

1 **Estimate of sediment pickup rate with densimetric Froude number**

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12 **Abstract**

13 Experimental data collected in open channel flows show that the sediment pickup rate is
14 better correlated with the densimetric Froude number than the Shields number. An empirical
15 formula is then proposed for estimating the pickup rate. It does not involve the critical
16 condition for the incipient sediment motion, and therefore can be applied to sediment
17 entrainment of different stages including weak sediment motion. The result obtained in the
18 present study is also supported by several series of data reported in the literature.

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22 Introduction

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24 The pickup rate E is defined as the volume of the sediment entrained by flow over a unit bed
25 area and has a unit of m/s. Its dimensionless form can be defined as (van Rijn 1984)

$$E_* = \frac{E}{\sqrt{\Delta g D}} \quad (1)$$

26 where $\Delta = (\rho_s - \rho)/\rho$ with ρ_s and ρ denoting the grain and fluid density, respectively, g is the
27 gravitational acceleration and D is the grain diameter. A pickup function that describes
28 sediment entrainment rate can be integrated as part of a bedload theory. For example,
29 Einstein (1950) proposed a probability-based pickup function in the derivation of bedload
30 formula. The bedload theory as a whole has been verified with various laboratory and field
31 data; however the pickup function included in the theory has been seldom calibrated
32 independently.

33 Laboratory pickup experiments have been reported only in a few publications. Van
34 Rijn (1984) designed a sediment lift and conducted a series of pickup experiments in open
35 channel flows. Based on the data collected, he proposed the following pickup function

$$E_* = 0.00033 D_*^{0.3} T^{1.5} \quad (2)$$

36 where D_* is the dimensionless grain diameter defined as $(\Delta g / \nu^2)^{1/3} D$ with ν being the
37 kinematic viscosity of fluid, and $T = (\tau_* / \tau_{*c}) - 1$, with τ_* and τ_{*c} denoting the Shields number
38 and its critical value for incipient sediment motion, respectively. By noting that τ_{*c} can be
39 expressed as a function of D_* , Eq. (2) shows that E_* generally varies with τ_* and D_* . Van Rijn
40 (1984) also reported that the prediction using the pickup function proposed by Einstein (1950)
41 is very poor, and some other formulas apply only for either small or large sizes of sediment.
42 Okayasu et al. (2010) observed pickup rates over a small area of mobile bed that was located
43 at 10 cm downstream a fixed bed in open channel flows. They employed a laser distance

44 meter to measure the change in the bed elevation and thus the pickup rate. Other relevant
45 studies include Damgaard et al. (1997) and Dey and Debnath (2001), who investigated
46 pickup rates in steep channels. A recent theoretical attempt is due to Zhong et al. (2011), who
47 derived a pickup function by integrating particle velocity over all possible upward motions.
48 In addition, Fernandez Luque (1974) applied a photography technique to measure the
49 deposition rate, which is equal to the pickup rate when bedload transport is in equilibrium.

50 In the present study, a sediment lift similar to van Rijn (1984) was designed to
51 measure pickup rates of three sizes of uniform sediments in open channel flows. An empirical
52 formula is then developed by correlating the dimensionless pickup rate with the densimetric
53 Froude number and the dimensionless grain diameter. The result shows that a better
54 correlation is associated with the densimetric Froude number rather than the Shields number.

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57 **Experiments**

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59 Experiments were conducted in a slope-adjustable flume, 14 m long, 0.6 m wide and 0.6 m
60 deep. Flow rates were measured with an electro-magnetic current meter with precision of 0.3
61 l/s. The test section was 9 m downstream from the channel entrance (Fig. 1). The flow depth
62 remained close to 20 cm and the water temperature varied in the range of $28\pm 1.5^\circ\text{C}$ for all
63 experiments.

64 To measure sediment pickup rate, a sediment lift was designed and installed at the
65 bottom of the flume, through a rectangular opening located in the centre of the channel bed,
66 which measured 1.5 cm in the streamwise direction and 10 cm in the lateral direction (see
67 Fig. 1). The system included a piston that was installed inside a rectangular cylinder and
68 connected with an electro-motor. By varying the speed of the piston, one could control the

69 rate of sediment supply. To operate the lifting system at very low entrainment rates, a gear
70 box was also employed so that the piston could move as low as 0.2 mm/min. The
71 displacement of the piston was measured by a dial gage with a precision of 0.01 mm. The
72 product of the measured displacement and the area of the bed opening A yielded the bulk
73 volume V of sediment grains entrained by flow. The sediment pickup rate E was then
74 calculated as $V(1-\varepsilon)/(AT)$, where ε is the porosity of sediment bed and T is the time of
75 observation.

