y) - (b \cdot x - a \cdot y) \cdot (-a \cdot z - c \cdot x) \\ (b \cdot x - a \cdot y) \cdot (-c \cdot y + b \cdot z) - (c \cdot y + b \cdot z) \cdot (b \cdot x + a \cdot y) \\ (c \cdot y + b \cdot z) \cdot (-a \cdot z - c \cdot x) - (-a \cdot z - c \cdot x) \cdot (-c \cdot y + b \cdot z) \end{bmatrix}. \end{aligned}$$

Taking cross product of $\mathbf{T}_{12}$ and $\mathbf{T}_{34}$,

$$\mathbf{T}_{12} \times \mathbf{T}_{34} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

From the above derivation, we have shown that the resultant torque $\mathbf{T}_1$ - $\mathbf{T}_4$ indeed resides on the same plane. Referencing to Figure 4.12, if we are to view from the $-\mathbf{Z}$ direction, Figure 4.13 presents a visualization of this plane, Π_{torque} . Since we have identified that the torque by the respective stator poles are on the same plane, we can extend this important property to the resultant torque formulation of the spherical actuator. Moving forward, ensuing discussions will be referenced to this plane, Π_{torque} . Figure 4.14 presents a frontal view of this plane at the rotor home position.

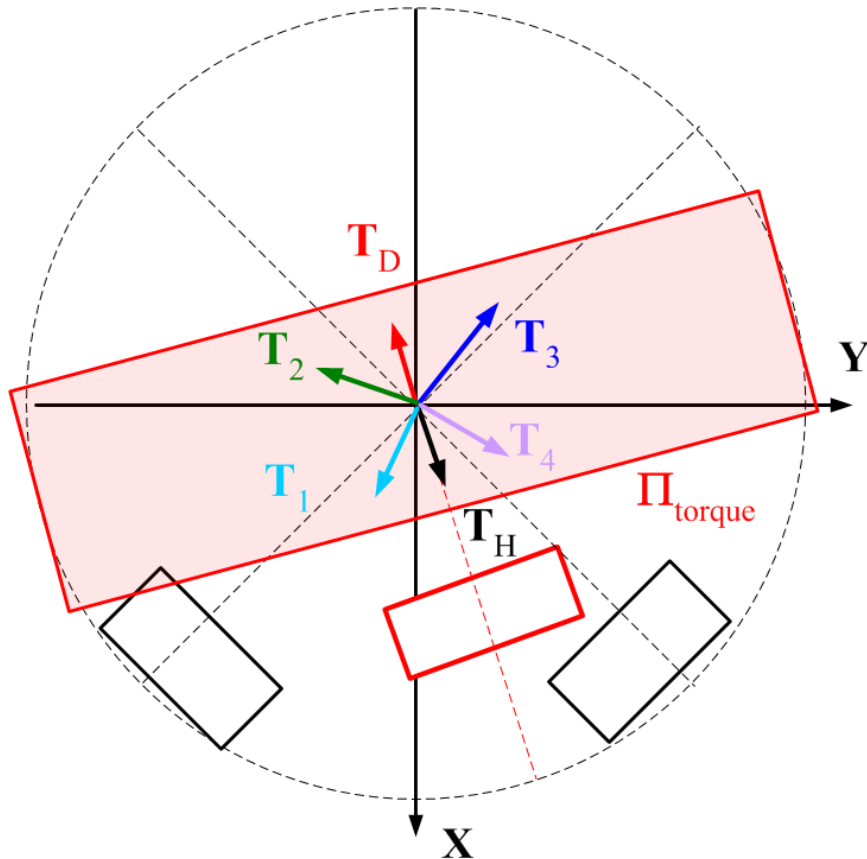


Figure 4.13. Visualization of torque plane.

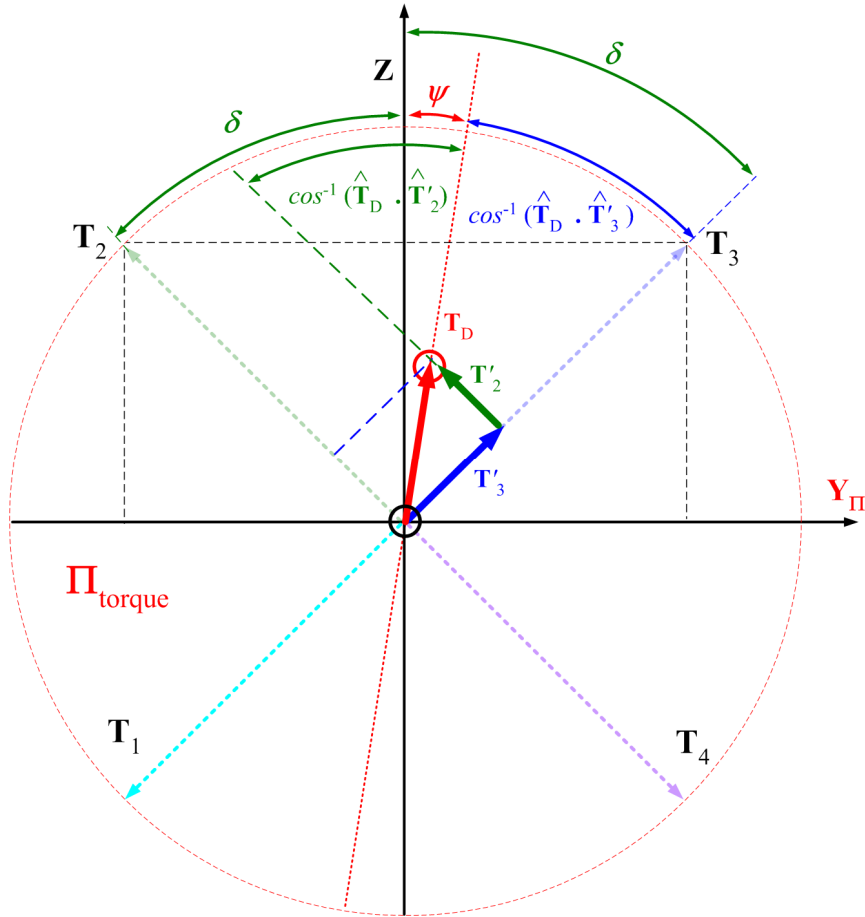


Figure 4.14. Visualization of resultant torque on plane, Π_{torque} .

We define the actuator home position with rotor pole $\mathbf{R}$ aligned with the stator $\mathbf{X}$ -axis. At this initial position, we will be able to acquire δ , define as the trajectory angle between the torque vector, $\mathbf{T}$ and the stator $\mathbf{Z}$ -axis

$$\delta = \cos^{-1}(\hat{T} \cdot \hat{Z}).$$

The desired trajectory path of the driving torque, $\mathbf{T}_D$ can be influence by the designated parameter, ψ . This angle relates the angular displacement of the driving torque vector, $\mathbf{T}_D$ from the $\mathbf{Z}$ -axis on plane, Π_{torque} . Angle ψ , in geometric sense controls the trajectory path of the rotor as illustrated in Figure 4.15.

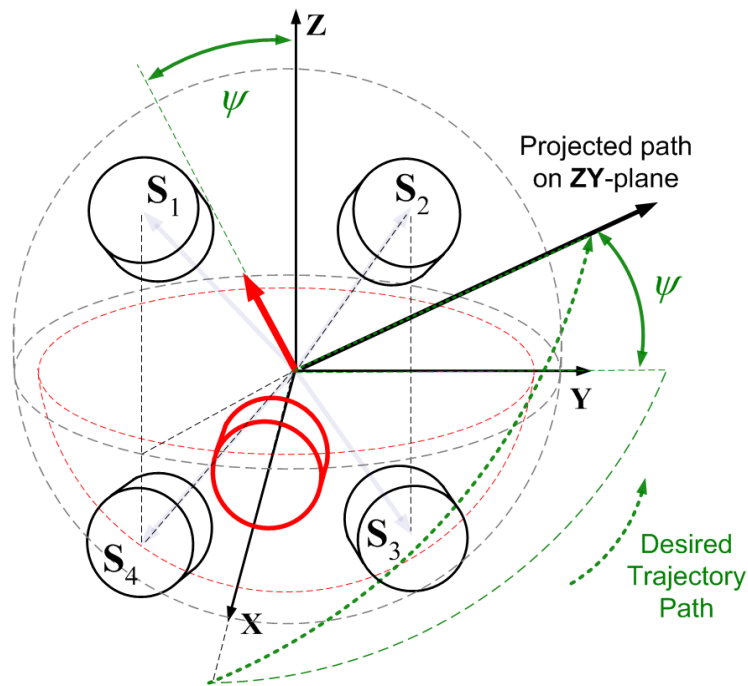


Figure 4.15. Visualization of trajectory angle ψ .

It is also important to realize that directional control of the driving torque, T_D is related to the magnitude of the contributing torque vectors, T'_2 and T'_3 . This can be easily comprehended by the fact that at every instantaneous point along the rotor pole trajectory, the torque vectors changes direction according to the shortest magnetic path between the magnetic dipoles. Figure 4.16 explains this observation. From the illustration, we ascertain that we are unable to regulate the direction of the resultant torque. The parameter which we can manipulate will be the input current of the individual stator poles which relates directly to the output torque. A simple illustration will be to direct the driving torque, T_D along the Z -axis. This can be achieved by simply energizing stator pole S_2 and S_3 with identical input current. Since the output torque T'_2 and T'_3 are equal in magnitude, the T_D will point along the Z -axis causing the spherical rotor to spin about the Z -axis. Hence, as long as the magnitude of the input current are held equal and constant, the rotor pole will move about Z -axis and stop when $T_D=0$. Physically, the rotor pole will move to a position in between the actuating stator pole S_2 and S_3 and remains stationary there. This is the position whereby the torque is zero with equal and opposite attraction forces restraining the rotor pole.

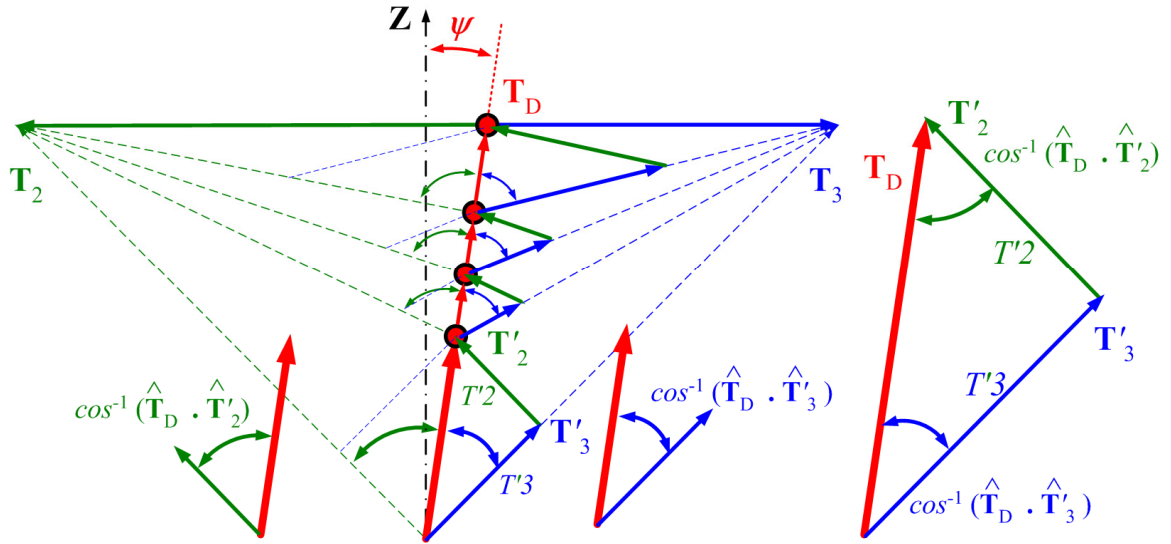


Figure 4.16. Trajectory path of rotor pole.

Thus, we have illustrated the key concept behind the control of this class of spherical actuators. The key factor is how we manipulate the magnitude of the actuating torque. We will now formulate the methodology in order to control the direction of the driving torque, $\mathbf{T}_D$. From Figure 4.14, adhering to the local coordinate system on the plane, Π_{torque} , we can establish position vectors for an arbitrary trajectory path with parameter ψ as,

$$\hat{\mathbf{T}}_2 = T_2 \begin{bmatrix} -\sin \delta \\ \cos \delta \end{bmatrix}, \quad \hat{\mathbf{T}}_3 = T_3 \begin{bmatrix} \sin \delta \\ \cos \delta \end{bmatrix}, \quad \hat{\mathbf{T}}_D = T_D \begin{bmatrix} \sin \psi \\ \cos \psi \end{bmatrix}$$

As the azimuthal and polar angles of the stator poles are constant, $T_2 = T_3$. $\mathbf{T}_D$ is defined by the parameter, θ , translating to the directional control of the spherical actuator. Referencing to Figure 4.17, in order to move the rotor from position 1 to position 2, we would need to acquire the actuating torque vectors, $\mathbf{T}'_2$ and $\mathbf{T}'_3$. This can be established by the vector sum

$$\mathbf{T}'_2 = \mathbf{T}_2 - \mathbf{T}_D, \quad \mathbf{T}'_3 = \mathbf{T}_3 - \mathbf{T}_D$$

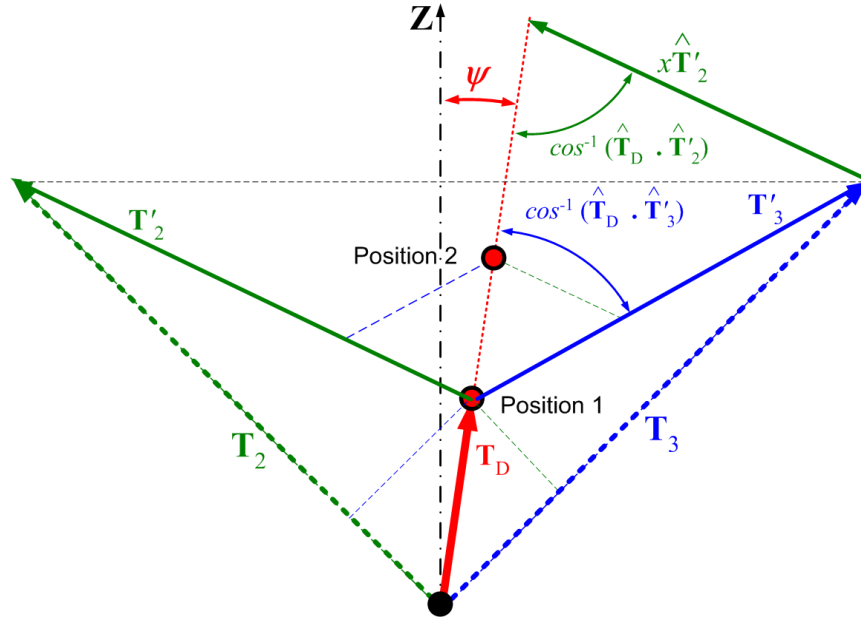


Figure 4.17. Determination of driving torque ratio.

We are interested in acquiring the ratio between the actuating torque T'_2 and T'_3 so as to ensure that the driving torque T_D is along the desired direction. This ratio can be determine geometrically by

$$\frac{T'_3}{\sin(\cos^{-1}(\hat{T}_D \cdot \hat{T}'_3))} = \frac{T'_2}{\sin(\cos^{-1}(\hat{T}_D \cdot \hat{T}'_2))},$$

and thus

$$\frac{T'_3}{T'_2} = \frac{\sin(\cos^{-1}(\hat{T}_D \cdot \hat{T}'_3))}{\sin(\cos^{-1}(\hat{T}_D \cdot \hat{T}'_2))}. \quad (4.6)$$

With the above correlation between the torque magnitude and position of the rotor pole, we establish the instantaneous torque ratio required to direct the rotor along the desired trajectory path. As this is a torque ratio, the only requirement on the input current will be that the resultant torque is able to overcome the inertial and frictional forces of the rotor. By discretizing the trajectory path, employing the analytical torque model, we can compute sets of input currents to the respective stator poles in order to map out the projected motion. We have expounded on the motion generation strategy in order to move the rotor. For orientation control, we need a countering torque to hold the rotor in the desired position. Previously, we have shown that all torque

vectors reside along the same plane. Since the rotor can be actuated by energizing stator pole, S_2 and S_3 , we will now exploit the remaining stator poles, S_1 and S_4 to generate the balancing torque required to hold the rotor in location. Figure 4.18 depicts the vector diagram for computing the holding torque, T_H . The approach duplicates what we have presented for the determination of T_D . Therefore, we establish the torque ratio for holding torque, T_H as,

$$\frac{T'_4}{T'_1} = \frac{\sin(\cos^{-1}(\hat{T}_H \cdot \hat{T}'_4))}{\sin(\cos^{-1}(\hat{T}_H \cdot \hat{T}'_1))} \quad (4.7)$$

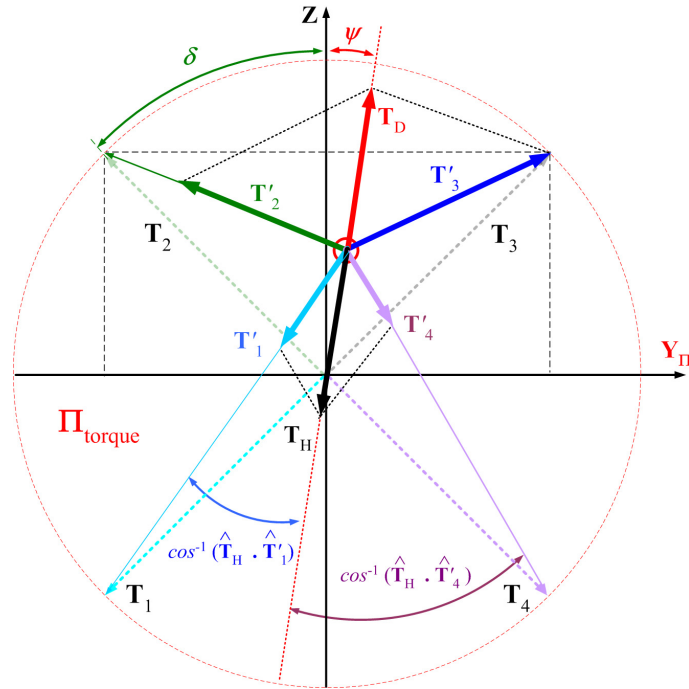


Figure 4.18. Illustration of holding torque, T_H .

From the preceding sections, we have presented the methodology to control the orientation of the spherical actuator. The proposed approach established the link between rotor position with the actuating torque generated by the stator poles during motion. Directional control is achieved by means of manipulating the input current to the stator poles in accordance with the computed ratio of the actuating torque. With the proposed strategy, multi-DOF motions can be generated and controlled using various pairs of stator poles.

4.7.1 VALIDATION OF PROPOSED MOTION CONTROL METHOD

The proposed control method was modeled using MATLAB for simulation. Rotor and stator poles are presented by their position vectors. In this simulation, we are able to validate and visualize what we have detailed in the preceding sections. Figure 4.19 presents the graphical presentation of the simulation model.

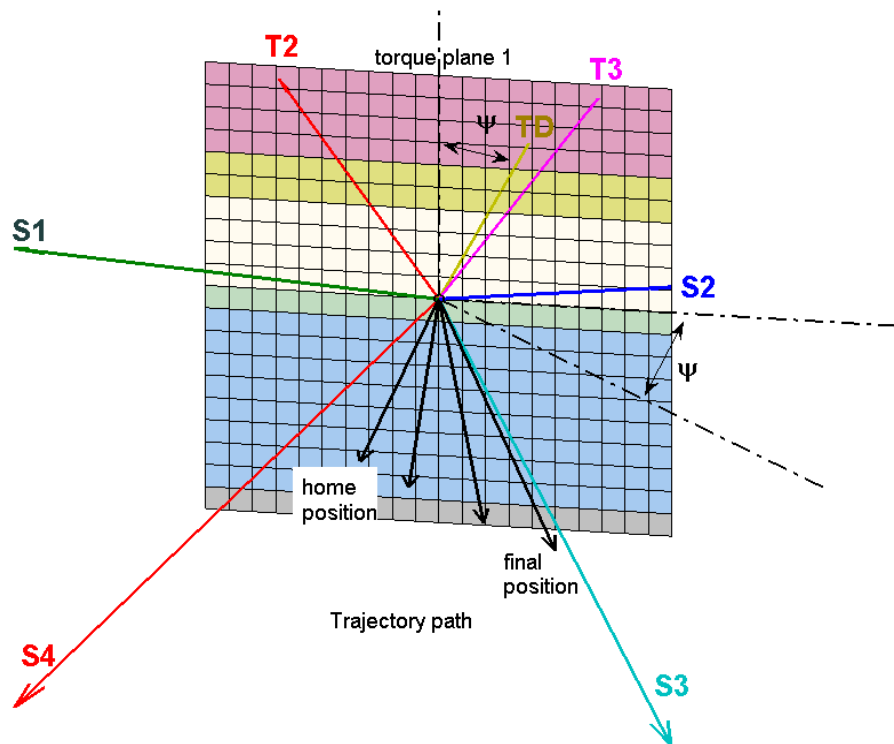


Figure 4.19. Simulation model for torque control.

