

Probabilistic Estimation of Plug-in Electric Vehicles Charging Load Profile

Nima H. Tehrani

Peng Wang

Nima H. Tehrani (e-mail: nimal@e.ntu.edu.sg) was with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798. He received his Ph.D. degree in 2014.

Peng Wang (e-mail: epwang@ntu.edu.sg) is an associate professor with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.

ABSTRACT

Plug-in Electric Vehicles (PEVs) are widely considered as a sustainable mode of transport by countries worldwide due to high efficiency and low or zero carbon emissions. However, PEVs will add significant additional load to the existing power distribution system and it will be a challenge to meet the new demand. In this study, probabilistic modelling has been presented to estimate the system-wide PEV charging load within domestic grids. U.S. national household travel survey data set has been utilized to quantitatively determine the mobility behavior of PEVs.

Uncertain nature of the problem in modelling and data preparation should be taken into account. Due to the existence of complex interdependencies between the system inputs, the problem definition leads to a multivariate uncertainty analysis problem. The modelling procedure is decomposed into two basic components: the modelling of the marginal distributions; and that of the stochastic dependence structure. In addition, Copula theory is presented for the multivariate modelling of dependent random variable. The results indicate that the PEVs can contribute to increase the load demand at certain hours, although the charging demand is very limited most of the time. Moreover, the probabilistic distribution of aggregated PEV charging demand is compared with that obtained by the Monte Carlo simulation. The numerical results have shown the effectiveness of the proposed methodology.

Keywords:

Load profile; charging demand; PEV; electric vehicle; probabilistic modelling

I. INTRODUCTION

Recently, the study of PEVs in a power system framework has been an active research topic. Due to the expected high penetration of PEVs in the near future, it is important to estimate the load they impose on the electrical grid. Several studies during the last few years have investigated this topic at both national and regional scales [1-4], however in the previous studies unlikely situations are represented. In [1], all PEVs are assumed to begin charging simultaneously at 5 p.m. or 10 p.m. In [5], off-peak electricity is consumed by the entire vehicle fleet. In uncontrolled charging scenario of [5], all vehicles leave home evenly between 8 and 9 a.m. and return home between 6 and 9 p.m. The electric energy consumption is often estimated without consideration of vehicle travel patterns. For example in [1, 4-6], all vehicles return home with the entire usable energy exhausted which leads to an overestimation of energy consumption. It is assumed in [7] that charging start time and initial State of Charge (SOC) are random variables and probability distribution of daily distance driven follows a normal distribution.

The results of a study carried out in 2006 showed that the U.S. national power grid has the technical potential to charge 73% of Light-Duty Vehicles (LDVs) [6]. Hence, sufficient generating capacity exists to charge large number of PEVs in the near future. However, it should be noted that PEVs are usually connected to local residential areas where distribution transformers are sometimes heavily loaded. Therefore, accurate and reliable models for the behavior of electric vehicle are required due to high penetration level of PEVs and their complex charging behavior. Although many studies are performed to model the operation of the coupling of PEVs and the power grid, there are still some shortcomings in the proposed models. In [8], a probabilistic constrained load flow is proposed considering both wind power generation and electric vehicles. In [9] and [10] the impact of charging electric vehicles in the distribution grid is analyzed. The power consumption of electric vehicles is estimated in [11]. In [12], two different algorithms are proposed to control the charge level of electric vehicles in deregulated electricity markets. Reference [13] presents a mid-term operation model to analyze the impact of PEVs in the Spanish power system. The recent paper [14] presents a robust optimization model to analyze the effect of including vehicle-to-grid facilities in small electric energy systems.

In fact, the power demand of PEV charging load depends on the number of vehicles, initial SOC, charging start time and its duration which are all Random Variables (RVs), and hence,

estimation of the demand is far from trivial. However, by evaluating the parameters and general pattern related to mobility behavior of vehicles which may be extracted from traffic habits, one can construct a stochastic model that estimates the actual behavior of the PEV charging load profile. In this study, the proposed probabilistic modeling (PM) employs statistical analysis, random simulation and queuing theory to find the distribution for the overall charging demand of PEVs. A comparison between PM and Monte Carlo simulation (MCS) is made in order to validate the effectiveness of the proposed modeling methodology which is supported by an analytical discussion.

II. TRANSPORTATION DATASET

The most comprehensive reference for transportation data is the National household travel survey (NHTS) sponsored by the U.S. Department of Transportation [15]. The NHTS website includes detailed transportation data from 1995 to 2009. The NHTS provides information to assist transportation planners and others who need comprehensive data on travel and transportation patterns in the U.S. Nonetheless, very few papers such as [16] have used the travel patterns in the U.S. obtained from NHTS. This study also utilizes the NHTS data set for PM of aggregated charging load. It is assumed that the advent of PEVs will not affect daily travel pattern and lifestyles in general; therefore, the driving behaviour of PEV owners will remain similar to the behaviour of drivers of conventional vehicles. With this assumption the parameters introduced will be valid for real-world travel patterns.

The 2009 NHTS data set contains data for 294,408 LDVs and 150,147 completed households in which at least 50% of the adults (age 18+) were interviewed. It consists of four large databases including household, person, vehicle and daily (travel day) trip level data. The 2009 NHTS was carried out by telephone interviews, and sampling was based on random digit dialing list of telephone numbers. This large pool of data is geographically distributed across the U.S. and reflects daily trip in a 24-hour period. For the purpose of PEV studies, this paper has primarily considered the data related to vehicles and daily trips. Therefore, two Microsoft Excel files of the 2009 NHTS: VEHV2PUB and DAYV2PUB are utilized to extract required information. 2009 NHTS codebook defines the attributes in all four datasets. Each household, person and vehicle in the 2009 NHTS has a unique ID number. These IDs are appropriate for linking any two data files [17].

Vehicle trips in District of Columbia and the 13 eastern states of U.S. which are covered by PJM Interconnection have been considered. Those who reported no day travel trip are eliminated. Thus, the relevant data sets of the 2009 NHTS are filtered out for this study from a large number of vehicle trips (more than 260,000). As the data pool is distributed over a broad enough portion of the U.S., it can be used for developing stochastic analyses that will be explained in details in the next section.

It is possible to extract the trips in progress by time of the day and day of the week from NHTS dataset, as required. The number of trips is found to have a peak in the morning when people leave their home and another peak in the afternoon which is mainly representative of education related trips and getting-back home trips. The latter is found to be more spread out than the morning peak. The average proportion of parked cars for weekdays and weekends can be obtained from this procedure, which is shown in Figure 1. The share of parked cars seems very high in general. High share of parked cars is due to the fact that many vehicles do not drive at all during a day. About 12 percent of the 60,282 individuals in useable households in the 2009 NHTS reported no day travel trips [17].

III. STOCHASTIC MODEL FOR PEV CHARGING PROFILE

Until the time that we have an active control over the period at which each vehicle starts to charge its battery, it is assumed that only the uncontrolled domestic charging scenario is available, i.e., charging starts at the moment when the PEVs are plugged in as they get back home. Suppose that the required infrastructure for grid connection is also available.

Mobility model which depicts the driving and parking behaviour of private cars is defined alongside the time axis of weekday. The departure time, arrival time, and trip distance for a set of trips are randomly generated based on the NHTS data which will be explained in the following. The time step was set to one hour according to the discretization of the mobility patterns.

Queuing theory is often used to mathematically analyze the effects of customers randomly arriving and being served by a system [18]. The process of EVs being randomly plugged in and charged by the grid fits this paradigm. Few studies employed the queuing theory to model PEVs charging demand [26], [27]. In this work, PEVs are also modelled as customers arriving in a queue to receive energy. To model the dynamics of the population; $M_t/G/\infty$ queue was used which has a non-homogeneous Poisson arrival process with deterministic time-dependent arrival-rate

function λ (assuming that the arrival rate equals the departure rate), and service times that are independent of arrival process, and infinitely many servers. Poisson arrivals means that the arrival time of one PEV carries no information of the arrival times of other PEVs and having infinitely many servers implies that PEVs do not interfere with each other. These are valid assumptions under the scenario of uncontrolled domestic charging.

