

Enhancing Robustness of Air Transportation Network

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Abstract

The design of robust air transportation networks is crucial for improving a network's ability to sustain/withstand failures and attacks. The difficulty lies in quantifying and optimizing the robustness. A comparison experiment shows that the total effective resistance, a spectral measure, is a promising measure satisfying the criteria that are intuitively predefined. Designing suitable strategies are the key challenges of this dissertation. Two strategies that are explicitly formulated for the minimization of the total effective resistance under several constraints are proposed to tackle the challenges.

First considered is the *flight route selection problem*, in which a set of routes is chosen from a set of candidate routes to minimize the total effective resistance of the network. The corresponding problem is a nonlinear combinatorial optimization problem. In order to enhance system performance and computational efficiency, two approaches are proposed to deal with the problem based on small, medium and large-scale networks in terms of airport number. For small/medium-scale networks, an efficient method is developed by using convex relaxation with the step-by-step rounding technique. For large-scale networks, a submodular greedy algorithm is proposed to allow the system to handle a network size far beyond the capabilities of the convex relaxation method and to guarantee a bounded optimality gap, based on the monotone submodular characteristic of the objective function. Numerical experiments are performed on some real air transportation networks of small, medium and large-scale.

A more general problem of creating robust network design under budget constraint, *the route budget allocation problem*, is also investigated. In this problem, both the edges to be added and their edge weights are allowed to be determined. The problem is solved exactly by a brute-force method to demonstrate its difficulty in obtaining an optimal solution. In order to achieve better computational efficiency, a convex relaxation method is proposed for the medium-scale networks, making use of the problem properties. For large-scale networks, a clustering-based convex relaxation method is proposed, in which the network dimensions can be reduced via the selection of critical airports. Three metrics, namely hub connectivity, number of flights, and passenger volume, are combined to define the hub hierarchy, and then the Gaussian mixture model is chosen to cluster the airports hierarchically.

These two problems and their associated algorithms are tested on three levels of air transportation networks of small, medium, and large-scale. The outcomes indicate the trade-off between the performance and computation time of these algorithms. Accordingly, the decision makers in different levels of organization could select the suitable algorithms.

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Chapter 1

Introduction

Air transportation is one of the most important and critical industries in today's regional and global activities. Together with other transportation industries, aviation has provided great convenience in people's daily life and is crucial to the globalization of the economy. To sustain the highly efficient operations of airline activities, various kinds of infrastructures and operations should be provided with a high capacity for tolerance [44]. Infrastructures are always the precondition of all types of operations and activities. The air transportation network is one of the most vital ingredients that play a critical role [18]. It is responsible for the transit of millions of passengers and goods each day, a significant enabler of the global economy, and makes the world more accessible than it has ever been. The statistic in Figure 1.1 represents the annual growth in the global air traffic passenger demand between 2005 and 2017. By 2017, traffic is estimated to grow by another 7.4 percent. Globally, passenger air travel is expected to maintain positive growth up to 2030. The revenue of the

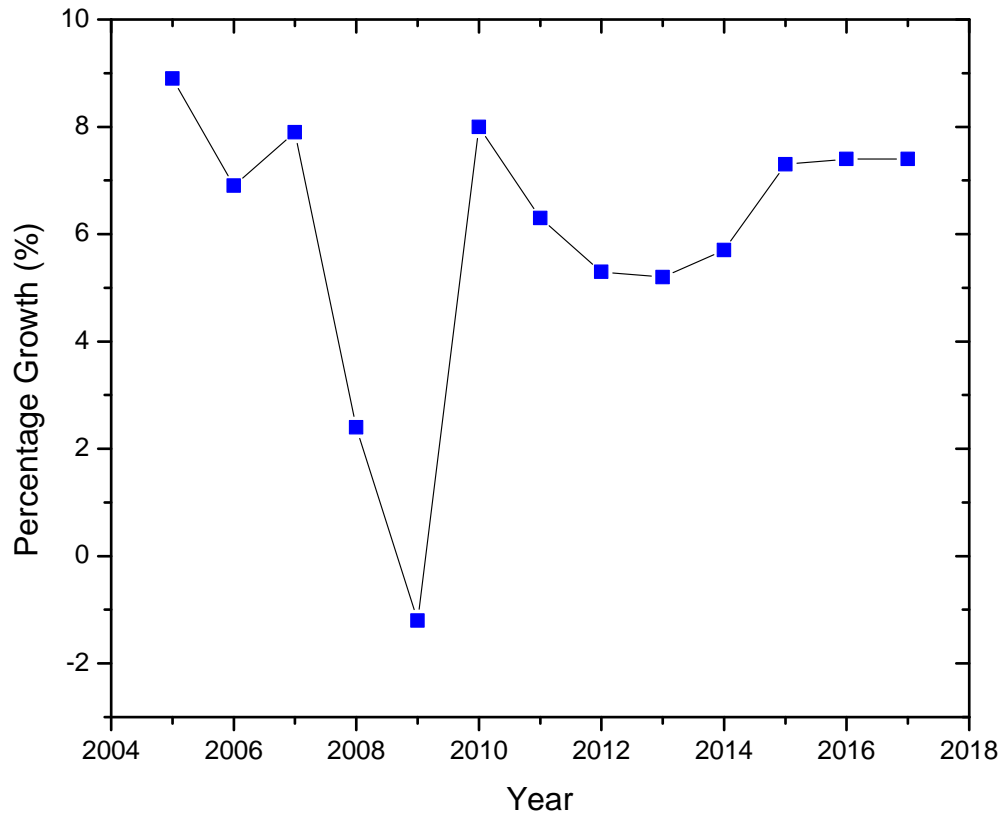


Figure 1.1: Annual growth in global air traffic passenger demand from 2005 to 2017.

global airline industry has been progressively rising in the recent decades, with the revenue in 2017 being almost double of that in 2005 (Figure 1.2). The International Air Transport Association predicts that in 2017, the global aviation industry's net profit will reach \$29.8 billion. Nearly 4 billion travelers and 55.7 million tons of cargo are expected to utilize the air transportation network [6].

Although current air transportation networks are successfully supporting aviation activities, the continuous growth of the airline industry has placed an enormous strain on the infrastructure of the air transport system. The goal of this dissertation is to design quan-

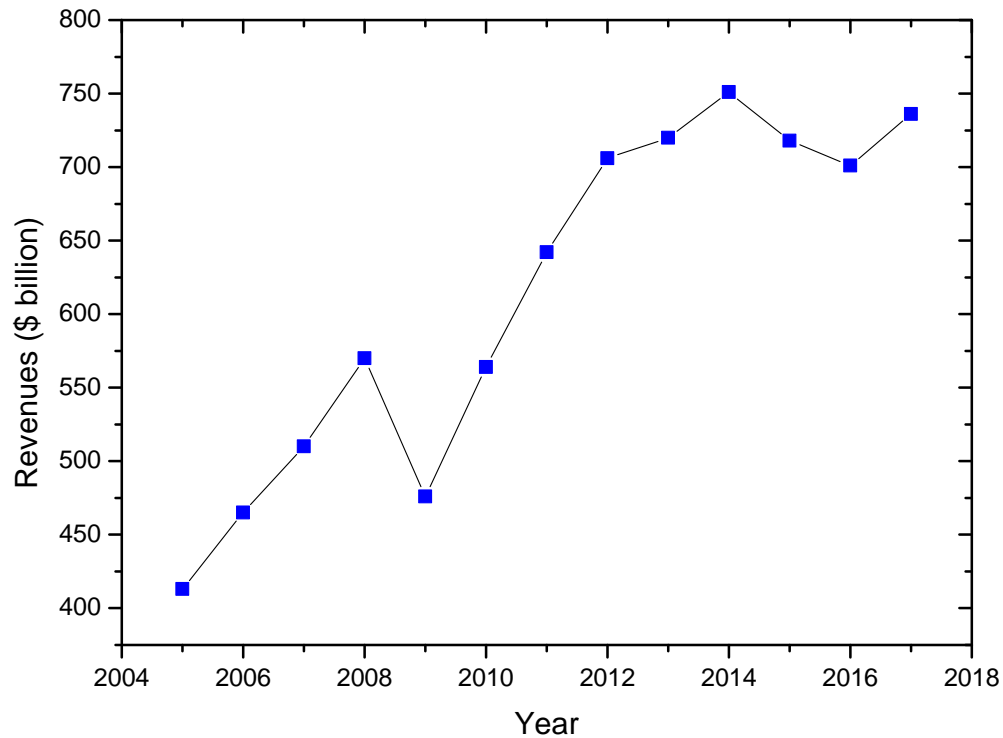


Figure 1.2: Global revenue vs year.

titative approaches for improving the robustness of the air transportation network through analysis, modeling, and optimization.

1.1 Motivation

Nowadays, it is more common to experience flight delays and cancellations. The national on-time arrival performance during 2016 in US is only 81.42%, with 17.41 % of all flights getting delayed by more than 15 minutes, and another 1.17 % getting cancelled [75]. The primary causes of flight delays and cancellations include aircraft maintenance problems,

fueling, extreme weather, airline glitches, air traffic congestion, late arrival of aircrafts, and security issues. These incidents and also aviation disasters have substantial effects on the overall air transportation system. For example, low clouds in the vicinity of O'Hare airport could reduce its flight landing rates from 100 per hour to just 72, in turn causing national flight delays and cancellations [32]. The failure and inefficiency of the air transportation system might be enormously costly economically; for instance, the eruption of Icelandic Eyjafallajokull on March 12, 2014 forced the European air traffic to go out of service for 30 days and, finally cost airlines more than a billion dollars [76]. Another example was in 2014, a communications contractor assigned to the Aurora FAA radar center set fire to the Chicago Air Route Traffic Control Center, grounding more than 2,000 flights in Chicago [1].

The increasing demand of aviation resources and the fragility of the air transport system make us rethink our current air transportation network. Can we employ a better system with high tolerance? However, the solution to this problem is far from simple. As the topology development of current air transport networks is determined by many historical "accidents", which were caused by geographical, political and economic factors, they are not designed to maximize the tolerant ability. To answer the question, the first thing is to characterize and optimize the topology of the global air transportation network. A network with an excellent structure property is essential for the operation of the overall system. For example, it is conventional that one aircraft has multiple continuous flight tasks in a single day, i.e., one plane flies from Singapore (SIN) to Kuala Lumpur (KUL) in the morning and then flies from Kuala Lumpur to Bangkok (BKK) in the afternoon. In the evening, the same aircraft is

scheduled to fly from Bangkok to Hong Kong. In this practical case, if one leg is delayed or cancelled due to weather, network design or other issues, all the consequent flight legs will also be affected. For example, inclement weather, such as thunderstorm or hurricane in the vicinity of BKK, or some maintenance problems with the aircraft before its departure; all these unexpected events will make it impossible for the flights to BKK and HKG to be on time. Several strategies are available to alleviate such problems. If there is a flight between KUL and HKG, the passengers who transfer from BKK to the destination HKG could take the direct flight from KUL to HKG. Another strategy is enhancing the strength of the route within KUL and BKK, by providing more backup aircrafts and advanced instrument landing systems. All of these strategies improve the robustness of the air transportation system for efficient operations.

In short, it is necessary to formulate a scheme to design a robust air transport network, that is resilient to bad weather, equipment shortages, airspace management, ground delay program, and other emergencies [50, 23]. This is the major motivation of this thesis.

1.2 Robustness of an Air Transportation Network

Air Transportation Network

The past decade has witnessed the rapid development of the emerging discipline of network science for applications in air transportation networks [8, 31, 35]. An air transportation

network can be characterized as a system of systems (SoS); namely, multiple, heterogeneous systems that operating independently but also interacting on various levels and through various networks [23, 21]. This kind of network can be used to describe the route map of an airline company, alliance network or even the most complex worldwide air transportation network. An air transportation network is composed of a set of airports and flight routes connecting them. Since the flight routes between airports are two-way routes, the air transportation network can be viewed as a simple undirected weighted graph. In practical operations, air transportation networks are very often influenced by various types of natural or man-made attacks on edges or nodes. An important aspect of air transportation networks is to design a robust network that is capable of carrying passengers between any two airports via one link or multiple links in the event of an unpredictable node or link failure. Structural robustness, the basis of the error tolerance of the compound systems, is one of the most fundamental characteristics representing how a network maintains its functions under uncertain attacks [2]. Specifically, an air transportation network with high robustness should be capable of mitigating the impacts of the possible failure of the node or edge.

The total effective resistance is used to measure the air transport network, the definition of the measure is based on the complete spectrum of the Laplacian matrix, and it is a function of the Laplacian eigenvalues. Compared to other measures such as connectivity, distance, betweenness, clustering coefficient, and algebraic connectivity [30, 52, 29, 82, 81, 80, 53, 34, 67, 96, 8, 60, 23, 11], total effective resistance has advantages in measuring both the local and global properties of robustness [42, 28]. The detailed definition and property

of the measure will be discussed in Chapter 2.

Total Effective Resistance and Network Robustness

In graph theory or a complex network, the network robustness refers to its resilience to failures and attacks on network nodes or edges [15]. There are a large number of measures to represent the network robustness in the literature. Each measure might reflect specific aspects of the graph. To represent and optimize the robustness of the air transportation network, the first problem we encounter is which measure should be adopted. The selected robustness measure needs to be consistent with our intuitions below. In this chapter, It is assumed that the robustness satisfies the following two criteria:

- *the robustness is minimal when the network is partitioned, and maximal when it is completely connected;*
- *the robustness increases when edges are added into the network;*

The first criterion represents the global property of the graph, and the second one reflects the local property. The advantages of the total effective resistance regarding the above two criteria over other measures are demonstrated by the following two examples. In the first one, it is started with creating five different topological graphs with four nodes and then compare the values of 11 commonly used measures (see Figure 1.3). Intuitively, the robustness of these graphs decreases from left to right. These 11 robustness measures are node degree k_v , edge degree k_e , diameter d_{\max} , average distance \bar{d} , average node betweenness \bar{b}_v , average

edge betweenness \bar{b}_e , maximum edge betweenness b_e^{\max} , clustering coefficient C , algebraic connectivity λ_2 , number of spanning trees ξ , and total effective resistance R_{tot} ; detailed definitions of these measures are described in [23, 87].

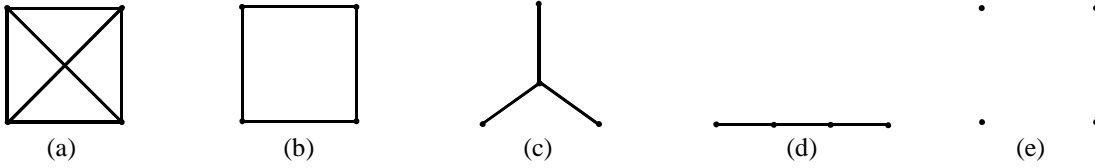


Figure 1.3: Five different topological graphs with four nodes

Table 1.1: The values of graph robustness measures for the five graphs in Figure 1.3.

<i>Graph Type</i>	k_v	k_e	d_{\max}	\bar{d}	\bar{b}_v	\bar{b}_e	b_e^{\max}	C	λ_2	ξ	R_{tot}
G_a	3	3	1	1	3	1	1	1	4	16	3
G_b	2	2	2	1.33	2	2	2	0	2	4	5
G_c	1	1	2	1.50	2.25	3	3	0	1	1	9
G_d	1	1	3	1.67	2.50	3.33	4	0	0.59	1	10
G_e	0	0	∞	∞	—	—	—	0	0	0	∞

Table 1.1 summaries the results of the robustness measures in terms of the 11 graph robustness measures for the five graphs in Figure 1.3. Let's conduct further analysis of the results. The clustering coefficient C is not a good indicator of graph robustness as its value remains unchanged for graphs G_{b-e} . The average vertex betweenness \bar{b}_v fails to reveal the monotone properties of these five graphs. The vertex connectivity k_v , the edge connectivity k_e , and the number of spanning trees ξ are indistinguishable for graphs $G_{c,d}$. The diameter d_{\max} is indistinguishable for graphs $G_{b,c}$. The maximum edge betweenness b_e^{\max} and algebraic

connectivity λ_2 can successfully distinguish these five figures. However, these two measures fail to reflect the edge addition in some cases. For example, if an edge was randomly added into graph G_c , these two measures would remain unchanged. It seems that the average distance \bar{d} and the average edge betweenness \bar{b}_e are good robustness measures. However, the definitions of these two measures are based on the shortest paths; they do not incorporate the characteristics of alternative paths that are also important. The only measure that gives the desired evaluation for the five graphs is the total effective resistance R_{tot} . $R_{\text{tot}} \rightarrow 0$ for a complete graph with infinite large weight; $R_{\text{tot}} = \infty$ for a partitioned graph. When one or several edges are added, R_{tot} increases. This measure successfully represents the global and local characteristics of graph. Hence, we measure and optimize the robustness of the air transportation network by computing its total effective resistance, which is considered as the most successful measure that satisfies the predefined criteria.