An arbitrary trajectory path denoted by the position vectors in black simulates the displacement of the rotor pole. These four vectors is a function of the azimuthal and polar angles that simulate points in space where the rotor pole will map from its home position. This trajectory path can be presented by the driving torque, T_D which makes an angle, ψ , from the Z -axis. At each trajectory position, torque vectors T_2 and T_3 are computed to ascertain its direction as depicted in Figure 4.20. As proven mathematically, actuating torque vectors resides on the same plane. Our simulation model validates this again as shown in Figure 4.21.

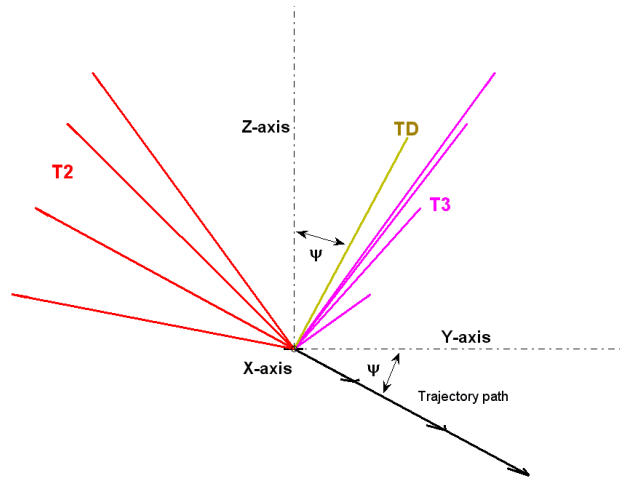


Figure 4.20. Computation of actuating torque vectors.

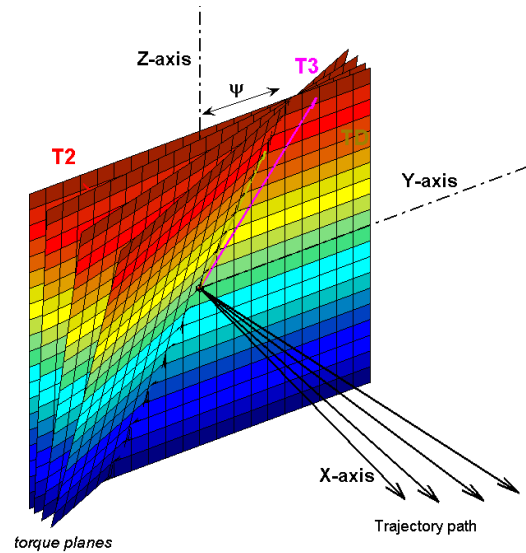


Figure 4.21. Actuating torque planes.

The above results reinforce the viability of the proposed control methodology. Since the simulation has shown that at each rotor position, the torque planes vary in direction due to the shortest path taken by the magnetic dipole, by acquiring the respective actuating torque ratio at each plane, we can ensure and maintain the desired rotor trajectory. According to the proposed strategy, rotor trajectory is a function of angle, ψ . Hence, this translates to how at each position, along the respective torque plane, we can control the actuating torque ratio. Figure 4.22 demonstrate this important factor.

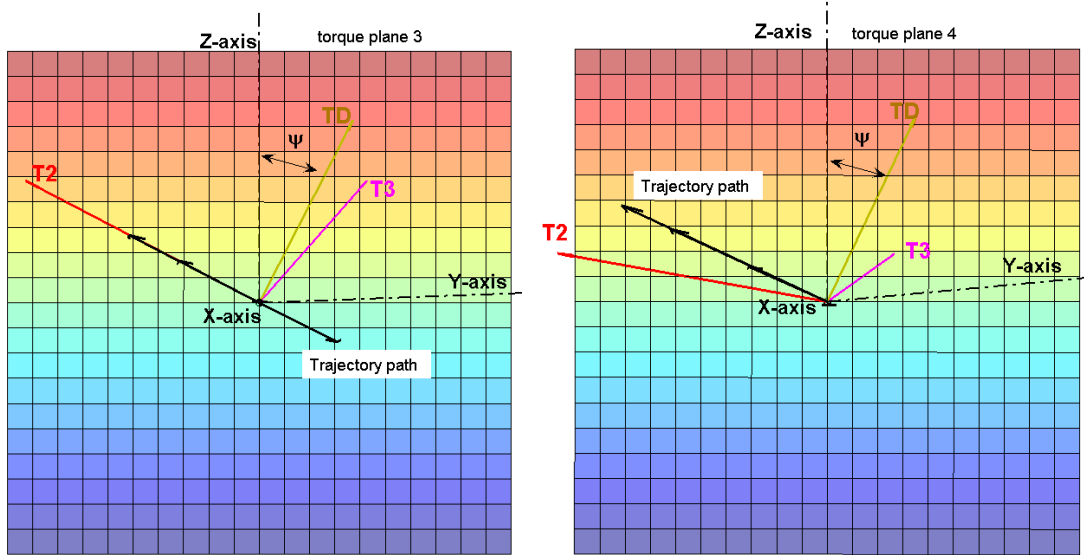


Figure 4.22. Verification of angle ψ .

The viability of the proposed control scheme was demonstrated by the simulation model. From the simulation results, we have verified and validated the approach for the control of the spherical actuator.

4.8 KINEMATICS OF THE SPHERICAL ACTUATOR

The rotor which consists of rare earth permanent magnets, rotor housing and spherical bearing is considered as rigid body which is free to rotate about the bearing center. Setting the bearing center as the absolute global frame, $\{XYZ\}$ the **Z-Y-Z** Euler angles representation maps the rotor frame, $\{xyz\}$ to this reference. At the initial rotor orientation, $\{XYZ\}$ coincides with $\{xyz\}$ as depicted in Figure 3.1. The speed and direction of the rotor can be represented by angular velocity vector, ω_{rotor} with reference to $\{xyz\}$ as shown,

$$\omega_{rotor} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (4.8)$$

The angular velocity vector of a rotating body as rates of the set of **Z-Y-Z** Euler angles is

$$\dot{\Theta}_{Z'Y'Z'} = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}. \quad (4.9)$$

The equation that relates angle set velocities to equivalent angular velocity vector is

$$\omega_{rotor} = E_{Z'Y'Z'}(\Theta_{Z'Y'Z'})\dot{\Theta}_{Z'Y'Z'} \quad (4.10)$$

where $E_{Z'Y'Z'}$ is a Jacobian relating Z - Y - Z Euler angle set velocity vector to the angular velocity vector, ω_{rotor} ,

$$E_{Z'Y'Z'} = \begin{bmatrix} 0 & -s\alpha & c\alpha s\beta \\ 0 & c\alpha & s\alpha s\beta \\ 1 & 0 & c\beta \end{bmatrix}. \quad (4.11)$$

The inverse of the above is therefore

$$\dot{\Theta}_{Z'Y'Z'} = E^{-1}_{Z'Y'Z'}(\Theta_{Z'Y'Z'})\omega_{rotor} \quad (4.12)$$

where

$$E^{-1}_{Z'Y'Z'} = \begin{bmatrix} -c\alpha \cot\beta & -s\alpha \cot\beta & 1 \\ -s\alpha & c\alpha & 0 \\ \frac{c\alpha}{s\beta} & \frac{s\alpha}{s\beta} & 0 \end{bmatrix}.$$

The above equations provide the kinematic mapping between the rotor angular velocity vector and the rates of change of the Euler angles.

4.9 DYNAMICS OF THE SPHERICAL ACTUATOR

Consider the rotor as a rigid body pivoted at the bearing center, rotation about the fixed point at certain angular velocity, the total angular momentum of the rotor, L is

$$L_i = \sum_j I_{ij} \omega_j \quad (4.13)$$

where I_{ij} is the moment of inertia, ω_j the angular velocity, corresponding to the respective axis with i and $j = 1$ to 3. Torque, $\mathbf{T}$ is equal to the rate of change of angular momentum,

$$\left(\frac{d\mathbf{L}}{dt} \right) + \boldsymbol{\omega} \times \mathbf{L} = \mathbf{T} \quad (4.14)$$

With respect to the rotor frame, $\{xyz\}$, moment of inertia tensor of the rotor, $\mathbf{I}$ is constant. Hence, we can use $\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$ to write the equation of motion as,

$$\mathbf{I}\dot{\boldsymbol{\omega}}_{rotor} + \boldsymbol{\omega}_{rotor} \times (\mathbf{I}\boldsymbol{\omega}_{rotor}) = \mathbf{T}_{rotor} \quad (4.15)$$

Extending the components of the total torque,

$$\mathbf{T}_{rotor} = \mathbf{T}_{mag} + \mathbf{T}_{grav} + \mathbf{T}_{fric} \quad (4.16)$$

where I is the 3×3 inertia matrix, $\boldsymbol{\omega}_{rotor}$ is the 3×1 angular vector and $\mathbf{T}_{mag}$, $\mathbf{T}_{grav}$ and $\mathbf{T}_{fric}$ are 3×1 vectors representing electromagnetic, gravitational and frictional torques. During the design stage, the rotor mass distribution was ensured to be axially symmetric with respect to z -axis and the center of mass was aligned to spherical bearing center using counter weights. Taking z -axis as one of the principle axes of the moment of inertia tensor we have,

$$I_{xx} = I_{yy} \text{ and } I_{xy} = I_{xz} = I_{yz} = 0$$

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{xx} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

In terms of coordinate axes that lie along the principle axes of I , the components of the Euler equation of rotational motion are,

$$I_{xx} \dot{\omega}_x + (I_{zz} - I_{yy}) \omega_y \omega_z = T_x$$

$$I_{yy} \dot{\omega}_y + (I_{xx} - I_{zz}) \omega_z \omega_x = T_y$$

$$I_{zz} \dot{\omega}_z + (I_{yy} - I_{xx}) \omega_x \omega_y = T_z$$

Henceforth,

$$\mathbf{I} \dot{\boldsymbol{\omega}}_{rotor} + \mathbf{V} \boldsymbol{\omega}_{rotor} = \mathbf{T}_{mag} + \mathbf{T}_{grav} + \mathbf{T}_{fric} \quad (4.17)$$

where

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{xx} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \text{ and } \mathbf{V} = \begin{bmatrix} 0 & I_{zz} \omega_z & -I_{yy} \omega_y \\ I_{xx} \omega_z & 0 & -I_{zz} \omega_x \\ 0 & 0 & 0 \end{bmatrix}$$

4.10 CHAPTER SUMMARY

In the foregoing sections of this chapter, the motion generation methodology employed by the spherical actuator prototype was illustrated. The feasibility of the proposed scheme was simulated and verified accordingly. Subsequently, the kinematics and dynamics of the spherical rotor was discussed and presented. With these models, we establish the mapping between the angular velocity vectors with the rates of change of the Euler angles and the dynamic behavior of the rotor.

CHAPTER 5

ORIENTATION SENSING METHODOLOGY

To realize the design of a reliable and robust closed-loop control system, it is imperative to have position feedback signals. This chapter introduces a novel method in acquiring the orientation of the spherical rotor. The uniqueness of the proposed scheme exploits the **B**-field which is inherent within the magnetic system and utilizes them for the determination of the rotor position. Ensuing sections details this sensing methodology. The feasibility of this method was validated through simulation.

5.1 PRELIMINARIES

The novelty behind the proposed sensing methodology fully utilizes the characteristics of the magnetic field generated by the source and the sensing output data provided by the magnetic field sensors. The main objective is to acquire the orientation of the rotor at any instantaneous time. Since the spherical actuator pivots about a single point at the origin, we can exploit existing known parameters by vector manipulations for the determination of the rotor orientation. Normalization is another advantage of the proposed method as the derivation only deals with the direction vectors. 3-axis Hall sensors are employed to sense magnetic field density in 3-D space. The hurdle lies in how to relate the magnetic field to be sensed with the rotor orientation. It is even more challenging if the magnetic field distribution does not possess any identifiable or recurring pattern. Hence, it will be insufficient to only identify the measurand but a need to propose a way to configure the field distribution

so as to correlate to the rotor orientation. Understanding the physical constraints and characteristics of various magnetic sources, we propose that a source that generates axial-symmetric field would be ideal. In view of that, ring magnet was recommended as the source in our orientation sensing system.

5.2 METHODOLOGY

The current spherical prototype is designed to generate simultaneous spin motion with positional orientation determined by any convenient set of Euler angles. Kinematics equations provide the mapping between the rotor angular velocity vector and the rates of change of the Euler angles. In view of orientation sensing, what we are interested in essence will be the instantaneous position of the rotor shaft that defines the orientation of rotor. The physical system of the proposed orientation sensing system consists of a ring magnet as the source and two 3-axis Hall sensors. The role of the ring magnet is to generate axial-symmetric field distribution about its axial axis while the 3-axis Hall sensor detects the **B**-field. Figure 5.1 presents the cross-sectional view of the proposed sensing system. As highlighted, the utilization of the ring magnet is to supply and ensure a symmetrical magnetic field distribution about its axial axis. Thus translating to an important and crucial characteristic of the field distribution whereby the field distribution on the radial plane of the ring magnet is identical. Thus, we the following assumption,

Magnetic field from ring magnet is assumed to be axial-symmetric. (A.3)

This axial-symmetric property will essentially collapse a three-dimensional distribution into two-dimensional profile and will be verified empirically in the subsequent section. Visualizing this property from Figure 5.1, we can in fact compute the field distribution on the **YZ** plane and in essence, fully described the field distribution in three-dimensional space. Hence, the following section will illustrate how this planar field distribution can be acquired analytically. Sensing trajectory shown in Figure 5.1 plots the relative path that the sensor maps with the magnetic field generated from the ring magnet when the rotor rotates about its **X**-axis. With the

rotor at the home position, the x -axis of the 3-axis Hall sensor is aligned with rotor Y -axis.

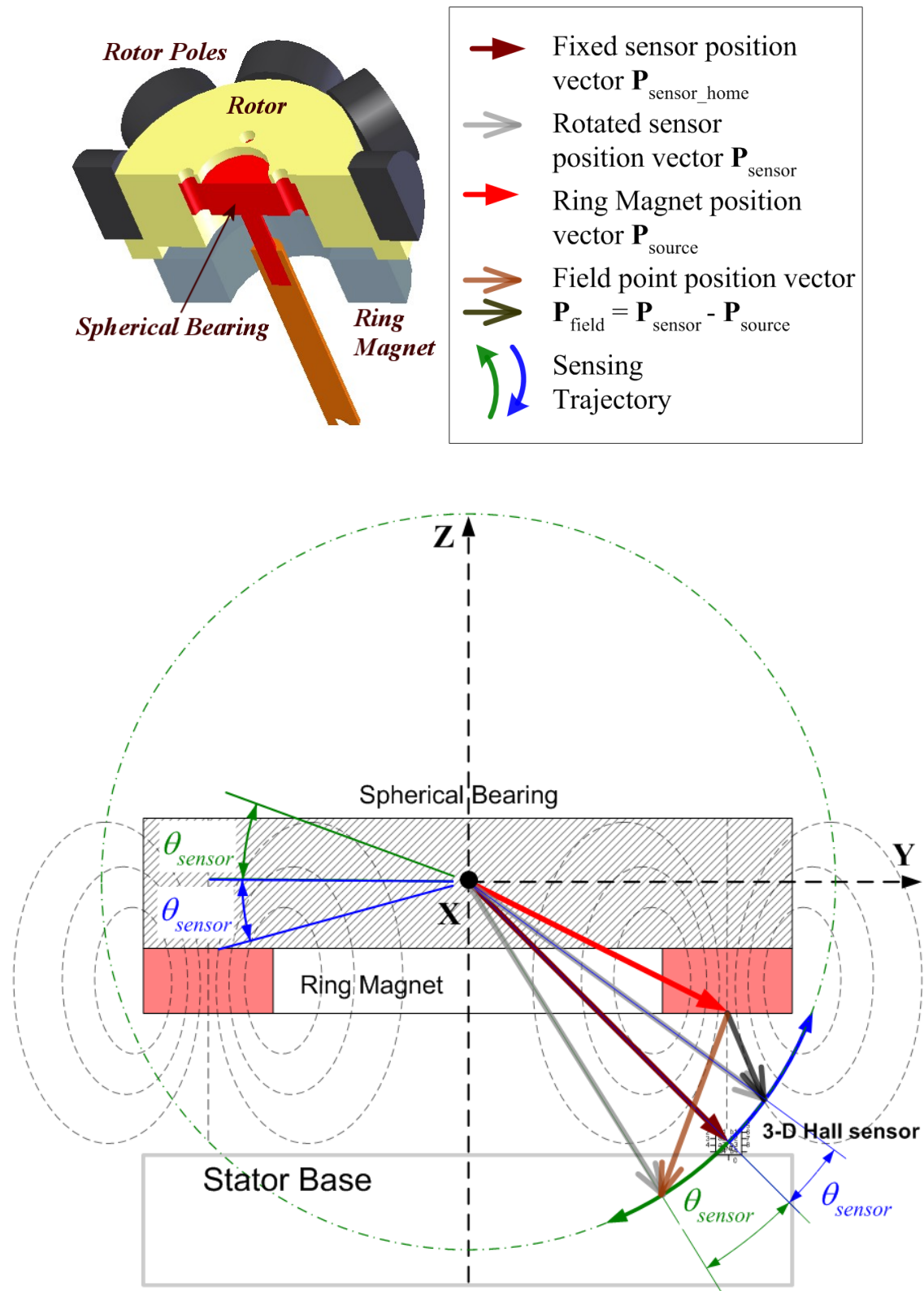


Figure 5.1. Cross section view of spherical actuator with orientation sensing system.

At this position, we define the following position vectors in Table 5.1,

Position vectors	Description
$\mathbf{P}_{\text{sensor_home}}$	Home position vector of Hall sensor
$\mathbf{P}_{\text{source}}$	Home position of the source (Ring magnet)
$\mathbf{P}_{\text{sensor}}$	Rotated position vector of Hall sensor
$\mathbf{P}_{\text{field}} = \mathbf{P}_{\text{sensor}} - \mathbf{P}_{\text{source}}$	Position vector of field point
θ_{sensor}	Rotor rotation angle about X -axis

Table 5.1. Position vectors of sensing parameters.

A field point is defined as a point in space relative from the ring magnet acting as the source. The idea behind formulating the positional vectors listed above is to acquire the relative field point, $\mathbf{P}_{\text{field}}$, viewing from the source. This relative vector can be acquired from its vectorial relationship with $\mathbf{P}_{\text{sensor}}$ and $\mathbf{P}_{\text{source}}$. Initiating from the home position to any final position of the rotor with a rotation angle of θ_{sensor} , the relative motion of $\mathbf{P}_{\text{sensor}}$ is related directly to θ_{sensor} . This signifies that we can correlate the **B**-field sensed by the Hall sensor with the actual rotation angle of the rotor. Therefore, the association of **B**-field along the sensing trajectory with θ_{sensor} allows us to establish the vital relationship between these two parameters for positional sensing using Hall sensors. The objective so far is to obtain the field distribution profile and correlate to the rotor rotation angle. With this relationship cemented, we can either generate a Look-Up Table (LUT) or perform curve fitting between the two quantities plotted. Thereafter, by placing additional sensors, we will be able to ascertain the orientation of the spherical actuator. The acquisition of the **B**-field data can be achieved through experimental or theoretical means. By the experimental approach, **B**-field data from the Hall sensors will be recorded while the rotor rotates about its **X**-axis and hence materializing their relationship. The theoretical approach requires a mathematical model to describe the **B**-field of the ring magnet. In this research work, an analytical magnetic field model was formulated for ring-type magnet in computing magnetic field point in space as shown in the Figure 5.2 below. This generic model is applicable to any ring-type magnets with no dimensional constraint.