A. Dependence Structure via Copula Approach

In most of the power system statistical studies, joint normal distribution is used to model dependency between normally distributed RVs but in some cases the marginal distribution of RVs do not have necessarily normal distribution. In order to model dependency of correlated non-normal distributions we should utilize a general method to find stochastic dependency of RVs. The mathematical function named copulas can be adopted to model the stochastic dependency of two measured data and obtain joint probability distribution of them. Marginal distribution of RVs can be coupled (or jointed) to their multivariate distribution using copulas [19].

Copulas are functions that characterize dependencies among variables and can be used to create distributions that model correlated multivariate data. Copulas model dependency of correlated non-normal distributions by coupling multivariate distribution functions to their one-dimensional marginal. Alternatively, copulas are multivariate distribution functions whose one-dimensional marginal are uniform on the interval $[0,1]$ [20].

According to Sklar's theorem [25] for a given joint distribution function F_{XY} of two RVs X and Y with marginal distribution F_X and F_Y (i.e, CDF of X and Y respectively), there exists a function C namely a copula such that

$$F_{XY}(x, y) = C(F_X(x), F_Y(y)) \quad (1)$$

If F_X and F_Y are invertible, then for $F_X(x) = u$ and $F_Y(y) = v$ one can write

$$C(u, v) = F_{XY}(F_X^{-1}(u), F_Y^{-1}(v)) \quad (2)$$

It should be noted that F_X and F_Y are CDFs of X and Y . Equation (2) can be used to find the joint distribution of X and Y . Copula method is used to develop a set of random variables with similar stochastic behavior of the empirical data but joint to each other with particular correlation.

Copula method couples two or more variables to produce another distribution function. To obtain the set joint random variables using copula, the level of dependency between them should

be measured. The strength of the dependency between RVs can be measured using product-moment correlation or linear correlation ρ . The linear correlation in actual domain fails to measure accurate level of dependency for non-normal distribution. In order to avoid the effect of marginal distribution on dependency structure, the RVs X and Y in actual domain are transferred to their rank domain using CDF transform. The transferred RVs to rank domain retain the dependence structure of the original variables. The product moment correlation for the respective ranks is calculated.

Suppose $U = F_X(x)$ and $V = F_Y(y)$ are corresponding rank of X and Y respectively. The rank correlation ρ_r between two RVs X and Y is defined as linear correlation between respective ranks of these X, Y (i.e. U, V), hence one can write:

$$\rho_r(X, Y) = \rho(F_X(X), F_Y(Y)) \quad (3)$$

Then, the measured rank correlation is used as input of copula function. The copula function is utilized to generate a set of stochastic variables in rank domain which are correlated to each other by measured rank correlation. The developed variables using copula are transformed back into their original domain by applying the inverse-CDF transformation of RVs. In the present context, copulas will be adopted to find the joint probability of home arrival time, daily travelled distance and home departure time. In order to generate the required random samples, the aforementioned RVs are modelled by probability density functions fitted to them which will be explained in details in the following.

BESTMILE developed by Oak Ridge National Lab, gives out the best estimate of annual miles driven by each vehicle. This study calculates daily mileage by dividing *BESTMILE* by 365. Histogram of daily mileage is shown in Figure 2, where the average daily mileage is found to be 29 km which is consistent with the formerly reported 33 km average miles driven in major U.S. metropolitan areas [21]. In practice there is always some charge left in the vehicles' batteries when they get back home which is directly related to their daily traveled distance. In order to estimate the SOC upon PEV arrival, the daily mileage data from NHTS dataset is employed, which is found to fit Weibull distribution as follows

$$f_{dmi}(d|a, b) = ba^{-b}d^{b-1}e^{-\left(\frac{d}{a}\right)^b} I_{(0, \infty)}(d) \quad (4)$$

The assessment whether the mileage data fits a Weibull distribution is conducted graphically and is depicted in Figures 3 and 4. Through maximum likelihood estimation [22] with confidence degree $1 - \alpha = 95\%$; Weibull distribution with *mean* = 29.9273, *Variance* = 594.305 and estimated parameter $a = 32.039, b = 1.23451$ is found to fit best with the data. The fit between the actual data and Weibull distribution is shown in Figure 4. In order to validate the above hypothesis, a two-sample Kolmogrov-Smirnov goodness-of-fit test (K-S test) [23] is performed to compare the distribution of the values in the two data vectors. The result accepts the null hypothesis which confirms that two samples are from the same continuous distribution at the 5% significance level.

Estimation of SOC when a PEV arrives is carried out as in [7]. It is assumed that the SOC of a PEV drops linearly with the distance of travel and adopt the idea of fully charged battery at $t = 0$, if a fully charged PEV drives d_{max} mile on electricity, then the residual battery charge of a vehicle after being driven d miles would be:

$$E_0 = 100. \left(1 - d/d_{max}\right) \quad (5)$$

The probability distribution of initial battery SOC at the end of the day is derived using the daily mileage data, and is shown in Figure 5. Obviously it follows the similar trend shown as Figure 2. Based on derived densities formula, the pdf of initial SOC is found. Suppose $d_{max} = 100$, then

$$f_{SOC}(E_0) = b a^{-b} (100 - E_0)^{b-1} \exp\left(\left(\frac{E_0 - 100}{a}\right)^b\right) \quad (6)$$

Same procedure is applied to find the marginal distribution for departure and arrival time. Weibull PDF is found to be the most appropriate function to be fitted to home departure time RV. Generalized expected value pdf with shape parameter k , scale parameter σ , and location parameter μ , is fitted to the home arrival time as follows

$$f_{arr}(t) = \frac{1}{\sigma} \left(1 + k \frac{(t - \mu)}{\sigma}\right)^{-\left(1 + \frac{1}{k}\right)} e^{-\left(1 + k \frac{(t - \mu)}{\sigma}\right)^{-\left(\frac{1}{k}\right)}} \quad (7)$$

When modelling dependent random variables using copula function, the following steps should be followed:

- 1) Find the most appropriate function to be fitted for each of the RVs
- 2) Transform datasets to the corresponding uniform space

- 3) Transform unified data via adopted copula family such as normal copula, t -copula and Archimedean copula family (e.g. Frank copula, Gumbel and Clayton). The best fit which can model more accurately stochastic characteristics of RVs can be found by comparing the original data distribution and simulated results using K-S test. The most promising copula function is then selected
- 4) Obtain correlation matrix of the mentioned datasets
- 5) Finally, the extracted copula can be utilized to generate correlated samples.

B. The $M_t/G/\infty$ Model

Consider the queuing model shown in Figure 6. Assume that the $M_t/G/\infty$ system started empty in the distant past, i.e., at $t = -\infty$ and let S be a generic service-time RV and let G be its CDF. Let S_e be a RV with the associated stationary excess CDF:

$$G_e(t) \equiv pr(S_e \leq t) \equiv \frac{1}{E[S]} \int_0^t (1 - G(u)) du, t \geq 0 \quad (8)$$

Let $N(t)$ represent the number of busy servers (PEVs charging from the grid) at time t and let $m(t) = E[N(t)]$. The main $M_t/G/\infty$ result is that $N(t)$ has a Poisson distribution with mean

$$m(t) = \int_0^\infty (1 - G(u)) \lambda(t - u) du = E \left[\int_{t-S}^t \lambda(u) du \right] = E[\lambda(t - S_e)]E[S] \quad (9)$$

Note that the time-dependent mean $m(t)$ is related to the instantaneous offered load $\lambda(t)E[\mu]$. From (9), the congestion as described by $m(t)$ lags behind $\lambda(t)$ (the peak of m will come after the peak of λ). The term $\int_{t-S}^t \lambda(u) du$ describes the cumulative arrival rate during a service period before time t . The expectation is then the expected number of arrivals during a random service period before t .