The measure that is highly related to our work is the algebraic connectivity λ_2 , the second smallest eigenvalue of the Laplacian matrix of the graph [86]. There are a handful of works in the literature on using the algebraic connectivity to study the air transportation network. The measure is able to determine the connectivity of a graph by showing whether $\lambda_2 > 0$. Despite the algebraic connectivity, λ_2 has proved to be an excellent measure of the air transportation network; it is still a kind of global measure for the connectivity of an entire network and may not be able to capture the local characteristics of a network. In contrast, the total effective resistance successfully reflects these two types of properties.

In the second experiment, we compare these two measures on the real air transportation

network of Tigerair Australia. This air transportation network consists of 14 airports, *i.e.*, Sydney (SYD), Brisbane (BNE), Cairns (CNS), Melbourne (MEL), Adelaide (ADL), Alice Springs (ASP), Coolangatta (OOL), Perth (PER), Coffs Harbour (CFS), Darwin (DRW), Hobart (HBA), Maroochydore (MCY), Mackay (MKY), and Proserpine (PPP), and the 21 routes connecting them. Any pairs of these airports can reach each other through one or a set of flight routes in this network.

The comparison experiment begins with using the existing network to create a weighted air transport network. The network is generated by randomly assigning three types of weights, $w_{i,j} = \{1, 2, 3\}$, representing the strength of the route. A larger weight assignment indicates that the route is stronger, and a smaller weight means it is more susceptible to a breakdown. The routes are then randomly added to the existing network. We denote the total effective resistance and algebraic connectivity of the existing network as R_{tot} and λ_2 . The simulation results shown in Figure 1.4 indicate that the total effective resistance and algebraic connectivity are monotone with the addition of edges. The addition process terminates with the minimal value of R_{tot} and maximal value of λ_2 . The similarity between these two measures proves that the total effective resistance is capable of reflecting the robustness of the network similar to algebraic connectivity. In addition, compared with algebraic connectivity, the total effective resistance can also reveal a gradual change with the addition of each edge. For example, when increasing k from 21 to 45, the algebraic connectivity remains unchanged, while the total effective resistance gradually decreases. In summary, it is more promising to utilize the total effective resistance to measure and optimize

the global as well as local characteristics of the air transportation network at the same time.

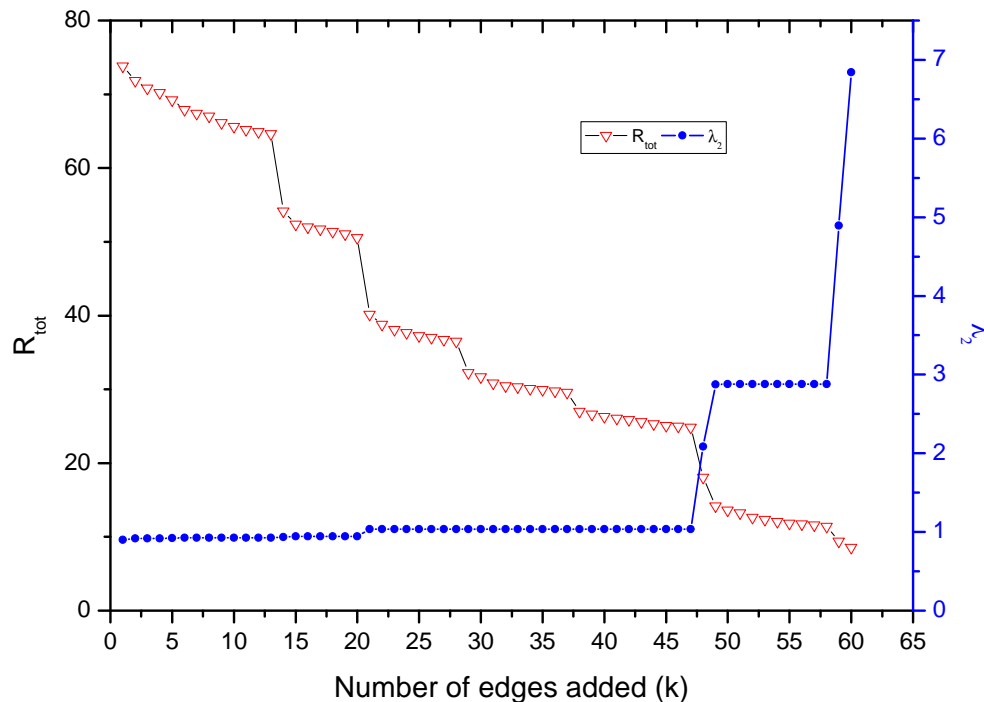


Figure 1.4: $R_{\text{tot}}, \lambda_2$ in terms of random route addition for Tigerair Australia

1.3 Research Gaps

In the literature, most of the works evaluate the topology of the air transportation network using various kinds of robustness measures. The findings and results are often qualitative instead of being quantitative. In particular, these outcomes are not really helpful in enhancing the existing network. A limited number of works use optimization methods to enhance the robustness of air transportation in a systematical manner; however, the robustness measures might not be promising due to their inability of representing the basic

properties of the air transportation network. The state-of-the-art works use the algebraic connectivity to optimize the network under different constraints [85],[86]. Even these works successfully get several results; however, their real-life applications might be limited due to the critical drawbacks of the algebraic connectivity. Moreover, the algorithms used to solve the total effective resistance related optimization problem are not applicable to a problem with the same objective function but different constraints; this has motivated us to design new algorithms to efficiently solve the specific problem. Especially, we should take into consideration the network scale, which plays a critical role in increasing the problem difficulty.

In summary,

1. there are no research works that focus on the application of the total effective resistance into the air transportation network;
2. consequently, no works solve the flight route selection problem and route budget allocation problem with the promising new robustness measure;
3. no algorithms are systematically designed to efficiently solve the two problems with a different network scale; and,
4. it is necessary to provide a general framework to define the hub hierarchy and use it to solve the large-scale problem.

1.4 Main Contributions

This research advances the state-of-the-art in several aspects that are important to the robustness enhancement of the air transportation network, and we briefly describe the main contributions of the research in the following paragraphs, please refer to Chapter 5 for a more detailed contribution summary.

The study introduces the total effective resistance into air transportation networks in the first time. We design two critical criteria that the robust measures should satisfy and carefully check 11 important measures through the five standard graphs in Section 1.2. Meanwhile, to illustrate the advantages over the state-of-the-art parameter algebraic connectivity, we make a simulation-based comparison on the Tigerair Australia network, and the results demonstrate that the total effective resistance is a more promising measure that reflects the global, local and the weighted route property.

The research first uses the total effective resistance to solve the flight route selection problem and route budget allocation problem. The optimization results provide a new insight and choice for improving the current operating network.

Compared with the works in [85],[86], in which algebraic connectivity is used to optimize the robustness of the air transportation network, our work has great advantages. As we have shown in Section 1.2, the total effective resistance is a more promising measure than the algebraic connectivity for the air transport network. The constraints of our proposed problems are the same with in [85],[86], the differences lie in the objective function, so the

results of this study will be more reasonable and applicable in general. Meanwhile, in [85], only the small-scale problem is solved regardless of the large-scale problems that make sense in practical applications.

However, the performance of the algorithms we proposed for the large-scale problem is guaranteed, and it can be used in more general cases in which more information of airports can be included. However, the algorithm proposed in [86] could only account the distance information into the model. In conclusion, we have provided two algorithms to systematically solve the problem of small, medium and large-scale flight path selection.

In addition, compared with the work in [28], in which they solve a small weight assignment problem, however, we solve the problem of adding edges as well as the problem of adding edges and determining the weight simultaneously. Hence, the algorithms in this research are very different. Moreover, the adaptive ability of our proposed algorithm is strong, and users can select the algorithm according to the scale and characteristics of the problem.

1.5 Organization of the Dissertation

The remaining of the thesis is organized into the following manner: Chapter 2 begins with a detailed description of the definition of the total effective resistance and the related properties used in this thesis and then makes a literature review from four different research directions, including relevant robustness measures, robustness analysis and optimization of the air transportation network, the works related to operations research, and the works

related to effective resistance. Chapter 3 introduces the flight route selection problem, in which a set of routes are chosen from a set of candidate routes to minimize the total effective resistance. Then we propose a convex relaxation to solve the small-scale networks. For the large-scale problem, we make use of the submodularity property of the utility function to design a greedy algorithm. In Chapter 4, we introduce the route budget allocation problem. The problem is solved exactly by the brute-force method to demonstrate its difficulty in obtaining an optimal solution. To achieve better efficiency for medium-scale networks, a convex relaxation method using the problem properties is introduced. For large-scale networks, the clustering-based convex relaxation method is further developed and exploited to reduce the dimensions of the network via selecting critical airports. Three metrics are combined to define the hub hierarchy, and then the Gaussian mixture model is chosen to cluster the airports hierarchically. Chapter 5 concludes the work of this thesis and discusses the potential research direction.

1.6 Summary

In this chapter, we began with the motivation of this research, and concluded that designing a robust air transportation network is critically required and has practical value. To select a robust measure to represent the network robustness in the air traffic region, we introduced two criteria and a comparison experiment with five graphs to check the 11 measures that existed in the literature. We then decided to select the total effective resistance as the

robust measures in this research. We compared the total effective resistance with algebraic connectivity, the state-of-the-art measure, on the real air transportation network of Tigerair Australia. We have concluded that it is more promising to utilize the total effective resistance to measure and optimize the global and local characteristics of the air transportation network at the same time. We also mentioned the research gaps and contributions in brief.

Chapter 2

Effective Resistance and Literature Review

In this chapter, we start with the description of the graphical model of the air transportation network, then present a detailed review of the definition of the effective resistance and how its associated properties will be used in the following chapters. The second part is centered around the literature reviews that are related to our work. The literature reviews are from four different perspectives, namely relevant robustness measures, structural robustness analysis and optimization of air transportation networks, air transportation network design using the operations research method, and the evaluation and optimization of various types of networks using effective resistance.

2.1 Effective Resistance

Graphical Model of Air Transportation Network

A graphical model can be used to describe the route network of an airline company [95]. Similarly, the worldwide air transportation and alliance network consisting of airlines and alliances could also be represented by the graphical model.

An air transportation network can be viewed as a weighted graph $G(V, E, W)$, where the node set V represents airports, the edge set E represents flight routes between nodes and W denotes edge weights encoding the information about edges, such as flight cancellation rates and numbers of flights [16]. The edge weights increase the generalization and the flexibility of the graphical model. As shown in the OpenFlights Database 2012 [58], about 99.44% (36002/36203) of the flight routes in the global air transport network are bidirectional, and one-way routes usually exist between small cities. Moreover, it is more beneficial to improve the robustness of the network of critical hubs, which are all connected by bidirectional routes. Hence, we consider modeling air transportation networks as undirected graphs. Assume $|V| = n$ and $|E| = m$, vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ represents the airports, and edge set $E = \{e_1, e_2, \dots, e_n\}$ denotes the flight routes. Each edge $e = \{v_i, v_j\}$ is associated with a weight $w_{i,j}$. $w_{i,j}$ is nonnegative, and equals to 0 if no edge exists between v_i and v_j [45].

The edge weight represents the link strength, which is used to describe how likely a link is to fail. A stronger link is assigned to a larger edge weight while a smaller edge weight

is assigned to the link that is more likely to break down. In this study, we quantitatively represent the edge weight by flight cancellation rate. A route with a higher cancellation rate is assigned with a lower weight. Since we are only concerned with the undirected network without a self-loop, then $w_{i,j} = w_{j,i}$ and $w_{i,i} = 0$. Let the weight matrix G be defined in the following manner:

$$W(G) = \begin{bmatrix} 0 & w_{1,2} & \dots & x_{1,n} \\ w_{2,1} & 0 & \dots & w_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,1} & w_{n,2} & \dots & 0 \end{bmatrix}$$

and

$$X(G) = \begin{bmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n \end{bmatrix}$$

with $x_i = \sum_{j=1}^n w_{i,j} = \sum_{u \in N(v_i)} w_{v_i,u}$, and x_i is the sum of the weights of all the edges connected to the node v_i , where $N(v_i)$ is the neighborhood of v_i .

The matrix $L(G) = X(G) - W(G)$ is called the *Laplacian matrix* of the weighted network G . This indicates that the entry $L_{i,j}(G)$ of Laplacian matrix $L(G)$ for a weighted graph G is

$$L_{i,j}(G) = \begin{cases} -w_{i,j}, & \text{if } i \neq j \text{ and edge exists between } v_i \text{ and } v_j; \\ 0, & \text{if } i \neq j \text{ and no edge exists between } v_i \text{ and } v_j; \\ d_i, & \text{if } i = j. \end{cases} \quad (2.1)$$

Suppose edge e connects node v_i and v_j , we define $h_e \in \mathbf{R}^n$ as $(h_e)_i = 1$, $(h_e)_j = -1$, and all other entries 0. Let $A \in \mathbf{R}^{n \times m}$ denote the incidence matrix of the graph G , *i.e.*, $A = [h_1, h_2, \dots, h_m]$, then we define the Laplacian matrix L of the weighted graph G as

$$L = \sum_{e=1}^m w_e h_e h_e^T = A \mathbf{Diag}(W) A^T \quad (2.2)$$

where W is the nonnegative weight vector whose components are associated with corresponding edges and $\mathbf{Diag}(W) \in \mathbf{R}^{m \times m}$ is the diagonal matrix constructed from W . The following properties are associated with the Laplacian matrix $L(G)$ for a weighted network G [54],

- $L(G)$ is symmetric, singular and positive semidefinite.
- The smallest eigenvalue is 0, and all the other eigenvalues, $0 < \lambda_2 \leq \dots \leq \lambda_n$, are real and positive.

Definition of Effective Resistance

We will start with stating of the formal definitions of the pairwise effective resistance. In addition to electrical network analysis, the effective resistance appears in many applications and fields, such as continuous-time averaging networks and Markov chains. Now, we present the mathematical definition of the effective resistance.

Definition 1. Let $G = (V, E, W)$ be a weighted network on n vertices, L be the weighted Laplacian matrix, and l be a solution of the equation,

$$Ll = t_i - t_j \quad (2.3)$$

where t_i is the i th unit vector, with the value of i th position is 1 and 0 elsewhere. The effective resistance $R_{i,j}$ between a pair of nodes v_i and v_j is

$$R_{i,j} = l_i - l_j \quad (2.4)$$

The summation of the effective resistance between all the distinct node pairs is the **total effective resistance** R_{tot} .

$$R_{tot} = \frac{1}{2} \sum_{i,j=1}^n R_{i,j} = \sum_{i<j} R_{i,j} \quad (2.5)$$

The total effective resistance is a scalar quantity that measures the connectivity of the network or the large level of the network. These measures appear in many cases; in an electronic network, where R_{tot} is related to the average power dissipation of the circuit excited by random currents. In Markov chains, the total effective resistance also comes up, and R_{tot} is a scaling factor that is equal to the average commuting time of a Markov chain with edge weights w .

The effective resistance between a pair of nodes i and j , $R_{i,j}$, is the electrical resistance measured across these two nodes when the network represents an electrical circuit with each edge considered as a resistor with an electrical conductance of $w_{i,j}$. In other words, $R_{i,j}$ is the potential difference that appears across terminals i and j when a unit current source is

applied between them. $R_{i,j}$ is relatively smaller when the nodes i and j are connected with more paths associated with higher conductance edges. The total effective resistance R_{tot} is the sum of $R_{i,j}$ between all distinct pairs of nodes (i, j) , which can be reduced by adding more edges or increasing the edge conductance. More interpretations of effective resistance are available in [28]. It has been shown in [42, 83] that the total effective resistance can quantify a number of important properties and performance metrics of a network, such as coherence, consensus rate, and robustness. The air transportation network is an analogy to the electrical circuit network, where a route with a failure rate of $1/w_{i,j}$ can be regarded as a resistor with a conductance of $w_{i,j}$. A smaller $R_{i,j}$ means that nodes i and j are associated with routes of lower failure rates. Since $R_{tot} = \sum_{i < j} R_{i,j}$, it can be used to measure the robustness of the entire network.

