

## Editorial

# Optimization Algorithms Combining (Meta)heuristics and Mathematical Programming and Its Application in Engineering

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Complex optimization problems can be tackled by means of mathematical programming methods as well as by means of (meta)heuristic methods. On the one hand, mathematical programming methods give us a guarantee of optimality while (meta)heuristic methods do not. On the other hand, heuristic methods can handle large and complex optimization problems while mathematical programming methods tend to fail as the size of the optimization problem increases. Thus, it makes sense to combine these two strategies to obtain better solutions to the problem that is being addressed. During the last two decades or so, algorithms that either include mathematical programming solvers into (meta)heuristic frameworks or include (meta)heuristic concepts within mathematical programming methods have demonstrated to be very effective in solving large complex optimization problems. These hybrid algorithms are also called *matheuristics*. These kinds of algorithms have been successfully applied to a wide range of optimization problems arising in engineering.

In this special issue, we aimed to highlight those new approaches that take advantage of the main features of both mathematical programming and heuristic algorithms to solve challenging optimization problems. We received 129 submissions from all around the world. From these, only 25 articles were accepted after a rigorous peer-reviewed process, that is, a 19% acceptance rate. In the following, we briefly introduce each paper and try to organise them based on their main focus.

Lagrangian relaxation (LR) based algorithms were one of the topics we include in this special issue. In the paper “An Improved Lagrangian Relaxation Algorithm for the Robust Generation Self-Scheduling Problem” P. Che et al. addressed the robust generation self-scheduling problem under electricity price uncertainty which is reformulated as a MINLP problem. Authors combine an LR approach and linear programming algorithms to approximately solve this problem. LR is also considered in the paper “A Hybrid Epigraph Directions Method for Nonsmooth and Nonconvex Constrained Optimization via Generalized Augmented Lagrangian Duality and a Genetic Algorithm” by W. P. Freire et al. In this case, authors combine the generalized augmented Lagrangian duality approach and genetic algorithms. The proposed approach is applied to a set of optimization problems from mathematics and mechanical engineering.

Global optimization algorithms were also considered within this special issue. In the paper “Multiple-Try Simulated Annealing Algorithm for Global Optimization” W. Shao and G. Guo propose an algorithm that combines simulated annealing and the multiple-try metropolis algorithm. The proposed algorithm has a rapid decreasing schedule while guaranteeing global optimum values. In the paper “A Modified Priority-Based Encoding for Design of a Closed-Loop Supply Chain Network Using a Discrete League Championship Algorithm” by J. G. Nahr et al., a novel league championship algorithm (LCA) with a modified priority-based encoding is applied to find a near-optimal solution.

Authors propose new operators for the LCA to search the discrete space. Their algorithm is applied to a very difficult problem in logistics. In the paper “Global Optimization for Generalized Linear Multiplicative Programming Using Convex Relaxation” Y. Zhao and T. Zhao present a simple yet efficient algorithm that combines a new convex relaxation method and the well-known branch and bound scheme with some accelerating techniques. They applied their algorithm to the generalized linear multiplicative programming problem. In “An Improved Shuffled Frog Leaping Algorithm and Its Application in Dynamic Emergency Vehicle Dispatching” X. Duan et al. present an improved shuffled frog leaping algorithm which uses the probability model of estimation of distribution algorithm to avoid locally optimal solutions. The proposed algorithm is applied to a routing problem arising in health-systems logistics. In the paper “Optimum Assembly Sequence Planning System Using Discrete Artificial Bee Colony Algorithm” Ö. Özmen et al. present a computer program developed based on a matrix-based approach and the discrete artificial bee colony algorithm, which determines the optimum assembly sequence among numerous feasible assembly sequences (FAS). The assembly sequences of three-dimensional parts are first coded using the matrix-based methodology and the resulting FAS are assessed and the optimum assembly sequence is selected according to the assembly time optimization criterion using the artificial bee algorithm. The paper “A Position-Level Global Optimization Inverse Kinematic Solution Algorithm for Dual Redundant Robots Based on Motion Characteristics” by J. He presents an inverse kinematics optimization algorithm for PRRPR-S redundant degrees of freedom camera robot. This paper analyses the motion characteristics, in Genetic Mix method, and then proposes a Simplify Mix (SM) method that, according to the authors, can stably converge to the global optimal solution in a short time. In “An Objective Penalty Function-Based Method for Inequality Constrained Minimization Problem” by S. Lian et al., a method to globally solve inequality constrained minimization problem based on penalty functions is presented. Finally, a review on global optimization methods to address problems in natural gas transmission pipelines is presented in “Operation Optimization of Natural Gas Transmission Pipelines Based on Stochastic Optimization Algorithms: A Review” by X. Wu et al.