In [18], various polynomial approximations for $m(t)$ were introduced which are based on assumption that the arrival rate function λ is a polynomial or can be approximated by a polynomial. Based on Taylor series expansion for λ ,

$$R_n(t) = (-1)^{n+1} E[\lambda^{(n+1)}(t - S_e^{(n+2)})] \frac{E[S^{n+2}]}{(n+2)!},$$

$$m(t) = \sum_{j=0}^n (-1)^j \frac{\lambda^{(j)}(t) E[S^{j+1}]}{(j+1)!} + R_n(t), \quad n \geq 0 \quad (10)$$

Equation (10) gives an explicit probabilistic expression for the congestion measure and remainder term. The probability of charging k vehicles at the same time in an $M_t/G/\infty$ queue, denoted as $pr(k)$, is given by:

$$pr(k) \approx \frac{1/n!}{\sum_{i=0}^{n-1} \frac{1}{(i!)} + \frac{1}{(1-\frac{1}{n})n!}} \quad k = 0, 1, 2, \dots, n \quad (11)$$

where n is the maximum number of vehicles. The charging time for vehicle i to absorb power follows the exponential distribution with mean t_μ but truncated within a certain range $[T_{min}, T_{max}]$:

$$t_{c_i} = \begin{cases} T_{min} & t_{c_i} \leq T_{min} \\ -t_\mu \cdot \ln(U) & T_{min} \leq t_{c_i} \leq T_{max} \\ T_{max} & t_{c_i} \geq T_{max} \end{cases} \quad (12)$$

where U is a random number between 0 and 1. Based on the trips in progress the normalized arrival rate function of vehicles at each hour is found and the associated polynomial fit is used in the above equation. Charging process of PEV battery is exponential function over time. Instantaneous charging status of PEV is simulated by the following exponential formula [8];

$$E_c(t_c) = E_{max} \left(1 - e^{-\epsilon t_c / t_{max}} \right) + E_0 \quad (13)$$

Accordingly, the power demand of a PEV at any instant of charging is:

$$P_c = P_{max} \cdot \epsilon / t_{max} \cdot e^{-\epsilon t_c / t_{max}} \quad (14)$$

For a linear approximation of PEV Charging Demand (CD), the same procedure as [3] is followed for recharging cycle. According to the current PEVs on the market, the average battery size is assumed to be 24 kWh with a consumption of 0.23kWh/mi.

The energy demand of each vehicle to be fully charged is

$$E_c = E_c(t_c) - E_0 = E_{max} \left(1 - e^{-\epsilon t_c / T_{max}} \right) \quad (15)$$

If there is $n_c (= m(t))$ charged vehicles at time t :

$$Q_c = n_c \cdot P_{max} \cdot \epsilon / T_{max} \cdot e^{-\epsilon t_c / T_{max}} \quad (16)$$

IV. SIMULATION & ANALYSIS

First of all, using copula functions the population size of the input data is increased by generating correlated samples. The parameters of the fitted PDFs to all three random variables are given in Table 1.

In the following, the results are depicted which shows the scatter plots of initial dataset in Figure 7. Normal copula is a common choice for modeling the dependence structure between variables but due to the fact that the t -copula presents more observations in the tails than the normal copula, it is more suitable for modelling. Correlation matrix of the mentioned data sets employed in this study is obtained as follows:

$$\rho = \begin{bmatrix} 1 & 0.38 & 0.29 \\ 0.38 & 1 & -0.02 \\ 0.29 & -0.02 & 1 \end{bmatrix}$$

Therefore, the final scatter plot with marginal histogram between departure and arrival time is shown as an example in Figure 8. The initial and the final 3D scatter plots of random variables are shown in Figures 9(a) and 9(b) respectively.

Having estimated the PEV arrival for charging and the power demand at different instants of charging process, the power demand for the charging can be readily estimated. It should be noted that each charging PEV will be drawing power for the maximum period of T_{max} . For this example, a full charge of the $24kWh$ battery system with $T_{max} = 8 h$ and $3 kW$ average capacity is adopted. Li-ion batteries currently used do usually not tolerate charge rate above 1C. A charge rate of 1C means charging at a rate equivalent of full capacity reached within an hour. Li-ion batteries should have lower charge rates, especially to prolong battery lifetime, but here the restriction is the capacity limit set by the domestic power connection. Thus, the maximum power is $P_{max} = 3 kW$ and the constant parameter ϵ is calculated at a value of 10.5197, assuming that a full battery system of PEV absorbs 97% of maximum power capacity approximately in the third part of the maximum charging time.

The penetration of PEVs is certainly the most uncertain aspect of the charging model. It is common to assume the number of electric vehicles penetration and to work on different scenarios. For example, in [1,2,24] penetration factors between 0% and 100% are treated. In this study the simulation is performed for 1000 PEVs. It is assumed that charging starts upon arrival of the PEV

and the battery is fully charged upon departure. The flow chart for the complete simulation procedure for each time-frame is shown in Figure 10. Random simulation is applied to calculate charging load at each time/iteration for comparison. The procedure of this random simulation is outlined as follows:

1. Set k the lowest value such that $U > pr(k)$ according to (11)
2. Randomly generate PEV driven mile and the corresponding SOC according to (4) and (6);
3. Calculate required recharge energy;
4. Randomly generate charging time according to (12);
5. Calculate CD according to (15);
6. Accumulate total CD

A. Numerical Simulation

By estimating hourly PDF of charging load demands, planners are able to produce as many as demand samples and then can be efficiently applied in a probabilistic distribution system planning procedure. As an example, Figures 11 (a)-(c) demonstrates demand distributions during three hours for a fleet of 1000 PEVs in addition to the normal PDFs fitted to them.

Table 2 shows the characteristics of fitted normal distribution to estimated CD from hour 17 to midnight. Figure 12 shows how the CD is varying gradually as the time goes on. To model the superposition of the demand caused by the PEVs at each hour t , it should be noted that the power demand at any instant is the sum total of the power drawn by PEVs which started charging in the previous hours following the flow chart illustrated in Figure 10. To calculate charging power demand, the initial SOC of the arriving PEVs is incorporated by mapping the SOC distribution and the appropriate charging process at that level of SOC. The probability of number of vehicles for a specific SOC at each hour is known. Thus, the power drawn at each instant of charging and the duration to full charge will depend on the level of SOC. Both these quantities will decrease for higher levels of SOC at the time of arrival.

Figure 13 demonstrates 50 load scenarios for 1000 PEVs generated regarding the extracted PDFs besides their average scenario. Although, the charging load varied from simulation to simulation, it is found that they stay within a narrow band, which reflects steady driving and charging patterns.

The peak CD is observed in the evening hours when the PEV arrival rate is also highest and it is going to contribute to the daily peak of the load. The maximum CD is found to lie between 3.32 MW and 2.66 MW.

For MCS, the charging demands are directly generated from the random simulation procedure in Section IV. The sample population $N_s = 10000$, which is enough for the convergence of MSC in this case. As shown in the figure 14, the empirical CDF curves obtained from MCS is approximated well by the results of probabilistic approach based on the proposed methodology. The probability plots and the associated normal kernel fits are shown in figure 15 for both PM and MCS.

B. Discussion

Assume that the plug-in probability of all the vehicles are identical and equal to p . In this case, probability distribution of the number of the plugged-in vehicles is a binomial random variable

$$pr(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0, 1, \dots, n \quad (17)$$

where k is the number of plugged-in vehicles and n is the number of electric vehicles. When n is sufficiently large, a good approximation can be given by normal distribution as

$$N(np, np(1-p)) \quad (18)$$

Meanwhile, normality is preserved by linear transformation, i.e. if X is a normal random variable with mean μ and variance σ^2 , and if a, b are scalars, then the random variable $Y = aX + b$ is also normal, with mean and variance $E[Y] = a\mu + b$, $var(Y) = a^2\mu^2$. For and identical power capacity P_{max} , the probability distribution of CD can be obtained as follows:

$$CD \sim N(P_{max}np, P_{max}^2 np(1-p)) \quad (19)$$

The CDF of (19) is given as

$$F(x) = 1/2 \left[1 + erf \left(\frac{x - P_{max}np}{P_{max}\sqrt{2np(1-p)}} \right) \right] \quad (20)$$

where $erf(\cdot)$ is the *Gauss error function* and is twice the integral of the Gaussian distribution with mean of 0 and variance of $1/2$. Thus, the probability that the CD is larger than a certain amount x is

$$pr(CD \geq x) = 1 - F(x) = 1/2 \left[1 - erf \left(\frac{x - P_{max}np}{P_{max}\sqrt{2np(1-p)}} \right) \right] \quad (21)$$

In practice, vehicles would have different plug-in probabilities and power capacities but still by clustering the vehicles into groups with similar specific plug-in probability and power capacity and considering the fact that the summation of two independent normal random variables is also normally distributed as follows

$$X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2) \quad (22)$$

The overall probability distribution of CD can be estimated by summation of each group. Therefore, it is expected that the pdf of CD within each hour follows a normal distribution which can be used to generate scenarios of PEV fleet demand required by the system planners. It should be noted that the obtained distribution is only valid for a specific time instance and CD is varying gradually as the time goes on.