In this thesis, an air transportation network with a small total effective resistance corresponds to a more robust network, and a large total effective resistance corresponds to a less robust network.

Theorem 1. [28] *Let $G = (V, E, W)$ be a weighted graph, L be the Laplacian matrix for G , and L^\dagger be the pseudo inverse matrix of L , the total effective resistance can be formulated as*

$$R_{tot} = n \mathbf{tr}(L + \mathbf{1}\mathbf{1}^T/n)^{-1} - n = n \mathbf{tr}L^\dagger = n \sum_{i=2}^n \frac{1}{\lambda_i}. \quad (2.6)$$

where symbol $\mathbf{1}$ denotes the vector with all entries 1.

Theorem 2. [28] *The pairwise total effective resistance does decrease when edges are removed. In other words, if H is an arbitrary subgraph of network G , then $R_{tot}(G) \leq R_{tot}(H)$.*

2.2 Related Work

The four areas of studies are related to this thesis are listed as follows: 1) relevant robustness measures, 2) structural robustness analysis and optimization of air transportation networks, 3) air transportation network design using operations research method, and 4) evaluation and optimization of various types of network using the effective resistance.

Relevant Robustness Measures

In the relevant literature, several other measures also partly reflect the characteristic of robustness. A number of works take into account the vulnerability of a transportation system, where a sudden event may occur and thus reduces the performance of the network components or significantly impacts the need for service. Berdica defined vulnerability as a susceptibility to interruptions, which can significantly reduce network services or reduce the ability to use a particular network link or route at a given time. Networks that cannot quickly recover from service outages are thought to be more vulnerable to attacks than those that recover faster but experience less overall interruptions [10]. Jenelius *et al.* believed that the vulnerability of transport networks was the consequence of a single or multiple link failures. Although there were many attempts to examine the vulnerability of the literature, the vulnerability of transport networks was still a rather vague term, lacking clear definitions and quantitative methods [38].

Because the vulnerability is often used qualitatively, quantitative measures of reliability

have been used to gain insight into a system's level of vulnerability. Husdal linked vulnerability and reliability from a cost-benefit perspective, with vulnerability being the cost and reliability being the benefit value [37]. Husdal argued that vulnerability is equivalent to non-reliability in certain circumstances. Dayanim argued that it was mandatory to incorporate reliability criteria into the network design processes so as to meet disaster recovery requirements. A variety of reliability measures have been implemented for transportation systems to measure their intended functions under uncertainties.

Another related measure is flexibility. Goetz and Szyliowicz argued that flexibility can be useful in dealing with uncertainty. Although it was mainly used in the analysis of manufacturing systems, some researchers have been considering its the application in the evaluation of transportation systems. Cho defined the capacity of flexibility as the ability of the transportation network to expand its capacity to adapt to changes in the requirements while maintaining satisfactory performance levels [22]. Morlok and Chang extended this definition from the perspective of external changes in travel requirements and network capabilities [55]. Sun, Turnquist, and Nozick further measured flexibility in a more complex problem setting, taking into account future traffic patterns, deterioration of services, and random requirements [74].

Resilience, which highly related to robustness measures, has been proposed to measure the performance of the engineering system [21]. For example, resilience was defined as the number of failures of a computer network to maintain the connection. For the supply network, resilience was described as the ability to handle externalities and restore normal

functioning [64]. Bartolacci used traffic efficiency, which was defined as the percentage of total traffic that the network can manage, as an elastic measure of the telecommunications network [74]. McManus defined organizational resilience as the function of system awareness, identification and management of the most critical system components and adaptability [64].

In this research, the robustness, the basis of the error tolerance of the complex systems, is one of the most fundamental characteristics representing how a network maintains its function under uncertain disruption. It measures the capacity of being able to continue functioning in the presence of internal and external challenges without fundamental changes to the original system. Specifically, an air transportation network with high robustness should be capable of mitigating the impacts of possible failure of nodes or edges.

Structural Robustness Analysis

Since the small-world phenomenon [43] and scale-free characteristic [9] were discovered, a increasing number of researchers have explored the applications of network theory into the air transportation network (also known as airport network, air transport network) of different airlines, regions, and countries. Guida and Maria investigated the topology of the Italian airport network as a small-world network without evidence of “communities” [30]. Malighetti *et al.* studied the connectivity of the European and alliance’s air transportation network [52]. They employed a time-dependent minimum path approach to calculate the minimum travel time, flight times, and waiting times between each pair of airports within

the network. The same technique was applied to the comparative analysis of the airport connectivity in China, Europe, and the US [60]. Sun *et al.* made an investigation of the temporal evolution of the seven central indicators of the air route network and the airport network, and the results were beneficial to better understanding of the network dynamics [73]. They also used the similarity scores computed by functionally independent metrics to assess the structural similarity of air navigation route networks [72]. Wang *et al.* studied the air transport network of China; they measured the overall network structure by degree distribution, average path length, clustering coefficient, and evaluated the nodal centrality of the individual airport by degree, closeness and betweenness. The results indicated that the air transport network of China exhibited some small-world network properties [82]. Zhang *et al.* studied the evolution of the airport network and traffic flow in China to find that the topology has remained steady for the past several years [96]. Bagler found that the Indian airport network is a small-world network and exhibited a disassortative mixing pattern [8]. Saper and Parekh showed that the Indian airport network was both scale-free and small-world through analyzing its centrality measures [67]. After investigating the structure and evolution of the Brazilian airport, Rocha concluded that its structure is dynamic, contracted on the route, but increased in number of passengers and quantity of goods [65]. In [90], Xu and Harriss adopt a methodology of the weighted complex network to study the air transportation network of a the US domestic intercity passenger. The results showed that the hub cities were inclined to form an interconnected traffic pattern with high flow, and they also tended to link with small cities. Wuellner *et al.* analyzed the individual structures of

seven passenger airlines in the United States. It is observed that the densely interconnected network had a strong resilience for the targeted removal of the airport (node) and accidental removal of the route (edge) [89]. They introduced two network rewiring schemes known as “diamond” and “chain”, which improved the flexibility of the perturbation at different levels while meeting the overall demand for flights and gates. Earnest built an agent-based model of the international air transportation system, which employed a genetic algorithm to identify the small disruptions that produced cascading network failures [25]. Mehta *et al.* have applied technologies in complex networks to describe the structure and dynamics of the US airport network over the past few years, showing that some properties associated with the network appeared stationary over time. However, others exhibited a high degree of variation [53]. Reggiani *et al.* made a complexity analysis of Lufthansas airline network. The results also indicated that the relevance of various network measures for understanding the changing patterns in airline network configurations [63]. They found that the airport network around the world is a small-world network with a power-law decaying betweenness centrality and degree distribution. The analysis results indicate that the cities with high betweenness tended to play a more critical role in maintaining network connectivity than cities with a high degree. Guimera and Amaral presented the exhaustive analysis of the world-wide airport network. They found that the world-wide airport network was a small-world network with a power-law decaying degree and betweenness centrality distributions. Meanwhile, the most connected cities (largest degree) were typically not the most central cities (largest betweenness centrality), demonstrating that nodes with high betweenness tend to

play a more important role in keeping networks connected than those with high degree [31]. Bonnefoy discovered that the air transportation network was not its scale-free and was in fact scalable due to the capacity constraints at major airports. However, the network became scale-free when multiple airports were aggregated into single nodes [12]. Bonnefoy also used the degree distribution of the existing light jet network to understand the potential impacts of very light jets [13]. Lacasa *et al.* presented a model of an air transportation network from the complex network modeling viewpoint [46]. Each node was weighted according to its load capacity, and the links were weighted according to the Euclidean distance separating each pair of nodes. They embedded their model in an European air transportation network, (proven to be scale-free), formed by 858 relevant airports and 11170 flight routes. Jia and Jiang analyzed the structural properties of the unweighted and weighted US airport network [39], and the results indicated that the network exhibited scale-free, small-world, and disassortative mixing properties. Moreover, there existed a remarkable relationship between the topological measurements in the unweighed network and the traffic measurements in the weighted network. In [40], they explored the evolution of the US airport network from 1990 to 2000. The analysis showed that the network preserved its scale-free, small-world, and disassortative mixing properties over time. Moreover, the stable cities formed the backbone of the network over time and their structural similarity showed regularity. Lamanna and Longo presented a study on the stability analysis of the air transportation network [47]. The results showed that it was not easy to enhance the efficiency of the networks that are with the scale-free and small-world properties. The analysis of the air network in Southeastern

Europe showed that the routes between two airports with a higher number of flights were not always the most critical ones in the case of a failure. Wu *et al.* correlated the network topology with the weighted complex networks [88]. They calculated the robustness of the interrelated scale-free networks using correlation, denoting the strength of the correlations between weight and degree, and then found the characteristic of the most robust networks. Finally, they compared the simulation results with the global airport network and found good agreement.

Structural Robustness Optimization

A small group of researchers has explored the structural optimization of air transportation networks. After analyzing the connectivity and concentration of Lufthansa network, Reggiani *et al.* proposed a multiple-criteria analysis to implement a strategic configuration of the airline network pattern [63]. Cai *et al.* applied the genetic algorithm to design the actual route network [17]. They formulated the problem as a bi-objective formulation, and a memetic algorithm was proposed to tackle it. An empirical study using real data from the current air route network of China showed that the algorithm had outperformed an existing approach to the crossing way-points location problem as well as three well-known multi-objective evolutionary algorithms. Moreover, the methods not only managed to reduce the cost of the current air route network but also improved the airspace safety. Redondi *et al.* evaluated the impact of adding new routes to the connectivity of airport network on module

identification techniques [62]. Simulated annealing was designed to validate the existence of highly interconnected modules, a set of airports with very high internal links, and a weak connection with the rest of the network in the European aviation network. The results showed that when new routes were added to airports with relatively unconnected modules, the improvements in the average number of steps required to reach other airports in the network were the largest.

Some works focused on maximizing the algebraic connectivity to enhance the robustness of the air transportation network. Vargo *et al.* explored the effectiveness of the heuristic algorithm and SDP relaxation in increasing the algebraic connectivity [79]. To investigate the underlying regional structures in the US airport network, a modification to an existing subgraph extraction algorithm was implemented; hence, the localized structures could be optimized independently and reconnected via a backbone network to achieve a superior network performance. Wei *et al.* made an in-depth investigation of using algebraic connectivity to optimize the air transportation. In [85], they proposed three approaches, namely modified greedy perturbation algorithm, the weighted tabu search, and the relaxed semi-definite programming with rounding techniques. These methods were designed to optimize the algebraic connectivity of the network through adding or deleting the flight routes. A case study of two actual air transport networks showed the feasibility of the methods. The flight route addition problem was extended by adding the leg number constraint, and binary semi-definite programming with the cutting plane method was designed to provide the optimal solution [56]. Two heuristic algorithms were proposed to find the optimal solution in a small-scale network

and near optimal solution in large-scale network, including the tabu search and 2-opt search heuristic algorithm. In [86], the third algebraic connectivity maximization problem with an operating cost constraint was formulated. In this problem, the number of edges and each edge weight needed to be determined. A relaxed SDP method with a golden-section search was developed to solve both variables at the same time. For the large-scale networks, the cluster decomposition method was utilized.

Network Design Using Operations Research Methods

There was also a body of works related to the robustness of a network design by using operations research techniques. They were used for particular instances and applications (e.g., the schedule for an airline or hub design). Aykin introduced a capacitated hub-and-spoke network design problem, in which the capacity between airports was limited. This problem was under a network policy that allowed direct (uninterrupted) and hub connection services between nodes. To solve the problem, a branch and bound algorithm and a heuristic algorithm that partitioned the solution set based on the hub locations were proposed [7]. Hsu and Wen developed a model that is capable of predicting urban passenger flow, designing airline networks, and determining the flight frequency of a single airline using grey theory and multi-objective programming [36]. In the case of hub congestion, Camargo *et al.* solved the problem of designing a multiple distribution hub-and-spoke network. A nonlinear mixed integer programming model was proposed, and the congestion model was configured

as a convex cost function. A Benders decomposition algorithm had been deployed, and it successfully solved the standard data set instance with up to 81 nodes [19]. To proactively handle hub disruptions, a reliable hub-and-spoke network design model was proposed, and the selection of alternate hubs and alternative routes were taken into account. To solve the problem of obtaining a reliable network design, the Lagrange relaxation method and branch constraint method were designed to solve these nonlinear mixed integer formulations. The optimal solution obtained was superior to the commercially leading nonlinear integer programming package [4]. Yang proposed two models to determine the hub position and service network under capacity constraints. The first model included the total number of passengers through the airport, while the second one only accounted for the transshipment flow. Both models considered the coexistence of services and the most possible services. This provided maximum flexibility for the construction of airline service networks [91]. Alibeyg *et al.* extended several families of classical hub location problems and formulated a set of profit oriented hub network design problems. These problems included deciding whether the origin/destination nodes should be served and the different kinds of routes that should be activated [3]. Kian and Kragar studied the hub location problem under power law congestion cost and transformed the problem into two problem and improved the methods based on the effective reduction of inequalities [41]. Lederer and Nambimadom made significant contributions to the design of airline networks. They used idealized models to analyze network schedule selection, which was the best choice for airlines to design their networks and schedules to maximize profits [48]. Campbell and O’Kelly reviewed the study of the hub

location of the past 25 years, especially in the field of transportation. In the review, they also discussed the inadequacy of the hub location research in depth [20].

Effective Resistance

There was a handful of literature on the application of effective resistance in various types of network. Ghosh *et al.* studied the problem of allocating edge weights over a given weighted network to minimize the total effective resistance. The the path had the maximum of the total effective resistance, while the complete graph had the least [28]. Van Mieghem *et al.* showed the qualitative relationship between the effective resistance and graph assortativity, a correlation of the similarities of nodes sharing a link [78]. The simulation results showed the effective graph resistance enhanced with the increase of assortativity. Wang *et al.* derived the upper and lower bound of the total effective resistance in the scenario of edge addition and deletion, then proposed several heuristic algorithms to improve the effective resistance [83]. Ellens *et al.* associated the optimal edge addition for graphs with the given number of vertices and diameter to minimize the total effective resistance [26]. Kimon and George examined modeling and the design of effective control interfaces, for human operators, of unmanned aerial vehicle (UAV) swarms [77]. Specifically, the open loop \mathcal{H}_2 norm of the network is selected as a performance metric for the reasoning regarding the effective human-swarm interaction. The role of topological features of the network is highlighted in the context of this metric and is related through the effective resistance of the corresponding

electrical network. The research used the edge rewiring method to solve the small-scale network optimization problem.

By the nature of its construction, effective resistance can only be computed in undirected graphs and yet, in several areas of its application, directed graphs arose as naturally (or more naturally) than undirected ones. In [92], Young *et al.* proposed a generalization of effective resistance to directed graphs that preserves its control theoretic properties in relation to consensus-type dynamics. Then, they proceeded to analyze the dependence of the algebraic definition on the structural properties of the graph and the relationship between the construction and a graphical distance. The results made possible the calculation of the effective resistance between any two nodes in any directed graph and provided a solid foundation for the application of effective resistance to the problems involving directed graphs. In [93], they continued to use the theory developed to compute the effective resistances in some prototypical directed graphs. This exploration highlighted cases in which the notion of the effective resistance for directed graphs behaved analogously to the experience from undirected graphs as well as cases where it behaved in unexpected ways.

2.3 Summary

In this chapter, we started with the description of the graphical model of the air transport network, and then presented a detailed review of the definition of effective resistance and the associated basic principles. Then, we presented the literature review that was highly related

to our research in four aspects, namely relevant robustness measures, structural robustness analysis and optimization of air transportation networks, air transportation network design using operations research method, and evaluation and optimization of various types of network using effective resistance.