Three articles included in this special issue present novel gradient-based strategies. The paper “A Three-Term Conjugate Gradient Algorithm with Quadratic Convergence for Unconstrained Optimization Problems” by G. Wu et al. presents a three-term WYL conjugate gradient algorithm for which both global and linear convergence is proved. In “An Adaptive Gradient Projection Algorithm for Piecewise Convex Optimization and Its Application in Compressed Spectrum Sensing” T. Wang et al. propose an Adaptive Gradient Projection (AGP) algorithm to solve the piecewise convex optimization in signal sparse representation. To find a sparser solution, AGP provides an adaptive step size to move the iteration solution out of the attraction basin of a suboptimal sparse solution and enter the attraction basin of a sparser solution. A modified Hestenes and Stiefel conjugate gradient algorithm is presented in “A Conjugate Gradient

Algorithm under Yuan-Wei-Lu Line Search Technique for Large-Scale Minimization Optimization Models” by X. Li et al. Authors claim that their algorithm has global convergence for nonconvex functions and that the new search direction possesses not only a sufficient descent property but also a trust region feature.

Papers on multiobjective optimization were also included in this special issue. In the paper “Multiobjective Optimization for a Wireless Ad Hoc Sensor Distribution on Shaped-Bounded Areas” A. Céspedes-Mota et al. present multiobjective differential evolution algorithm to solve a problem arising in networks optimization. The differential evolution algorithm is combined with the Prim-Dijkstra and the Hungarian algorithms to improve its efficiency. Authors define objectives as maximising coverage area of the network and minimizing energy consumption. In “Grey Relational Bidirectional Projection Method for Multicriteria Decision Making with Hesitant Intuitionistic Fuzzy Linguistic Information”, Y. Zang et al. propose a comparison method of hesitant intuitionistic fuzzy linguistic term sets. Based on this comparison method, the authors propose the grey relational bidirectional projection method for dealing with MCDM problems. The vector of weights is computed by solving a nonlinear optimization model.

Algorithms for nonlinear optimization problems have been also published in this special issue. In “Filled Function Method for Nonlinear Model Predictive Control”, H. Degachi et al. present a framework to solve a nonlinear model predictive control for the Hammerstein model. They use a filled function to approach the global optimum of the problem and then find it using local search strategies. In the paper “Heuristic Determination of Resolving Controls for Exact and Approximate Controllability of Nonlinear Dynamic Systems”, A. Zh. Khurshudyan aims to determine, heuristically, control functions providing exact and approximate controllability of dynamic systems with nonlinear state constraints. Using a recently developed approach based on Green’s function method, the controllability analysis of nonlinear dynamic systems, in general, is reduced to nonlinear integral constraints with respect to the control function. In the paper “Local Search Algorithms for the Beam Angles’ Selection Problem in Radiotherapy” G. Cabrera-Guerrero et al. deal with a nonlinear problem arising in radiotherapy for cancer treatment. In their framework, the authors combine local search strategies and an interior point method.

Image processing algorithms are also part of this special issue. In “Reconstruction of Medical Images Using Artificial Bee Colony Algorithm” N. A. Rusdi et al. combine an artificial bee colony algorithm and the Douglas Peucker algorithm to reconstruct medical images. In “Patch Based Collaborative Representation with Gabor Feature and Measurement Matrix for Face Recognition”, Z. Xu et al. propose a patch based collaborative representation method for face recognition via Gabor feature and measurement matrix. Authors claim that their method can solve the problem of the lack of accuracy for the linear representation of the small sample size in face recognition. In “Local Negative Base Transform and Image Scrambling” by G. Xiong et al. a new class of scrambling algorithms for image encryption and hiding is

obtained by exploiting negative integer as the base of number representation to express the natural numbers.

Finally, other more-specific topics were also included in this special issue. For instance, neural networks for circular cone programming are presented in “A Projection Neural Network for Circular Cone Programming” by Y. Zhang and H. Liu. A deep learning classification algorithm is introduced in “A New Generalized Deep Learning Framework Combining Sparse Autoencoder and Taguchi Method for Novel Data Classification and Processing” by A. M. Karim et al. A MILP problem arising in energy systems is modelled and solved in “Allocation of Distributed Energy Systems at District-Scale over Wide Areas for Sustainable Urban Planning with a MILP Model” by Y. Ok and M. Atak. Finally, a cooperative MIMO multirelay network is implemented in “Generalized Beamforming Design for Cooperative MIMO Multirelay Networks with Infinite Constraints and Imperfect CSI” by H. Yu et al.

As guest editors, we deeply hope that this special issue can be a step forward in the development of optimization algorithms that combine (meta)heuristics and mathematical programming.

### **Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this article.

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