V. CONCLUSION

Integration of the vehicular loads into the probabilistic distribution system planning problems has been addressed in this paper. A stochastic modelling framework has been thoroughly elaborated that can be efficiently employed in order to take into account the uncertainty attributes of the plug-in electric vehicles. A probabilistic approach based on queuing theory is presented to estimate the system wide battery charging power demand of PEV fleet. The exponential charging profile of the battery has been used to extract the power demand at different instances of the charging period. NHTS dataset has been utilized in several ways to probabilistically quantify the status of PEVs to indicate whether they are 'parked' or 'on the move'. The daily mileage data was also used for multiple purposes. For example, the driving data was utilized to measure the arrival rates of PEVs for charging at different hours of the day.

Since the model variables are characterized by a stochastic behavior and are correlated, a multivariate distribution function was built by means of copula function and the respective marginal empirical distributions, which in turn allowed the computation of the hourly system load.

The results indicate that in the fixed electricity rate environment, PEVs can contribute to increase the load demand at certain hours, although the CD is very limited most of the time. Furthermore, these high CDs are found to decrease significantly when the short commuting mileage is considered which leads to significant levels of SOC of the PEV batteries when arriving for charging. The numerical results have shown the effectiveness of the proposed methodology compared to MCS.

VI. NOMENCLATURE

| | |
|------------|---|
| α | Confidence level |
| λ | Mean rate of vehicles' arrival |
| ρ | Rank correlation matrix |
| ϵ | Battery charging constant |
| d | Daily distance |
| d_{max} | Maximum mileage with a fully charged battery |
| E_0 | Initial SOC of the battery |
| E_c | Instantaneous energy status of PEV battery |
| E_{max} | Maximum Energy level of storage battery |
| f_{SOC} | Probability density function of initial SOC |
| f_{dmi} | Probability density function of daily mileage |
| f_{arr} | Probability density function of arrival time |
| G | Cumulative function of servicing time RV |
| $m(t)$ | Congestion measure at time t |
| P_{max} | Maximum battery power |
| P_c | Instantaneous power absorbed from the grid |
| pr | Plug-in probability |
| S | Service time RV |
| t_{c_i} | Time for i^{th} PEV to absorb power |
| T_{max} | Maximum charging time |
| T_{min} | Minimum charging time |

VII. ACRONYMS

| | |
|------|-----------------------------------|
| CD | Charging demand |
| CDF | Cumulative distribution function |
| MCS | Monte Carlo simulation |
| NHTS | National household travel survey |
| PDF | Probability distribution function |

| | |
|-----|--------------------------------|
| PEV | Plug-in electric vehicle |
| PJM | Pennsylvania, Jersey, Maryland |
| PM | Probabilistic modeling |
| SOC | State of charge |
| RV | Random variable |

REFERENCES

- [1] S. Hadley and A. Tsvetkova, "Potential Impacts of Plug-in Hybrid Electric Vehicles on Regional Power Generation," Oak Ridge National Laboratory Report, ORNL/TM- 2007/150, January 2008.
- [2] P. Denholm and W. Short, "An Evaluation of Utility System Impacts and Benefits of Optimally Dispatched Plug-In Hybrid Electric Vehicles," National Renewable Energy Laboratory, Golden, CO, Tech. Rep. NREL/TP-620-40293, Oct. 2006.
- [3] S. Rahman and G. B. Shrestha, "An investigation into the impact of electric vehicle load on the electric utility distribution system," *Power Delivery, IEEE Transactions on*, vol. 8, pp. 591-597, 1993.
- [4] A. Hajimiragha, C. A. Cañizares, M. W. Fowler, and A. Elkamel, "Optimal transition to plug-in hybrid electric vehicles in Ontario, Canada, considering the electricity-grid limitations," *IEEE Trans. Ind. Electron.*, vol. 57, pp. 690–701, Feb. 2010.
- [5] S. Letendre and R. A. Watts, "Effects of plug-in hybrid electric vehicles on the Vermont electric transmission system," in *Proc. Transportation Research Board 88th Annu. Meeting, Washington, DC, Jan. 2009*.
- [6] M. Kintner-Meyer, K. Schneider, and R. Pratt, "Impacts assessment of plug-in hybrid vehicles on electric utilities and regional US power grids, Part 1: Technical Analysis," Pacific Northwest National Laboratory, 2006.
- [7] Q. Kejun, Z. Chengke, M. Allan, and Y. Yue, "Modeling of Load Demand Due to EV Battery Charging in Distribution Systems," *Power Systems, IEEE Transactions on*, vol. 26, no. 2, pp. 802-810, 2011.
- [8] J. G. Vlachogiannis, "Probabilistic Constrained Load Flow Considering Integration of Wind Power Generation and Electric Vehicles," *Power Systems, IEEE Transactions on*, vol. 24, no. 4, pp. 1808-1817, 2009.
- [9] K. Clement-Nyns, E. Haesen, J. Driesen, "The Impact of Charging Plug-In Hybrid Electric Vehicles on a Residential Distribution Grid," *Power Systems, IEEE Transactions on*, vol. 25, no. 1, pp. 371-379, 2010.
- [10] L. P. Fernandez, T. G. S. Roman, R. Cossent, C. M. Domingo, P. Frias, "Assessment of the Impact of Plug-in Electric Vehicles on Distribution Networks," *Power Systems, IEEE Transactions on*, vol. 26, no. 1, pp. 206-213, 2011.
- [11] D. Wu, D. C. Aliprantis, K. Gkritza, "Electric Energy and Power Consumption by Light-Duty Plug-In Electric Vehicles," *Power Systems, IEEE Transactions on*, vol. 26, no. 2, pp. 738-746, 2011.
- [12] N. Rotering, M. Ilic, "Optimal Charge Control of Plug-In Hybrid Electric Vehicles in Deregulated Electricity Markets," *Power Systems, IEEE Transactions on*, vol. 26, no. 3, pp. 1021-1029, 2011.
- [13] C. Fernandes, P. Frás, J. M. Latorre, Impact of Vehicle-to-Grid on Power System Operation Costs: The Spanish case study, *Applied Energy*, vol. 96, pp.194-202, 2012.
- [14] C. Battistelly, L. Baringo, A. J. Conejo, Optimal Energy Management of Small Electric Energy Systems Including V2G Facilities and Renewable Energy Sources, *Electric Power System Research*, vol. 92, pp. 50-59, 2012.
- [15] *U.S. Department of Transportation, Federal Highway Administration, 2009 National Household Travel Survey*. Available: <http://nhts.ornl.gov>
- [16] Z. Darabi and M. Ferdowsi, "Aggregated Impact of Plug-in Hybrid Electric Vehicles on Electricity Demand Profile," *Sustainable Energy, IEEE Transactions on*, vol. 2, no. 4, pp. 501-508, 2011.
- [17] "U.S. Department of Transportation, Federal Highway Administration, 2009 National Household Travel Survey user's guide," February 2011.
- [18] S. G. Eick, W. A. Massey, and W. Whitt, "The Physics of the M/G/infty Queue," *Operations Research*, vol. 41, pp. 731–742, 1993.

