

Revisited Vanoni-Brooks sidewall correction

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Abstract

The evaluation of the sidewall friction could be inconvenient in the implement of the Vanoni-Brooks sidewall correction procedure. Using the Colebrook-White equation and Nikuradse's pipe friction data, two explicit formulae are developed in this note for finding the sidewall friction factor. They are applicable for various sidewalls that are either hydrodynamically-smooth or rough, or in the transitional regime.

Key Words: Sidewall correction, Friction factor, Open channel flow, Pipe flow, Sediment transport

1 Introduction

In conducting experimental studies of open channel flow, the hydraulic resistance of the sidewalls of a laboratory flume may not be the same as that of the bed. For example, sidewall friction may differ significantly from bed friction in investigating sediment transport in a laboratory flume (e.g. Tang and Wang, 2009). In natural streams, flood flow could be subject to high bank roughness in the presence of lateral riparian buffers (e.g. Afzalimehr and Dey, 2009). To unify resistance results that are governed by specified bed configuration, removal of sidewall or bank effect, which is referred to as sidewall correction, is needed.

Among various approaches available in the literature (Brownlie, 1981; Cheng and Chua, 2005; Dey and Raikar, 2005; Einstein, 1942; Williams, 1970), the correction procedure proposed by Vanoni and Brooks (1957) is the most widely used. The Vanoni-Brooks sidewall correction was derived by applying the Colebrook-White pipe friction equation or the Moody diagram to evaluation of sidewall friction. The assumptions made in the derivation include that the average velocity and energy slope are considered the same for the sidewall and bed. For a rectangular open channel, Vanoni and Brooks proposed the bed shear stress be computed as follows:

$$f_b = f + \frac{2h}{B}(f - f_w) \quad (1)$$

$$r_b = \frac{f_b}{f} r \quad (2)$$

$$\tau_b = \rho u_{*b}^2 = \rho g r_b S \quad (3)$$

where f_b is the bed friction factor, f_w is the wall friction factor, $f (= 8grS/V^2)$ is the bulk friction factor, h is the flow depth, B is the channel width, r is the hydraulic radius, r_b , u_{*b} , and τ_b are the bed-related hydraulic radius, shear velocity and shear stress, respectively, ρ is the fluid density, g is the gravitational acceleration, V is the cross-sectional average velocity, and S is the energy slope. The Vanoni-Brooks correction basically provides a way to predict the bed-related hydraulic radius, r_b , in the presence of sidewalls. For extremely wide channels, i.e. $B \gg h$, the prediction simply yields the result of $r_b = h$.

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It is noted that to evaluate f_w may be an inconvenient step for implementing the Vanoni-Brooks sidewall correction. However, for sidewalls that are hydrodynamically smooth, the procedure for finding f_w is simplified, and its value can be read from an empirical curve using the ratio, Re/f , where $Re = 4Vr/\nu$, where ν = kinematic viscosity of fluid. Vanoni and Brooks (1957) mentioned that their approach also applies for other sidewall conditions, but did not provide relevant details that are relatively complicated.

This note is presented here with two purposes. First, the Vanoni-Brooks approach to the calculation of f_w is generalized for various sidewalls, which are either hydrodynamically rough or smooth or in the transitional regime. To facilitate the computerization of the correction, explicit equations are developed to replace graphical relations including the curve of f_w against Re/f provided by Vanoni and Brooks (1957). Second, for the case that the Nikuradse's friction data derived for sand-roughened pipes is preferable to the Moody diagram, this note also provides an alternative equation for the evaluation of f_w .

2 Calculation of sidewall friction factor f_w based on Colebrook-White equation

The Colebrook-White equation, though derived from pipe flow measurements, serves as a practical tool to estimate the friction factor for turbulent open channel flows. It relates the friction factor f to the Reynolds number Re and equivalent roughness height k_s , as

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{Re \sqrt{f}} + \frac{k_s}{3.7D} \right) \quad (4)$$

where $D (= 4r)$ is the hydraulic diameter and $Re = VD/\nu$. To perform the sidewall correction, Eq. (4) is rewritten as

$$\frac{1}{\sqrt{f_w}} = -2 \log \left(\frac{2.51}{Re_w \sqrt{f_w}} + \frac{k_{sw}}{3.7D_w} \right) \quad (5)$$

where the subscript “w” denotes the variables that are sidewall-related. Equation (5) has two unknowns, f_w and r_w . However, by noting $V = V_w$ and $S = S_w$, as assumed by Vanoni and Brooks (1957), we get $Re_w = (Re/f)f_w$ and $D_w = (D/f)f_w$. Then, Eq. (5) can be reformulated as an equation including only a single unknown, f_w , i.e.