76 Three sets of uniform sediments were tested, with $D = 0.23, 0.44$ and 0.86 mm, and ρ_s
77 $= 2650$ kg/m³. The corresponding bed porosity measured was 0.44, 0.44 and 0.41,
78 respectively. To have consistent bed roughness through the channel, the same grains as filled
79 inside the sediment lift were also glued onto the entire bottom of the flume. To load sediment
80 into the lift, sediment grains was first submerged in water in a separate glass container and
81 then stirred in order to release air bubbles. Next, the grains were poured inside the lift with a
82 funnel beneath the water surface. Finally, the grains inside the slot were briefly stirred and
83 smoothed flat.

84 During the measuring of the pickup rate, the speed of the piston was adjusted slowly
85 to avoid any formation of hump or pit over the bed area of observation. It was considered
86 acceptable only when a flat surface with moving grains was observed over the area of
87 observation. It should be mentioned that the upward speed of the piston was much smaller
88 than the fall velocity of the sediment particles being picked up, so the lift would not affect
89 significantly the hydrodynamic force acting on the particles and thus the pickup rate. The
90 time for pickup rate measurement was taken as long as possible to avoid significant temporal
91 changes. For weak grain movement, the pickup rate was measured as long as 12 minutes and
92 for high transport the measurement took less than a minute for all the grains in the lift to be
93 entrained. Each run was repeated at least five times under the same flow condition to make

94 sure that reliable values were recorded. The results were considered acceptable only when
95 the deviation of each reading from the average was less than 15 %. The standard deviation of
96 the measurements was computed and then normalized by their mean as an indicator of the
97 variation in the pickup rate measurements. The normalized standard deviation had an average
98 of 7 % for all the experiments and a maximum of 22 % for the very large pickup rates for
99 which all the sediment loaded in the lift were entrained in less than a minute.

100 To perform each test, the flume was first filled with water from the downstream end
101 of the flume. Then, the pump was turned on to increase the flow rate gradually. At the same
102 time, the tailgate was also adjusted until approximately a uniform flow was observed. Finally,
103 the sediment pickup rate and the flow velocity were measured separately. Flow velocity
104 measurements were carried out prior to or after the pickup measurements using a 3D down-
105 looking Acoustic Doppler Velocimeter (ADV), which was positioned at the centre of the
106 flume right above the lift slot. To provide sufficient seeding material inside the flow, an air
107 bubble generator was positioned 1 m upstream the ADV location. It consisted of a mesh
108 made from 0.2 mm stainless steel wire, which was connected to a direct current to generate
109 tiny bubbles by means of electrolysis. For each velocity profile, 18-20 points were sampled
110 from the bed towards the water surface at a rate of 50 Hz, each for 3 minutes. The raw data of
111 flow velocity were processed using software WinADV. The shear velocity u_* was estimated
112 by fitting the velocity profile measured in the near-bed zone (about 25% of the flow depth) to
113 the logarithmic law. The boundary Reynolds number $Re_* = u_* D / \nu$ varied from 2.4 to 38.7,
114 showing that the bed condition encountered in the experiments was either hydraulically
115 smooth or transitional.

116 In total, 61 experiments were completed in the present study for a range of low and
117 high sediment pickup rates. The pickup rate E varied from 4.7×10^{-7} - 7×10^{-4} m/s, the

118 Reynolds number Re from $4.7 \times 10^4 - 2 \times 10^5$ and the Froude number Fr from 0.21-0.66. A
119 summary of the experimental data is presented in Table 1.

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122 **Comparison with previous studies**

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124 The measured pickup rates are compared with those predicted using van Rijn's (1984)
125 formula [i.e. Eq. (2)], as shown in Fig. 2. It can be observed that the experimental
126 measurements are not well predicted by van Rijn's formula. The predictions generally agree
127 with the measurements for $D = 0.44$ mm, but the predicted pickup rates become much greater
128 than (up to ten times) the measurements for $D = 0.23$ mm and 0.86 mm. In addition, there are
129 five cases that cannot be predicted using van Rijn's formula because their bed shear stresses
130 are lower than the critical value for incipient sediment motion. Significant discrepancies were
131 also observed when comparing the data with the formulas by Fernandez Luque (1974) and
132 Nakagawa and Tsujimoto (1980).