Table 2.1: A summary of the studies related to robustness evaluation of air transportation network

<i>Reference</i>	<i>Network Type</i>	<i>Robustness Measure</i>	<i>Weighted</i>
[30]	Italian	Degree and betweenness	N
[52]	European	Time dependent minimum path	Unknown
[60]	China, Europe, and USA	Time dependent minimum path	Both
[73]	Air navigation route network	Global and local node metrics	Y
[72]	European	Degree, weighted degree, clustering, betweenness, closeness, weighted betweenness, and weighted closeness	Both
[82]	China	Degree distribution, average path length, and clustering coefficient	N
[96]	China	Average, ingoing and outgoing degree, average shortest path length, diameter, and clustering	Y
[65]	Brazilian	Betweenness, total number of routes, and connections	Both
[8]	India	Degrees, degree-correlations, clustering coefficients, average shortest path length, and average clustering coefficient	Both
[67]	India	Strength, degree, closeness, betweenness, efficiency	Y
[90]	USA	Average path length, clustering coefficient, betweenness, and degree	Y
[89]	Seven USA largest passenger carriers	Number of nodes and edges, mean node degree, geodesic and clustering coefficient, degree assortativity and Gini coefficients, and skewness of degree distribution	Both

[40]	USA	Degree, betweenness, clustering, and average shortest path	Y
[47]	Southeastern Europe	Degree, diameter, clustering, efficiency, and shortest path	Y
[88]	USA	Node degree	Y

Table 2.2: A summary of the studies related to the robust optimization of air transportation network

<i>Reference</i>	<i>Problem</i>	<i>Robustness Measures</i>	<i>Optimization Methods</i>
[63]	Configure the network pattern	Connectivity and concentration	Multiple-criteria analysis
[89]	Network resilience improvement	Size of the largest connected component and relative global travel cost metric	Two rewiring schemes
[17]	Optimal route design	Total airline cost and total flight conflict	Memetic algorithm
[62]	Route addition	Network connectivity	Module identification techniques
[79]	Algebraic connectivity maximization	Algebraic connectivity	Tabu search and SDP relaxation
[85, 86]	Algebraic connectivity maximization	Algebraic connectivity	Modified greedy perturbation algorithm, weighted tabu search, and relaxed SDP

Chapter 3

Flight Route Selection Problem

In Chapter 1, the total effective resistance is demonstrated to be a good measure to represent the robustness of the air transportation network, in this chapter, we try to minimize the total effective resistance by adding a set of candidate routes.

Compared to the existing literature, the research for the first time adopts the total effective resistance to optimize the robustness of air transportation networks. To solve the corresponding *flight route selection problem*, a difficult integer nonlinear programming problem, we develop two methods for different network scales to balance the trade-off between optimality performance and computational efficiency. An interior-point method based on convex relaxation and duality gap is developed for small/medium-scale networks, which achieves a better optimality performance within acceptable time. An accelerated greedy algorithm based on proved monotone submodularity is developed for large-scale networks, which can substantially reduce the computation time in deriving a good solution with a guar-

anteed optimality gap. Three case studies from real practice are performed to demonstrate the application of the proposed methods.

The rest of this chapter is organized as follows. In Section 3.1, we generally describe the flight route selection problem. In section 3.2, we describe the convex relaxation method, which uses an approximate relaxation to achieve guaranteed performance. In Section 3.3, we prove the monotone submodular property of the total effective resistance, a greedy algorithm and its accelerated version are correspondingly developed to improve the computational efficiency with a guaranteed optimality gap further. In Section 3.4, the numerical experiments on three real air transportation networks with small, medium, and large-scale are provided. Section 3.5 makes a chapter summary.

3.1 Problem Formulation

Flight Route Selection Problem

Now we can describe the flight route selection problem. Since many airlines have already set up certain air transportation networks to provide services in local or worldwide regions, it is not necessary to create an entirely new air transportation network from scratch. Most likely in practice, we will face a flight route selection problem, in which a few extra routes are selected from a set of potentially available and viable routes and added into the existing network to improve the robustness. We would like to develop an optimal and efficient way

in selecting edges.

Let (V, E_0) represent the existing air transportation network with the initial edge set E_0 consisting of the existing routes between the airports in V and let E_P denote the set of extra edges for improving robustness. The objective is to minimize the total effective resistance R_{tot} of the new network with k edges selected from the given edge set E_P and added into the existing network (V, E_0) . Assume $|E_P| = p \geq k$ and $E_P = \{1, \dots, p\}$. The total effective resistance R_{tot} can be effectively computed through the weighted Laplacian matrix. Then the corresponding augmented Laplacian matrix of the new network can be expressed as follows,

$$L = L_0 + \sum_{e=1}^p y_e w_e h_e h_e^T$$

where L_0 is the Laplacian matrix of the existing network and y_e is a Boolean variable to indicate whether the edge e is selected from E_P . Finally, the flight route selection problem can be formulated as,

$$\begin{aligned} & \underset{y_1, \dots, y_p}{\text{minimize}} && R_{tot}(L_0 + \sum_{e=1}^p y_e w_e h_e h_e^T) \\ & \text{subject to} && \mathbf{1}^T y = k, \\ & && y_e \in \{0, 1\}, \quad e = 1, \dots, p. \end{aligned} \tag{3.1}$$

where $y = [y_1, \dots, y_p]^T$ and $R_{tot}(\cdot)$ is a function of a Laplacian matrix. The problem (3.1) is a Boolean-convex problem because the objective function is convex in $0 \leq y \leq 1$ as shown in Theorem 3 in Section 3.2 and the constraint is linear.

3.2 Convex Relaxation

The flight route selection problem in (3.1) is an integer nonlinear programming problem. Clearly, a brute force method of enumerating all $\binom{p}{k}$ route combinations is not practical in solving the problem unless k and p are small. In this section, we will develop an interior-point algorithm based on the analysis of the duality gap of its convex relaxation.

Convex Relaxation of Flight Route Selection Problem

The original flight route selection problem (3.1) can be relaxed by replacing the original binary variables with the continuous variables $y_e \in [0, 1]$ for $e = 1, \dots, p$ as follows,

$$\begin{aligned} & \underset{y_1, \dots, y_p}{\text{minimize}} && n \mathbf{tr}(L_0 + \sum_{e=1}^p y_e w_e h_e h_e^T + \mathbf{1}\mathbf{1}^T/n)^{-1} - n \\ & \text{subject to} && \mathbf{1}^T y = k, \end{aligned} \tag{3.2}$$

$$0 \leq y_e \leq 1, \quad e = 1, \dots, p$$

This relaxed flight route selection problem (3.2) can be proved to be convex through the following theorem. (For the continuity of reading, all the proofs are placed in the Appendix in this chapter.)

Theorem 3. *The total effective resistance function $R_{\text{tot}}(y) = n \mathbf{tr}(L(y) + \mathbf{1}\mathbf{1}^T/n)^{-1} - n$ is convex in y , where $L(y) = L_0 + \sum_e y_e w_e h_e h_e^T$.*

Based on Theorem 3, the objective function of the relaxed problem (3.2) is convex. Combining it with the fact of the linear constraints, we can show that the relaxed problem

(3.2) is a convex programming problem. It can be further proved that the problem (3.2) is also a semidefinite programming problem (SDP). Although it can be solved using SeDuMi [70], a popular SDP solver, we can develop an interior-point algorithm based on the duality gap of the problem (3.2), which is much faster than solving it as an SDP problem.

Dual Problem of Relaxed Flight Route Selection Problem

In this section, we formulate the dual problem of the relaxed flight route addition problem for deriving the duality gap.

First, we introduce a new variable $X = L_0 + \mathbf{1}\mathbf{1}^T/n + \sum_{e=1}^p y_e w_e h_e h_e^T$ and reformulate the problem (3.2) as the following problem with the variables $y \in \mathbf{R}^p$ and $X \in \mathbf{S}^n$ (the set of symmetric $n \times n$ matrices):

$$\begin{aligned}
& \underset{y}{\text{minimize}} && n \text{tr} X^{-1} - n \\
& \text{subject to} && X = L_0 + \sum_{e=1}^p y_e w_e h_e h_e^T + \mathbf{1}\mathbf{1}^T/n, \\
& && \mathbf{1}^T y = k, \\
& && 0 \leq y \leq 1.
\end{aligned} \tag{3.3}$$

Second, we introduce the Lagrange multipliers λ for $y \geq 0$, μ for $y \leq 1$, ν for $\mathbf{1}^T y = k$, and Λ for $X = L_0 + \sum_{e=1}^p y_e w_e h_e h_e^T + \mathbf{1}\mathbf{1}^T/n$ to form the Lagrangian of the problem (3.3)

as follows:

$$\begin{aligned}
& L(y, X, \Lambda, \lambda, \mu, \nu) \\
&= \mathbf{tr}(\Lambda(X - L_0 - \sum_{e=1}^p y_e w_e h_e h_e^T - \mathbf{1}\mathbf{1}^T/n)) \\
&\quad + n\mathbf{tr}X^{-1} - \lambda^T y + \mu^T(\mathbf{1}^T y - k) + \nu^T(y - 1) - n. \\
&= (\sum_{e=1}^p y_e(-w_e h_e \Lambda h_e^T - \lambda_e + \mu_e + \nu_e)) - (1/n)\mathbf{1}\Lambda\mathbf{1}^T \\
&\quad + \mathbf{tr}(nX^{-1} + X\Lambda) - \mathbf{tr}(\Lambda L_0) - \mathbf{1}^T \mu - \nu k - n.
\end{aligned}$$

where $\lambda \in \mathbf{R}^p$, $\mu \in \mathbf{R}^p$ are non-negative, $\nu \in \mathbf{R}$, and $\Lambda \in \mathbf{S}^n$.

The Lagrange dual function g is given by

$$g(\Lambda, \lambda, \mu, \nu) = \inf_{y, X} L(y, X, \Lambda, \lambda, \mu, \nu).$$

Since the minimization of L is bounded only if

$$\lambda_e - \mu_e - \nu_e + w_e h_e \Lambda h_e^T = 0,$$

the Lagrange dual function g holds that

$$g(\Lambda, \lambda, \mu, \nu) = \begin{cases} 2\mathbf{tr}(n\Lambda)^{1/2} - (1/n)\mathbf{1}\Lambda\mathbf{1}^T - \mathbf{tr}(\Lambda L_0) - \mathbf{1}^T \mu - \nu k - n, & \text{if } \lambda_e - \mu_e = \nu_e - w_e h_e \Lambda h_e^T, \\ \infty, & \text{otherwise.} \end{cases}$$

To justify the equality, we note that $\mathbf{tr}(nX^{-1} + X\Lambda)$ is unbounded below, as a function of X , unless $\Lambda \succeq 0$; when $\Lambda \succ 0$, the unique X that minimizes it is $X = (\Lambda/n)^{-1/2}$ and the corresponding minimal value is

$$\mathbf{tr}(nX^{-1} + X\Lambda) = \mathbf{tr}(n(\Lambda/n)^{1/2} + \Lambda(\Lambda/n)^{-1/2}) = 2\mathbf{tr}(n\Lambda)^{1/2}.$$

Finally, the dual problem of the relaxed flight route selection problem can be formulated as

$$\begin{aligned}
& \text{maximize} && 2\text{tr}(n\Lambda)^{1/2} - (1/n)\mathbf{1}\Lambda\mathbf{1}^T - \text{tr}\Lambda L_0 - \mathbf{1}^T\mu - \nu k - n \\
& \text{subject to} && \lambda_e = \mu_e + \nu - w_e h_e \Lambda h_e^T, \quad e = 1, \dots, p, \\
& && \lambda_e \geq 0, \mu_e \geq 0, \quad e = 1, \dots, p.
\end{aligned} \tag{3.4}$$

After eliminating the variables λ_e , it can be further equivalently reduced to the problem (3.5) below,

$$\begin{aligned}
& \underset{\mu, \nu}{\text{maximize}} && 2\text{tr}(n\Lambda)^{1/2} - (1/n)\mathbf{1}\Lambda\mathbf{1}^T - \text{tr}(L_0\Lambda) - \mathbf{1}^T\mu - k\nu - n \\
& \text{subject to} && w_e h_e^T \Lambda h_e \leq \nu + \mu_e, \quad e = 1, \dots, p, \\
& && \mu_e \geq 0, \quad e = 1, \dots, p.
\end{aligned} \tag{3.5}$$

where $\Lambda \in \mathbf{S}^n$, $\mu \in \mathbf{R}^p$, and $\nu \in \mathbf{R}$.

Duality Gap

The dual problem (3.5) itself is a convex programming problem with $\Lambda \in \mathbf{S}^n$, $\mu \in \mathbf{R}^p$, and $\nu \in \mathbf{R}$. The Lagrange multipliers can be eliminated because the optimal value is obviously $\mu_e + \nu = \max_e(w_e h_e^T \Lambda h_e)$. Since the dual problem (3.5) is a maximization problem, we have $\mu = 0$ and $\nu = \max_e(w_e h_e^T \Lambda h_e)$. Since the primal problem (3.3) has only linear equality and inequality constraints, the convex Slater's condition can be satisfied when $y = (k/p)\mathbf{1}$. Hence the optimal gap between the primal and dual problem is zero. If X^* is the optimal solution of the primal problem (3.3), then

$$\Lambda^* = n(X^*)^{-2}, \quad \nu^* = \max_e(w_e h_e^T \Lambda^* h_e), \quad \mu = 0$$

are the optimal solution of the dual problem (3.5). Conversely, if Λ^* is the optimal solution of the dual problem, then $X^* = (\Lambda^*/n)^{-1/2}$ is the optimal solution of the primal problem (3.3).

From the analysis above, a duality gap δ can be derived by Theorem 4 below, which is useful to measure the suboptimality of any feasible solution y . An interior-point algorithm can be accordingly developed based on the duality gap δ .

Theorem 4. *Let $L = L_0 + \sum_{e=1}^p y_e w_e h_e h_e^T$ and $S = (\Lambda, \mu, \nu)$ denote a feasible solution of the dual problem (3.5), where $\Lambda = n(L + \mathbf{1}\mathbf{1}^T/n)^{-2}$, $\nu = \max_e (w_e h_e^T \Lambda h_e)$ and $\mu = 0$. And y is the edge vector associated with S . Then the duality gap δ with respect to S is*

$$\delta = -\max_e (R_{tot} + k w_e^{-1} \frac{\partial R_{tot}}{\partial y_e} - n \text{tr}(L_0(L + \mathbf{1}\mathbf{1}^T/n)^{-2})).$$

Interior-Point Algorithm

In this section, we develop an interior-point algorithm, namely, a barrier method, with the help of the duality gap δ in Theorem 4 to efficiently solve the relaxed flight route selection problem (3.2).

We adopt logarithmic barrier functions to implicitly include the inequality constraints in the problem (3.2) in the objective function and approximately formulate the problem (3.2) as follows:

$$\begin{aligned} & \underset{y}{\text{minimize}} && \psi(y) = R_{tot}(y) - \gamma \sum_{e=1}^p (\log(y_e) + \log(1 - y_e)) \\ & \text{subject to} && \mathbf{1}^T y = k \end{aligned} \tag{3.6}$$

where γ is a small positive scalar and called the barrier parameter that controls the quality of approximation. Let \tilde{y} and y^* denote the optimal solution of the problem (3.6) and (3.2) respectively. As shown in the Section 11.2 of [14], \tilde{y} is at most $2p\gamma$ suboptimal for the relaxed flight route selection problem (3.3), that is,

$$R_{\text{tot}}(\tilde{y}) - R_{\text{tot}}(y^*) \leq 2p\gamma. \quad (3.7)$$

It provides an opportunity to achieve y^* by solving a sequence of decreasing values of γ until $\gamma \leq \epsilon/2p$ which guarantees an ϵ -optimal solution of the original problem (3.2). In the barrier method, we can use the duality gap δ to construct the sequence of decreasing values of γ by setting $\gamma = \beta\delta/2p$, where β is some constant within $(0, 1]$. Combining it with (3.7), it holds that

$$R_{\text{tot}}(\tilde{y}) - R_{\text{tot}}(y^*) \leq 2p * \beta\delta/2p \leq \delta. \quad (3.8)$$

We start with the initial solution as $y = (k/p)\mathbf{1}$. In each iteration, we apply the Newtons' method to compute the search direction Δy_{nt} as

$$\Delta y_{nt} = -(\nabla^2\psi)^{-1}\nabla\psi + \left(\frac{\mathbf{1}^T(\nabla^2\psi)^{-1}\nabla\psi}{\mathbf{1}^T(\nabla^2\psi)^{-1}\mathbf{1}}\right)(\nabla^2\psi)^{-1}\mathbf{1}.$$

in which the gradient of ψ is

$$(\nabla\psi)_e = -n\mathbf{tr}(\bar{L}_E(y))^{-1}h_{e,w}h_{e,w}^T\mathbf{tr}(\bar{L}_E(y))^{-1} + \frac{\gamma}{y_e} - \frac{\gamma}{(1-y_e)}, \quad e = 1, \dots, p.$$

and the Hessian of ψ is

$$\begin{aligned} \nabla^2\psi &= 2n(A_w^T(\bar{L}_E(y))^{-2}A_w) \circ (A_w^T(\bar{L}_E(y))^{-1}A_w) \\ &\quad - \gamma \text{diag}\left(\frac{1}{y_1^2} + \frac{1}{(1-y_1)^2}, \dots, \frac{1}{y_p^2} + \frac{1}{(1-y_p)^2}\right) \end{aligned}$$

where \circ denotes the Hadamard product, $h_{e,w} = w_e^{\frac{1}{2}}h_e$, $A_w = [h_{1,w} \cdots h_{p,w}]$, and $\bar{L}_E(y) = L_0 + \sum_e y_e w_e h_e h_e^T + \mathbf{1}\mathbf{1}^T/n$. We then compute the step size $s \in (0, 1]$, and update y as $y + s\Delta y_{nt}$. The search process will be terminated when the duality gap δ is reduced to an acceptable value.