- [19] G. Papaefthymiou and D. Kurowicka, "Using Copulas for Modeling Stochastic Dependence in Power System Uncertainty Analysis," *Power Systems, IEEE Transactions on*, vol. 24, no. 1, pp. 40-49, 2009.
- [20] R. B. Nelsen, *An Introduction to Copulas*, Springer, New York, 1999.
- [21] "Assessment of Plug-in Electric Vehicle Integration with ISO/RTO Systems," KEMA, Inc. and ISO/RTO Council, 2010.
- [22] F. M. Dekking *et al.*, *A Modern Introduction to Probability and Statistics*. London, U.K.: Springer-Verlag, 2005, pp. 313-328.
- [23] G. W. Corder and D. I. Foreman, *Nonparametric Statistics for Non-Statisticians*. New York: Wiley, 2009, pp. 26-35.
- [24] D. Dallinger, D. Krampe, and M. Wietschel, "Vehicle-to-Grid Regulation Reserves Based on a Dynamic Simulation of Mobility Behavior," *Smart Grid, IEEE Transactions on*, vol. 2, no. 2, pp. 302-313, 2011.
- [25] A. Sklar, Fonctions de r'epartition 'a n dimensions et leurs marges, Publications del'Institut de Statistique de L'Universit'e de Paris, (1959), pp. 8:229-231.
- [26] G. Li and X. Zhang, "Modeling of Plug-in Hybrid Electric Vehicle Charging Demand in Probabilistic Power Flow Calculations," *Smart Grid, IEEE Transactions on*, vol. 3, no. 1, pp. 492-499, 2012.
- [27] M. Alizadeh, A. Scaglione, J. Davies, and K. S. Kurani "A Scalable Stochastic Model for the Electricity Demand of Electric and Plug-In Hybrid Vehicles," *Smart Grid, IEEE Transactions on*, vol. 5, no. 2, pp. 848-860, 2014.