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135 **Correlation based on densimetric Froude number**

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137 In the following, two kinds of correlations are performed, one based on the dimensionless
138 grain diameter D_* and the Shields number τ_* and the other based on D_* and the densimetric
139 Froude number F_* defined as

$$F_* = \frac{U}{\sqrt{\Delta g D}} \quad (3)$$

140 where U is the depth-averaged flow velocity. The densimetric Froude number describes the
 141 bulk fluid force relative to the submerged grain weight. In comparison, the Shields number
 142 quantifies the bed shear force relative to the submerged grain weight. It is noted that F_* has
 143 been often used in the prediction of local scour (Hager 2007; Hong et al. 2013). When
 144 correlating with the Shields number τ_* and the dimensionless sediment diameter D_* , the
 145 dimensionless pickup rate E_* can be expressed in the power form,

$$E_* = a_1 D_*^{b_1} \tau_*^{c_1} \quad (4)$$

146 where a_1 , b_1 and c_1 are constants. It is noted that van Rijn (1984) presented a similar τ_* -based
 147 correlation, but involving the critical shear stress. Similarly, for the F_* -based correlation, E_*
 148 can be expressed as

$$E_* = a_2 D_*^{b_2} F_*^{c_2} \quad (5)$$

149 where a_2 , b_2 and c_2 are constants.

150 To quantify the goodness of fit of the proposed power functions, the Pearson product-
 151 moment correlation coefficients (i.e. Pearson's R) were calculated. Fig. 3 presents the
 152 variation of R_1^2 with b_1 , where R_1 is the correlation coefficient of $\log(E_*/D_*^{b_1})$ and $\log(\tau_*)$. It
 153 can be seen that R_1^2 achieves its maximum 0.82 at $b_1 \approx 1.8$. Also plotted in Fig. 3 is the
 154 variation of R_2^2 with b_2 , where R_2 is the correlation coefficient of $\log(E_*/D_*^{b_2})$ and $\log(F_*)$.
 155 By using F_* , the R^2 -value increases by 15% and R_2^2 achieves its maximum 0.94 at $b_2 \approx 2.5$.
 156 The improvement in the correlation can be further appreciated by comparing the relation of
 157 $E_*/D_*^{1.8}$ against τ_* (see Fig. 4) and that of $E_*/D_*^{2.5}$ against F_* (see Fig. 5). Furthermore, the
 158 relation presented in Fig. 5 can be described using the following empirical equation,

$$E_* = 0.0001 D_*^{2.5} F_* \exp\left(-\frac{40}{F_*}\right) \quad (6)$$

159 It is noted that Eq. (6) does not involve any critical condition for incipient sediment motion,
160 and thus is applicable for low pickup rate.

161 Finally, the data collected in the present study [and thus Eq. (6)] are further compared
162 with the experimental results reported in the previous studies by Fernandez Luque (1974),
163 van Rijn (1984) and Okayasu et al. (2010). Fernandez Luque (1974) conducted experiments
164 using natural and artificial grains with $D = 0.9\text{-}3.3$ mm and $\rho_s = 1340\text{-}4580$ kg/m³. Van Rijn's
165 (1984) experiments involved five sizes of sand grains with $D = 0.13\text{-}1.3$ mm, while Okayasu
166 et al.'s (2010) study was limited to a single size of sand with $D = 0.31$ mm. All these data are
167 presented in Fig. 6, in the form of $E_*/D_*^{2.5}$ against F_* . It shows that almost all the data points
168 follow the same trend as given by Eq. (6).

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171 **Summary**

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173 A series of experiments were completed in the present study to measure sediment pickup rate
174 in open channel flows. The measurements cannot be well predicted using the previous
175 formulas. Correlation analyses suggest that the dimensionless pickup rate is better associated
176 with the densimetric Froude number than the commonly-used Shields number. An empirical
177 pickup function is finally proposed to associate the dimensionless pickup rate with the
178 dimensionless grain diameter and the densimetric Froude number, which fit the data collected
179 in the present study and also those reported in the literature.

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216 kinetic theory." *Journal of Hydraulic Engineering-ASCE*, 137(2), 222-233,
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221 Table 1 Summary of experimental data