$$\frac{1}{\sqrt{f_w}} = -2 \log \left(\frac{2.51}{f f_w^{1.5}} + \frac{1}{f k_{sw}} \frac{1}{3.7 f_w} \right) \quad (6)$$

Equation (6) shows that f_w can be estimated by iteration with known values of Re/f and $D/(fk_{sw})$. To compute f_w explicitly, we consider two extreme conditions. First, for hydrodynamically smooth sidewalls, Eq. (6) reduces to

$$\frac{1}{\sqrt{f_{ws}}} = -2 \log \left(\frac{2.51}{f f_{ws}^{1.5}} \right) \quad (7)$$

where subscript “ws” denotes the smooth sidewall condition. Furthermore, Eq. (7) can be transformed in an explicit form,

$$f_{ws} = 31 \left[\ln \left(1.3 \frac{Re}{f} \right) \right]^{-2.7} \quad (8)$$

The transformation of Eq. (7) into Eq. (8) was made through curve-fit based on the logistical function, of which the details are given in Appendix. Equation (8) agrees with Eq. (7) with absolute errors less than 0.48% for $f_{ws} = 0.005$ to 0.05. The selected range of f_{ws} is considered to cover possible values encountered in practice, which is also much wider than that considered by Vanoni and Brooks (1957). It should be mentioned that Eq. (8), which quantifies f_{ws} as a function of Re/f for smooth sidewalls, is in excellent agreement with the curve provided by Vanoni and Brooks (1957).

Similarly, for hydrodynamically rough sidewalls, Eq. (5) reduces to

$$\frac{1}{\sqrt{f_{wR}}} = -2 \log \left(\frac{1}{3.7 \frac{D}{fk_{sw}} f_{wR}} \right) \quad (9)$$

where subscript “ wR ” denotes the rough sidewall condition. By performing transformation similar to that applied to Eq. (7), Eq. (9) can also be rewritten as an explicit function,

$$f_{wR} = 11.7 \left[\ln \left(7.6 \frac{D}{fk_{sw}} \right) \right]^{-2.5} \quad (10)$$

Eq. (10) agrees with Eq. (9) with absolute errors less than 0.59% for $f_{wR} = 0.005$ to 0.05.

To interpolate between f_{wS} and f_{wR} for sidewalls in the transitional regime, Eqs. (8) and (10) can be further combined in the power-sum form,

$$f_w^\alpha = f_{wS}^\alpha + f_{wR}^\alpha \quad (11)$$

where α is a function of D/fk_{sw} . Equation (11) applies generally for all kinds of sidewalls, which are either hydrodynamically smooth or hydrodynamically rough, or in between. Figure 1 shows that f_w -values

computed using Eq. (11) with $\alpha = 2 \left(\frac{d}{fk_{sw}} \right)^{0.1}$ are very close to those computed using Eq. (6) by iteration,

with absolute errors less than 3.4% for $Re/f = 10^4 - 10^{12}$.

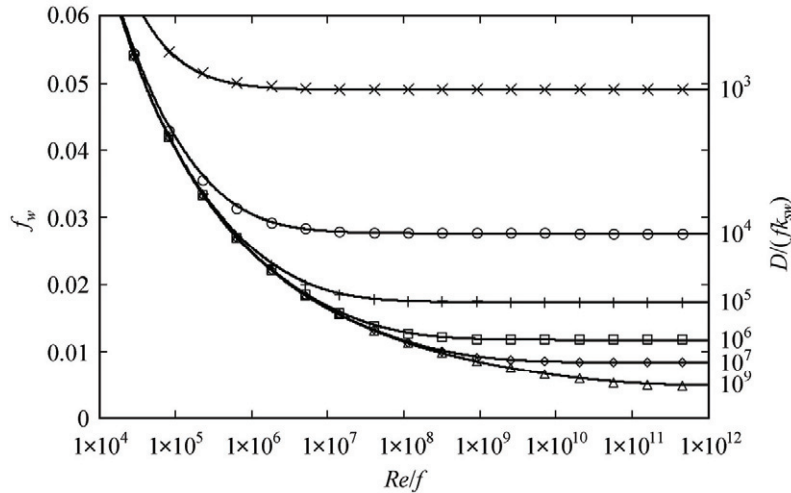


Fig. 1 Generalized function for finding f_w for various sidewall conditions. The symbols are computed by iteration using Eq. (6). The solid lines are computed explicitly with Eq. (11)