FIGURES

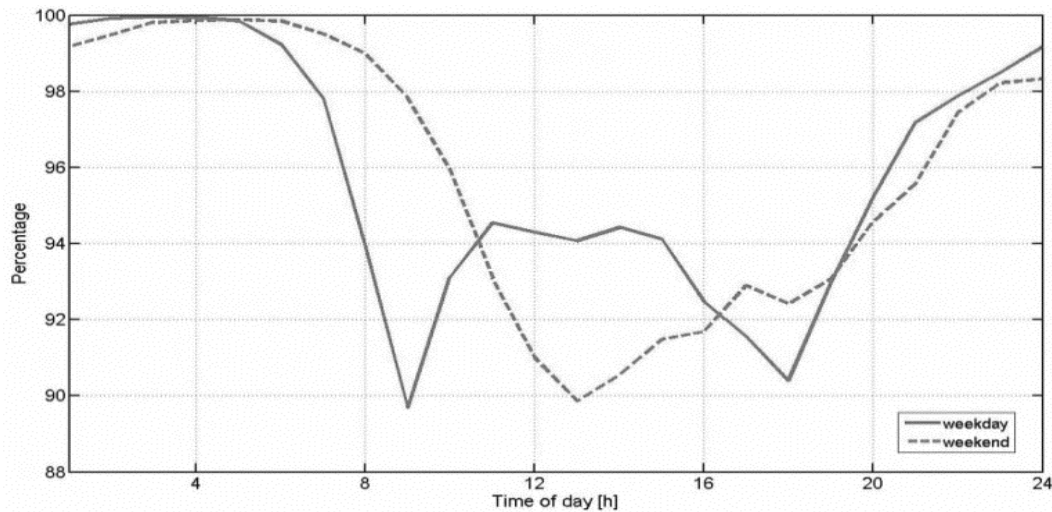


Figure 1. Average proportion of parked cars

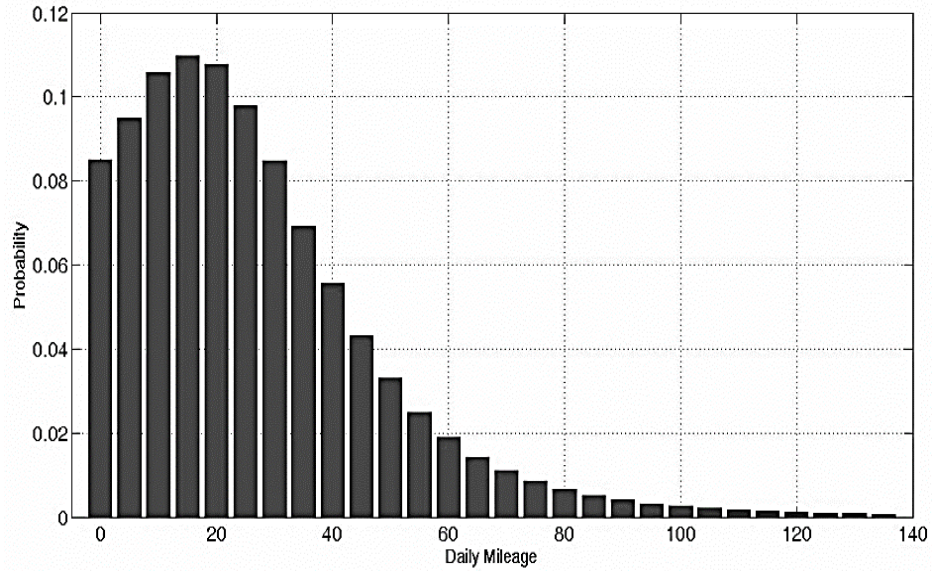


Figure 2. Histogram of daily mileage

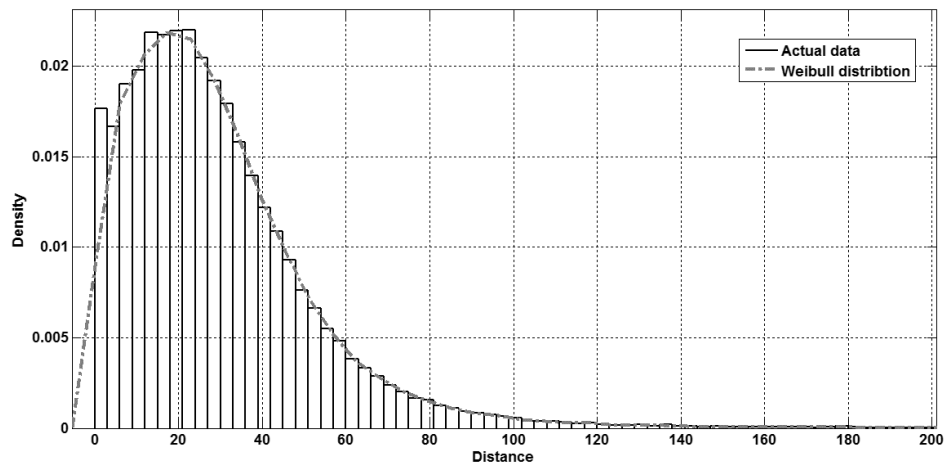


Figure 3. Probability density function of daily mileage

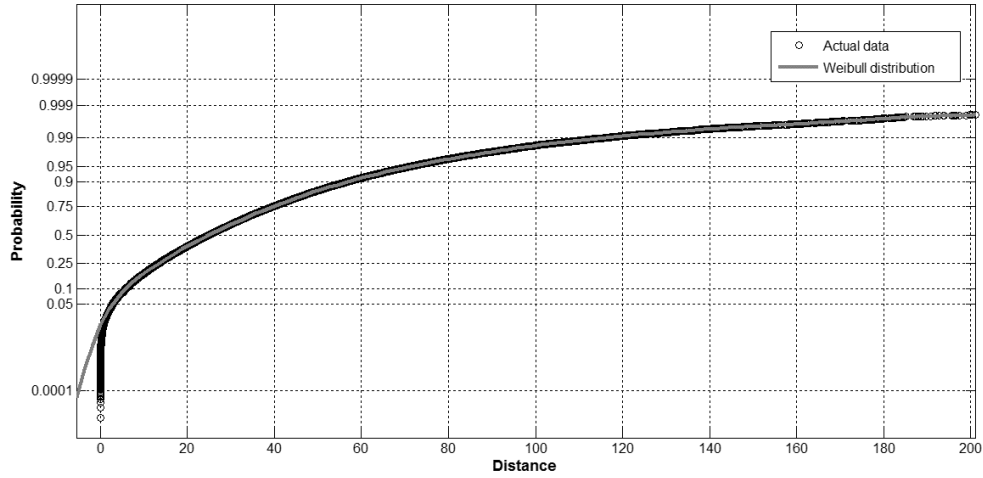


Figure 4. Test for Weibull distribution of daily mileage

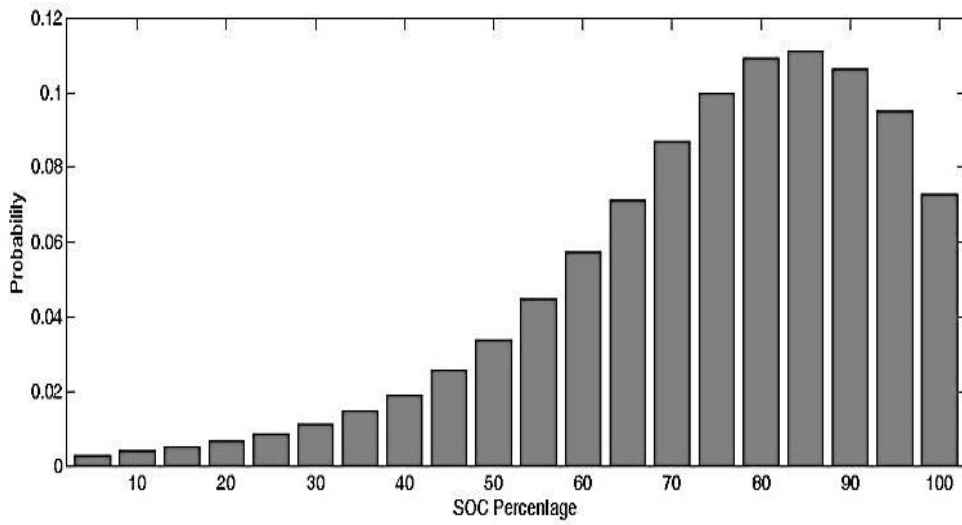


Figure 5. Probability density distribution of initial battery SOC

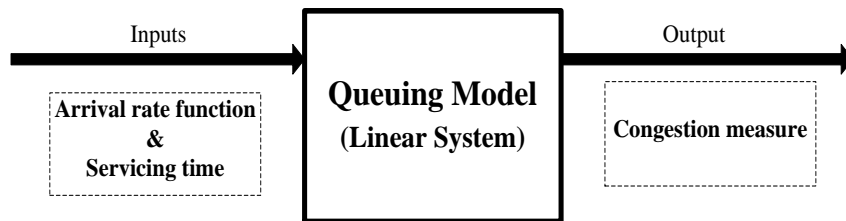


Figure 6. Congestion measure as a response of the queuing model

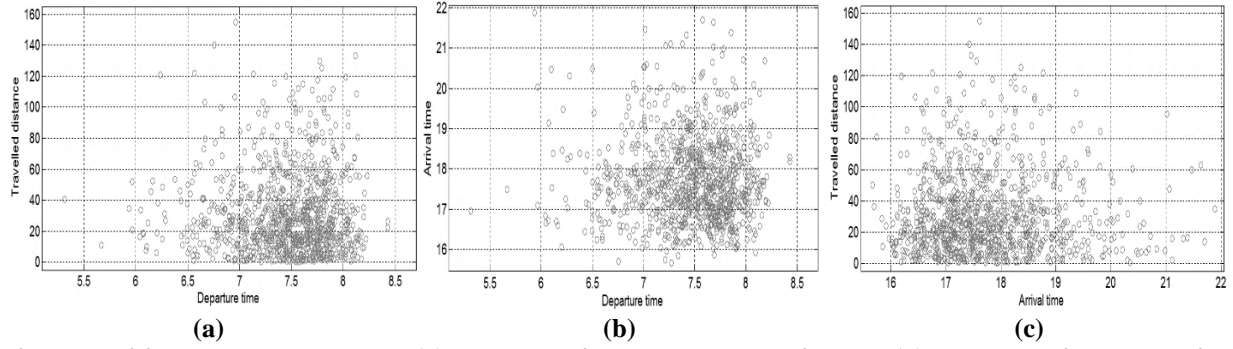


Figure7. Initial scatter plot between (a) departure time and travelled distance, (b) departure time and arrival time, (c) arrival time and travelled distance

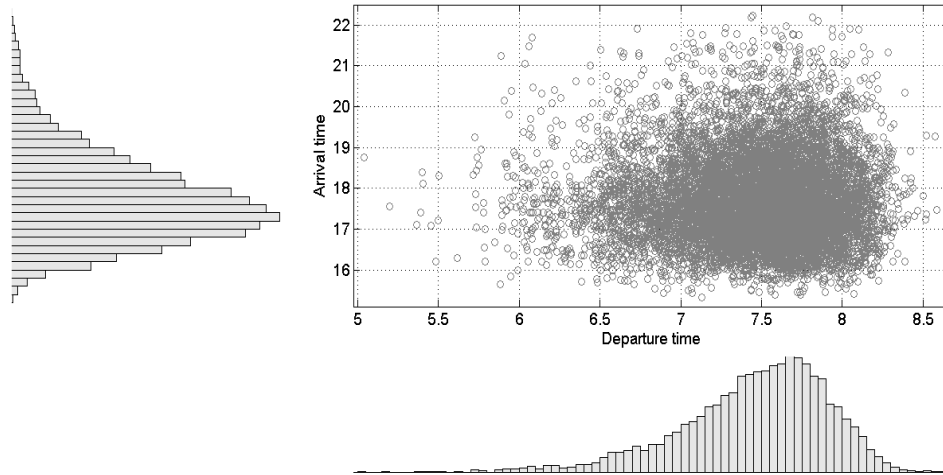
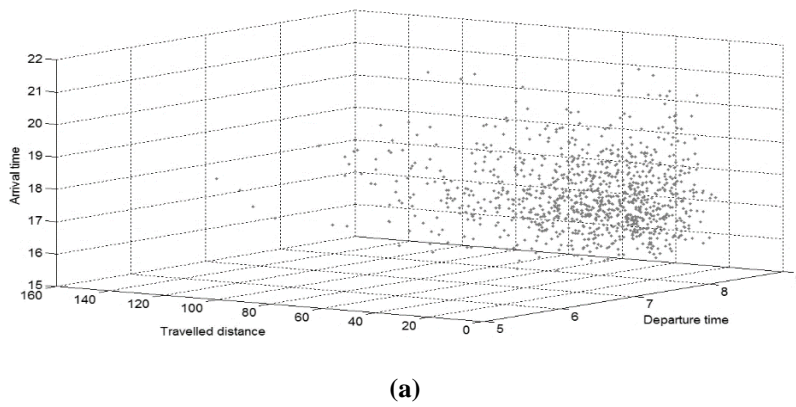
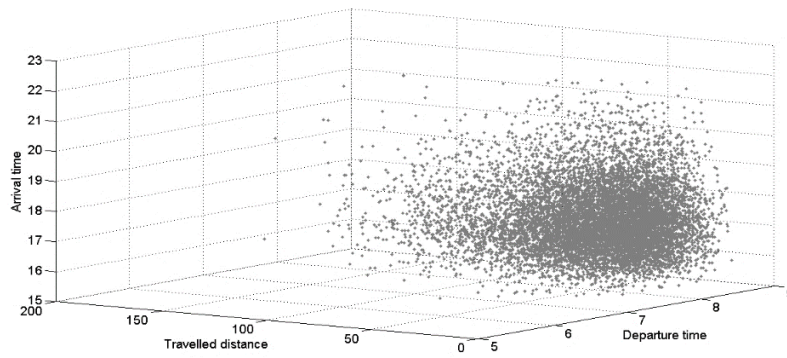


Figure 8. Final Scatter plot between departure time and arrival time in actual domain