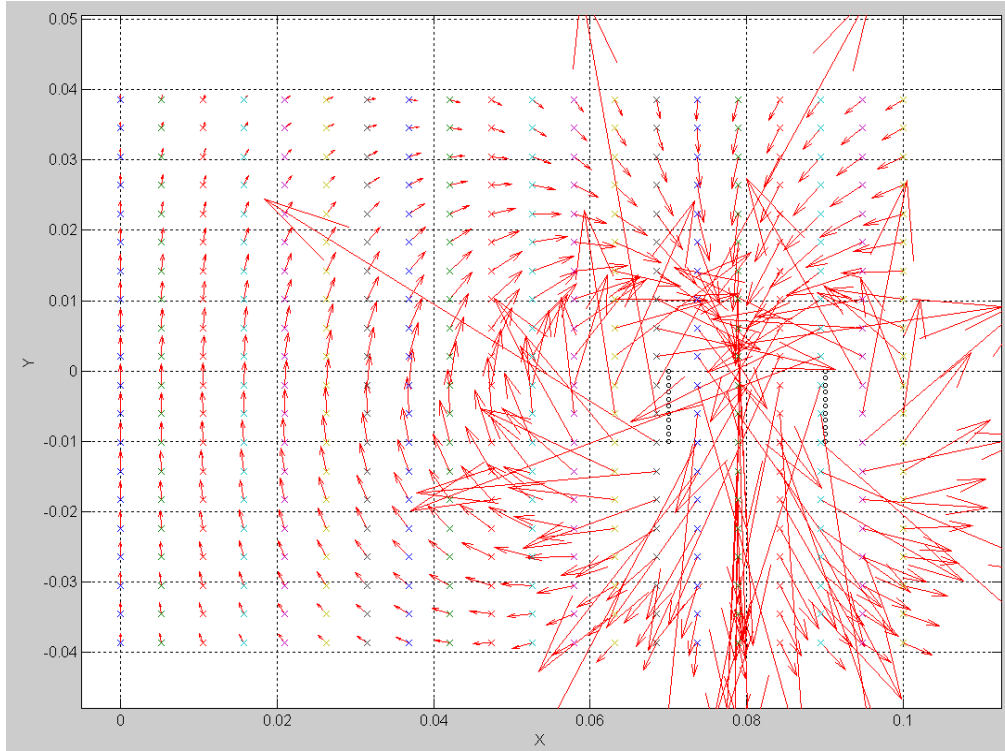


Figure 5.2. Analytical magnetic field model of a ring magnet.

Henceforth, with $\mathbf{P}_{\text{field}}$ as the input parameter, we can generate the same set of $\mathbf{B}$ -field data along the sensing trajectory as the experimental counterpart. By constructing the mathematical relationship between sensing parameters $\mathbf{B}_{\text{sensor}}$ and sensor output, θ_{sensor} , we have established the association between the sensor outputs with the actual rotor orientation. Equipped with this information will therefore allow direct encryption of the sensing parameter to rotor shaft orientation. Regardless whether the data is acquired experimentally or analytically, the correlation will be realized through curve fitting and hence establishing the mathematical relationship or simply forming a LUT for quick referencing. Validation of the above proposed scheme is conducted by building a numerical model and comparing it with our analytical results. Figure 5.3 presents the 3-D numerical model of the ring magnet. From this numerical model, we are able to extract the $\mathbf{B}$ -field distribution along the sensing trajectory. As indicated by the computed results, $\mathbf{B}$ -field fluctuates along the sensing trajectory with unique values on the path of interest. This observation confirms the feasibility of our proposed method for orientation sensing using Hall sensors.

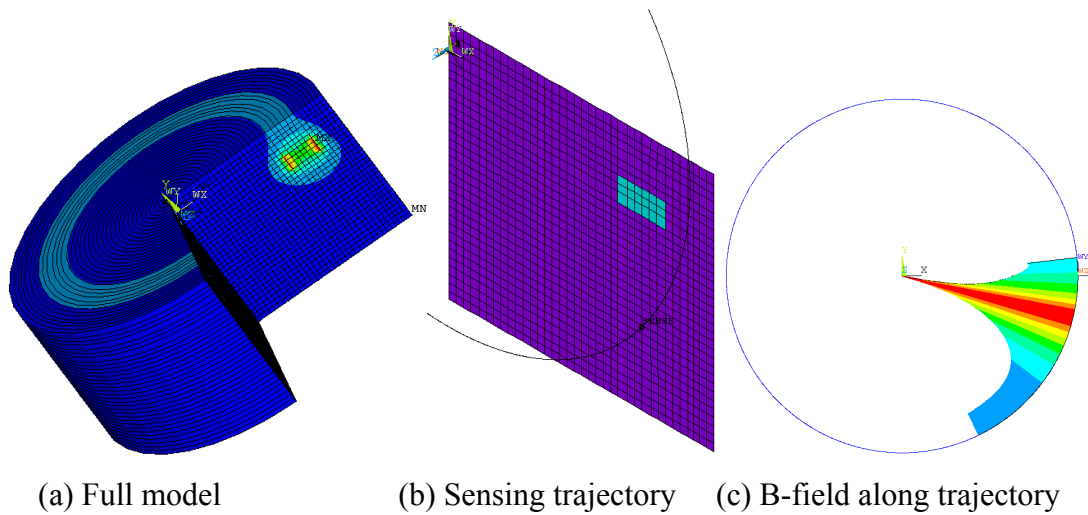


Figure 5.3. Numerical model to evaluate **B**-field along sensing trajectory.

An analytical model was also compiled to calculate the field data replicating exactly the same conditions as the numerical model was subjected to. Figure 5.4 summarizes the comparison between the numerical and analytical results along $\pm 20^\circ$ of the sensing trajectory with 0.1° resolution. Discrepancies between both the calculated results in terms of magnitude can be credited to computational limitation in terms of meshing and averaging of the nodal values of the numerical model. The important aspect of this evaluation clearly substantiates the viability of using the **B**-field data and correlating to the orientation of the rotor. The **B**-field distribution from both the computed results shares the same variational profile along the intended path with unique values corresponding to each respective rotational angle. As mentioned, curve fitting of the data points will establish the mathematical correlation between the two parameters or alternatively, a LUT can be constructed.

From the preceding discussions so far, we have shown and verified how we can relate the output from a Hall sensor to the orientation of the rotor. This establishes the anchoring aspect of our orientation sensing system. Extending this further, we understand that in real-life application, there are other external factors, example, stray fields, manufacturing errors or even inconsistency in materials that may affect the accuracy of our theoretical modeling of the **B**-field.

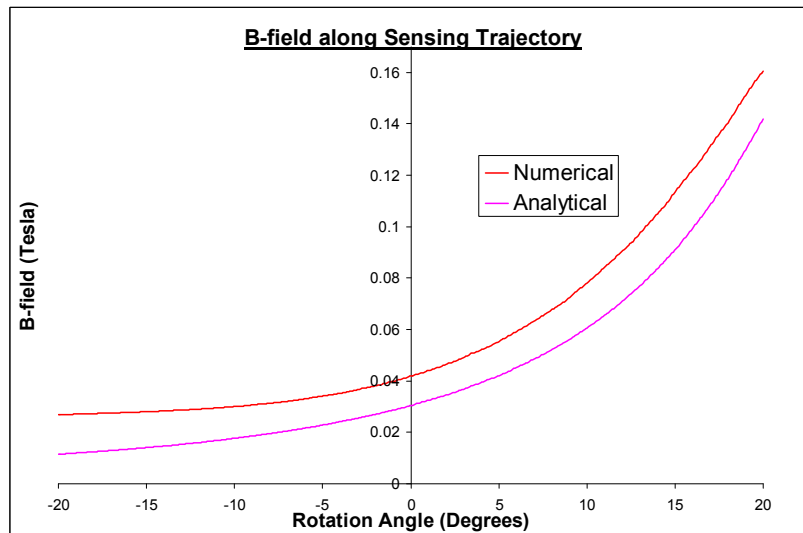
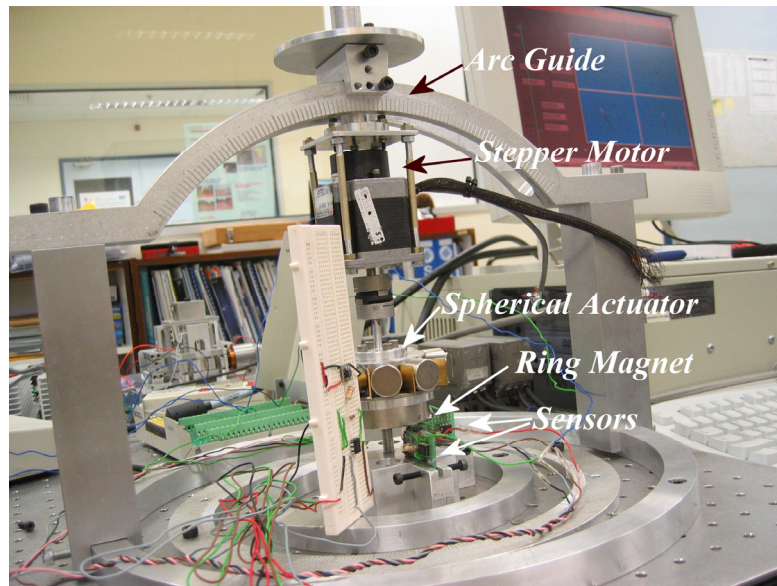


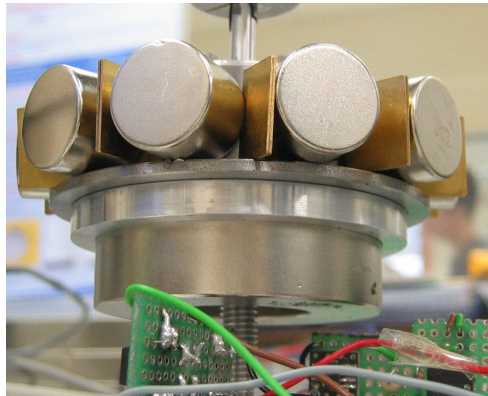
Figure 5.4. Comparison of **B**-field results.

Hence, to minimize these potential hindrances, we can in fact construct the relationship between the sensing and rotor parameters through experimental means. By acquiring **B**-field distribution along the radial plane rotation about the orthogonal axis of the rotor, we will be able to establish their correlation. Figure 5.5 below presents an experimental setup to acquire the **B**-field distribution as cited. The testbed is designed to accommodate the spherical actuator with the proposed sensing approach by incorporating tilt angle control and spin capability about the **Z**-axis. Control of the pan or tilt angle is made available from this platform, which facilitates the calibration phase of the sensors.

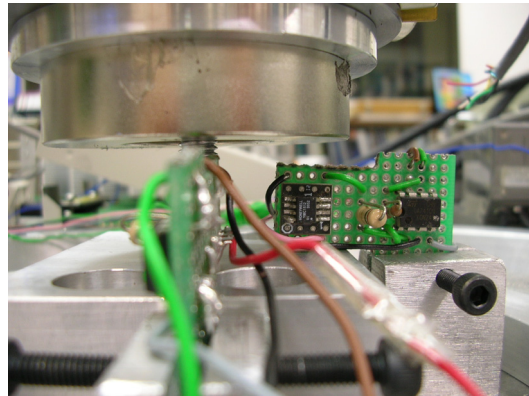
Equipped with the information for the radial **B**-field distribution on a single plane is inadequate to ascertain the spherical actuator orientation. As highlighted in the beginning, we can fully exploit the geometric nature of the spherical actuator to obtain its orientation by vector manipulation. The prerequisite in employing this strategy lies in the determination of any two vectors residing on the rotor **XY** plane. Figure 5.6 illustrates this precondition schematically. If at any instantaneous time, we can acquire any two direction vectors lying on the **X'Y'** plane, we can perform cross product of these vectors to locate **Z'**, which itself points to the orientation of the output shaft. Therefore, the following sections will be dedicated in expounding how we can obtain these two direction vectors from magnetic field sensors.



(a) Orientation sensing using AMR sensors.



(b) Rotor with ring magnet installed.



(c) AMR sensors below ring magnet.

Figure 5.5. Experimental setup for **B**-field measurement.

Before embarking further, Table 5.2 summarizes a list of the known, sensed and derived parameters that we will be using for subsequent elaborations.

Parameters	Description	Known	Sensed	Derived
$\hat{\mathbf{p}}_{\text{sensor}}$	Direction vector of sensor	×		
$\alpha_{zyx} \beta_{zyx}$	Alpha and Beta Euler angle of the local coordinate frame $\{x'''y'''z'''\}$	×		
$\mathbf{B}_{\text{sensor}}$	Magnetic field density vector in sensor local coordinate frame $\{x'y'z'\}$		×	

$\hat{\mathbf{P}}_{\text{rotor}}$	Direction vector of rotor on $z'''x'''$ plane			×
$\hat{\mathbf{P}}_{\text{rotor_home}}$	Direction vector of intersection line between $z'''x'''$ and $z''x''$ plane			×
$\hat{\mathbf{n}}_{z'''x'''}$	Normal direction vector to $z'''x'''$ plane			×
$\hat{\mathbf{n}}_{z''x''}$	Normal direction vector to $z''x''$ plane			×
θ_{sensor}	Actual angle between $\mathbf{P}_{\text{rotor_home}}$ and $\mathbf{P}_{\text{rotor}}$ on $z'''x'''$ plane sensed by Hall sensor			×
γ_{zyx}	Gamma Euler angle of the local coordinate frame $\{x'''y'''z'''\}$			×

Table 5.2. List of known, sensed and derived parameters.

5.2.1 DETERMINATION OF ROTOR ORIENTATION $\hat{\mathbf{P}}_{\text{rotor}}$

According to the **Z-Y-X** Euler angle convention, the position of the sensor can be established by the respective Euler angles α_{zyx} , β_{zyx} and γ_{zyx} ,

$$\hat{\mathbf{P}}_{\text{sensor}} = \mathbf{R} \cdot \hat{\mathbf{P}}_{\mathbf{X}} \quad (5.1)$$

where $\mathbf{R}$ is the rotation operator with the known Euler angles and $\hat{\mathbf{P}}_{\mathbf{X}}$ denotes the direction vector of the global **X**-axis. Output $\mathbf{B}_{\text{sensor}}$ can be acquired directly from the Hall sensor. Our goal is to infer from $\mathbf{B}_{\text{sensor}}$ so as to determine the direction vector $\hat{\mathbf{P}}_{\text{rotor}}$ in the global coordinate frame $\{\mathbf{XYZ}\}$. As expounded earlier in the formulation of the LUT, we are mapping the θ_{sensor} data with the tilt angle between the home and rotated position of the **XY** plane. This set of data is valid for rotation about the orthogonal radial axis along the axial plane of the ring magnet. For an arbitrary rotor rotation, the **B**-field sensed by the sensor may not lie on the plane perpendicular to the sensor. Hence it is important to realize this geometric attribute during the derivation process. As highlighted, the intentional use of a ring magnet is to provide axial-

symmetric magnetic field distribution about the axial axis along the radial plane of the magnet. This can be visualized in the schematic below.

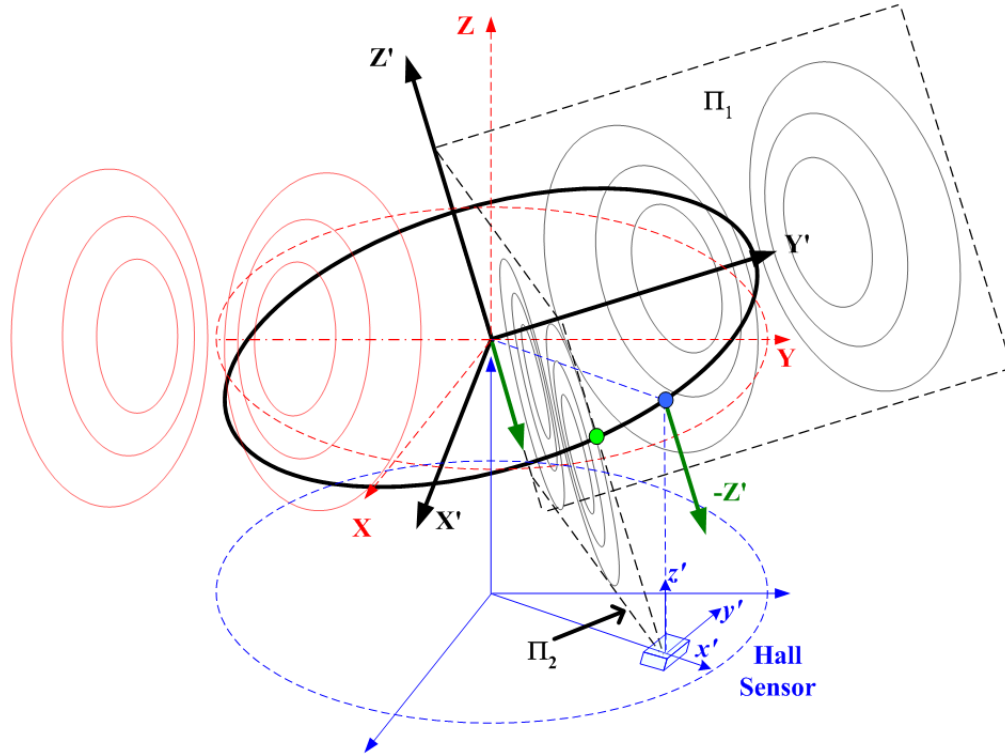


Figure 5.6. Illustration of actual B-field sensed by Hall sensor.

The dotted red lines denote the rotor plane and magnetic field generated from the ring magnet at home position. The black lines correspond to the final position of the rotor plane. At any rotated position of the rotor, the **B**-field distribution will be identical on the radial plane, Plane Π_1 , as it revolves about the axial axis, Z' as shown in Figure 5.6. Due to this axial-symmetric nature of the magnetic field, at any rotated position of the rotor, the direction of the **B**-field picked up by the Hall sensor will be normal to $X'Y'$ plane and parallel to the rotor shaft direction indicated by the green free vector, $-Z'$. Hence, we conclude that at any arbitrary sensor location, the **B**-field detected is not on the sensor $z'x'$ plane, rather, the **B**-field in fact resides in the tilted plane normal to the rotor $X'Y'$ plane as outlined by plane Π_2 . From this geometric constraint, we understand that the **B**-field sensed by the Hall sensor lies on Π_2 which is normal to the rotor $X'Y'$ plane. Therefore, by referencing $|\mathbf{B}_{\text{sensor}}|$ with the LUT, we are actually acquiring θ_{sensor} that relates directly to the angle between the initial XY plane and final position of the rotor $X'Y'$ plane along the tilted plane, Π_2 .

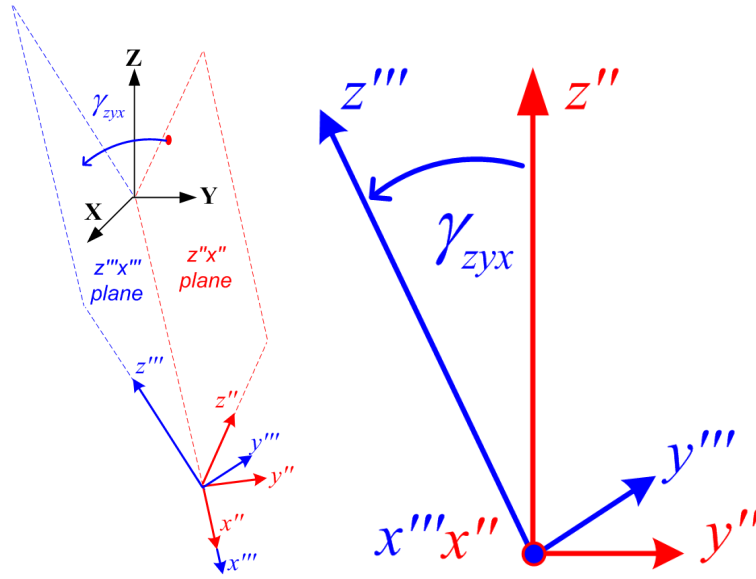


Figure 5.8. Dihedral angle γ_{zyx} .