3 Calculation of sidewall friction factor f_w based on Nikuradse’s pipe friction data

Nikuradse (1933) established the classical boundary friction database for both smooth and sand-roughened pipes over a wide range of Reynolds numbers. The friction factor derived from Nikuradse’s data differs from the Moody diagram in that Nikuradse’s data show a “dip” in the friction factor for turbulent pipe flows between fully-smooth and fully-rough regimes (Fig. 2). The dip phenomenon has been attributed to the pipe wall roughened by well-sorted sand particles, and it may die out in commercial pipes with non-uniform distribution of roughness elements (Colebrook, 1939). Similar transitional effects were also observed recently for other regular roughness configurations (Jimenez, 2004).

Equation (11) describes the monotonic change in f_w , but cannot predict the dipped or reduced f_w , as shown by Nikuradse’s data. Because the difference between the Moody diagram and Nikuradse’s friction data is solely limited to the transitional regime, the formulae used to predict f_{wS} and f_{wR} , [i.e. Eqs. (8) and(10)] still apply, while the interpolation in between needs to be modified. To consider the dipped f_w , f_{wS} and f_{wR} are combined in the following multiplicative form,

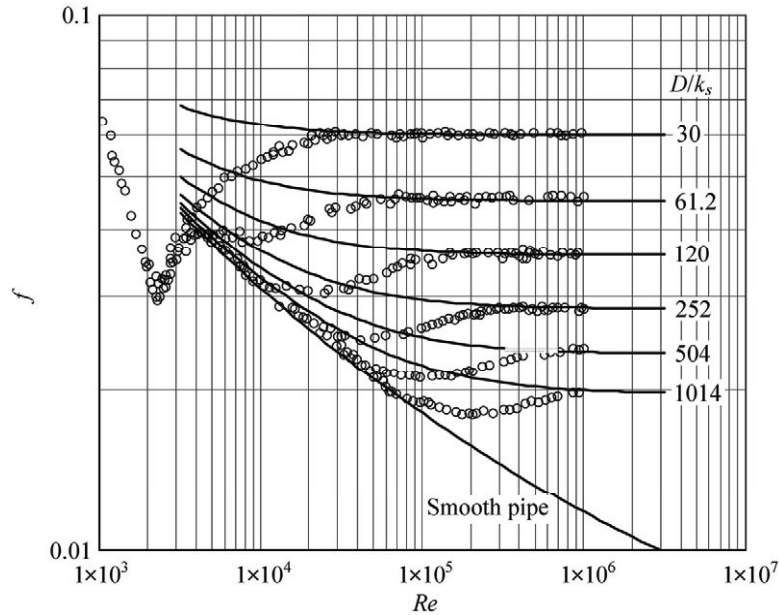


Fig. 2 Dipped friction factor in the transitional regime in comparison with the Moody diagram. The symbol of circle denotes Nikuradse's (1933) data. The solid lines are computed using Eq. (4)

$$f_w^\beta = f_{wS}^\beta f_{wR}^{1-\beta} \quad (12)$$

where $\beta = \frac{1}{1 + [Re/(80D/k_{sw})]^2}$. The interpolation given by Eq. (12) was used early by Cheng (2008) for

describing the Nikuradse's friction factor that is dipped in the transitional regime. As Eq. (11), Eq. (12) is applicable for finding f_w for all kinds of sidewalls. Figure 3 provides comparisons of four sets of f_w -curves, which were computed using Eqs. (11) and (12), respectively. It can be observed that the reduction in f_w (denoted by the dashed line) is significant for small $D/(fk_{sw})$ but limited to the transitional regime.