(b)

Figure 9. (a) Initial 3D scatter plot of three random variables in actual domain, (b) Final 3D scatter plot of three random variables in actual domain

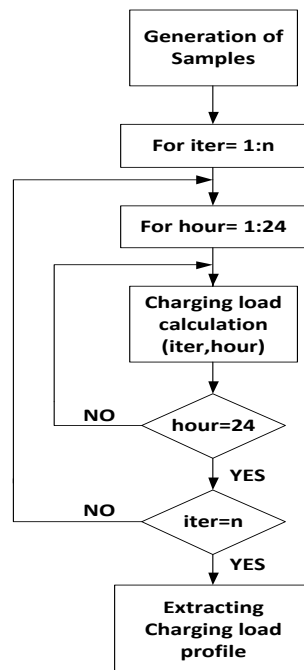
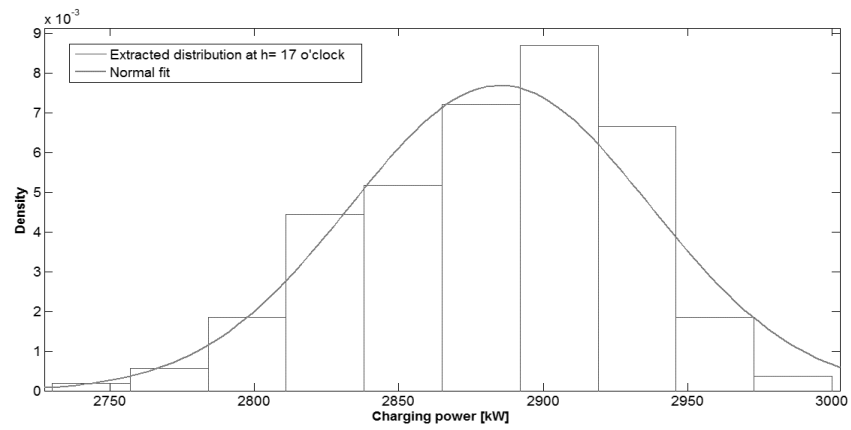
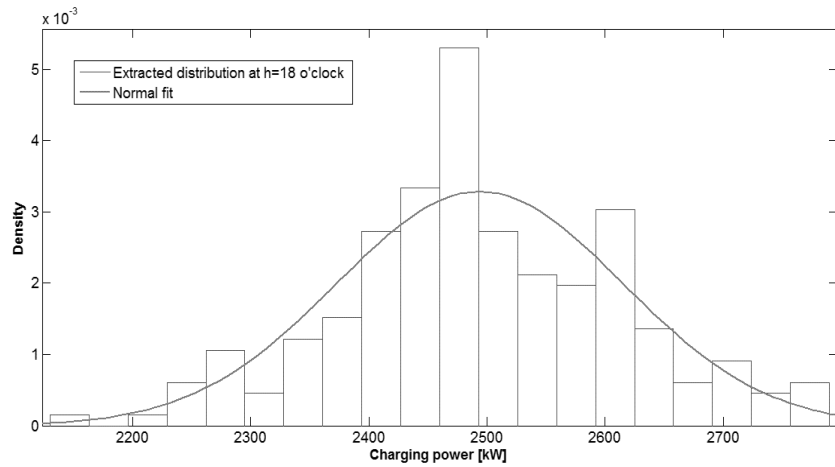


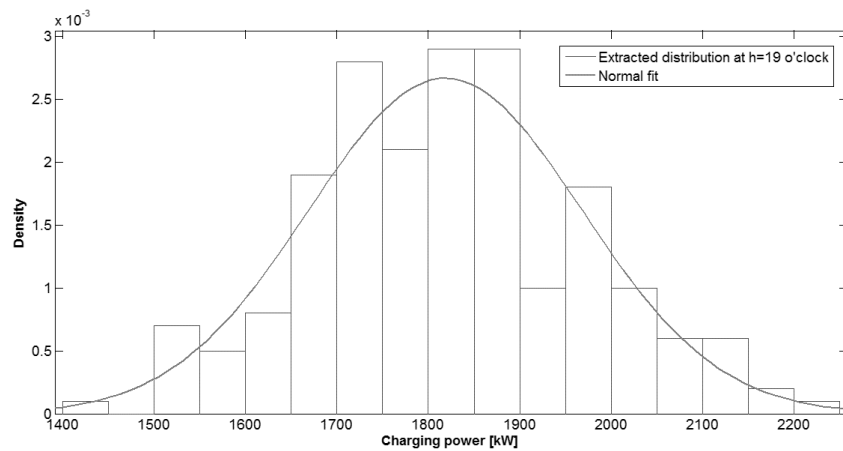
Figure 10. Flow chart illustrating the procedure for the whole day



(a)



(b)



(c)

Figure 11. Samples of the distributions of PEVs demand power along with the fitted PDFs at (a) h=17, (b) h=18 and (c) h=19