If we are to scrutinize the geometric constraint of the spherical actuator, one would realize that rotational motion is pivoted about its origin and hence translating to the use of direction vectors for determining orientation. Referencing from Figure 5.7, adhering to the **Z-Y-X** Euler angles convention, we observed that we are able to describe the orientation of $z'''x'''$ plane with prior knowledge of the α_{zyx} , β_{zyx} and γ_{zyx} angles. As of $\hat{\mathbf{P}}_{\text{sensor}}$, α_{zyx} and β_{zyx} are known parameters. The issue will be to determine the γ_{zyx} angle. Since the sensor is sensing the axial-symmetric **B**-field of the source, i.e, the ring magnet, we concur that at any instantaneous rotor position, the direction of the **B**-field being sensed is always normal to the rotor **X'Y'** plane. Therefore, γ_{zyx} is essentially the dihedral angle between $z''x''$ and $z'''x'''$ plane depicted in Figure 5.8. We can compute γ_{zyx} by taking the inner product of the normal vectors between $z''x''$ and $z'''x'''$ plane. Hence we have

$$\hat{\mathbf{n}}_{z''x''} = z'' \times x'', \quad (5.2)$$

$$\hat{\mathbf{n}}_{z'''x'''} = x''' \times \mathbf{B}_{\text{sensor}}. \quad (5.3)$$

Consequently we acquire,

$$\gamma_{zyx} = \cos^{-1} \left(\frac{\hat{\mathbf{n}}_{z''x''} \cdot \hat{\mathbf{n}}_{z'''x'''}}{|\hat{\mathbf{n}}_{z''x''}| |\hat{\mathbf{n}}_{z'''x'''}|} \right). \quad (5.4)$$

As we are aware of the fact that the spherical actuator is free to rotate about its origin. This condition entails the need for directional properties of the γ_{zyx} . This essential data can again be inferred from the parameter, $\mathbf{B}_{\text{sensor}}$. According to Figure 5.6, the 3-axis Hall sensor is aligned with its x -axis along the radial direction. Thus, at any rotor position, the direction vector of $\mathbf{B}_{\text{sensor}}$ will either fall in the second and fourth quadrant or the first and third quadrant of the sensor coordinate frame. Figure 5.9 illustrates this precondition. In summary, the direction of γ_{zyx} can be determined by the following conditions,

$$\text{sgn } B_y = \text{sgn } B_z \Rightarrow \gamma_{zyx} \text{ is } -\text{ve}, \quad (5.5)$$

$$\text{sgn } B_y \neq \text{sgn } B_z \Rightarrow \gamma_{zyx} \text{ is } +\text{ve}, \quad (5.6)$$

$$B_y = 0 \Rightarrow \gamma_{zyx} = 0. \quad (5.7)$$

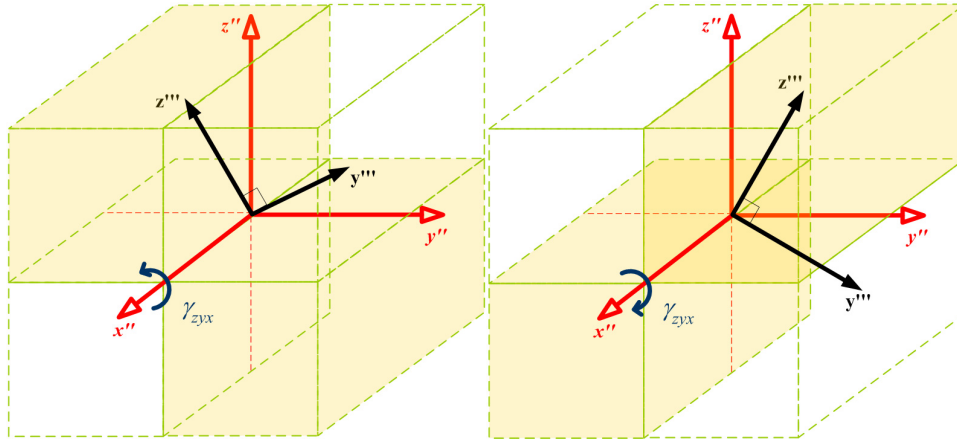


Figure 5.9. Condition for dihedral angle γ_{zyx} .

With all three Euler angles known, α_{zyx} , β_{zyx} and γ_{zyx} , we, in fact, establish the plane that contains the origin and sensor position and is normal to the tilted rotor $\mathbf{X}'\mathbf{Y}'$ plane as illustrated in Figure 5.10. Our objective is to locate $\hat{\mathbf{P}}_{\text{rotor}}$ in the global coordinate frame $\{\mathbf{XYZ}\}$. Prior to that, from the above sensing parameter, θ_{sensor} and computed information, we are only able to determine the direction vector of ${}^{x'''}y''{}^{z'''}\hat{\mathbf{P}}_{\text{rotor}}$ in the local coordinate frame $\{x''y''z''\}$ with reference to ${}^{x'''}y''{}^{z'''}\hat{\mathbf{P}}_{\text{rotor_home}}$. To ascertain $\hat{\mathbf{P}}_{\text{rotor_home}}$, we recognize that it is geometrically the line of intersection between the global $\mathbf{XY}$ plane and the $z''x''$ plane.

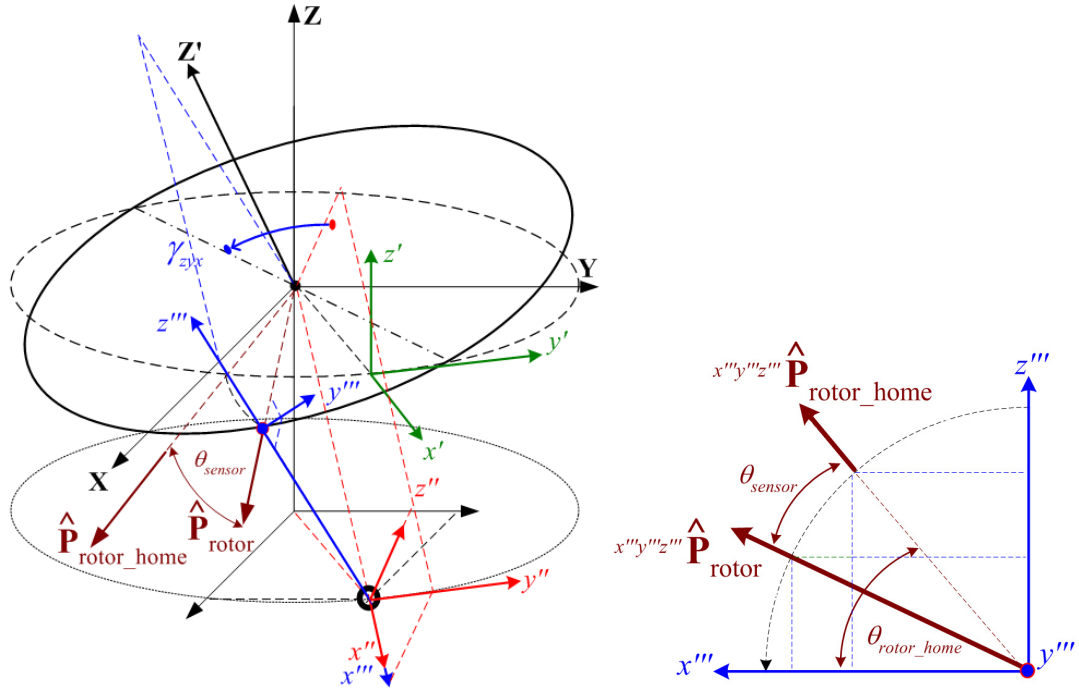


Figure 5.10. Locating $\hat{\mathbf{P}}_{\text{rotor}}$ in local coordinate frame $\{x'''y'''z'''\}$.

Thus, we compute $\hat{\mathbf{P}}_{\text{rotor_home}}$ from

$$\hat{\mathbf{P}}_{\text{rotor_home}} = y''' \times \mathbf{Z}. \quad (5.8)$$

Subsequently we obtain ${}^{x'''y'''z'''}\hat{\mathbf{P}}_{\text{rotor_home}}$ by

$${}^{x'''y'''z'''}\hat{\mathbf{P}}_{\text{rotor_home}} = {}_{XYZ}^{{}^{x'''y'''z'''}}\mathbf{R} \cdot \hat{\mathbf{P}}_{\text{rotor_home}}. \quad (5.9)$$

With this acquired information, according to Figure 5.10, we are able to determine,

${}^{x'''y'''z'''}\hat{\mathbf{P}}_{\text{rotor}}$ with reference to ${}^{x'''y'''z'''}\hat{\mathbf{P}}_{\text{rotor_home}}$ as

$${}^{x'''y'''z'''}\hat{\mathbf{P}}_{\text{rotor}} = \begin{bmatrix} \cos(\theta_{\text{rotor_home}} - \theta_{\text{sensor}}) \\ 0 \\ \sin(\theta_{\text{rotor_home}} - \theta_{\text{sensor}}) \end{bmatrix}. \quad (5.10)$$

Therefore mapping back to global coordinate frame, we have

$$\hat{\mathbf{P}}_{\text{rotor}} = \mathbf{R}_{\text{XYZ}} \cdot {}^{x''''y''''z''''} \hat{\mathbf{P}}_{\text{rotor}} \quad (5.11)$$

where $\mathbf{R}_{\text{XYZ}}$ has the input Euler angles as α_{zyx} , β_{zyx} and γ_{zyx} . From the above, we have described the extraction of a direction vector of the rotor from the $\mathbf{B}$ -field furnished by a 3-axis Hall sensor. Hence, we can deduce the rotor orientation by the cross product of two such direction vectors as shown in Figure 5.11.

$$\hat{\mathbf{P}}_{\text{rotor}} = \hat{\mathbf{P}}_{\text{rotor_2}} \times \hat{\mathbf{P}}_{\text{rotor_1}} \quad (5.12)$$

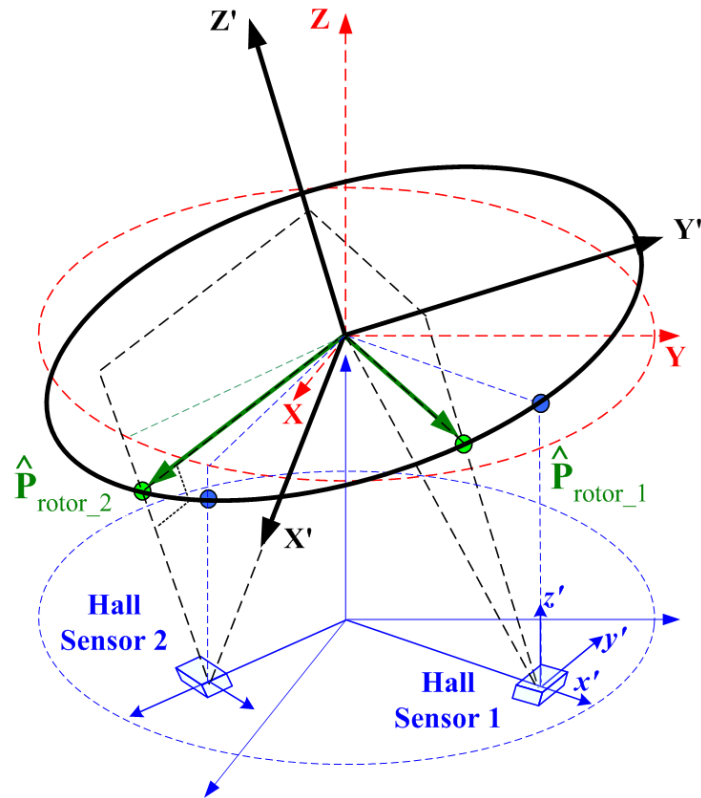


Figure 5.11. Determination of rotor orientation from two known direction vectors.

From the foregoing discussions, we have presented the methodology behind the proposed orientation sensing scheme. In summary, we can categorize the sensing algorithm into two modules. The calibration module, where axial-symmetric magnetic field of the source are being consolidated along the rotation angle of the rotor. This set of calibration data will be used to reference $\mathbf{B}_{\text{sensor}}$ during operation. The second

component will be the sensing algorithm where it describes how the rotor position can be inferred from the sensor output, $\mathbf{B}_{\text{sensor}}$. Figure 5.12 summarizes the proposed sensing methodology.

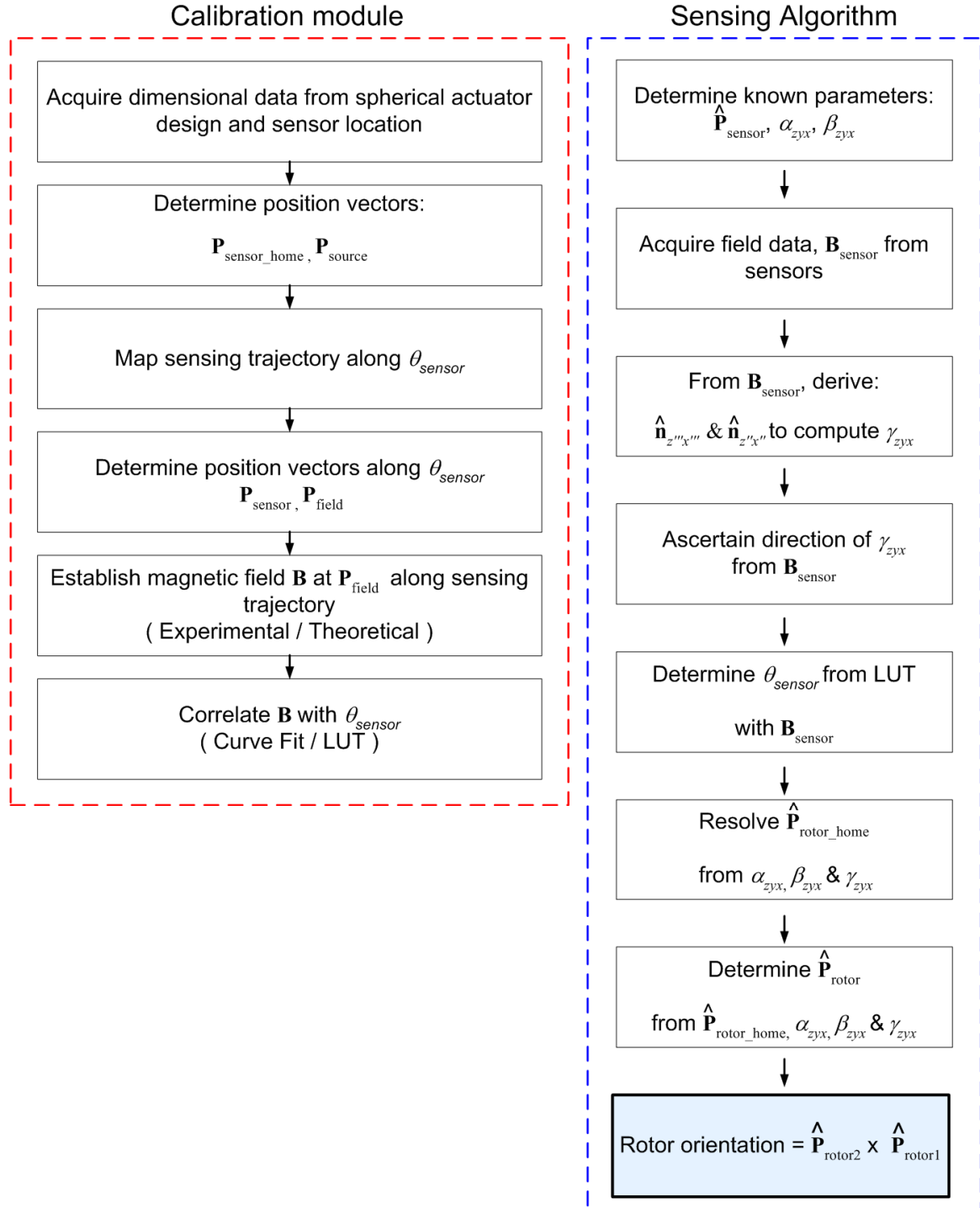


Figure 5.12. Orientation sensing methodology.

5.3 VALIDATION OF SENSING METHODOLOGY

In order to verify the feasibility of the proposed orientation sensing method, a simulation model was built using Matlab. First, the rotor is presented by a plane pivoted at the origin with rotation defined by the **Z-Y-Z** Euler angles convention. Next, the magnetic sensors positions are positioned below the rotor plane replicating the actual detection of the **B**-field from the ring magnet. **B**-field distribution data can be computed using our analytical field model as explained before. In this simulation, the assimilated **B**-field by the sensors is taken from the precondition that the sensor is sensing the field along the plane normal to the rotated rotor plane as described previously. A graphical representation of the simulation model is presented in Figure 5.13. The brown bold line with a circle symbolizes the rotor shaft with the rotated rotor plane represented by the blue plane as shown. From the simulated results, the green circle denotes the actual point in space with the inputs of the known, sensed and derived parameters described in the preceding sections. As shown and verified, this point lies on the rotated plane of the rotor and therefore validates the proposed sensing methodology.

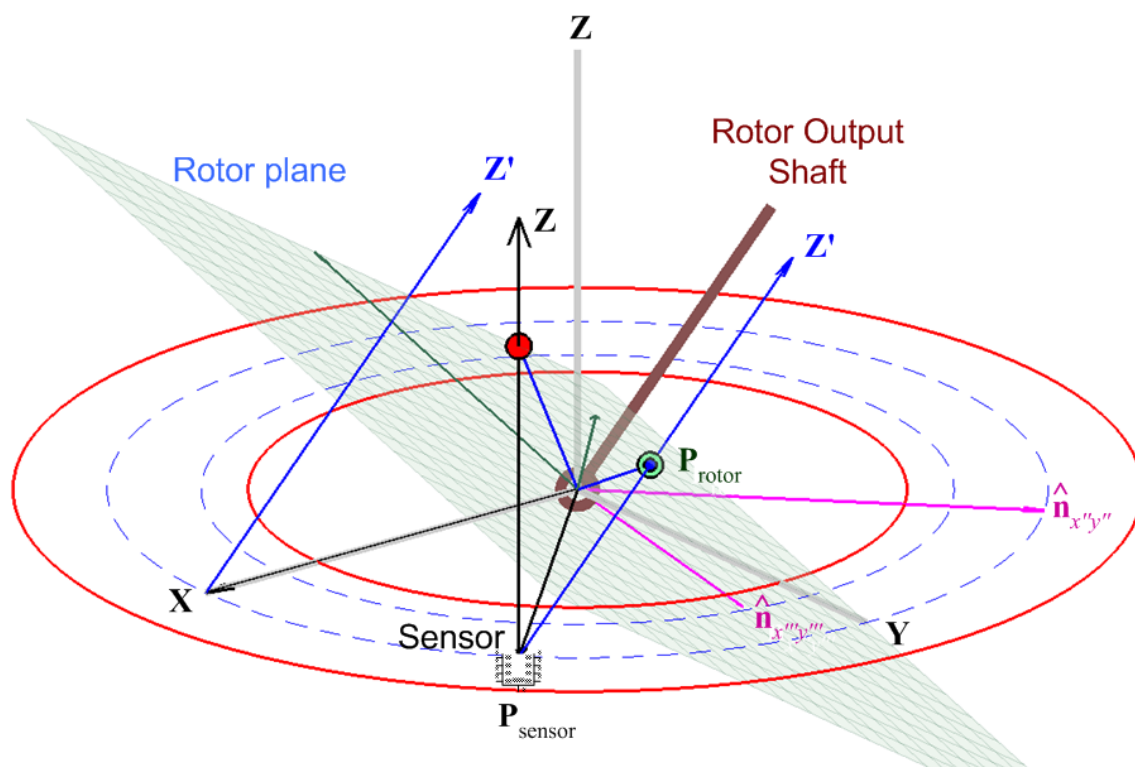


Figure 5.13. Simulation model.

Experimental verification was conducted using Anisotropic Magneto-Resistive effect (AMR) sensors due to their availability. These AMR sensors detect directional change in the magnetic field. Two AMR sensors were deployed onto the testbed as shown in Figure 5.5. Analogue output signals from sensors are acquired through data acquisition card and interfaced to a PC for computations. Axial-symmetry property of the magnetic field was verified by the sensors as it records identical field distribution when rotor spins about its **Z**-axis and therefore validated Assumption (A.3).

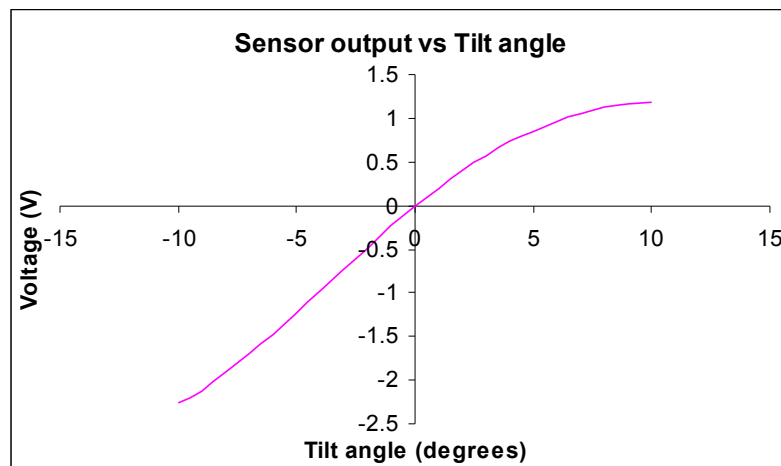


Figure 5.14. Tilt angle of rotor with reference to AMR sensor output.