Run	Grain diameter D (m)	Flow rate (m^3/s)	Flow depth (m)	Average velocity U (m/s)	Shear velocity u_* (m/s)	Pickup rate E (m/s)
1	0.00023	0.036	0.200	0.27	0.010	5.59E-07
2	0.00023	0.038	0.204	0.32	0.013	9.31E-07
3	0.00023	0.036	0.199	0.31	0.014	1.86E-06
4	0.00023	0.039	0.199	0.34	0.014	2.23E-06
5	0.00023	0.047	0.200	0.37	0.013	3.26E-06
6	0.00023	0.044	0.205	0.37	0.017	3.45E-06
7	0.00023	0.042	0.199	0.36	0.018	3.65E-06
8	0.00023	0.047	0.205	0.40	0.021	7.36E-06
9	0.00023	0.050	0.205	0.42	0.020	9.32E-06
10	0.00023	0.053	0.205	0.44	0.022	1.35E-05
11	0.00023	0.056	0.202	0.47	0.020	2.95E-05
12	0.00023	0.058	0.202	0.49	0.025	2.98E-05
13	0.00023	0.061	0.202	0.51	0.025	4.77E-05
14	0.00023	0.064	0.201	0.54	0.026	5.51E-05
15	0.00023	0.067	0.200	0.57	0.028	9.43E-05
16	0.00023	0.069	0.200	0.59	0.031	1.25E-04
17	0.00023	0.072	0.202	0.59	0.031	1.47E-04
18	0.00023	0.081	0.199	0.61	0.030	1.50E-04
19	0.00023	0.075	0.201	0.61	0.032	2.01E-04
20	0.00023	0.084	0.199	0.68	0.032	2.35E-04
21	0.00023	0.086	0.202	0.72	0.033	2.78E-04
22	0.00023	0.089	0.201	0.70	0.035	2.94E-04
23	0.00023	0.092	0.200	0.76	0.037	3.78E-04
24	0.00044	0.040	0.207	0.34	0.013	4.71E-07
25	0.00044	0.042	0.200	0.36	0.013	1.83E-06
26	0.00044	0.039	0.201	0.33	0.013	1.89E-06
27	0.00044	0.044	0.200	0.38	0.016	4.91E-06
28	0.00044	0.051	0.200	0.43	0.018	2.34E-05
29	0.00044	0.053	0.200	0.46	0.019	2.82E-05
30	0.00044	0.057	0.207	0.48	0.019	4.04E-05
31	0.00044	0.056	0.200	0.47	0.018	2.93E-05
32	0.00044	0.062	0.200	0.53	0.021	6.13E-05
33	0.00044	0.068	0.201	0.58	0.023	8.70E-05
34	0.00044	0.068	0.200	0.58	0.025	9.79E-05
35	0.00044	0.064	0.201	0.53	0.020	9.82E-05
36	0.00044	0.071	0.200	0.59	0.026	1.71E-04
37	0.00044	0.074	0.200	0.64	0.027	2.14E-04
38	0.00044	0.079	0.200	0.67	0.028	2.85E-04
39	0.00044	0.081	0.200	0.70	0.028	2.97E-04
40	0.00044	0.084	0.200	0.70	0.031	3.59E-04
41	0.00044	0.089	0.200	0.76	0.032	4.28E-04
42	0.00044	0.091	0.202	0.78	0.032	5.04E-04
43	0.00086	0.046	0.206	0.40	0.020	4.95E-07
44	0.00086	0.050	0.205	0.43	0.022	1.40E-06
45	0.00086	0.053	0.204	0.46	0.022	1.47E-06
46	0.00086	0.056	0.204	0.49	0.023	2.97E-06
47	0.00086	0.028	0.110	0.43	0.025	6.15E-06