The interior-point algorithm is summarized in Algorithm 1 below. The final solution \tilde{y} obtained by Algorithm 1 satisfies the termination condition $\delta \leq \varphi R_{\text{tot}}(\tilde{y})$ and the condition in (3.8) that $R_{\text{tot}}(\tilde{y}) - R_{\text{tot}}(y^*) \leq \delta$, which implies

$$\frac{R_{\text{tot}}(\tilde{y}) - R_{\text{tot}}(y^*)}{R_{\text{tot}}(y^*)} \leq \frac{\varphi}{1 - \varphi}.$$

Thus, Algorithm 1 can guarantee to derive an edge vector for the relaxed flight route selection problem (3.2) with a relative optimality gap less than $\varphi/(1 - \varphi)$.

Algorithm 1 Interior-Point Algorithm

Input: relative tolerance $\varphi \in (0, 1)$, $0 < \beta < 1$.

- 1: Initialize $\tilde{y} \leftarrow (k/p)\mathbf{1}$.
 - 2: **while** $\delta > \varphi R_{\text{tot}}(y)$ **do**
 - 3: Set $\gamma \leftarrow \beta\delta/p$.
 - 4: Compute Newton step $\delta\tilde{y}_{nt}$ for $\psi(\tilde{y})$ by solving
 - 5: $\Delta\tilde{y}_{nt} = -(\nabla^2\psi)^{-1}\nabla\psi + (\frac{\mathbf{1}^T(\nabla^2\psi)^{-1}\nabla\psi}{\mathbf{1}^T(\nabla^2\psi)^{-1}\mathbf{1}})(\nabla^2\psi)^{-1}\mathbf{1}$.
 - 6: Find the step length s by backtracking line search.
 - 7: $\tilde{y} \leftarrow \tilde{y} + s\Delta\tilde{y}_{nt}$.
 - 8: **end while**
 - 9: **return** $R_{\text{tot}}, \tilde{y}$.
-

Rounding Techniques

The interior-point algorithm is developed to solve the relaxed problem whose solutions \tilde{y} are not necessarily 0-1 integers. Rounding methods can be applied to construct a good solution from \tilde{y} , in which $\hat{y}_e = 1$ for some k edges and $\hat{y}_e = 0$ for the other $p - k$ ones. The following three rounding techniques have been implemented and compared, and the one with the best performance is adopted in this chapter.

- *Greedy*: Choosing k biggest elements \hat{y}_e from the relaxed optimal solution \tilde{y} to form \hat{y} .
- *Random*: Random rounding is a powerful method in the design and analysis of approximation algorithms, and determines a feasible solution which is provably not too far away from the best-possible solution. During implementation, \tilde{y} is normalized first, and then the normalized values are set as the probability value of P_e . For each $e \in \{1, \dots, m\}$, $\hat{y}_e = 1$ with probability P_e and $\hat{y}_e = 0$ otherwise. Then k elements are randomly chosen according to the probability distribution. Multiple iterations are required, and the best values are kept.
- *Step by Step*: In each step, we select the maximal element from \tilde{y} and update the Laplacian by adding this edge in the approximate flight route selection problem. Then we resolve this problem and iterate till k edges are added. This rounding method needs to solve the problem (3.6) for k times. When n or k are big, the computation time might be quite long.

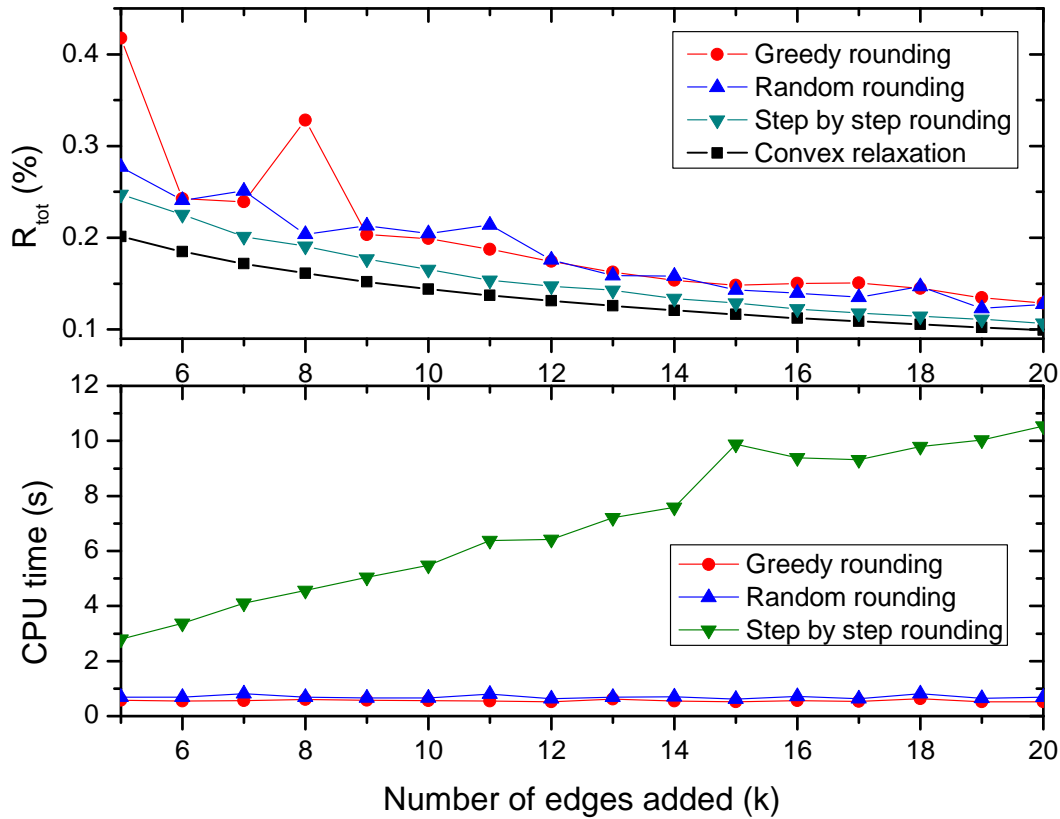


Figure 3.1: The performance and running time of three rounding techniques of k for flight route selection problem. The results are represented for a 30-node graph.

We use numerical simulations to compare the performance and the computation time of these three rounding strategies. A scale-free network with nodes $n = 30$ is generated and then used as the current existing networks. Three types of weights $w_{i,j} = \{1, 2, 3\}$ are randomly assigned to the edges in E_0 and E_P with uniform distribution. These results are presented in Figure 3.1 with the performance and the running time as the functions of k . In addition to these three rounding techniques, the lower bound obtained by the convex relaxation is also plotted for the reference.

It can be observed from Figure 3.1 that when edges are gradually added to the network G , the total effective resistance decreases monotonically. Step by step rounding technique always provides the best system performance as the number of edges k increases. Although its computation time is the worst, it is still acceptable for a relatively small/medium-scale problem. Hence we choose step by step rounding strategy to select k edges from \tilde{y} for a better optimality performance. All the steps are finally summed up in Algorithm 2, which leads to a good approximation of the optimum.

Algorithm 2 Interior-point algorithm with step by step rounding technique

Input: weighted graph $G(V, E_0)$, w_e for each $e \in E_P$.

- 1: Solve problem (3.6) with k .
 - 2: **for** $i = 1$ to k **do**
 - 3: $j \leftarrow \arg \max_e \{y_e | y_e \leq 1\}$.
 - 4: $\hat{y}_j \leftarrow 1$.
 - 5: Solve problem (3.6) with $k - i$ (**Algorithm 1**).
 - 6: **end for**
 - 7: **return** R_{tot}, \hat{y}
-

For a given set of nodes, the problem is solved for different values of k and the efficiency of the algorithm is computed by the following ratio of lower bound $R_{tot}(y^*)$ to approximate optimal total effective resistance, $r = 100 \times \frac{R_{tot}(y^*)}{R_{tot}(\hat{y})} \%$.

The results are illustrated in Figure 3.2. It can be observed that, although the results are relatively far from the lower bound for small values of k , the objective value quickly approaches to the lower bound of the approximate flight route selection problem as k increases.

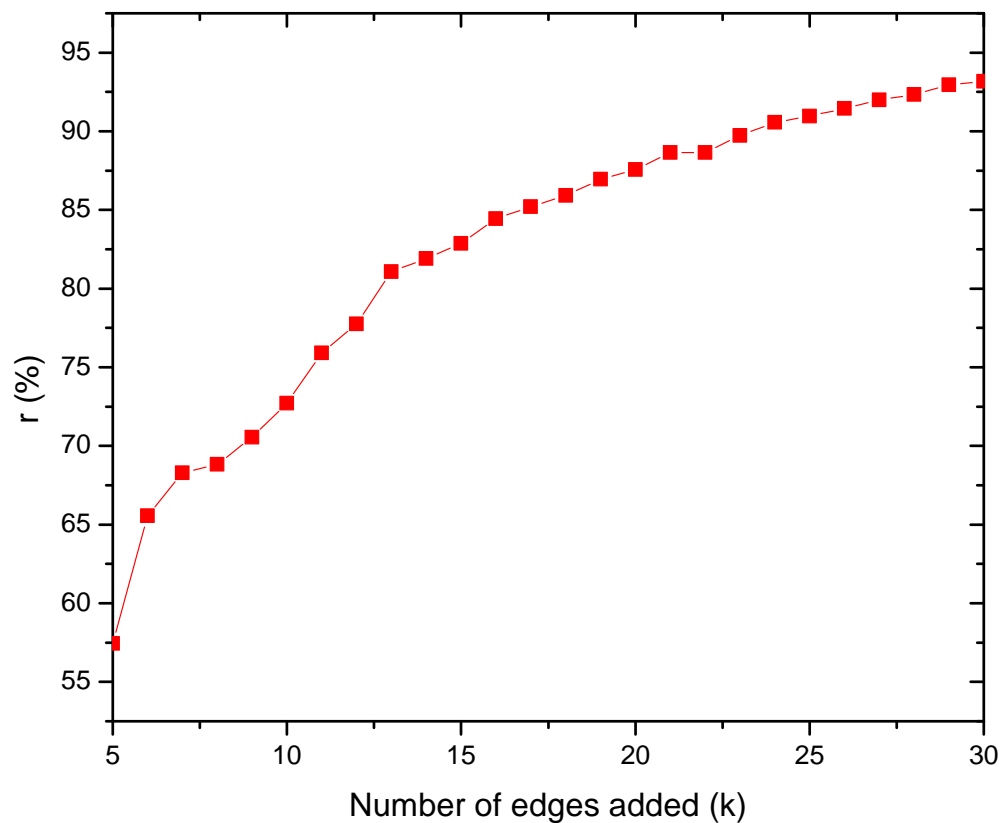


Figure 3.2: Value r for different values of k . The results are from step by step rounding strategy for a 30-node graph.

3.3 Submodular greedy algorithm

The application of convex relaxation method with step by step rounding technique is still subject to the curse of dimensionality, albeit having a better optimality performance and computational efficiency for small/medium-scale networks. The most time-consuming step in Algorithm 2 is solving the problem (3.6). As shown in Figure 3.3, the computation time exponentially increases with the growth of the number of nodes n . Since there are $O(n^2)$ variables in the relaxed problem for a fully-connected graph with n nodes, a large number

of relaxed problems are required to be solved and it becomes computationally intractable on a regular workstation for $n \geq 70$. To approach a large-scale problem that contains several hundred nodes, we will further develop a submodular greedy algorithm which provides a promising optimality performance within practical time limits.

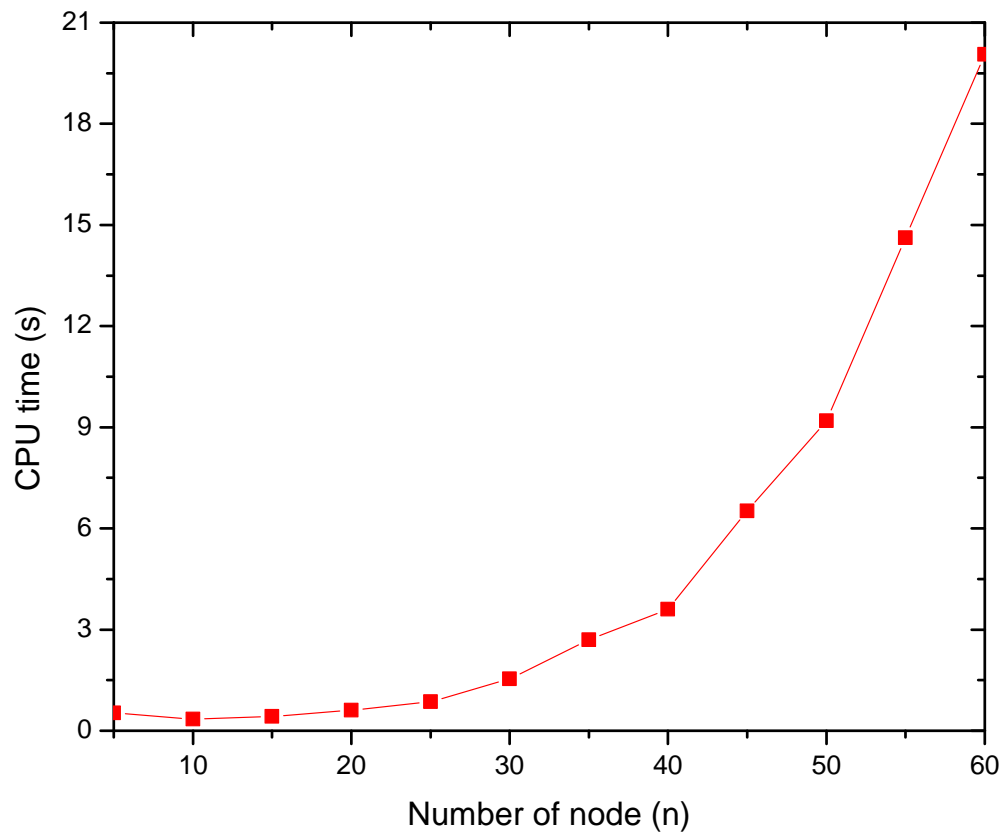


Figure 3.3: CPU Time in solving a convex relaxation problem for n -node graph

Submodularity of R_{tot}

The flight route selection problem (3.1) can be regarded as a set function problem (3.9) below if let $f(S) = -R_{tot}(S)$.

$$\begin{aligned} & \underset{S \subseteq E}{\text{maximize}} && f(S) \\ & \text{subject to} && |S| = k \end{aligned} \tag{3.9}$$

where $f : 2^E \rightarrow \mathbf{R}$ is a set function that assigns a real number to each subset of E . It can be proved that the set function $f(S) = -R_{tot}(S)$ is monotone submodular as shown in Theorem 5 below.

Theorem 5. *Given a graph $G(V, E_0)$, The set function $f(E) = -R_{tot}(E_0 \cup E)$ is monotone submodular for $E \subseteq E_P$, that is, for any $A \subseteq B \subseteq E_P$ and $e \in E_P \setminus B$,*

$$f(A \cup \{e\}) - f(A) \geq f(B \cup \{e\}) - f(B),$$

and

$$f(A) \leq f(B)$$

The submodularity is such a promising property of the set functions with solid theoretical background [51] and far-reaching applications [49, 59, 33]. In combinatorial optimization, submodularity is the counterpart of convexity (concavity) in continuous optimization.

The submodularity exhibits a natural characteristic of diminishing gains, *i.e.*, adding an element e to a larger set gives a smaller marginal benefit than adding one to a smaller subset. It enables a greedy algorithm to efficiently obtain a good solution with a guaranteed optimality gap as shown in Theorem 6 below.