Figure 5.14 presents a calibration graph for the AMR sensor employed for the experimental setup. For the initial calibration, sensor output signals are acquired at 1° angular step with rotation angle of $\pm 11^\circ$. From the graph, we can correlate the sensor output with the rotor tilt angle analytically or by the use of LUT as was discussed in the preceding sections.

In addition, a third AMR Sensor 3, was deployed along the center plane of the rotor with the intention of exploring the measurement of the rotor spin angle. Figure 5.15 depicts the setup and a plot of the sensor output at various tilt angles when the rotor spins about its **Z**-axis. Observation shows that truncations of the signal occur at incremental steps of the tilt angle. The consistent field pattern detected by sensor 1 and 2 at various tilt angles when the rotor is spinning has again attests to the axial-symmetric field properties of the ring magnet.

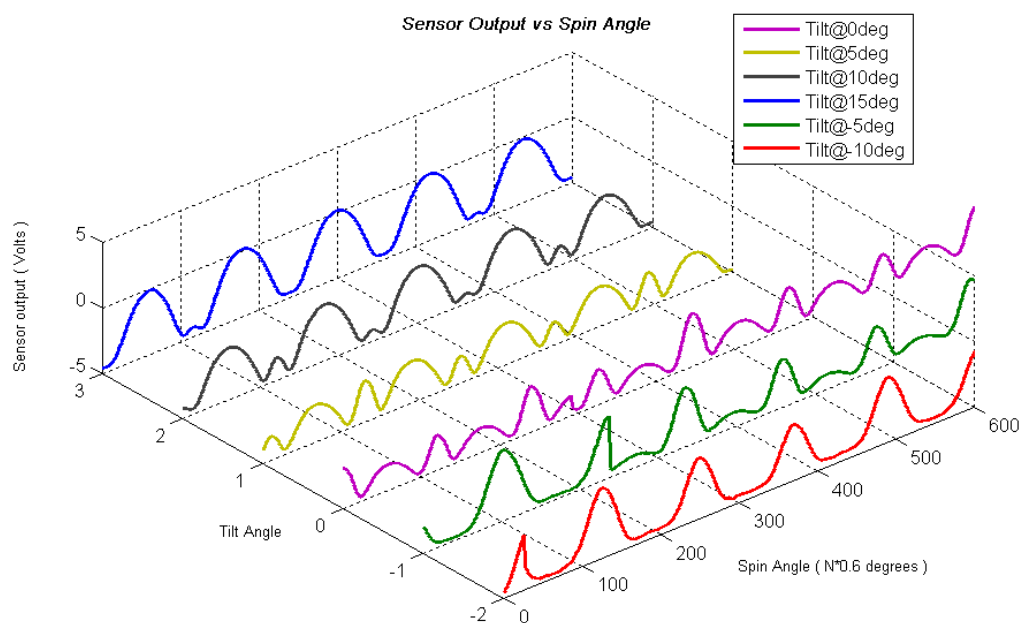
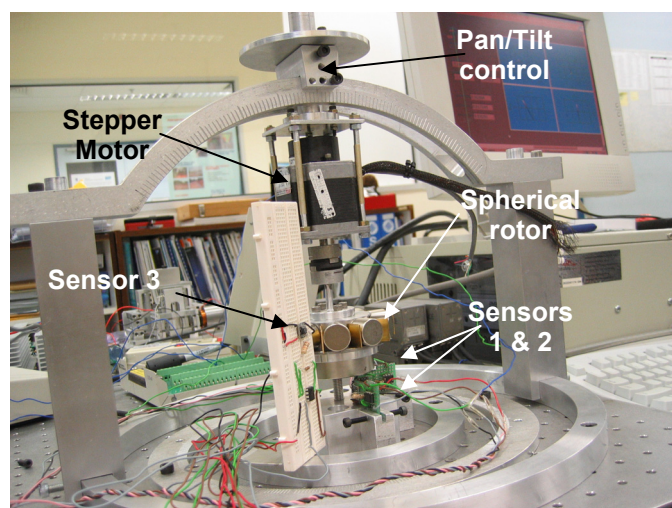


Figure 5.15. AMR sensor output at different tilt angles.

A LabVIEW program was compiled to acquire the rotor tilt angle versus the AMR sensor outputs as plotted in Figure 5.14. The basic function of the program was to initialize the calibration and sensing module and carry out computation and interpolation of the rotor orientation from the signals from the sensors. Upon completion, the orientation and angular velocity of the rotor shaft are displayed graphically as shown in Figure 5.16.

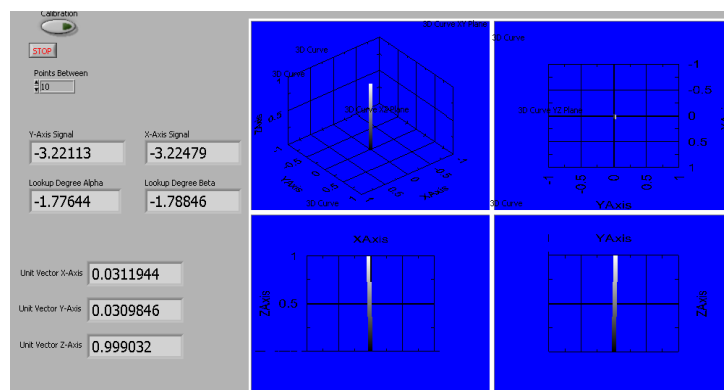
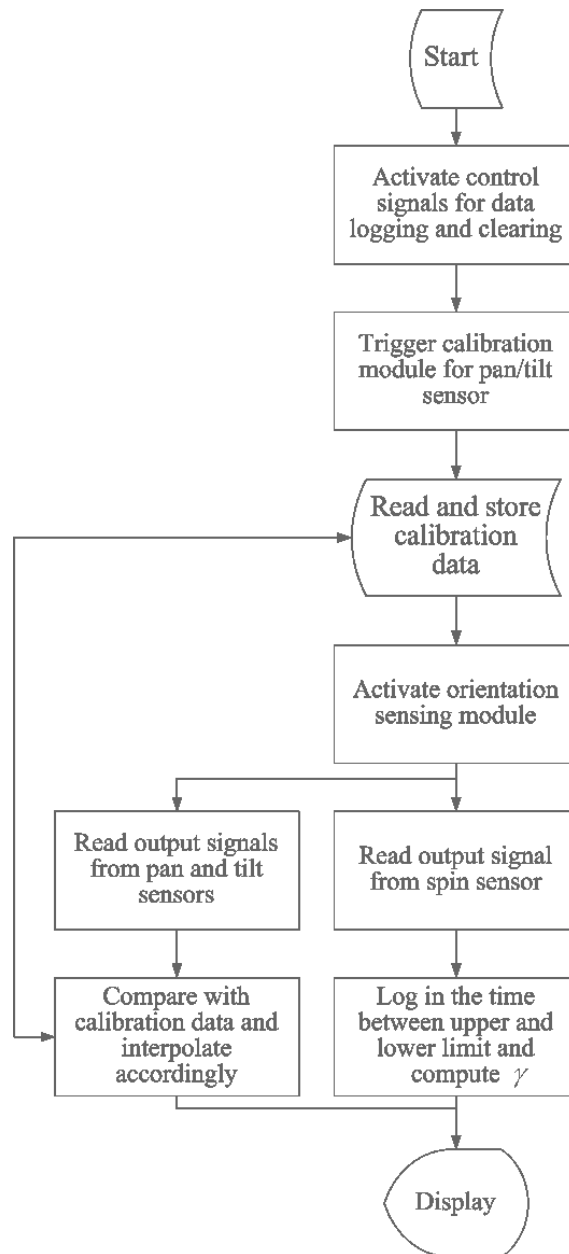


Figure 5.16. Graphical User Interface (GUI) of orientation sensing system.

5.4 CHAPTER SUMMARY

A novel approach in orientation sensing for spherical actuator is presented in this chapter. The contactless sensing strategy employed by the proposed scheme utilizes magnetic field sensors that detect flux density of the magnetic source. This approach overcomes teething issues like surface processing translating to high fabrication cost as encountered by other spherical sensing methods. The originality of the intended methodology exploits on the knowledge of magnetic field vector at the point of interest and correlating the sensors output to the actual sensing parameters. Hence providing the mathematical relationship between the actual rotor orientation to the sensors output which can be implemented for real-time control. The main features of this unique method are:

- Contactless sensing translating to reduction in inertial and frictional loads.
- Utilization of existing magnetic field as measurand translating to simplicity in design.
- Preconditioning of surface to be sensed is not necessary avoiding the complexities during surface preparation.
- Orientation can be determined with only two units of sensors resulting in cost efficiency.

We have presented the orientation sensing methodology and also proven the feasibility of this approach. Future work will be focused on the application of 3-axis Hall sensors onto the existing prototype. Impending challenges that we foresee will be the repeatability and detection limits of the Hall sensors. With successful validation of the proposed theoretical model to experimental measurements, this approach possesses great potential for a low cost orientation sensing system applicable for spherical actuators.

CHAPTER 6

NUMERICAL MODEL AND DISCUSSIONS

In today's context, Finite Element Analysis (FEA) is the most popular numerical method in solving an extensive range of engineering problems. This powerful method is versatile and readily adaptable to a wide variety of applications. Numerous commercially available FEA software packages are available that enable users to solve complex system without having to develop specific algorithms. FEA software ANSYS was utilized in this work to model and analyze the proposed spherical actuator. The advantage of a numerical model is its flexibility in conducting various type of analysis based on varying preconditions. Moreover, optimization with respect to different criterion can be conducted. This chapter will present a numerical model of the prototype constructed for electromechanical and magnetic field analysis. In the ensuing sections, the numerical model will be use to validate the magnetic field and torque model proposed in the preceding chapters.

6.1 NUMERICAL MODEL

In ANSYS, the solution of magnetic field problems is commonly obtained using potential functions. There are two kinds of potential functions, the magnetic vector potential and the magnetic scalar potential. These functions are used depending on the problem to be solved. Factors affecting the choice of potential include field dynamics, field dimensionality, source current configuration, domain size and discretization. In the case of our spherical actuator, analysis will be conducted in static field theory

which is independent of time. This restrictive theory applies to a wide range of important phenomenon involving steady currents and stationary charge distributions. Time dependent elements in the Maxwell's equations are regarded as negligible,

$$\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial \mathbf{B}}{\partial t} = 0.$$

From this, we can uncouple the field equations into magnetostatic and electrostatic equations,

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \quad \text{(Magnetostatic)}$$

$$\begin{aligned} \nabla \times \mathbf{E} &= 0 \\ \nabla \cdot \mathbf{D} &= \rho \end{aligned} \quad \text{(Electrostatic)}$$

$$\begin{aligned} \mathbf{B} &= \mu_0 (\mathbf{H} + \mathbf{M}), \\ \mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P}. \end{aligned} \quad \text{(Constitutive relations)}$$

These uncoupled equations alone with their constitutive relations can be solved independently for the magnetic and electric fields. Hence, the scalar potential method will be utilized in solving the 3-D magnetostatic field. The solution algorithm in ANSYS solves magnetostatic problems by satisfying the Maxwell's equations and the constitutive relation listed above. There are basically three different solution strategies that ANSYS offer. They are the Reduced Scalar Potential (RSP), the Difference Scalar Potential (DSP) and the General Scalar Potential (GSP) strategy. Each strategy is formulated to solve various problems scenario according to the elements residing in the problem domain. For our case in specific, we have basically three regions of interest. Namely, the conducting region at the spherical bearing (Region I), the nonconducting permeable region of the permanent magnet rotor pole (Region II) and the free space region of the stator pole and the surrounding airgap and free space (Region III). The electromagnetic field regions are depicted in Figure 6.1. Hence, the DSP solution option was selected for the analysis. The element types used for the numerical model are SOLID98 and SOURC36. INFIN47 is employed to model the boundary surface of a 3-D unbounded field problem.

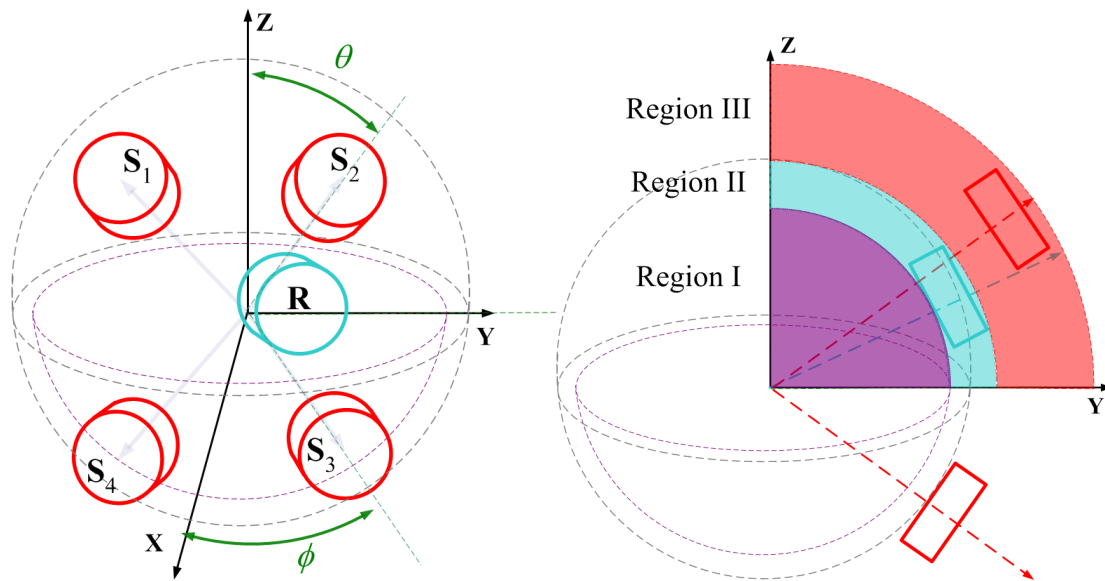


Figure 6.1. Electromagnetic field regions of spherical actuator.

Element Type	Descriptions
SOLID98	Model bearing and rotor pole
SOURC36	Model stator pole
INFIN47	Model boundary surface
Input parameters	Descriptions
I	Input current
n	Number of turns in stator coil
$Airgap$	Airgap between rotor and stator poles
$\mu, H_c, BH\text{-curve}$	Materials properties
$r_{Rotor}, r_{Stator}, l_{Rotor}, l_{Stator}$	Dimensions of rotor and stator poles
$R_{Rotor}, \theta_{Rotor} \text{ and } \phi_{Rotor}$	Spherical coordinates of rotor position
$R_{Stator}, \theta_{Stator} \text{ and } \phi_{Stator}$	Spherical coordinates of stator position

Table 6.1. Parameters of numerical model.

Table 6.1 summarizes the input parameters of our numerical model. From the formulated numerical model, we can relate key parameters listed in Table 6.1 to the magnetic and mechanical output of the spherical actuator. An APDL (ANSYS Parametric Design Language) code is created for the numerical model so as to facilitate the analysis of various configuration associated with the prototype design.

This will also forms the basis for future design optimization. Figure 6.2 presents the actual numerical model within the ANSYS environment.

Figure 6.3 charts the process algorithm undertaken in the construction and analysis of the numerical model. This parametric model is able to analyze various configuration of the spherical actuator and can be served as a reference to the analytical models proposed in this dissertation. In the ensuing portion of this chapter, the validation of the magnetic field derived from elliptical integrals and the newly formulated torque model based on the magnetic dipole moment will be presented.

6.2 VALIDATION OF MAGNETIC FIELD MODEL

Employing the above results for the field calculation of a single current loop, we are able to input them into mathematical software for computation. Using Matlab, a source code was written to compute the full three-dimensional field for a planar current loop. The software code has the following input parameters compiled in Table 6.2.

Matlab parameters	Symbolic representation	Description
I	I	Input current
ext_R	$\mathbf{x}'$	Current loop radius
r	$\mathbf{x}$	Field point
phi	ϕ	Azimuthal angle
theta	θ	Polar angle
not	none	Number of turns
nol	none	Number of layers
maxy	none	Limit of y field points
maxz	none	Limit of z field points
wire_thk	none	Wire thickness
res	none	Resolution between the field point limits

Table 6.2. Parameters of analytical model.

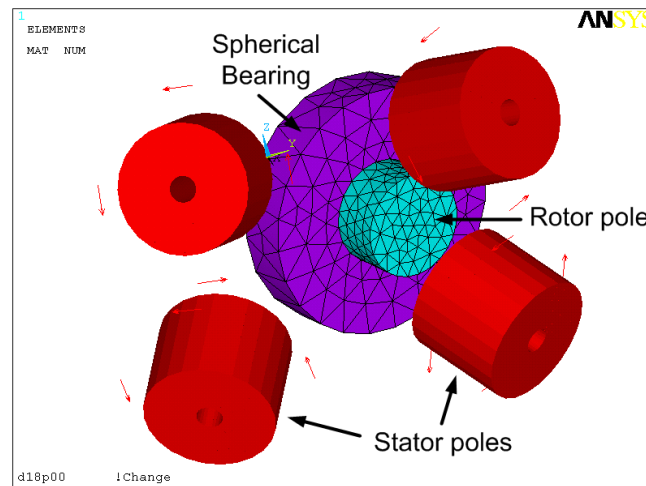


Figure 6.2. 3-D numerical model.

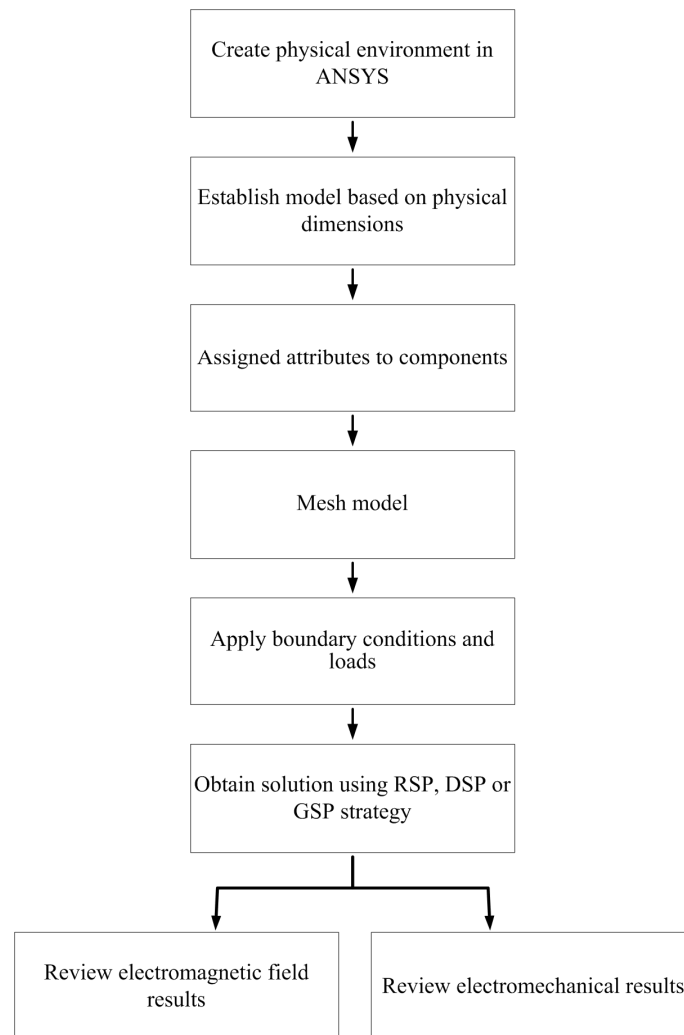


Figure 6.3. Electromagnetic field regions of spherical actuator.

From the analytical model, we are able to acquire the full field description of a current loop as shown in Figure 6.4. This is for the case of a current loop with radius 0.001m and a current input of $I = 5\text{A}$. The green dot in the diagram symbolizes the cross sectional area of wire and the **B**-field is represented vectorially in space by the red arrows. Again, due to symmetry as highlighted previously, full field is obtained with the computation on the **YZ** plane.

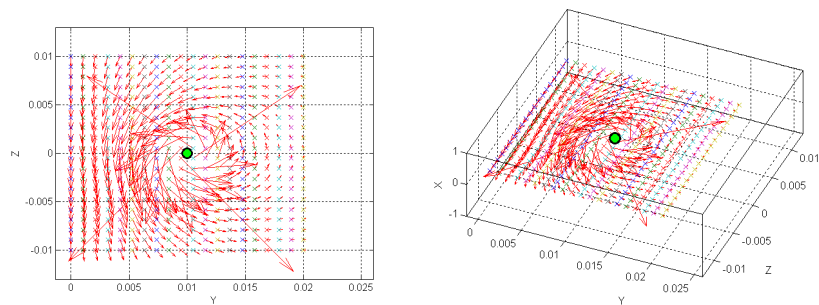


Figure 6.4. Graphical representation of **B**-field results from analytical model.