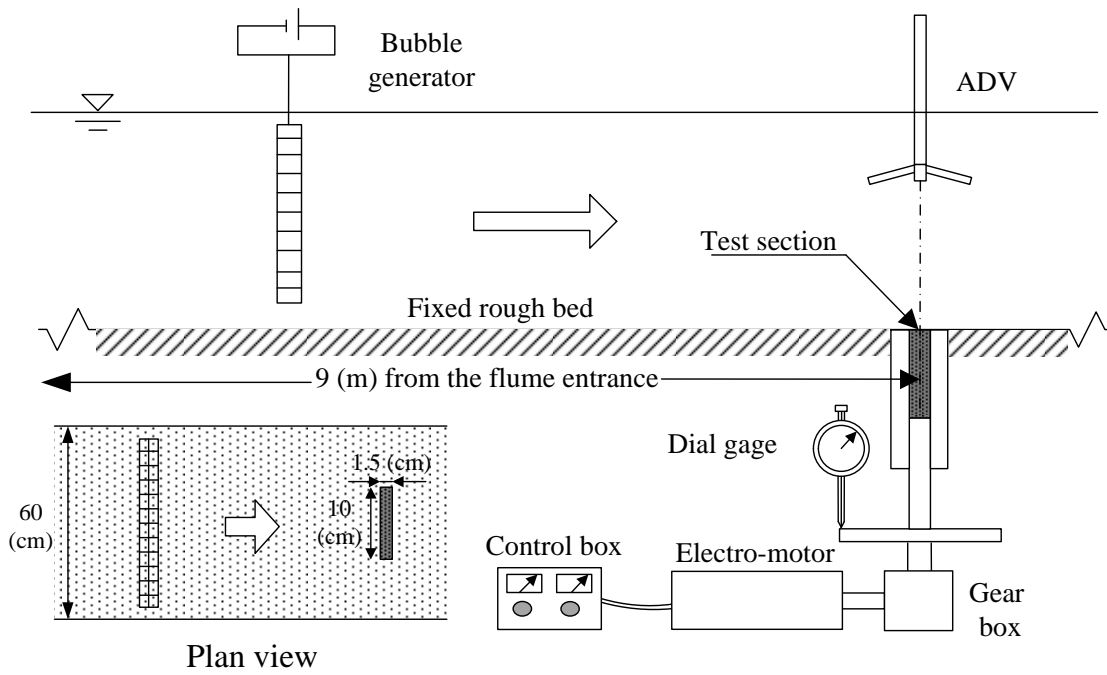
48	0.00086	0.058	0.203	0.50	0.025	8.23E-06
49	0.00086	0.061	0.202	0.52	0.027	1.30E-05
50	0.00086	0.064	0.202	0.55	0.027	1.50E-05
51	0.00086	0.067	0.201	0.58	0.029	2.17E-05
52	0.00086	0.060	0.200	0.51	0.029	2.55E-05
53	0.00086	0.071	0.200	0.59	0.030	6.52E-05
54	0.00086	0.072	0.201	0.62	0.031	6.09E-05
55	0.00086	0.035	0.109	0.52	0.031	7.03E-05
56	0.00086	0.078	0.200	0.69	0.034	1.05E-04
57	0.00086	0.083	0.202	0.72	0.035	1.30E-04
58	0.00086	0.093	0.200	0.76	0.037	2.00E-04
59	0.00086	0.046	0.111	0.68	0.039	3.55E-04
60	0.00086	0.108	0.200	0.93	0.041	4.50E-04
61	0.00086	0.118	0.207	0.99	0.045	7.01E-04

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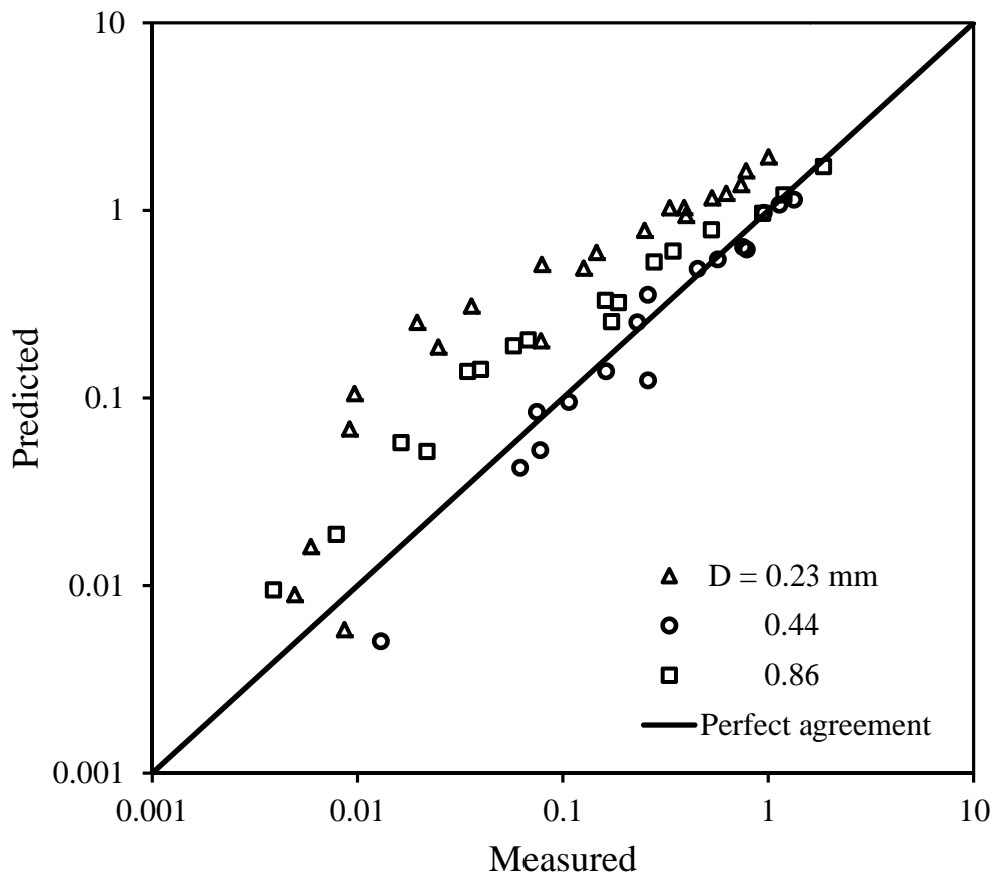
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233 Fig. 1. Experimental setup