Theorem 6. Let R_{tot}^{OPT} denote the function value of an optimal solution to problem (3.1) and R_{tot}^G denote the one of the solution derived by the greedy algorithm (Algorithm 3 below).

Then it holds that

$$R_{tot}^G \leq \left(1 + \frac{\alpha - 1}{e}\right) R_{tot}^{OPT}$$

where $\alpha = \frac{R_{tot}(E_0)}{R_{tot}^*}$ and R_{tot}^* is the minimal function value achieved by adding all the candidate edges.

Submodular Greedy Algorithm

As shown in Theorem 6, the greedy algorithm (Algorithm 3) can efficiently obtain a solution with a guaranteed optimality gap, in which Line 1 initializes a solution, Line 3 selects the edge from E_P which makes the maximal decrease in terms of R_{tot} , and Line 4 makes addition and deletion operations on E_S and E_P respectively.

Algorithm 3 Basic submodular greedy algorithm

Input: weighted graph $G(V, E_0)$, w_e for $e \in E_P$.

- 1: Initialize $E_S \leftarrow E_0$.
 - 2: **while** $|S| \leq k$ **do**
 - 3: $e \leftarrow \arg \max_{e \in E_P} [R_{tot}(E_S) - R_{tot}(E_S \cup e)]$.
 - 4: $E_S \leftarrow E_S \cup e$, $E_P \leftarrow E_P \setminus e$.
 - 5: **end while**
 - 6: **return** R_{tot} , S .
-

Algorithm 3 is only a basic greedy algorithm and it can be still computationally intensive if the marginal function in Line 3 cannot be efficiently evaluated and optimized, especially when facing a large-scale network with a huge $|E_P|$. With the help of Theorem 7 below,

instead of separately computing R_{tot} for two different networks, the marginal function in Line 3 of Algorithm 3 can be more efficiently evaluated by (3.10), which paves the way for the accelerated submodular greedy algorithm in Algorithm 4.

Theorem 7. *Given any connected weighted network $G(V, E)$ with weighted Laplacian matrix L_E , any edge $e \in E_P$ for addition with weight w_e , and incidence matrix h_e . Then*

$$R_{tot}(E) - R_{tot}(E \cup e) = \frac{nw_e}{\beta} \|(L_E + \mathbf{1}\mathbf{1}^T/n)^{-1}h_e\|^2 \quad (3.10)$$

where $\beta = 1 + w_e h_e^T (L_E + \mathbf{1}\mathbf{1}^T/n)^{-1} h_e$.

Although (3.10) in Theorem 7 simplifies the evaluation of the marginal function, it may still suffer from the computational burden resulted from computing $(L_{E_s} + \mathbf{1}\mathbf{1}^T/n)^{-1}$. Fortunately, since only one edge e will be added into E_s every iteration, after calculating $(L_{E_0} + \mathbf{1}\mathbf{1}^T/n)^{-1}$ in the first iteration, $(L_{E_s} + \mathbf{1}\mathbf{1}^T/n)^{-1}$ can be efficiently derived by performing a rank-1 update in the following iterations, which finally enables the accelerated submodular greedy algorithm developed in Algorithm 4.

In Line 2 of Algorithm 4, for the initial iteration, the inverse of $L_{E_0} + \mathbf{1}\mathbf{1}^T/n$ can be efficiently obtained by the conjugate gradient method [61] in the complexity of $\mathcal{O}(\kappa\sqrt{v})$, where κ is the number of nonzero entries in $(L_{E_0} + \mathbf{1}\mathbf{1}^T/n)$ and v is its condition number. Line 4 uses (3.10) to evaluate each edge in E_P and chooses the one with the largest marginal decrease, in which $(L_{E_s} + \mathbf{1}\mathbf{1}^T/n)^{-1}$ is commonly computed using the Sherman-Morrison formula [69, 71, 24] to perform the rank-1 updates to the inverse matrix derived in the previous iteration in the complexity of $\mathcal{O}(n^2)$. Line 5 and 6 just update $R_{tot}(E_S)$, E_S and

E_P . The total computational complexity of the accelerated submodular greedy algorithm is about $\mathcal{O}(\kappa\sqrt{v} + kn^2)$.

Algorithm 4 Accelerated submodular greedy algorithm

Input: weighted graph $G(V, E_0)$, w_e for $e \in E_P$.

- 1: Initialize $E_S \leftarrow E_0$.
 - 2: Compute $(L_{E_S} + \mathbf{1}\mathbf{1}^T/n)^{-1}$ with the conjugate gradient method and then let $R_{tot}(E_S) \leftarrow n\text{tr}(L_{E_S} + \mathbf{1}\mathbf{1}^T/n)^{-1} - n$.
 - 3: **while** $|S| \leq k$ **do**
 - 4: $e \leftarrow \arg \max_{e \in E_P} \frac{nw_e}{\beta} \|(L_{E_S} + \mathbf{1}\mathbf{1}^T/n)^{-1}h_e\|^2$.
 - 5: $R_{tot}(E_S) \leftarrow R_{tot}(E_S) - \frac{nw_e}{\beta} \|(L_{E_S} + \mathbf{1}\mathbf{1}^T/n)^{-1}h_e\|^2$.
 - 6: $E_S \leftarrow E_S \cup e$, $E_P \leftarrow E_P \setminus e$.
 - 7: **end while**
 - 8: **return** R_{tot} , S .
-

Numerical Results

We first compare the performance of the submodular greedy algorithm and the convex relaxation method with step by step rounding technique. The experiment begins with a relatively small scale-free network with 20 nodes. Figure 3.4 shows the results of applying these two algorithms for different k . It is observed that, although the convex relaxation with step by step rounding technique always provides a relatively better optimality performance, about 10% improvement in the case of $k = 20$, the greedy algorithm achieves a substantial reduction in computation time, about 95% decrease in the case of $k = 20$.

We further compare the computation time between the basic submodular greedy algorithm and the accelerated one. Both algorithms are applied to a scale-free network with n nodes. These two algorithms select fixed k edges in each case. Figure 3.5 shows the com-

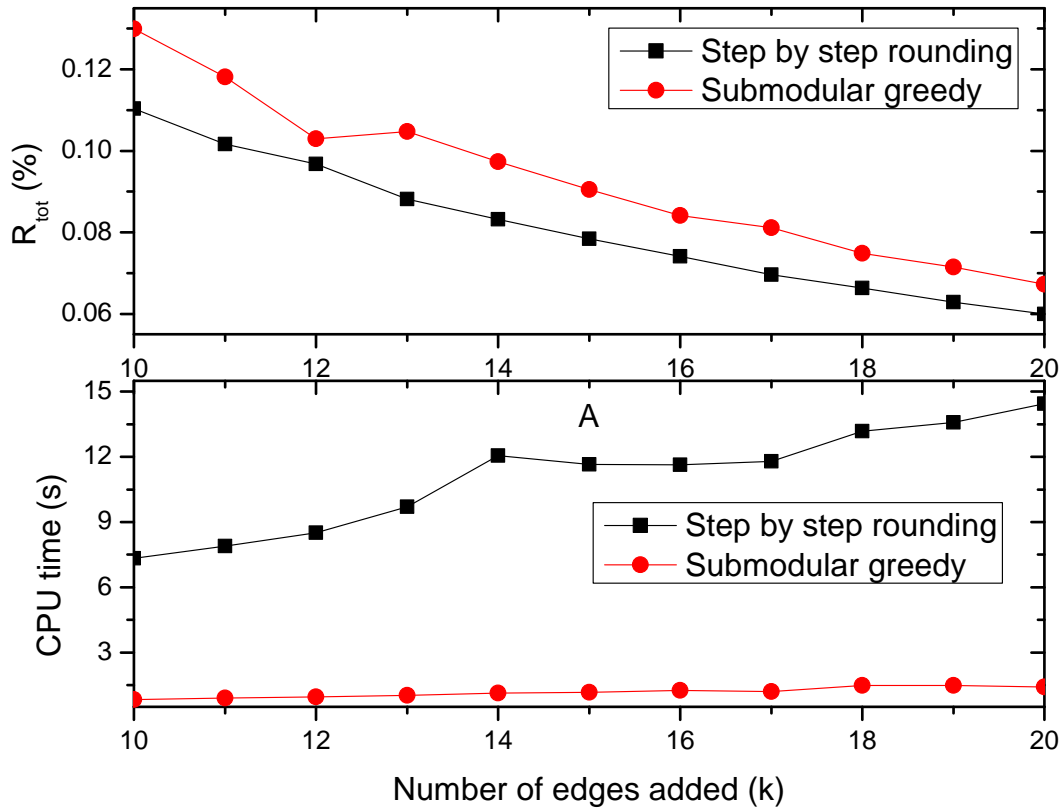


Figure 3.4: The comparison of performance and running time between step by step rounding technique and submodular greedy algorithm

putation time of these two algorithms for various network sizes. It can be easily seen that the running time of the basic submodular greedy algorithm is substantially increased around 150 nodes. The accelerated submodular greedy algorithm demonstrates a promising computational efficiency when the number of nodes is large than 200. For example, the accelerated algorithm exhibits a factor of 20 speed-up in the case of $n = 150$, and it's able to optimize the networks with up to thousands nodes in hours, with nearly millions of potential edges, which is far beyond the capability of convex relaxation methods.

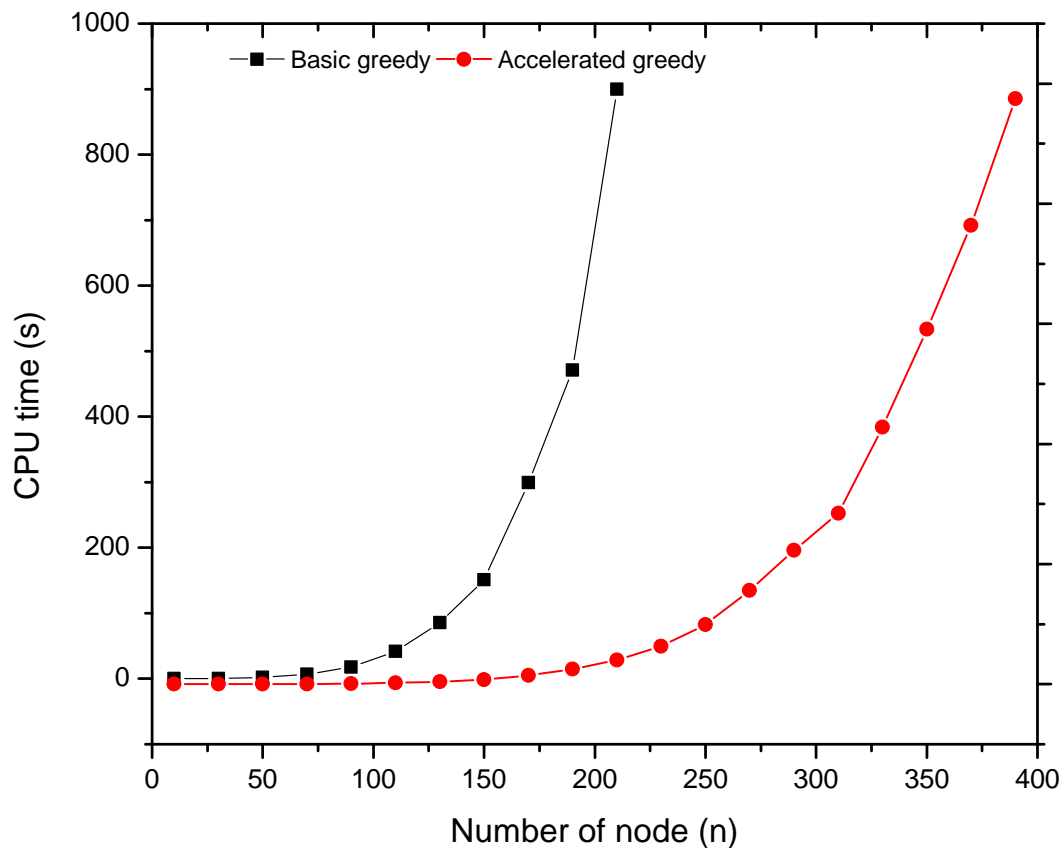


Figure 3.5: The comparison of running time between basic greedy algorithm and accelerated greedy algorithm

3.4 Case study

Small-scale Air Transportation Network Case: Drone Cargo

Network

The drone cargo network is studied to illustrate the application of convex relaxation with step by step rounding technique. The network changes constantly and its topology is crucial to the successful cargo delivery. The case study is about a drone cargo network of S.F.

Express, one of the largest logistic companies in China. Since the drone cargo network is currently in development, its scale is relatively small so far. The convex relaxation algorithm can achieve a better optimality performance within acceptable time. The diameter of the network is approximately to 50 km and the flight distance of current electric drone is limited by 30 km. It is necessary to set up transfer airports to provide batteries for drone to extend its flight distance. Besides the delivery function of each airport, different functional characteristics are associated with airports. These airports are classified into warehouse, transfer airport and final delivery airport. Batteries are backed up in the transfer airports to provide the power for drones. Setting up routes between each airport pair is impossible due to budget constraints. Besides getting the approval of flight routes from the Aviation Bureau, the telecommunication, antennae, and emergency area should also be built and associated with each route.

Since the drone cargo network is still under test and route failure rate data are not available, we cannot use the mechanism of mapping cancellation rates into different weights. Instead, we define the edge weight based on node degree commonly used to characterize the node centrality [94]. Generally speaking, the more routes the airport connects, the more important the airport is. Since more advanced equipments are usually equipped along the routes between critical hubs to facilitate a high demand, the routes connecting critical hubs with a higher node degree are less likely to fail, which implies a larger route weight. Therefore, we define the weight of route (i, j) as $w_{i,j} = f(\deg(v_i) + \deg(v_j))$, where function $f(\cdot)$ maps the sum of degree connectivity into weights $\{1,2,3\}$ based on a piecewise linear

Table 3.1: Top 5 and 10 routes to be added to S.F. drone cargo network

Top 5 <route, weight>	Top 10 <route, weight>	
<NanJiang-SiXia, 3>	<NanJiang-SiXia, 3>	<YouShi-PingShi, 2>
<SiXia-ZiYang,3>	<SiXia-ZiYang, 3>	<SiXia-SheXi, 3>
<SiXia-LongMu, 3 >	<SiXia-LongMu, 3>	<HeYuang-PingShi, 2>
<SiXia-SheXi, 3>	<ShuangXi-PingShi, 2>	<HeYuang-DaPing, 2>
<SiXia-ShiBaTang, 3>	<AnHe-PingShi, 2>	<SiXia-ShiBaTang, 3>

approximation.

In this case study, the convex relaxation method is applied to add $k = 5$ and $k = 10$ routes. Table 3.1 shows the selected routes and their associated weights in this two scenarios. It is noted that the distance of candidate routes is less than 30 km which is the drone's maximum flight duration. In a small added route set, edges tend to connect the low-degree airport and transfer airport. For example, the degree of the airports NanJiang, ZiYang, and LongMu is 1. Namely, only one route exists between each of these airports with others. If the route fails, drone cannot fly to the transfer airport by any routes or their combinations. With the increase of available route set, edges with higher weights slowly begin to connect the airports with high node degree.

Medium-scale Air Transportation Network Case: SF Airline

S.F. Express owns the largest cargo airline in China and its network is in medium size. The cargo network includes 40 airports and 114 bidirectional routes. The existing edges are weighted based on the failure rate computed from real operations. According to the data, the flight cancellation rate ranges from 0% to 9.6%. The existing edges with cancellation

Table 3.2: Top 5 and 10 routes to be added to S.F. air cargo network

Top 5 <route, weight>	Top 10 <route, weight>	
<LHW-CKG, 2>	<WUH-CKG, 2>	<CKG-CGO, 2>
<CKG-WUH, 2>	<LHW-CKG, 3>	<URC-KWE, 2>
<URC-KWE, 2 >	<URC-NKG, 2>	<CKG-SZX, 2>
<CKG-SZX, 2>	<CKG-PEK, 2>	<YNT-FOC, 2>
<URC-PVG, 2>	<CKG-DLC, 2>	<URC-PVG, 2>

rates in $[0, 3.2\%)$ are assigned a weight of 3, the ones with cancellation rates in $[3.2\%, 6.4\%)$ are assigned a weight of 2, and the ones with cancellation rates in $[6.4\%, 9.6\%]$ are assigned a weight of 1. The mechanism of mapping cancellation rates into different weights is motivated by the practical operations in the aviation industry. A flight route with a small weight indicates that it is prone to failure under disruptive situations, such as aircraft shortage or severe weather, and vice versa. In this case study, the greedy method is applied to add $k = 5$ and $k = 10$ routes. These results are illustrated in Table 3.2 with each edge name and its associated weight. It is noted the selected edges with weight 2 account for a large proportion, indicating those edges with medium strength are influential for the robustness improvement of overall robustness.