To extend this computational model further, we can consider the single current loop as the fundamental module with additional circuits through summation. Hence, to model a commonly found physical system like a solenoid, one will only need to sum the respective fundamental module based on the various dimensional and input parameters. Figure 6.5 illustrates the above mentioned rationale with a simple solenoid with a total of four turns. Moreover, we will also exploit the versatility of using this fundamental module for the field calculation of permanent magnets. The ensuing section will demonstrate these claims.

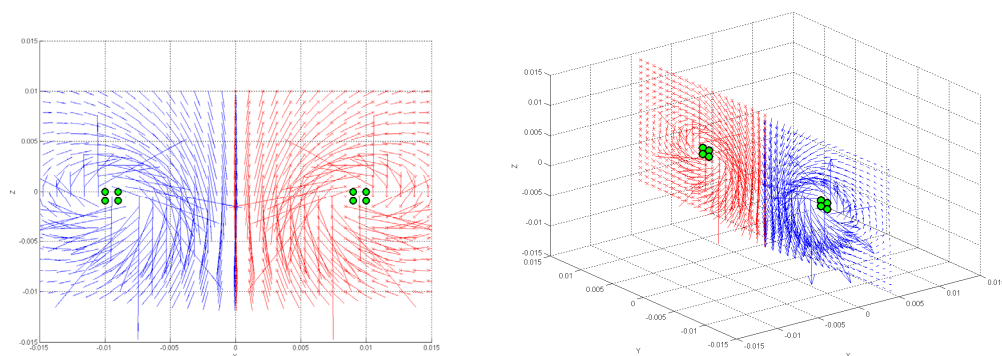


Figure 6.5. Graphical representation of **B**-field of a simple solenoid with $\text{not}=\text{nol}=2$.

6.2.1 MAGNETIC FIELD MODEL FOR CURRENT LOOP

Entailing from the above proposed analytical field model for a current loop using elliptic integrals, the next important step is to ascertain the accuracy of this new model. In order to validate this, results from our model will be compared with the verification manual compiled by ANSYS, Inc. The ANSYS test library and cases represent comparisons of ANSYS solutions with known theoretical solutions, experimental results, or other independently calculated solutions. A survey of the results comparisons in this manual shows an average accuracy within 1-2% of the target solution. Thus, the following section will illustrate the precision of our proposed analytical model as compared to ANSYS test case. Test case number 168 from ANSYS verification manual release 10.0 is singled out as it models the magnetic field in a nonferrous solenoid. Axial magnetic flux density was computed at the center, end and a distance of from the coil. With our proposed model, we will model this particular test case and compare our analytical results with ANSYS published results. A schematic of the problem sketch is depicted in Figure 6.6 below.

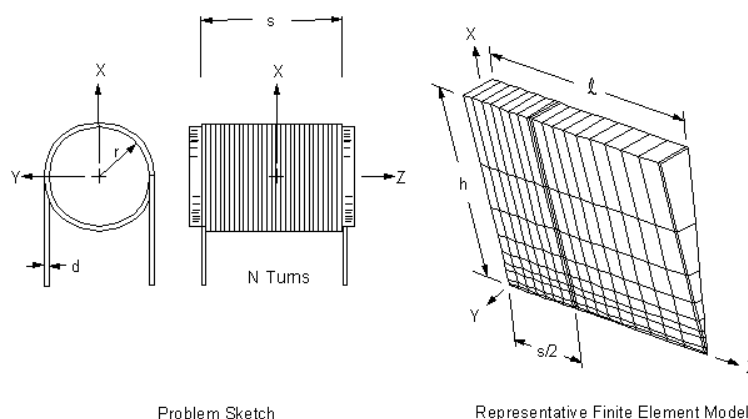


Figure 6.6. ANSYS VM168 magnetic field problem sketch.

Table 6.3 summarizes the magnetic field calculations by ANSYS and our analytical model with the target values [73].

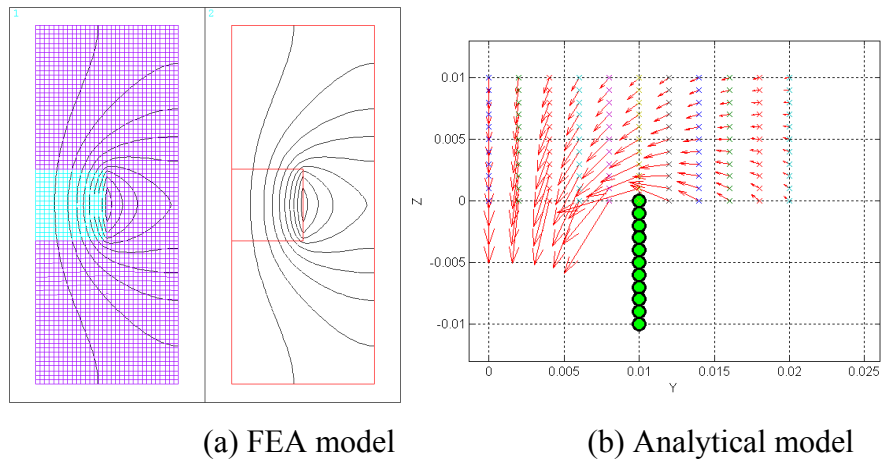
	Target	ANSYS	Ratio	Model	Ratio
B_z ($\times 10^{-6}$)T at $z = 0$	1120	1121	~ 1	1120	1
B_z ($\times 10^{-6}$)T at $z = 0.1905$ m	2.12	2.12	1	2.22	1.04

Table 6.3. Comparison of **B**-field.

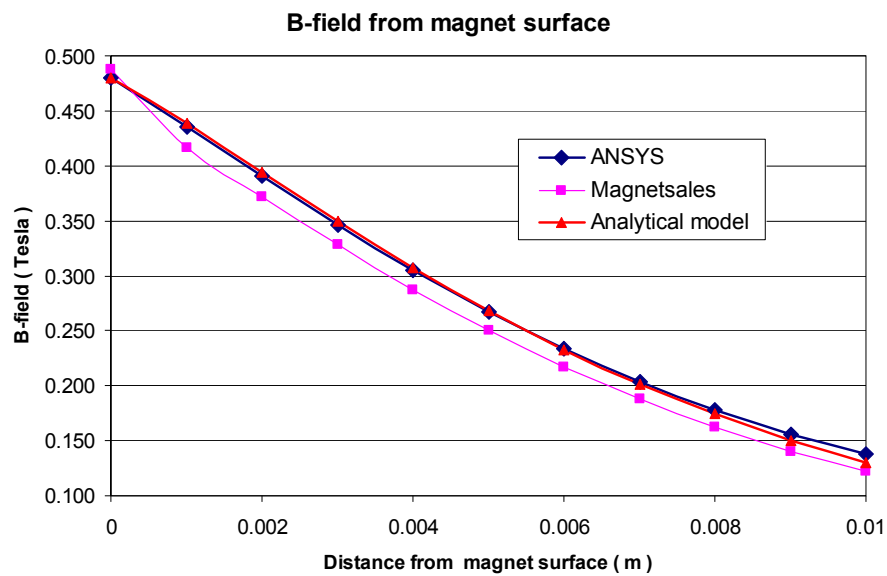
From the above results, we can conclude that the proposed analytical model is able to model the magnetic field of a solenoid to a high degree of accuracy as compared to the Finite Element Method using ANSYS and the target values. As the field model is derived from fundamental laws and solved directly using elliptical integrals explains the precision of the computed results. Hence, we have verified the validity of the proposed theoretical model.

6.2.2 MAGNETIC FIELD MODEL FOR PERMANENT MAGNET

What we have demonstrated so far was the capability and ability of the proposed analytical model in modeling the magnetic field of a current carrying physical system. Practically in all electromechanical systems, employments of permanent magnets are prevalent. Therefore, it would be ideal if the proposed model is also able to compute the field of permanent magnets as well. Hence, we build a cylindrical permanent magnet model using ANSYS and compare it with the values computed from our analytical model and also referencing it to a commercial magnetic field data [88]. The theory behind using our analytical approach for the modeling of permanent magnet was expounded in Chapter 2. From the current prototype setup, we will model the permanent magnet poles using a physical system of solenoid energized by the equivalent current density. Figure 6.7 presents the ANSYS model along with our analytical model and the computed **B**-field results from the surface along the axial direction of a 20x10mm permanent magnet. From the graphical standpoint, we observed that the analytical model accurately described the **B**-field of the permanent magnet as compared to the FEA model. Consequently, we have shown and proven the flexibility and accuracy of our analytical model in the computation of magnetic field for current loop and magnets. Many theoretical problems are not realistic in that the assumptions necessary to obtain a closed-form analytical solution make the mathematical model depart from a practical application problem.



Distance (m)	ANSYS (Tesla)	Magnetsales (Tesla)	Ratio	Model (Tesla)	Ratio
0.000	0.47941	0.4879	1.02	0.5033	1.05
0.001	0.43555	0.4163	0.96	0.4581	1.05
0.002	0.39069	0.3719	0.95	0.4111	1.05
0.003	0.34668	0.3284	0.95	0.3646	1.05
0.004	0.30534	0.2875	0.94	0.3202	1.05
0.005	0.26732	0.2501	0.94	0.2793	1.04
0.006	0.23355	0.2168	0.93	0.2424	1.04
0.007	0.20385	0.1875	0.92	0.2099	1.03
0.008	0.1782	0.1622	0.91	0.1816	1.02
0.009	0.1562	0.1404	0.90	0.1573	1.01
0.010	0.13744	0.1218	0.89	0.1364	0.99



(c) Axial **B**-field plot.

Figure 6.7. Analytical and numerical modeling of a 20x10mm permanent magnet.

By encompassing this analytic approach in the analysis of electromagnetic systems, not only are we able to formulate the magnetic field mathematical model in closed-form, the calculation time and efficiency is greatly enhanced as compared to the widely used numerical methods. Hence, we have resolved the fundamental but yet crucial element in the analysis of electromagnetic system.

6.2.3 MAGNETIC FIELD MODEL FOR HIGH PERMEABLE REGION

This section presents the verification with the inclusion of ferromagnetic material in the spherical actuator system. As described in Chapter 2, the method of imaging was utilized for modeling such region. This region corresponds to the ferrous material of the spherical bearing where the rotor poles are in contact. Figure 6.8 depicts the 3-D model. The objective is to replicate the exact physical equivalent system of the numerical model consisting of the magnet and iron using our analytical field model coupled with the method of imaging principle for the field computation. Figure 6.9 presents the computed axial magnetic field results from the numerical model and the analytical model. Visual inspection of the graphs demonstrates excellent fit between both models with maximum variation of 0.03 Tesla. This verification process attests to the accuracy of the proposed analytical field model employing the method of imaging principle.

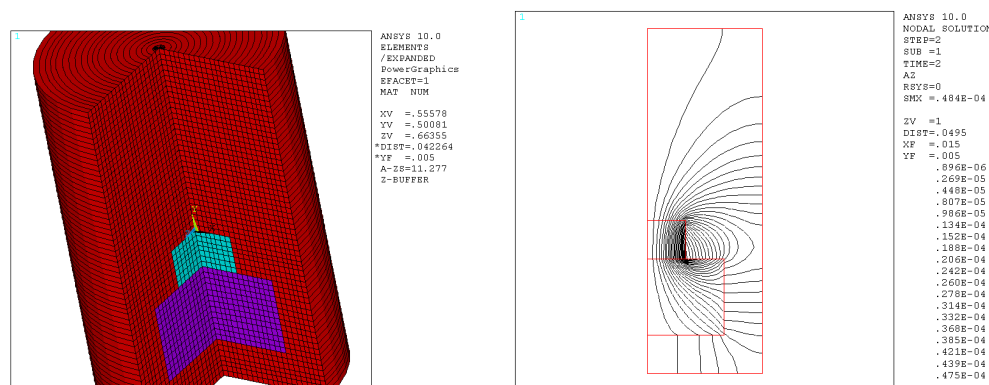


Figure 6.8. Numerical model consisting of high permeability elements.

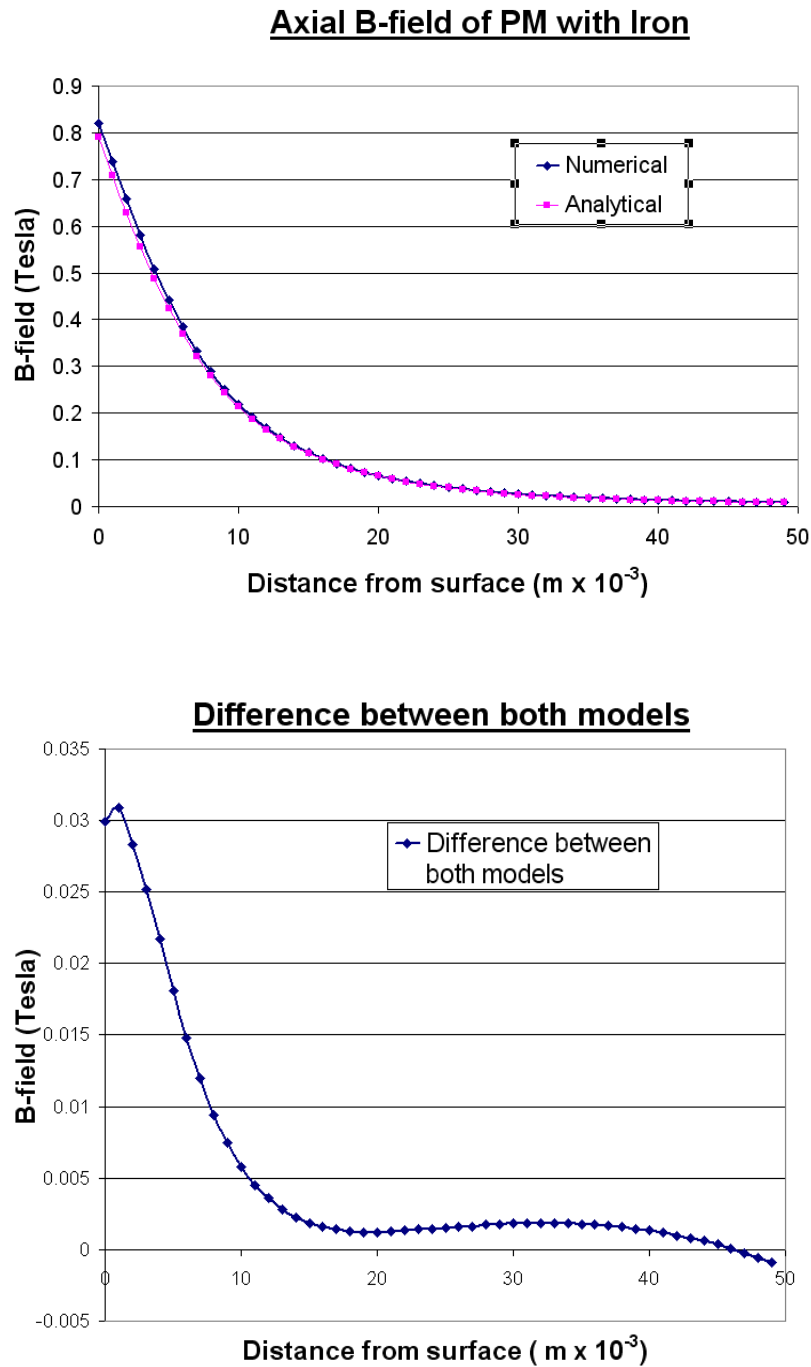


Figure 6.9. Comparison of results from numerical and analytical model.

6.3 VALIDATION OF TORQUE MODEL

In order to validate the theory behind the newly proposed torque model, comparison will be drawn against the results generated by the numerical model and experimental investigations. The electromechanical force outputs from ANSYS are computed at the

element level. Numerical calculation of magnetic forces in current carrying elements can be resolved by three approaches. By numerically integrating using Lorentz force law, the application of Maxwell stress tensor to the region of interest or alternatively, through the derivative of energy versus the displacement of the movable part commonly known as the Virtual work method. For our verification, we shall compare our analytical torque model with the Maxwell stress tensor approach and draw conclusions on the validity of our analytical model.

Concurrently, experimental investigation will also be performed using a skeleton model of the actual prototype as shown in Figure 6.10. This experimental setup allows manipulation of various configurations and positions of both rotor and stator poles. Output torque measurement of the rotor can be acquired at different stator pole position by the torque sensor.

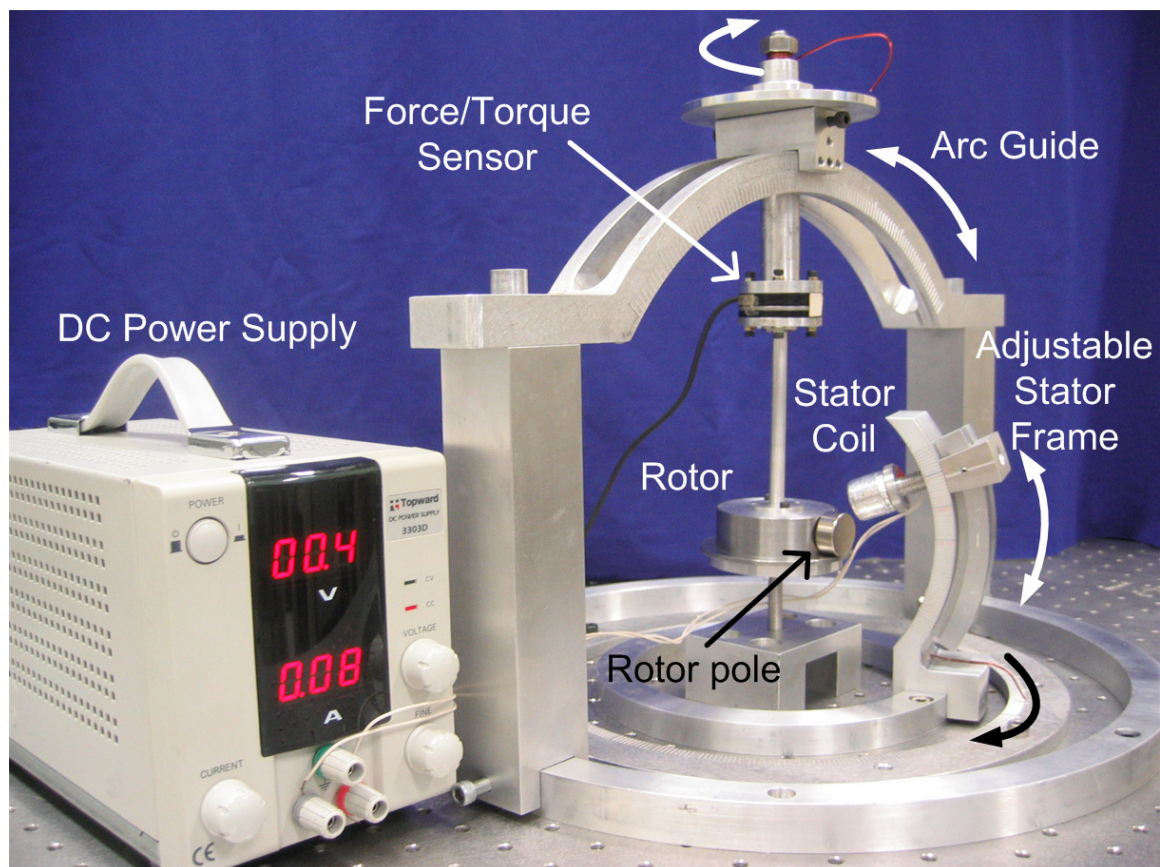


Figure 6.10. Experimental setup for torque measurement.

Since this is a 3-DOF actuator, we shall investigate the workspace within the reachable range of our prototype. As highlighted in the preceding chapters, the basic

module in motion generation for our prototype consists of four stator poles and one rotor pole. At any instantaneous position of the rotor, along with the input current by the stator poles, resultant torque will be induced onto the rotor and cause it to move. As it moves along an arbitrary trajectory, it will experience varying resultant torque profile. With the basic layout of the rotor and stator poles, we gathered that from the home position of the rotor where the rotor pole is aligned with the **X**-axis, trajectory paths are symmetrical within a quadrant enclosed by the **X**, **Y** and **Z**-axis. Moreover, within this quadrant, symmetry exists about the trajectory V illustrated in Figure 6.11. The above descriptions can be visualized from Figure 6.11.