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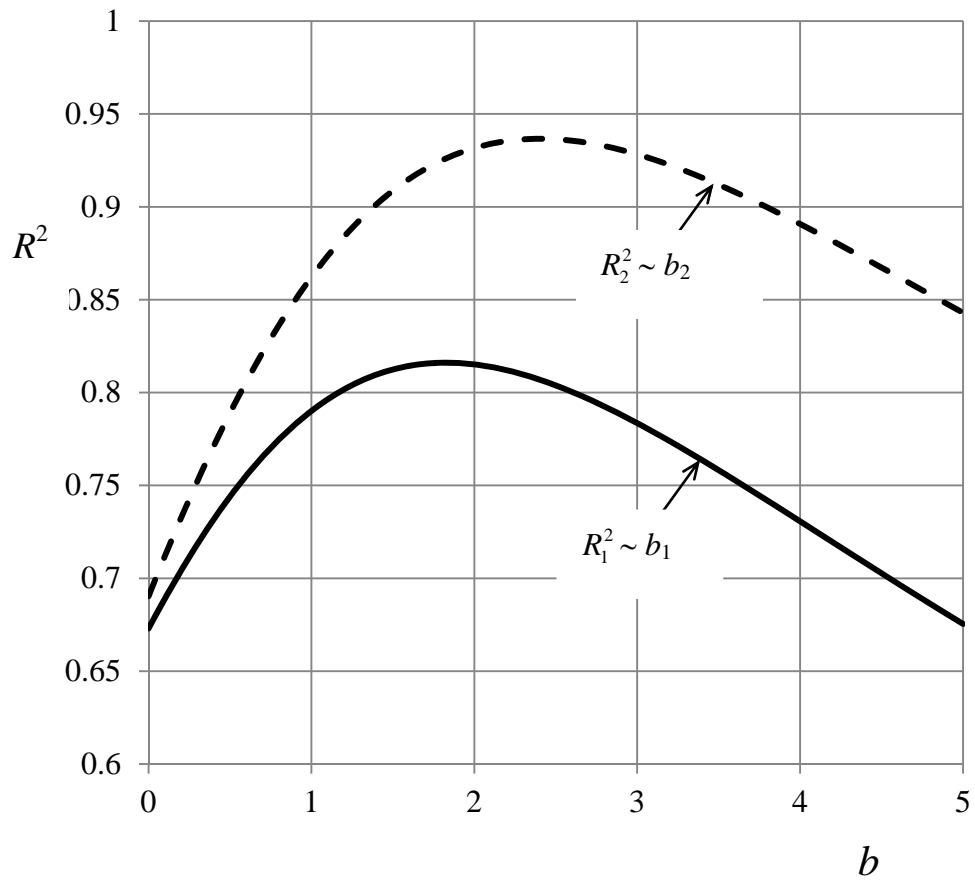
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242 Fig. 2. Comparison of measured pickup rates with predictions by van Rijn's formula