Large-Scale Air Transportation Network Case: Worldwide

Network

In this section, we study the application of the greedy algorithm for the worldwide air transportation network. The worldwide network supports the traffic of over three billion passengers traveling between more than 4000 airports on more than 50 million flights in a

year. Since the majority of the air transportation network network is based on hub-and-spoke network configuration, it is more beneficial and useful for the air transportation industry to improve the robustness of the network composed of critical hubs [7]. Based on the data for 2012 travel year from OpenFlights Database that contains 36203 routes between 3425 airports [58], 300 critical airports are selected in terms of node degree in the worldwide network as shown in Figure 3.6. There are 6736 routes existing in the current network of these 300 critical airports. The greedy submodular algorithm is used to choose k routes from 38114 candidate routes between these 300 airports. The result is illustrated in Figure 3.7, where the left y-axis is the relative R_{tot} defined as $R_{tot}(E_s)/R_{tot}(E_0)$. It can be observed that the computation time linearly increases and the network robustness measured by R_{tot} is continuously improved as k grows. When it reaches $k = 35$, the total effective resistance can be improved by 8.6% and the computation time is 2644s using MATLAB R2016a on a desktop computer with 2.70GHz Intel(R) Core(TM) i7 CPU and 8GB RAM in Windows 10 OS.

3.5 Summary

In this chapter, we show that the total effective resistance is an effective and reasonable measure, indicating the local and overall robustness of the air transportation network. The problem of selecting k flight routes, from among a set of candidate edges is formulated for the minimization of the total effective resistance, which is a difficult combinatorial problem. For

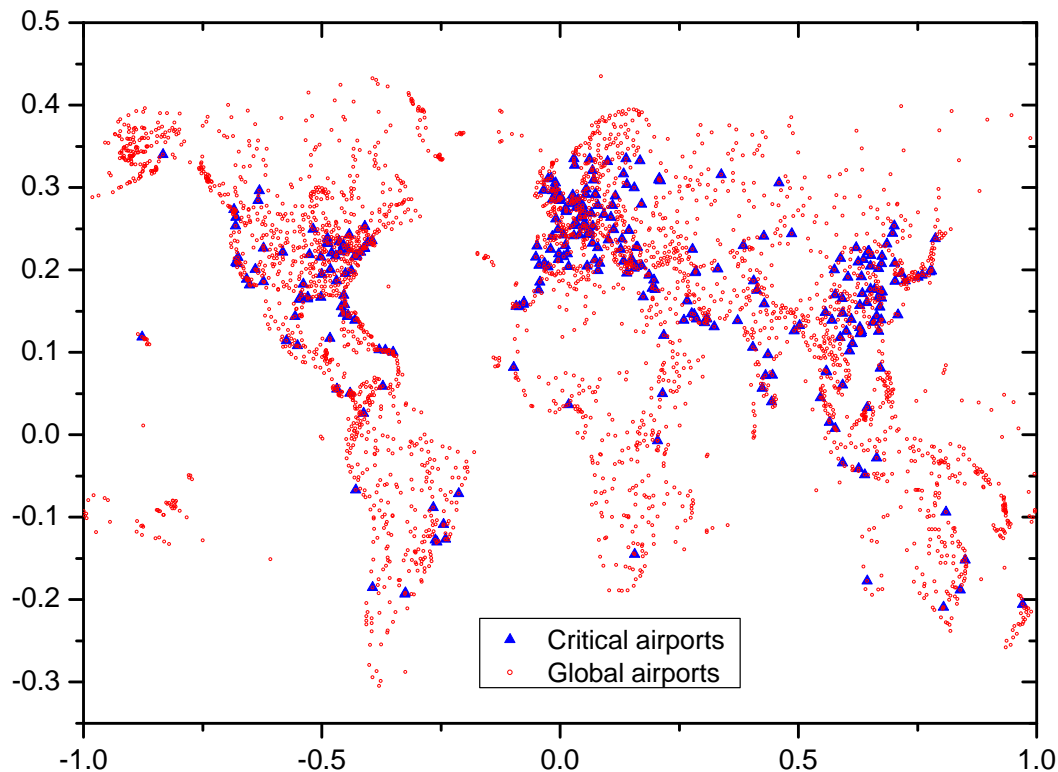


Figure 3.6: 300 Critical Airports of Worldwide Air Transportation Network

small-scale networks, we have shown that the convex relaxation with step-by-step rounding achieves a good optimality and an acceptable computation time. Then, we prove the submodular characteristics of the total effective resistance and propose an accelerated greedy algorithm based on matrix inverse updates, which allows scaling to the network size far beyond the capabilities of the convex relaxation method. Finally, numerical experiments are implemented on three real air transportation networks with small, medium, and large-scale.

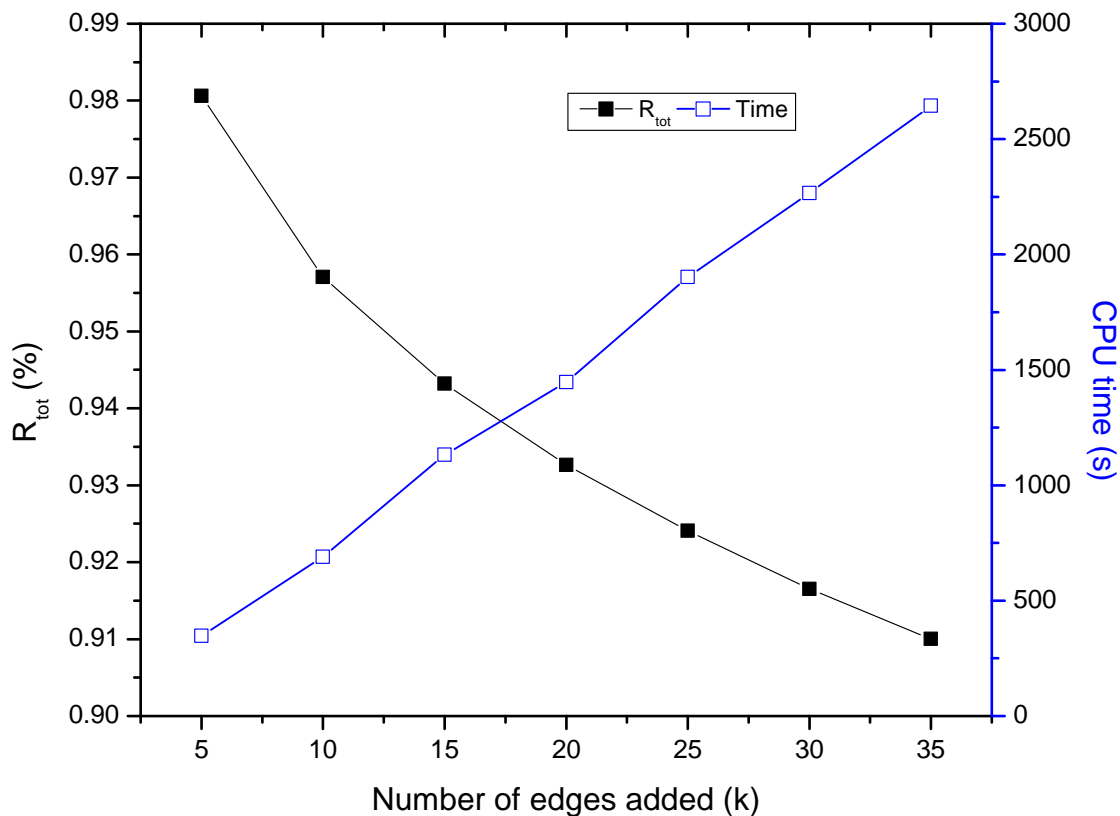


Figure 3.7: Relative R_{tot} and CPU time vs the number of edges added (k) for the 300 critical hubs of the worldwide air transportation network

3.6 Appendix

Proof of Theorem 3

Proof. The convexity property can be proved by verifying that the Hessian matrix of R_{tot} is positive semidefinite. To prove the property, we need to introduce the following identity function(see [14], section A.4.1),

$$\frac{\partial X^{-1}}{\partial t} = -X^{-1} \frac{\partial X}{\partial t} X^{-1}$$

where invertible symmetric matrix $X(t)$ is a differentiable function of the parameter $t \in \mathbf{R}$.

Define $\bar{L}(y) = L(y) + \mathbf{1}\mathbf{1}^T/n$ and $h_{e,w} = w_e^{\frac{1}{2}} h_e$, we can express the gradient as

$$\begin{aligned} \frac{\partial R_{tot}}{\partial y_e} &= -n \mathbf{tr}[\bar{L}(y)^{-1} \frac{\partial \bar{L}(y)}{\partial y_e} \bar{L}(y)^{-1}] \\ &= -n \mathbf{tr}[\bar{L}(y)^{-1} \frac{\partial(\bar{L}_0 + \sum_e y_e h_{e,w} h_{e,w}^T)}{\partial y_e} \bar{L}(y)^{-1}] \\ &= -n \mathbf{tr}[\bar{L}(y)^{-1} h_{e,w} h_{e,w}^T \bar{L}(y)^{-1}] \\ &= -n \|\bar{L}(y)^{-1} h_{e,w}\|^2. \end{aligned}$$

where $\|\cdot\|$ denotes the Frobenius norm, we have $\partial R_{tot}/\partial y_e \leq 0$, thus $R_{tot}(L(y))$ is a nonincreasing function of y , indicating a larger value of y leads to a better network in terms of R_{tot} .

Let $A_w = [h_{1,w} \cdots h_{p,w}]$, we can express the gradient as

$$\nabla R_{tot} = -n \text{diag}(A_w^T (\bar{L}_E(y))^{-2} A_w)$$

We now derive the Hessian matrix of $R_{tot}(y)$ from the derivative matrix,

$$\begin{aligned} \frac{\partial^2 R_{tot}}{\partial y_e \partial y_k} &= -n \frac{\partial}{\partial y_k} \|\bar{L}_E(y)^{-1} h_{e,w}\|^2 \\ &= 2n h_{e,w}^T (\bar{L}_E(y))^{-2} h_{k,w} h_{k,w}^T (\bar{L}_E(y))^{-1} h_{e,w} \end{aligned}$$

Using the definition of A_w , we can express the Hessian of $R_{tot}(y)$ as

$$\nabla^2 R_{tot} = 2n (A_w^T (\bar{L}_E(y))^{-2} A_w) \circ (A_w^T (\bar{L}_E(y))^{-1} A_w) \succeq 0$$

where \circ denotes the Hadamard product. The inequality above can be established because both the first component and second component of the product matrix are positive semidefinite. \square

Proof of Theorem 4

Proof. The solution $S = (\Lambda, \mu, \nu)$ is evidently feasible for the dual problem, so its dual objective value gives a lower bound \underline{R} on $R_{tot}(y)$, $\underline{R} \leq R_{tot}(y)$

$$\begin{aligned} \underline{R} &= 2\mathbf{tr}(n\Lambda)^{1/2} - (1/n)\mathbf{1}\Lambda\mathbf{1}^T - \nu k - \mathbf{tr}(\Lambda L_0) - \mathbf{1}^T \mu - n \\ &= 2n\mathbf{tr}(L + \mathbf{1}\mathbf{1}^T/n)^{-1} - \|(L + \mathbf{1}\mathbf{1}^T/n)^{-1}\mathbf{1}\|^2 - \mathbf{tr}(\Lambda L_0) - \max_e nk \|(L + \mathbf{1}\mathbf{1}^T/n)^{-1}h_e\|^2 - n \\ &= 2n\mathbf{tr}(L + \mathbf{1}\mathbf{1}^T/n)^{-1} - n\mathbf{tr}((L + \mathbf{1}\mathbf{1}^T/n)^{-2}L_0) - 2n - \max_e nk \|(L + \mathbf{1}\mathbf{1}^T/n)^{-1}h_e\|^2. \end{aligned}$$

In the second line, we use $(L + \mathbf{1}\mathbf{1}^T/n)^{-1}\mathbf{1} = \mathbf{1}$. Let δ denote the difference between this lower bound \underline{R} and the value of R_{tot} achieved by the selected edge vector y . There is a duality gap associated with y , using $R_{tot} = n\mathbf{tr}(L + \mathbf{1}\mathbf{1}^T/n)^{-1} - n$, the gap δ between the lower bound \underline{R} and the value of R_{tot} can be expressed as,

$$\begin{aligned} \delta &= R_{tot}(y) - \underline{R} \\ &= -n\mathbf{tr}(L + \mathbf{1}\mathbf{1}^T/n)^{-1} + \max_e nk \|(L + \mathbf{1}\mathbf{1}^T/n)^{-1}h_e\|^2 + n\mathbf{tr}(L_0(L + \mathbf{1}\mathbf{1}^T/n)^{-2}) + n \\ &= -\max_e (kw_e^{-1} \frac{\partial R_{tot}}{\partial y_e}) + n\mathbf{tr}(L_0(L + \mathbf{1}\mathbf{1}^T/n)^{-2}) - R_{tot} \\ &= -\max_e (R_{tot} - n\mathbf{tr}(L_0(L + \mathbf{1}\mathbf{1}^T/n)^{-2})) + kw_e^{-1} \frac{\partial R_{tot}}{\partial y_e} \end{aligned}$$

\square

Proof of Theorem 5

Proof. Taking any $A \subseteq B \subseteq E_P$ and $e \in E_P \setminus B$, we have

$$\begin{aligned}
f(A \cup \{e\}) - f(A) &= -R_{tot}(L_{A \cup E_0} + L_e) + R_{tot}(L_{A \cup E_0}) \\
&= -n\mathbf{tr}(L_{A \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)^{-1} + n\mathbf{tr}(L_{A \cup E_0} + \mathbf{1}\mathbf{1}^T/n)^{-1} \\
&= -n\mathbf{tr}[(L_{A \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)^{-1} - (L_{A \cup E_0} + \mathbf{1}\mathbf{1}^T/n)^{-1}] \\
&= n\mathbf{tr} \left[\frac{L_e}{(L_{A \cup E_0} + \mathbf{1}\mathbf{1}^T/n)(L_{A \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)} \right] \\
&\geq n\mathbf{tr} \left[\frac{L_e}{(L_{B \cup E_0} + \mathbf{1}\mathbf{1}^T/n)(L_{B \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)} \right] \\
&= -n\mathbf{tr}[(L_{B \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)^{-1} - (L_{B \cup E_0} + \mathbf{1}\mathbf{1}^T/n)^{-1}] \\
&= f(B \cup \{e\}) - f(B)
\end{aligned}$$

where the inequality satisfies since $L_{A \cup E_0} \preceq L_{B \cup E_0}$ and thus $(L_{A \cup E_0} + \mathbf{1}\mathbf{1}^T/n)^{-1} \succeq (L_{B \cup E_0} + \mathbf{1}\mathbf{1}^T/n)^{-1}$, $(L_{A \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)^{-1} \succeq (L_{B \cup E_0} + L_e + \mathbf{1}\mathbf{1}^T/n)^{-1}$. Therefore, we have

$$f(A \cup \{e\}) - f(A) \geq f(B \cup \{e\}) - f(B) \quad (3.11)$$

which implies that the set function $f(E)$ is submodular. It can be easily shown that $f(E)$ is also monotone increasing. Hence, $f(E)$ is a monotone submodular function. \square

Proof of Theorem 6

Proof. Let f^{OPT} be the value of an optimal solution to problem (3.9) and f^G be the value of a particular solution to (3.9). There exists a greedy algorithm which begins with empty

set, $S \leftarrow \emptyset$ and computes the loss $\Delta(e|S_i) = f(e|S_i) - f(S_i)$ for all elements $e \in E$ till the k th iteration. In each step, the algorithm selects the element with the highest loss

$$S_{i+1} \leftarrow S_i \cup \{\arg \max_e \Delta(e|S_i) | e \in E\}.$$

As shown in [57], if f is a monotone increasing and submodular function, then the greedy algorithm always produces a solution satisfying

$$\frac{f^{\text{OPT}} - f^G}{f^{\text{OPT}} - f(\emptyset)} \leq \left(\frac{k-1}{k}\right)^k \leq \frac{1}{e}$$

Combining it with Theorem 5, the greedy algorithm in Algorithm 3 results in

$$\frac{-R_{tot}^{\text{OPT}} + R_{tot}^G}{-R_{tot}^{\text{OPT}} + R_{tot}(E_0)} \leq \frac{1}{e} \quad (3.12)$$

where E_0 is the existing edges of the initial network. Rearranging the inequality (3.12), we have

$$\begin{aligned} R_{tot}^G &\leq \frac{1}{e} R_{tot}(E_0) + \left(1 - \frac{1}{e}\right) R_{tot}^{\text{OPT}} \\ &= \left(\frac{R_{tot}(E_0)}{e R_{tot}^{\text{OPT}}} + 1 - \frac{1}{e}\right) R_{tot}^{\text{OPT}} \\ &\leq \left(\frac{R_{tot}(E_0)}{e R_{tot}^*} + 1 - \frac{1}{e}\right) R_{tot}^{\text{OPT}} \\ &= \left(1 + \frac{\alpha - 1}{e}\right) R_{tot}^{\text{OPT}} \end{aligned}$$

The second inequality is satisfied because $R_{tot}^* \leq R_{tot}^{\text{OPT}}$ and R_{tot}^* is the optimal value obtained by adding all the candidate edges with given weights. \square

Proof of Theorem 7

Proof. Define $\bar{L}_E = L_E + \mathbf{1}\mathbf{1}^T/n$ and $h_{e,w} = w_e^{\frac{1}{2}}h_e$, it holds that

$$\begin{aligned}
 R_{tot}(E \cup e) &= n\mathbf{tr}(L_{E \cup e} + \mathbf{1}\mathbf{1}^T/n)^{-1} - n \\
 &= n\mathbf{tr}(L_E + \mathbf{1}\mathbf{1}^T/n + w_e h_e h_e^T)^{-1} - n \\
 &= n\mathbf{tr}(\bar{L}_E + h_{e,w} h_{e,w}^T)^{-1} - n
 \end{aligned} \tag{3.13}$$

Since the rank of $L_E + \mathbf{1}\mathbf{1}^T/n$ is n and invertible, $\beta = 1 + w_e h_e^T (L_E + \mathbf{1}\mathbf{1}^T/n)^{-1} h_e > 0$.