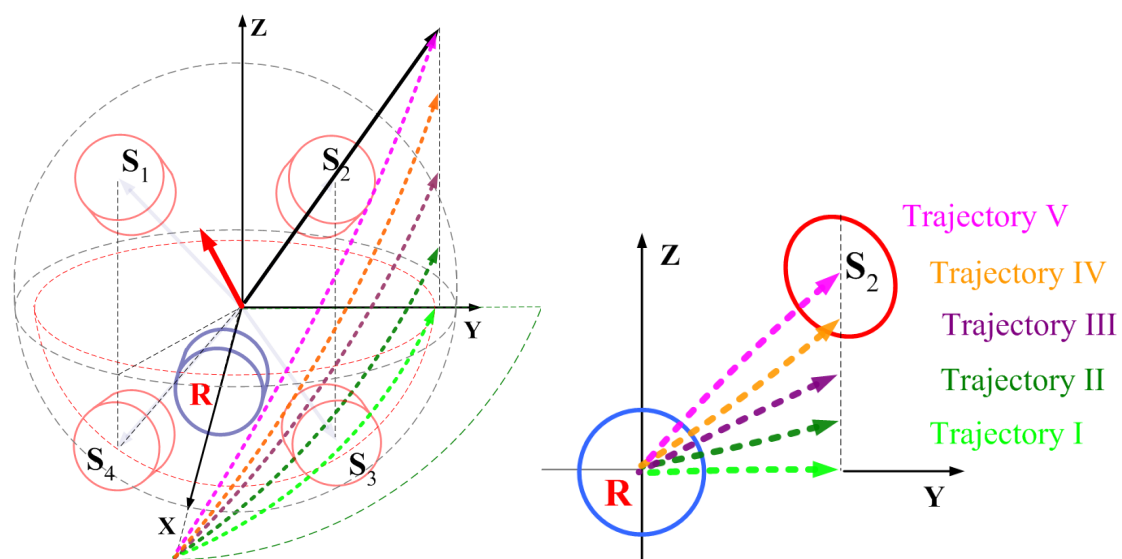


Figure 6.11. Trajectories of test cases.

Hence, we will explore the torque output of the rotor along five different trajectory paths in the quadrant which stator S_2 is residing. During this investigation, only S_2 is energized. The trajectories are defined by the spherical coordinate system with variation in the azimuthal (ϕ) and polar (θ) angles. Table 6.4 lists the start and end points of the respective trajectory paths. Each trajectory will commence from home position ($\phi = 0$, $\theta = 0$) to the specified end point.

Parameter	Azimuthal angle, ϕ (degrees)	Polar angle, θ (degrees)
R	0	0
S₂	30	20

Trajectory I	30	0
Trajectory II	30	5
Trajectory III	30	10
Trajectory IV	30	15
Trajectory V	30	20

Table 6.4. Trajectories position angles.

Along each trajectory, we will discretize the path evenly for evaluation. Figure 6.13 summarizes graphically the torque generated by both numerical and analytical models along with the experimental results. Resultant torque about the **X**, **Y** and **Z**-axis are denoted by T_x , T_y and T_z . T_r represents the magnitude of the total resultant torque. Based on this set of results, we are able to draw the following observations:

1. The trend lines of T_x , T_y , T_z and T_r computed by the numerical, analytical and empirical model exhibit similar profile. Presented in the figures are line plot connecting each data points from the empirical results. If nonlinear regression is employed to the set of data points, a smoother profile can be established. Based on this result, we are able to attest to the validity of our analytical torque model.
2. The analytical and empirical results exhibit a smoother torque curve about all three-axes as compared to the numerical model. In analytical magnetostatics, calculation of the net force or torque acting on a current carrying body is well established. Maxwell's equations ensure a unique field solution of uniform perfect accuracy throughout space for any well-posed problem. Consequently, the net force acting on the region is guaranteed unique and application of any valid force calculation method yields the required force exactly. However, in finite element based computational magnetostatics, difficulties are often encountered in obtaining consistent and accurate force calculations. Prior to the computation of force/torque acting on the body of interest, magnetic field data needs to be acquired. A fine and uniform meshing of the 3-D region of interest will ensure accurate and consistent computed fields. Encountered

during our analysis were two hindering factors that contributed to the inconsistent results generated by the numerical model,

i. The limit on the maximum number of elements available under the academic edition of ANSYS. This restriction constrains optimal mesh leading to coarse meshing especially in the airgap surrounding the rotor and stator poles. As highlighted, fine and uniform meshing are prerequisites for successful implementation of FEM analysis. This to a certain extent degrades the quality of our 3-D model and hence contributing to the inconsistency of the results.

ii. The uneven distribution of the air elements during 3-D meshing. This refers to the airgap surrounding the stator poles. Uniformly distributed meshing of the air element surrounding the stator pole is of paramount importance as it directly determines the magnetic field at elemental level. Furthermore, Maxwell Stress Tensor employed in the determination of magnetic torque relies heavily on the adjoining air element which again, demands for uniform and consistency meshing. As depicted in Figure 6.12, what we are able to manipulate will be the mesh distribution around the rotor pole and rotor circumference along the various trajectories. What we are unable to control was the consistency and uniformity of the air element surrounding the rotor pole as shown in Figure 6.12. The three-dimensional nature of the analysis further aggravates the regularity of the mesh. This inherently affects the consistency and accuracy of the numerical torque results.

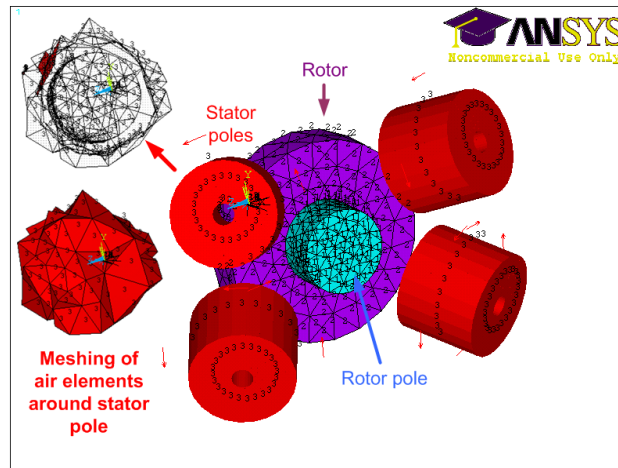


Figure 6.12. Meshing of air element surrounding stator pole.

Figure 6.14 presents the plot of T_r , for the respective trajectory paths. From the illustrations and results, we draw the following observations:

1. Analytical model shares identical torque profile with empirical results. The accuracy of the analytical model in comparison with the empirical data is observed to be $< 0.01\text{Nm}$. This reinforces the correctness and validity of the proposed analytical torque model.
2. The difference between the torque computed by both numerical and analytical model is not of absolute nature as the numerical model depends heavily on the mesh size which directly influence the precision of the numerical results.
3. Disparity between numerical and analytical models is more evident as the rotor pole gets closer to the stator pole (Trajectory IV and V). Geometrically, this coincides at positions when the rotor pole is close to the stator pole surface. At the circumference or edge of the cylindrical poles, the fringing **B**-field changes direction abruptly as shown in Figure 6.15.
4. The analytical torque model is able to accurately identify the maximum torque achievable. Maximum torque will occur when the stator pole experiences the highest magnetic field density acting onto it. Therefore at positions where the edge of the rotor pole nears the stator pole, the torque will be the maximum. Beyond the position of maximum torque, the **B**-field decreases and so will be the resulting induced torque. This is clearly shown in Figure 6.16 when the peak of the torque curve of the analytical model shifts

as it nears the stator pole and equates to zero when it is fully aligned with the axis of stator pole S_2 . Hence, this attests the validity of the proposed torque model based on the principle of magnetic dipole moment.

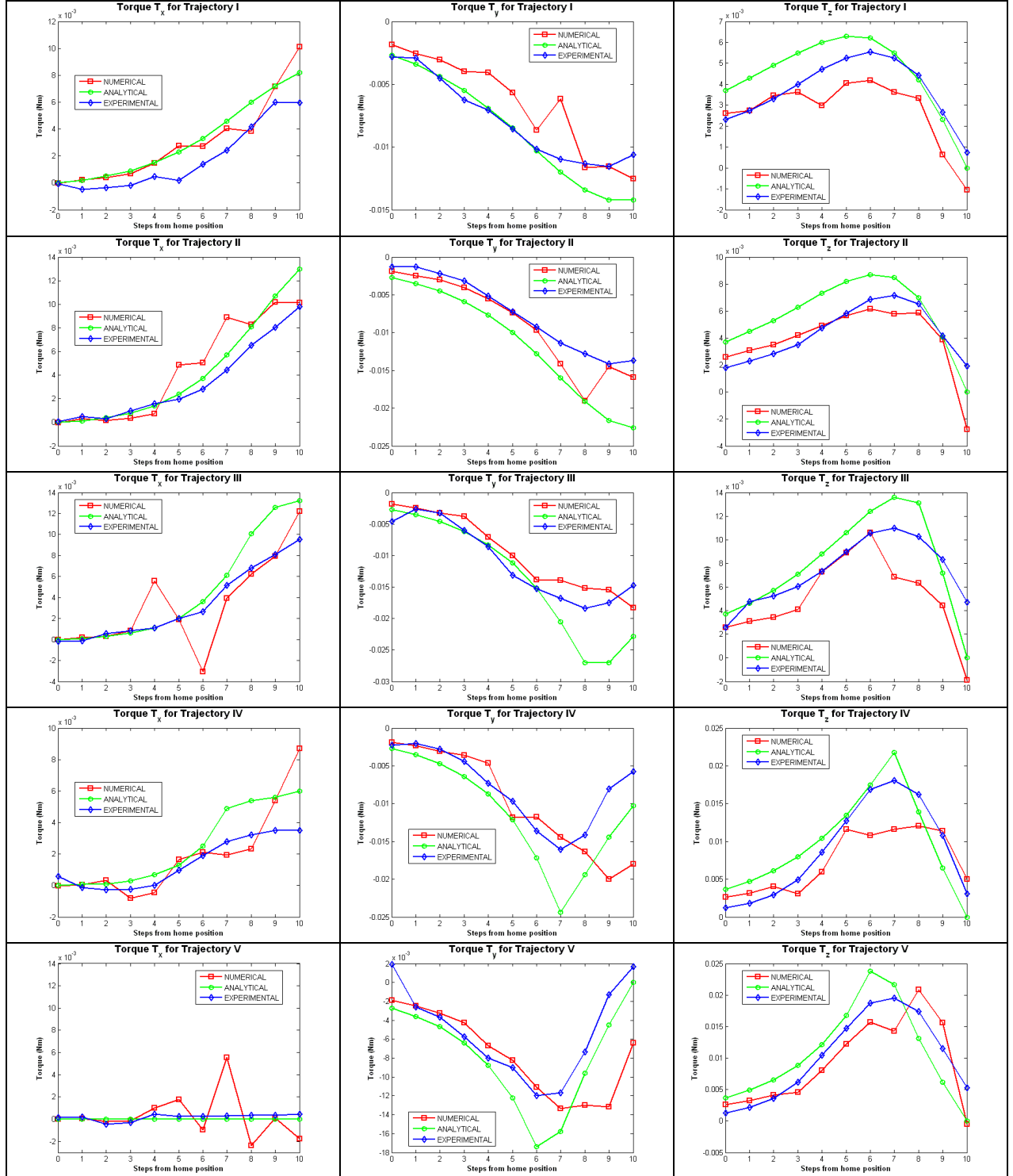


Figure 6.13. Comparison of torque about principal axes between numerical, analytical and empirical results.

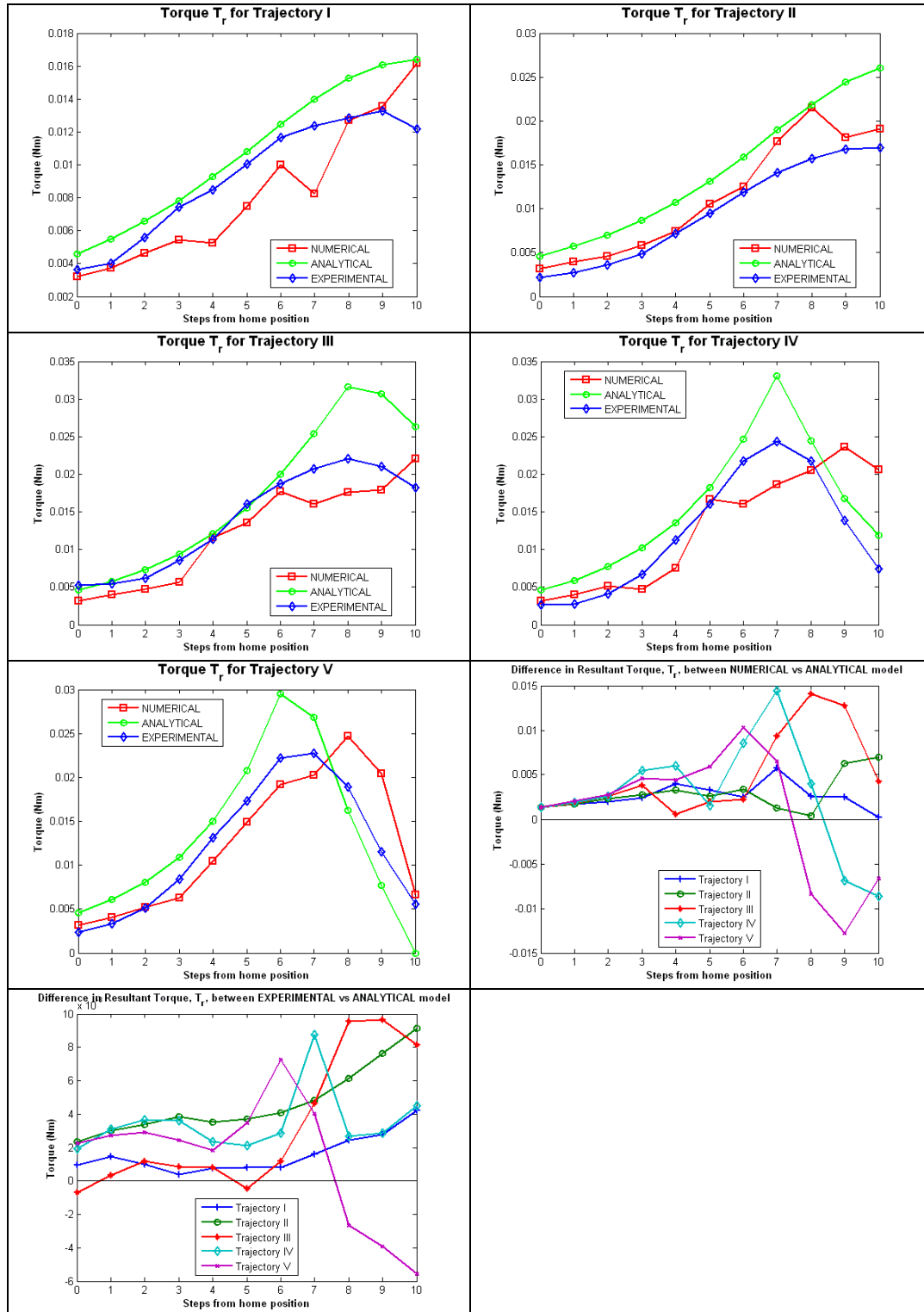


Figure 6.14. Comparison of resultant torque by numerical, analytical and empirical results.

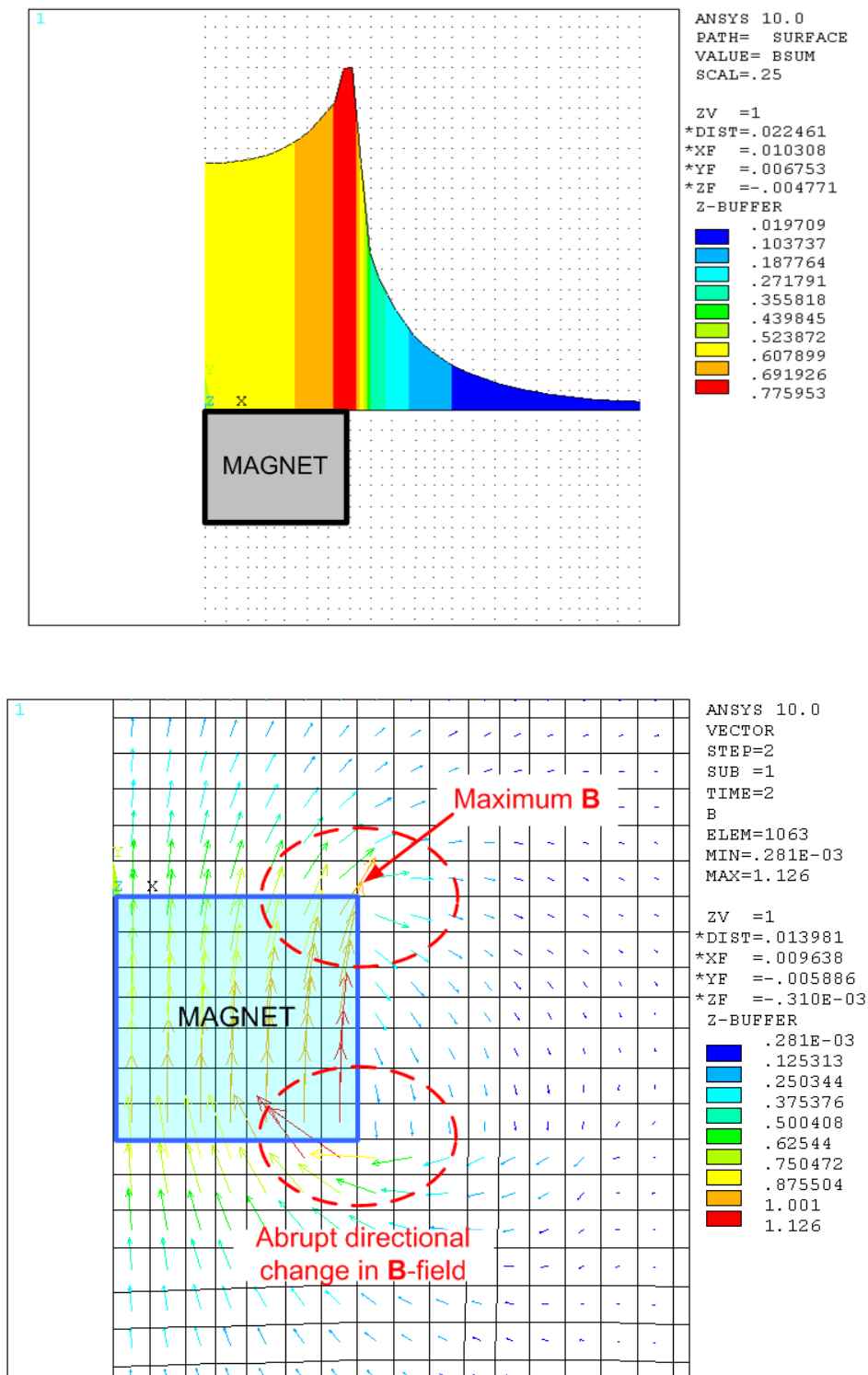


Figure 6.15. Fringing field at edge of magnet.