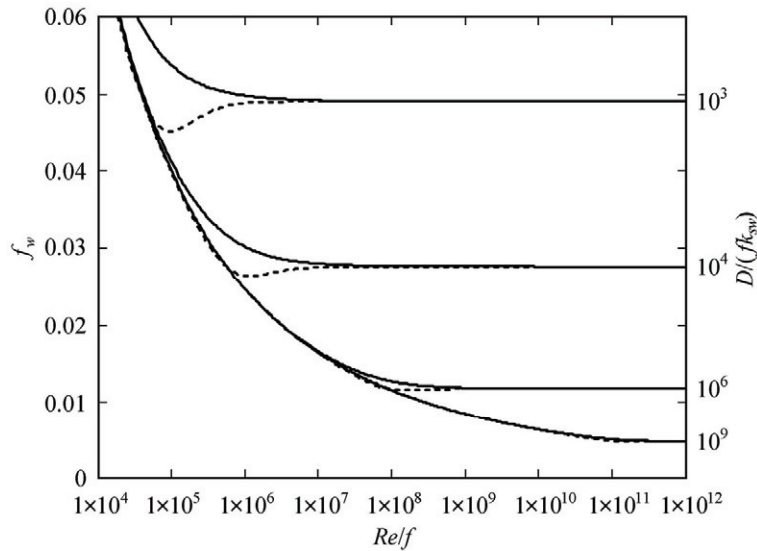


Fig. 3 Reduction of f_w in the transitional regime. The solid lines are computed based on the Colebrook-White equation. The dashed lines are computed based on Nikuradse's friction data

4 Conclusions

Explicit formulae are proposed for computing the sidewall friction factor. They are applicable to various sidewalls that are either hydrodynamically smooth or rough, or in between. The proposed approach is generally useful in simplifying the sidewall correction procedure developed by Vanoni and Brooks in 1957. This note also shows how the dipped friction factor in the transitional regime, as demonstrated by Nikuradse's pipe data, can be effectively taken into account for sidewall correction. It should be mentioned that the present approach was developed based solely on the Colebrook-White equation and Nikuradse's data, and could be further modified for other roughness configurations.

5 Appendix: transformation using the logistical function

First, use Eq. (7) to numerically compute a series of values of f_w and Re/f for a range of f_w , say, $f_w = 0.005-0.05$. Then, take $x = \ln(f_w)$ and $y = 1/\ln(Re/f)$. Next, perform best-fit with the logistical function defined as

$$y = \frac{a}{1 + b \exp(-cx)} \quad (13)$$

where a , b and c are constants. Finally, with optimized values of a , b and c , we get the following explicit functions,

$$f_w = \left(\frac{a}{b}\right)^{-1/c} \left[\ln\left(e^{-1/a} \frac{Re}{f}\right) \right]^{-1/c} \quad (14)$$

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References

- Afzalimehr H. and Dey S. 2009, Influence of bank vegetation and gravel bed on velocity and Reynolds stress distributions. *International Journal of Sediment Research*, Vol. 24, No. 2, pp. 236–246.
- Brownlie W. R. 1981, Reexamination of Nikuradse roughness data. *Journal of the Hydraulics Division-ASCE*, Vol. 107, No. 1, pp. 115–119.
- Cheng N. S. 2008, Formulas for friction factor in transitional regimes. *Journal of Hydraulic Engineering-ASCE*, Vol. 134, No. 9, pp. 1357–1362.
- Cheng N. S. and Chua L. H. C. 2005, Comparisons of sidewall correction of bed shear stress in open-channel flows. *Journal of Hydraulic Engineering-ASCE*, Vol. 131, No. 7, pp. 605–609.
- Colebrook C. F. 1939, Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws. *Proceedings of the Institution of Civil Engineers*, 12, pp. 393–422.
- Dey S. and Raikar R. V. 2005, Scour in long contractions. *Journal of Hydraulic Engineering-ASCE*, Vol. 131, No. 12, pp. 1036–1049.
- Einstein H. A. 1942, Formulas for the transportation of bed load. *Transactions American Society of Civil Engineers*, 107, pp. 561–577.
- Jimenez J. 2004, Turbulent flows over rough walls. *Annual Review of Fluid Mechanics*, 36, pp. 173–196.
- Nikuradse J. 1933, *Stromungsgesetze in rauhen Rohren*. *Forschung auf dem Gebiete des Ingenieurwesens*, Forschungsheft 361. VDI Verlag, Berlin, Germany (in German). (English translation: *Laws of flow in rough pipes*, NACA TM 1292, 1950).
- Tang L. M. and Wang X. K. 2009, Experimental study on three dimensional movements of particles I Effects of particle diameter on velocity and concentration distributions. *International Journal of Sediment Research*, Vol. 24, No. 2, pp. 159–168.
- Vanoni V. A. and Brooks N. H. 1957, *Laboratory studies of the roughness and suspended load of alluvial streams*. Sedimentation Laboratory, California Institute of Technology, Pasadena, Calif.
- Williams G. P. 1970, *Flume width and water depth effects in sediment transport experiments*. United States Geological Survey Professional Paper, 562-H.