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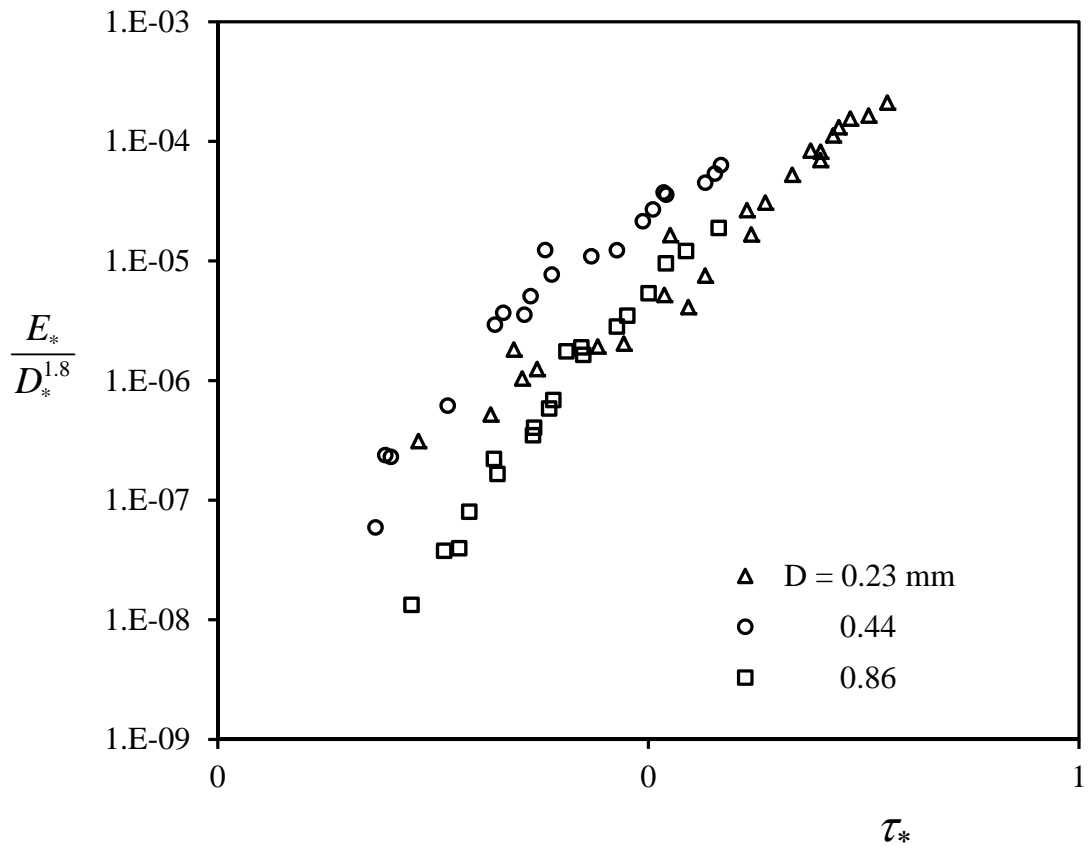
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253 Fig. 3. Variations of R^2 with the power b

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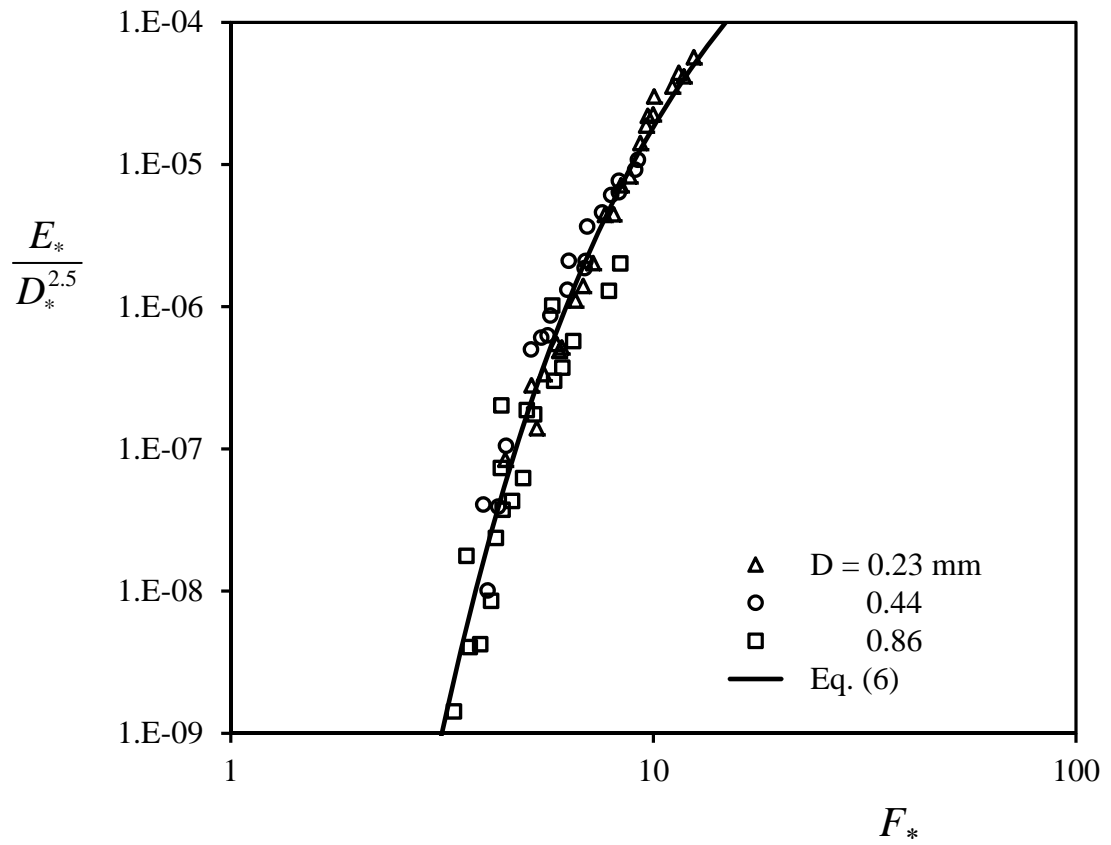
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264 Fig. 4. Variation of $E_*/D_*^{1.8}$ with τ_*