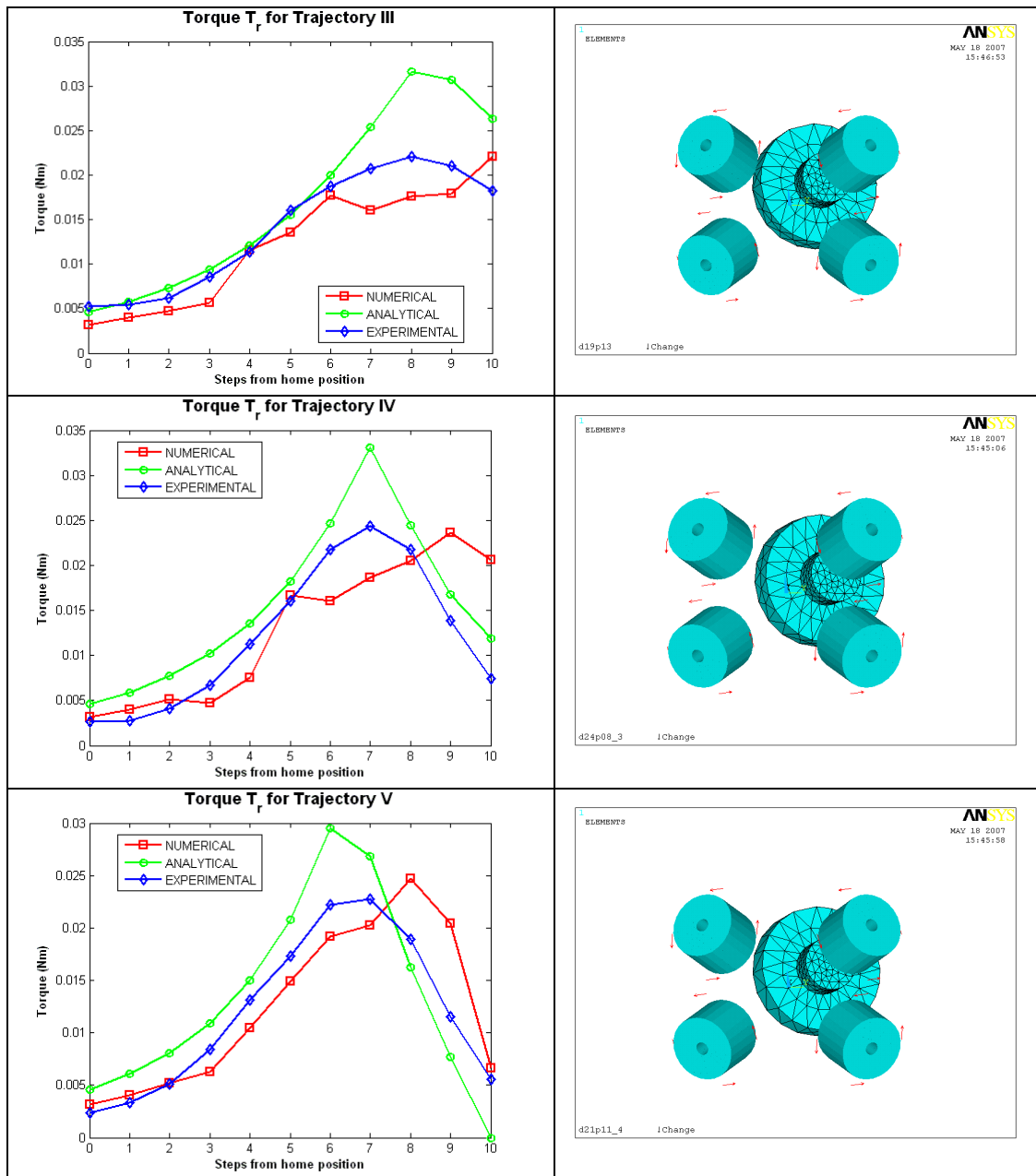


Figure 6.16. Orientation of rotor with maximum torque.

For calculating magnetic force and torque, Virtual Work and Maxwell's stress method are widely used in FEM formulation. Virtual Work method needs two finite element solutions and has numerical problems in distinguishing small changes in large values of energy. Maxwell's stress method involves integrating the magnetic stress on an imaginary closed surface whereby the results often depend on the choice of the surface. Essentially, each of these approaches use only a tiny percentage of the computed fields by selecting a small set of elements to define the region of integration. The enveloping region is frequently chosen to be as close to the body as

possible, while making little or no effort to ensure that the parts of the fields that are used have been computed accurately. Inaccuracies in the integrated fields are reflected in the final force calculations that lead to considerable inaccuracy within the computed result.

From the preceding results presented, we can conclude that good agreement can be drawn between the analytical and empirical data. Discrepancies can be seen between the numerical and analytical model. As highlighted, the employment of numerical model was to validate the analytical model. The unevenness or inconsistency of the numerical data points can be credited to the mesh quality and the limitation in the number of elements available under ANSYS academic edition. These are the two factors that impact and contributed to the inconsistency of the numerical results. By and large, from the graphical results, we are confident that by overcoming the above mentioned factors, we will be able to acquire a more accurate and consistent torque profile as reflected by the analytical model.

6.4 CHAPTER SUMMARY

From the preceding sections of this chapter, we have reviewed and assessed the accuracy of the magnetic field and the torque model proposed in this research effort. By comparing with numerical results, the accuracy of the magnetic field model derived by elliptical integrals has been authenticated to be exceptional. The main reason for its high precision lies in the fact that the magnetic field was postulated from first principle with the analytical solution derived by elliptical integrals. The versatility of the proposed magnetic field model is clearly shown to be able to model the three main regions commonly found in electromagnetic field analysis. The advantage of the proposed approach allows precise calculation of the **B**-field at a point in space efficiently as compared to the conventional numerical approach where the computation is performed for the entire region. Ensuing from the verification of the magnetic field model, by incorporating into the torque model, we are able to compute the resultant torque of the spherical actuator. This was verified against the numerical model and empirical results when the rotor was subjected to various trajectory paths. Comparisons of the computed results by the three approaches attest to the validity behind the analytical torque model. The torque variation about three principle axes deduced by the proposed model shares the same trend as compared to the numerical counterpart. Moreover, geometric interpretation of the torque curve with the position of the rotor with maximum torque reinforces the soundness of the theory behind the newly formulated torque model. In conclusion, the analytical models proposed in this dissertation are validated by its corresponding empirical and numerical models with results showing good fit between them. The novel approach using magnetic dipole moment to describe and determine resultant torque of permanent magnet spherical actuator has been verified in principle and this brings a new dimension to the analysis and design of such class of actuators.

CHAPTER 7

CONCLUSION

7.1 CONCLUSION AND CONTRIBUTIONS

This thesis describes the design and modeling of a MDOF spherical actuator. As the names implies, this class of actuators can perform MDOF motion within a compact structure. Being a direct drive system with high structural rigidity, the advantages and potential are immense especially in the field of robotics. MDOF motion generated from such configuration will not only ensure isotropic resolution in all direction but also improve the dynamics response of the entire system. Hence, the focus of this work involves a review of this cluster of actuators and identifying the key factors for successful design and implementation.

Chapter 1 clearly defines the motivation and objectives put forth in our work. An assessment on existing spherical actuator designs reveals three critical areas for further research and development. They are namely, theoretical modeling of the electromechanical system, a configurable and effective mechanical design and orientation sensing of the spherical rotor. Being able to resolve these three important areas will assist in the establishment of a systematic and holistic approach for the design and analysis of such actuators. As cited previously, theoretical modeling forms the cornerstone in the design and control of any type of systems. The function of the theoretical model is to bridge a correlation between the input and output parameters. Armed with the mathematical formulation relating the key parameters, one will be

able to perform analysis, evaluation and design optimization of the proposed design. For the spherical actuator described in this thesis, the underlying working principle for motion generation is based on the universal law of attraction between magnetic poles. Therefore, prior to the formulation of the torque model, the magnetic field distribution within the workspace is of paramount importance.

In any physical system where we are to perform electromagnetic analysis, we often need to solve Maxwell's equations. It is beyond doubt that Maxwell's equations can be solved directly for the various fields. However, one can also resolve the equations in a more palatable form by utilizing potential functions. The essence behind this approach is to rewrite the four coupled first-order field equations in terms of a pair of uncoupled second-order equations by making a change of variables to the vector and scalar potentials, $\mathbf{A}$ and ϕ [60-63]. This scheme usually employs numerical approach in the solving of the magnetic fields and hence increasing computational time. The approach taken in this work is to exploit special functions, the elliptical integrals in deriving the precise field computation for our model as detailed in Chapter 2. In our work, an analytical field model was derived from first principle and extended to the description of the magnetic field at a single point in space. This technique ensures computational time be kept to the minimum and paves the way for future real time control implementation.

In the study of magnetic field theory, we encounter boundary-value problems frequently. These problems arise naturally with constraints on the solution specified on the boundary of the region. Boundary-value problems can often be solved analytically when the boundary coincides with constant coordinate surfaces in an orthogonal curvilinear coordinate system. Using the method of separation of variables, the partial differential equation can be reduced to a system of ordinary differential equations. Pertaining to the magnetic model proposed in modeling scenarios of such nature, the method of image was introduced. This method allows direct computation of the field above the surface of interest by adhering to the equivalent boundary condition as described in Chapter 2. This scheme provides an efficient and accurate alternative in acquiring the magnetic field in the region required for electromechanical evaluation. For the modeling of permanent magnets, the current

model was employed. Field region of these physical systems are replaced with coils energized with equivalent current density. This approach permits the modeling of different dimensions and shapes of the rotor and stator poles simply by ensuring the geometric constraints are met. Comparison studies were conducted against numerical models for various electromagnetic regions and proposed approach with the result exhibiting high level of accuracy as presented in Chapter 6.

In an electromechanical analysis, while the field themselves are of interest, their interaction with charged matter is equally important. That is to say we need to know the electromagnetic force and torque imparted on the steady current in the region of concern. To determine these mechanical outputs, the behavioral study of charge q moving with velocity $\mathbf{u}$ through a region permeated by $\mathbf{E}$ and $\mathbf{B}$ -fields is imperative. Lorentz law governs the electromechanical interaction between moving charge and fields. Chapter 3 presents a new model based on the magnetic dipole moment principle. The formulation is derived from first principle of the magnetic dipole moment induced on a current carrying loop. By the conservation of potential energy, a magnetic dipole will align itself with the external magnetic field inducing a resultant torque. Applying this notion to the modeling of the spherical actuator, we will be able to acquire the resultant torque induced on the rotor by the stator poles. This is the first time this approach is implemented for torque analysis on an electromagnetic spherical actuator. To validate the proposed formulation, results from a numerical model with the same input parameters are constructed and compared. The outcome of the evaluation attests to the soundness and correctness of the proposed theory. The implication of the validated model shows that this novel method is applicable for electromechanical analysis of electromagnetic spherical actuators. In summary, a feasible torque model for this class of spherical actuators is formulated that enables us to analyze, optimize and control the actuator. The uniqueness and advantages of this newly proposed torque model can be summarized as follow,

Magnetic Dipole Moment Principle

The approach is validated systematically through the simulation results. This novel approach for the determination of resultant torque in three-dimensional space provides insights to the direction of the actuating torque and hence facilitates the understanding and establishment of motion planning of such actuators.

B-field computation

For the computation of electromechanical forces based on magnetic dipole moment principle, **B**-fields are calculated at current loop level as compared to conventional Lorentz force method which requires the full field description prior to the estimation of the electromechanical forces. Deriving from first principle and resolving in elliptical integrals, an analytical model was proposed for the determination of magnetic field. This significantly reduces computational time and increases the computation efficiency since only the essential elements are computed.

The actual spherical prototype design was presented in Chapter 4 and the specifications are summarized in Figure 7.1. The structure is simple and utilizes cylindrical permanent magnets and electromagnets as poles [74,76,78]. Air-core stator poles were employed so as to avoid non-linearity and saturation of conventional ferrous material. What differentiates from the other design significantly will be the support mechanism utilizing a unique spherical bearing. This novel arrangement minimizes frictional forces contributed from the support system due to its short moment arm design. Integration of balancing weight is another innovative aspect of the newly proposed spherical actuator. In this chapter, motion generation methodology which is an integrated and essential part in the design for this category of actuators was also detailed. The section explores systematically the underlying actuating principle and identifies clearly how directional control can be achieved. The proposed actuation scheme was validated through simulation.

The review in Chapter 1 indicates the lack of a viable and efficient orientation sensing system for spherical actuators. Therefore, Chapter 5 describes a potential orientation sensing method for the spherical actuator which is vital for closed-loop control. The methodology exploits axis-symmetric field generated from the source by detecting them using Hall sensors and hence acquiring the instantaneous orientation of the rotor. The advantages of such layout can be appreciated as there are no surface preparations required and only two magnetic sensors are required to determine the rotor orientation. Simulation of the algorithm was conducted and verified to be feasible [83,84]. Chapter 6 encompasses the details of the 3-D numerical model constructed in this work along with the validation results of the analytical models.

From the numerical model, a parametric code was compiled which can be utilized for future design optimization and investigation.

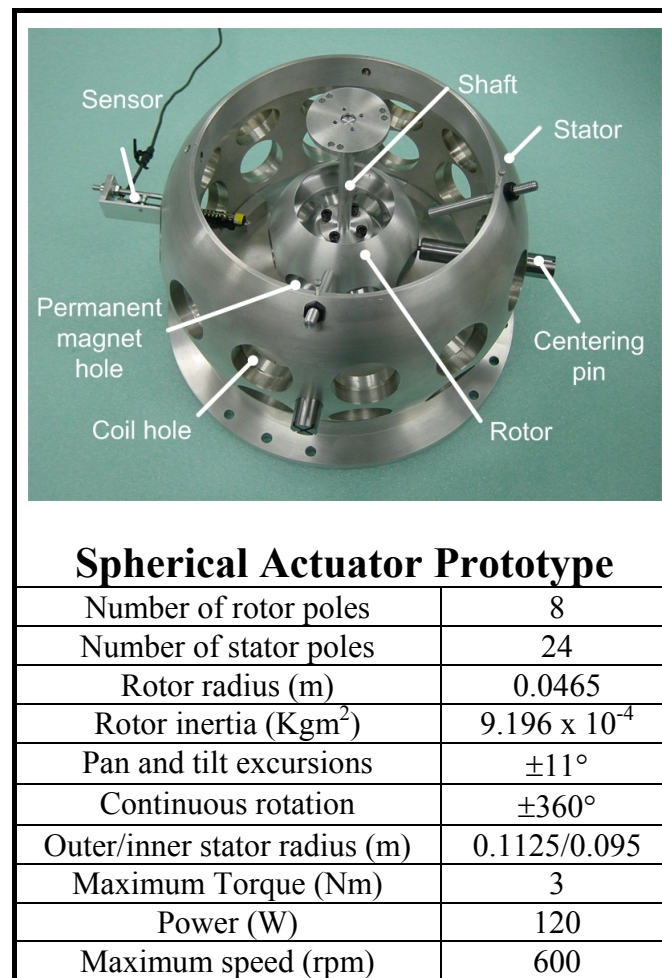


Figure 7.1. Specifications of spherical actuator prototype.

In conclusion, we have accomplished the objectives set forth in this research effort and covered the scope of study as targeted. Figure 7.2 summarizes the completed tasks. The main contributions of this interest work on electromagnetic spherical actuator can be summarized as follow:

1. An analytical magnetic field model derived from Biot-Savart Law and resolved using elliptical integral was proposed. This theoretical model is able to compute field distribution in three-dimensional space for various regions of a magnetic system to a higher level of accuracy.

2. The formulation of a new analytical torque model that fully describes the resultant torque of the proposed electromagnetic spherical actuator based on magnetic dipole moment principle. From the proposed model, the inverse torque relationship can be established for control implementation.
3. A new electromagnetic spherical actuator was designed, fabricated and assembled in this work.
4. Identifying and detailing the motion generation methodology for the electromagnetic spherical actuator.
5. Establishment of a 3-D numerical model for magnetic field and electromechanical modeling of the spherical electromagnetic actuator applicable for validation and future parametric studies.
6. Experimental investigation validating the proposed analytical torque model.
7. A potential orientation sensing method for the electromagnetic spherical actuator for the realization of closed-loop control.

From the above contributions, we have established critical formulations for the analysis and design of the electromagnetic spherical actuator. Design optimization and control can be realized based on the proposed analytical models. Motion planning strategy and the orientation sensing solution will pave the way for the control implementations.

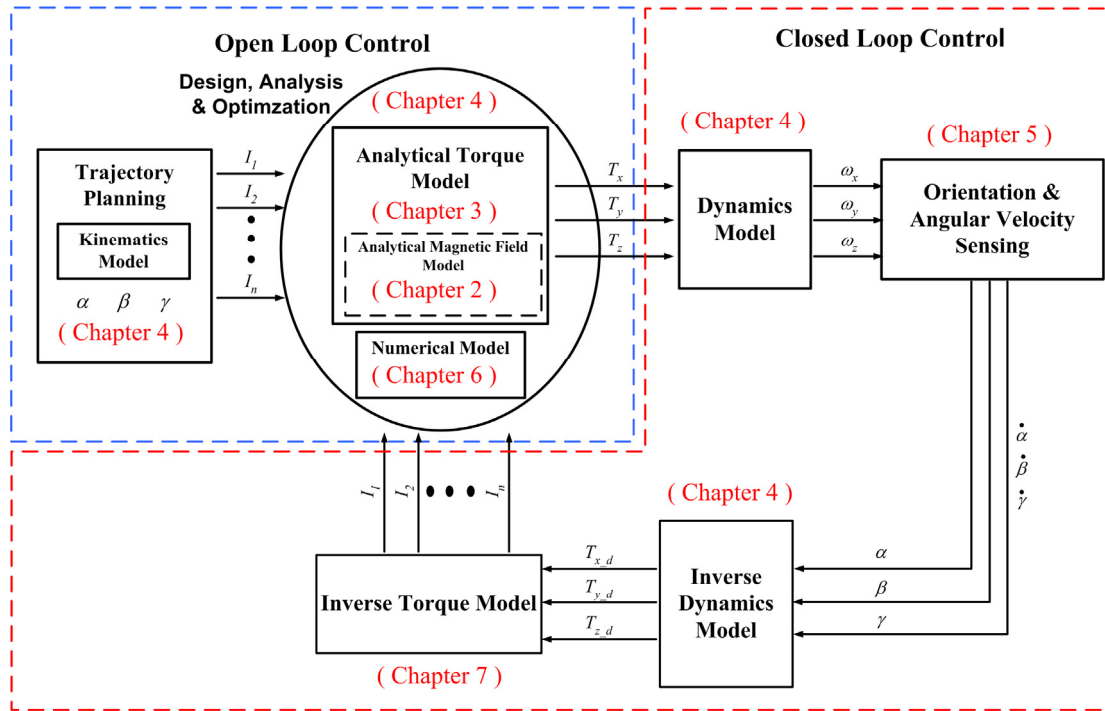


Figure 7.2. Accomplished tasks.

7.2 FUTURE WORK

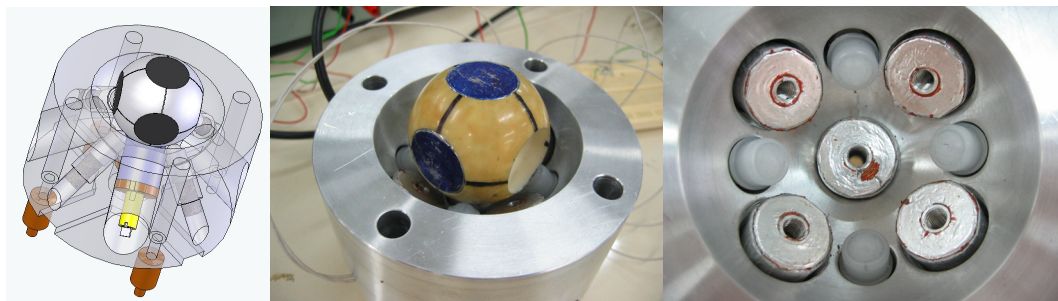
Impending work apart from the accomplished tasks in this project will be the hardware integration of the proposed method with the prototype. Prior to realizing this task, the power controller and design aspect needs to be investigated. The primary function is to allow individual control of the 24 channels of input current to the stator poles for actuation and motion generation. As the prototype is based on air core pole design, the achievable power to torque ratio is of lower magnitude as compared to system consisting of ferrous materials. This translates to the need for higher input current and hence, the need for a feasible power amplifier that can support the power requirement.

Higher input current will elevate the heat built up within the system due to coil resistance. This will be an issue during continuous operation of the spherical actuator as the internal parts of the prototype are not immune to heat. Overheating will result in the disintegration of the protective coating of the stator coil and thus shorting the wires. Rare earth magnets when subjected to temperature beyond its operating value

will be severely demagnetized. Hence, thermal analysis and management of the system is essential in maintaining the actuator power efficiency and operation.

With the available power inputs and controls, we will be able to demonstrate the positioning capabilities of our spherical actuator. Implementation of the proposed orientation system using 3-axis Hall sensors will be carried out in order to complete the closed-loop control for the spherical actuator. Eddy current investigation is another recommended area to be explored. As the strong magnetic fields transverse in space during motion, eddy currents will be induced within the stator structure. This frictional effect will inhibit the motion of the rotor. By modeling this phenomenon and incorporating into the dynamics of the actuator, precise control is attainable.

Existing spherical prototype has workspace limitation. Hence, a spherical ball motor using the same working principle was designed and fabricated as shown in Figure 7.3. The rotor of this spherical prototype is able to rotate freely about any axis as it is supported by transfer bearings. By energizing the stator coils below the spherical rotor, we are able to generate and control the direction of the rotation accordingly [81]. This interesting design will be incorporated into the realization of an omni-direction mobile platform.



Stator	
<i>Dimension</i>	Ø160x75mm
<i>Material</i>	Aluminum
<i>Pole material</i>	Aluminum/Steel
<i>Poles layout</i>	Total of 5 poles with 4 poles distributed 90° apart and angled at 42° from
Rotor	
<i>Dimension</i>	SØ = 52.4mm
<i>Material</i>	Aluminum
<i>Pole material</i>	NdFeB
<i>Poles layout</i>	4 x 25x10mm
<i>Windings</i>	Number of turns, $N_{\text{turns}} \sim 600$
	4xcastor balls

Figure 7.3. Spherical ball motor.

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