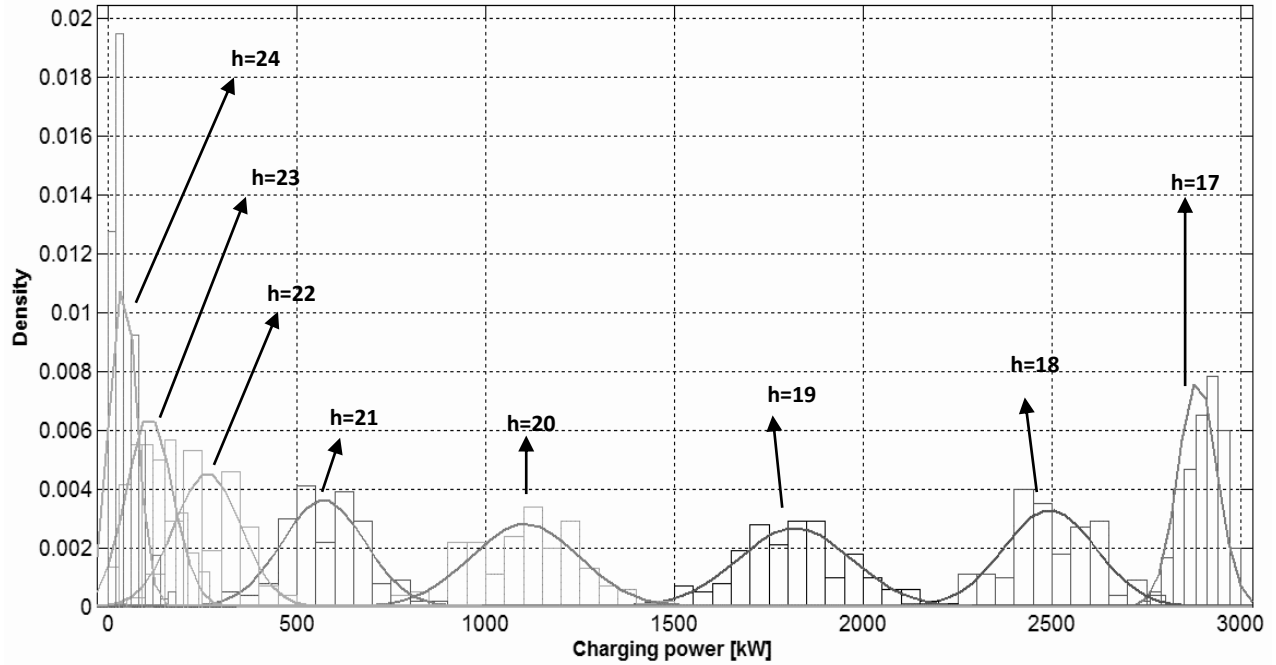


Figure 12. Probability distribution of charging demand from h=17 to h=24

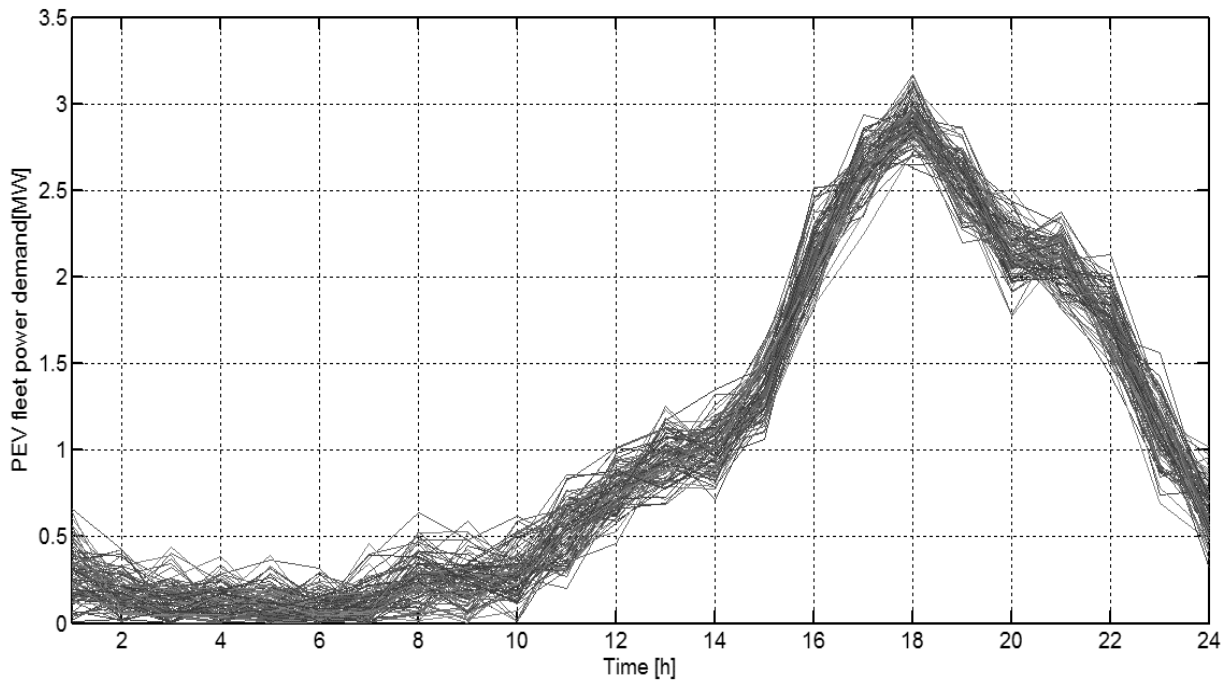


Figure 13. Uncontrolled domestic charging power demand for 1000 PEV

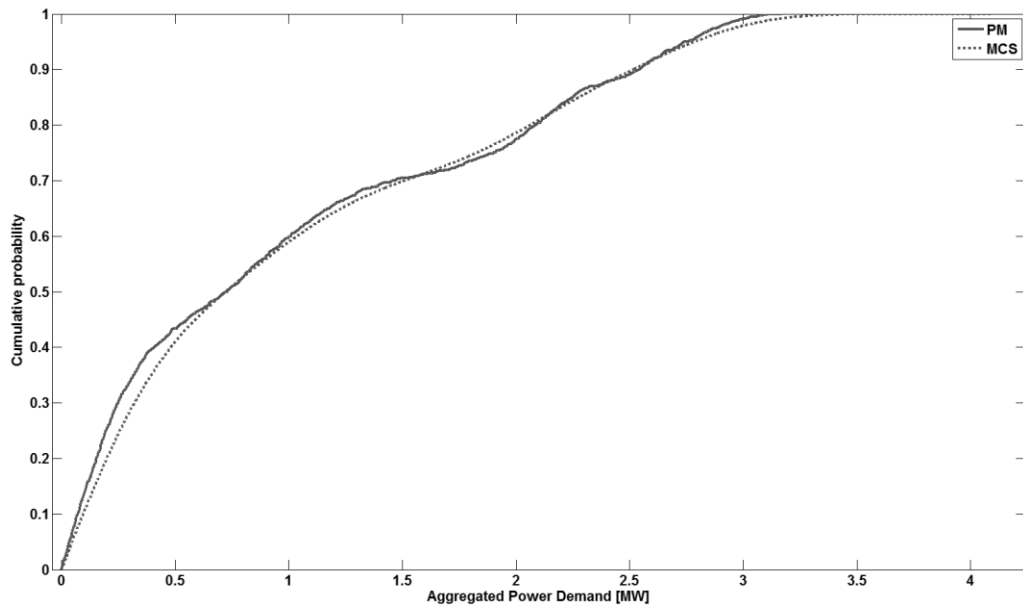


Figure 14. Empirical CDF curves obtained from PM and MCS

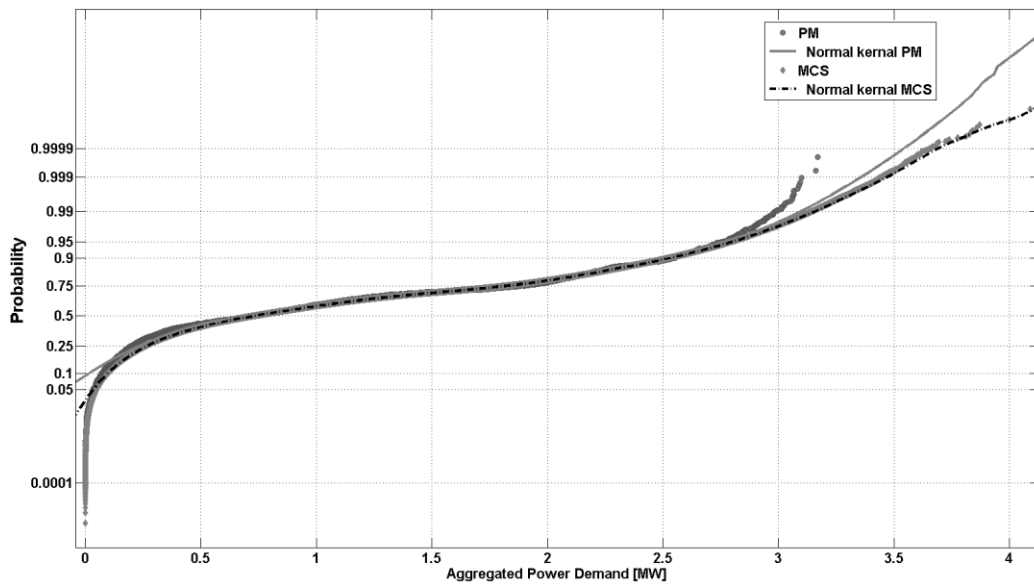


Figure 15. Probability plots and normal kernel fits for PM and MCS

TABLES

Table 1. The parameters of the fitted pdfs

| Random Variable | Fitted Distribution | Parameters |
|--------------------|---------------------------|--|
| Departure time | Weibull | $a = 7.67$ $b = 21.83$ |
| Travelled distance | Weibull | $a = 32.04$ $b = 1.23$ |
| Arrival time | Generalized extreme value | $k = -0.06$ $\mu = 17.3$ $\sigma = 0.85$ |

Table 2. Characteristics of normal distribution fit to estimated CD

| Hour | μ (Estimate) | σ (Estimate) | μ (Std. Err.) | σ (Std. Err.) | Log likelihood |
|-------------------|------------------|---------------------|-------------------|----------------------|----------------|
| 17 o'clock | 2885.1 | 51.8404 | 3.66567 | 2.6018 | -1072.92 |
| 18 o'clock | 2493.9 | 121.435 | 8.58677 | 6.09466 | -1243.16 |
| 19 o'clock | 1818.6 | 149.519 | 10.5726 | 7.50412 | -1284.77 |
| 20 o'clock | 1105.05 | 142.494 | 10.0758 | 7.15157 | -1275.15 |
| 21 o'clock | 570.6 | 110.338 | 7.80209 | 5.53771 | -1224 |
| 22 o'clock | 263.25 | 87.7063 | 6.20177 | 4.40185 | -1178.09 |
| 23 o'clock | 109.5 | 61.691 | 4.36221 | 3.09618 | -1107.72 |
| 24 o'clock | 40.35 | 36.1651 | 2.55726 | 1.81508 | -1000.91 |