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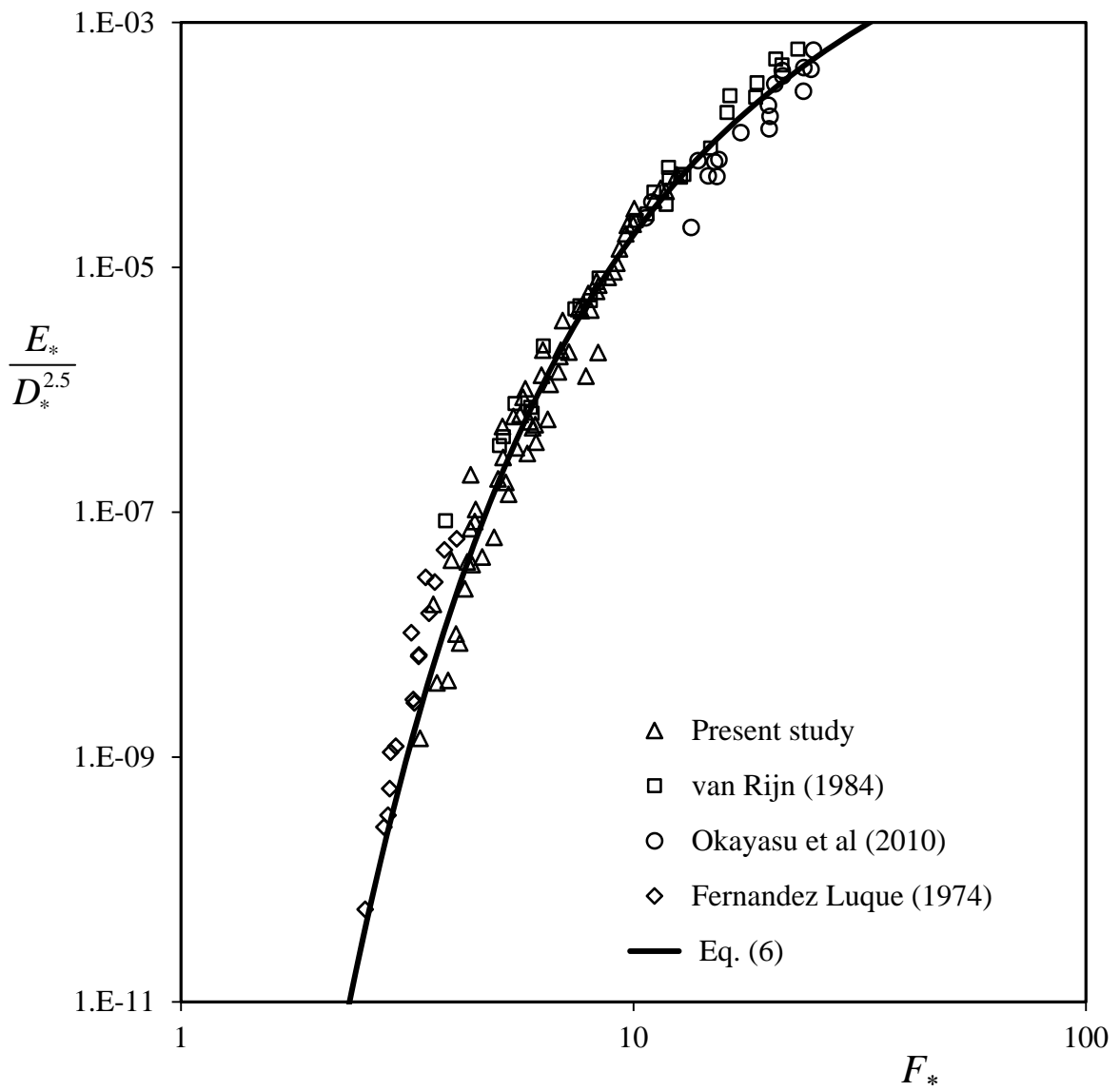
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275 Fig. 5. Variation of $E_*/D_*^{2.5}$ with F_*

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283 Fig. 6. Comparison of the present study with experimental data reported in the literature.

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