From the Sherman-Morrison formula [69], we have

$$(\bar{L}_E + h_{e,w} h_{e,w}^T)^{-1} = \bar{L}_E^{-1} - \frac{1}{\beta} \bar{L}_E^{-1} h_{e,w} h_{e,w}^T \bar{L}_E^{-1}$$

Using the equality above, we can reformulate equation (3.13) as

$$\begin{aligned}
 R_{tot}(E \cup e) &= n\mathbf{tr}(\bar{L}_E + h_{e,w} h_{e,w}^T)^{-1} - n \\
 &= n\mathbf{tr}(\bar{L}_E^{-1} - \frac{1}{\beta} \bar{L}_E^{-1} h_{e,w} h_{e,w}^T \bar{L}_E^{-1}) - n \\
 &= n\mathbf{tr}(\bar{L}_E^{-1}) - n - \frac{n}{\beta} \mathbf{tr}(\bar{L}_E^{-1} h_{e,w} h_{e,w}^T \bar{L}_E^{-1}) \\
 &= R_{tot}(E) - \frac{n}{\beta} \|\bar{L}_E^{-1} h_{e,w}\|^2 \\
 &= R_{tot}(E) - \frac{nw_e}{\beta} \|(L_E + \mathbf{1}\mathbf{1}^T/n)^{-1} h_e\|^2
 \end{aligned} \tag{3.14}$$

which implies (3.10). □

Chapter 4

Route Budget Allocation Problem

The goal of this chapter is to minimize the total effective resistance under the budget constraint. Each edge is associated with an operating cost that we assume it is a linear function of edge weight. Given the total budget, we need to determine the number of added edges and their corresponding weights. In this chapter, we propose several approximation methods to solve the *route budget allocation problem* for small, medium and large-scale with different performance considerations. Compared with the existing works, the primary contributions of this chapter are twofold. On the one hand, this is the first work that robustness optimization of air transportation network under budget constraint is formulated with the total effective resistance. On the other hand, we are the first to present clustering-based convex relaxation to achieving a better performance for large-scale network. We also propose a method to measure hub hierarchy.

The rest of this chapter is structured as follows. In Section 4.1, the problem is explicitly

defined and formulated mathematically. In Section 4.2, the problem is accurately solved for small-scale networks, and more efficient full algorithm, convex relaxation method, for the medium-scale network is presented. In Section 4.3, clustering-based convex relaxation is introduced for large-scale network. The measure of hub hierarchy is defined and combined with Gaussian mixture method (GMM) to form a full algorithm, GMM-based convex relaxation algorithm. In Section 4.4, two real air transportation networks including Jetstar Asia Airway and domestic American Airline are investigated.

4.1 Problem Formulation

The goal of the chapter is to minimize the total effective resistance of the network under a budget constraint. The budget constraint should be first explicitly defined to formulate the problem. In the complete graph, there is a total of $m = (n(n - 1))/2$ edge, each associates with a weight $w_{i,j}$ representing the link strength. Assume the link strength is a value within lower bound α and upper bound β , which explicitly expressed by $\forall (i, j) \in E, \alpha \leq w_{i,j} \leq \beta$, and $w_{i,j} = 0$ when there is no edge between v_i and v_j .

Another assumption defines route cost is a linear function of route strength. This assumption is intuitively right as high route strength incurs more cost. The hypothesis is also based on practical consideration. For example, the most effective method to prevent flight cancellation due to the mechanical and electrical problem is having more spare parts and backup airplane. Similarly, to avoid the flight cancellation incurred by flight crew is

recruiting more crew. As for the weather disturbances, loading more fuel will provide the aircraft more potential choices of rerouting or course reversal. All of these approaches could increase the link strength. However, an extra cost will also be introduced. We assume the total operating cost is limited by budget C , $\sum_{i,j} w_{i,j} c_{i,j} \leq C$, $c_{i,j}$ is the cost coefficient associated each route. The objective is to improve the network through allocating the edge budget, which is equal to minimize the total effective resistance under several constraints. In summary, the route budget allocation problem is explicitly formulated as,

$$\min_w R_{\text{tot}}(L(w)) \quad \text{s.t.} : \begin{cases} \sum_{i,j} w_{i,j} c_{i,j} \leq C \\ w_{i,j} \in \{0, [\alpha, \beta]\} \end{cases} \quad (4.1)$$

Exact Solution

To demonstrate the difficulty of origin problem (4.1), we first get the exact solution by brute force method. The idea here is we enumerate each possible configuration of the network we want to optimize and observe their performances. Consider the graph with n nodes, the potential configurations of the complete graph might be 2^m , where $m = \frac{n(n-1)}{2}$. In each configuration, the weight associated with each edge should be either 0 or in $[\alpha, \beta]$, we optimize the particular graph under corresponding constraints and then get the best result. Assume E_0 is the edge collection already selected in the graph, our goal is to solve the following problem,

$$\min_w R_{\text{tot}}(L(w)) \quad \text{s.t.} : \begin{cases} \sum_e w_e c_e \leq C \\ \alpha \leq w_e \leq \beta, \quad e \in E_0 \\ w_e = 0, \quad e \notin E_0 \end{cases} \quad (4.2)$$

This is equivalent to the weight assignment problem (see ([28]) for details), the problem can be solved by following semidefinite programming (SDP) formulation,

$$\begin{aligned} \min_w \quad & n \mathbf{tr} \mathbf{M} - n \\ \text{s.t.} \quad & c^T w \leq C \\ & \begin{bmatrix} \sum_e w_e h_e h_e^T + \mathbf{1}\mathbf{1}^T/n & I \\ & I & \mathbf{M} \end{bmatrix} \succeq 0 \\ & \alpha \leq w_e \leq \beta, \quad e \in E_0 \\ & w_e = 0, \quad e \notin E_0 \end{aligned} \quad (4.3)$$

where $\mathbf{M} \in \mathbf{S}^n$ (the set of symmetric $n \times n$ matrices) is the slack matrix. The solution of above SDP formulation can be obtained using the standard solver, for example, via SeDuMi [70] or standard barrier methods implemented by using gradient and Hessian formulas given in [28]. A custom interior-point algorithm making use of the duality gap is also proposed in [28], and its performance substantially exceeds the standard SDP formulation.

Table 4.1 shows optimization result for a toy example, a three node graph. There are three candidate edges for this graph. Let $e_1 = \{v_1, v_2\}$, $e_2 = \{v_1, v_3\}$, and $e_3 = \{v_2, v_3\}$ be the edges between different node pair. Three types of edge cost $c_1 = 1$, $c_2 = 2$ and $c_3 = 3$

are given, total cost budget C is 5. The weight bound parameters $\alpha = 0.5$ and $\beta = 3$. Eight distinct topologies exist with different edge combination. When only one edge is selected $k = 1$, the result is not available due to the disconnected graph, indicating the R_{tot} is infinite; when two or three edges are selected, it is possible to get an exact solution.

The optimization results expressed by (R_{tot}, k) for two graphs with different node size are shown in Figures 4.1 and 4.2. For each configuration in terms of k selected edges, the optimal value of R_{tot} is computed by solving the weight assignment problem (4.3). From the results it is noticed that the best robustness, minimum value of R_{tot} , does not reach at the maximal number of edges; therefore, the selection of the edges is related to the edge weights. In Figure 4.1, the minimal value of R_{tot} is reached when $k = 6$. In Figure 4.2 the optimal value is reached at $k = 10$. Assume the time complexity to solve the weight optimization problem is T , the whole process needs to solve $O(2^{n(n-1)})$ problems. Therefore, the complexity of the exact algorithm can be approximated as $O(T2^{n(n-1)})$, which is exponential in term of network size n . When the network size is large, it is impractical to exactly solve the problem. An algorithm solving the problem in an approximate manner is going to be designed.

Table 4.1: Optimization result for three node graph

k	Edge Selected	Edge Weight	R_{tot}
1	e_1	—	$+\infty$
1	e_2	—	$+\infty$
1	e_3	—	$+\infty$
2	e_1, e_2	$w_1 = 2.07, w_2 = 1.46$	2.33
2	e_1, e_3	$w_1 = 1.83, w_3 = 1.06$	2.99
2	e_2, e_3	$w_2 = 1.12, w_3 = 0.92$	3.96
3	e_1, e_2, e_3	$w_1 = 1.54, w_2 = 0.98, w_3 = 0.5$	2.18

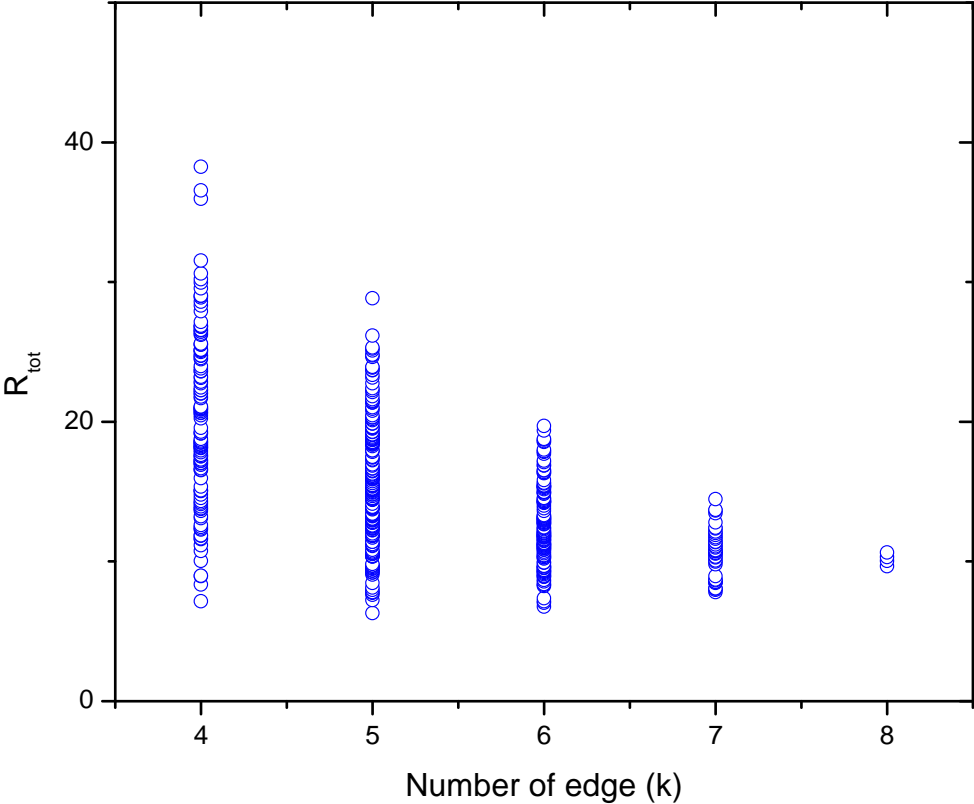


Figure 4.1: Results of (R_{tot}, k) for all configurations of $n = 5$ and $C = 5$

4.2 Convex Relaxation

To solve the problem (4.1), we introduce another binary variable y_e stating the fact that an edge e exists. We observe the fact $y_e = 1 \Leftrightarrow w_e \neq 0$, this is useful because the domain of w can be represented as, $\alpha y_e \leq w_e \leq \beta y_e$. The origin problem with one decision variable

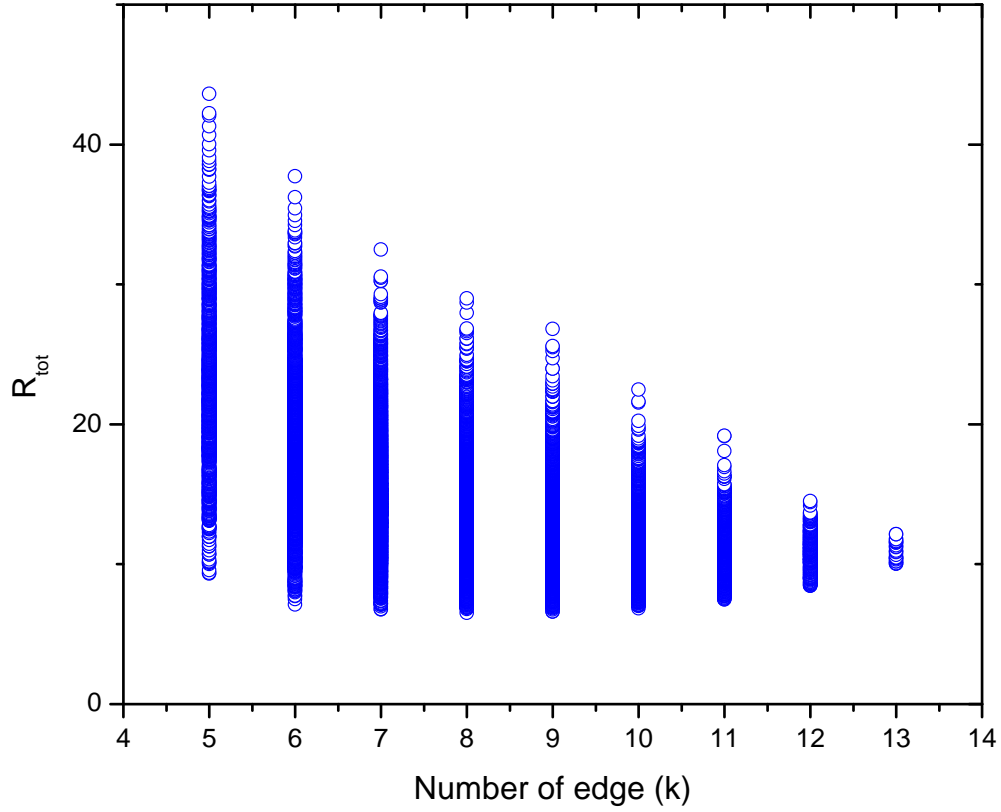


Figure 4.2: Results of (R_{tot}, k) for all configurations of $n = 6$ and $C = 10$

(4.1) is reformulated as a problem with two decision variables y, w .

$$\min_{y,w} R_{\text{tot}}(L(y, w)) \quad \text{s.t.} : \begin{cases} \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e \in \{0, 1\} \end{cases} \quad (4.4)$$

The main idea of the approximate algorithm is to take advantage of the convexity of the relaxed formulation of problem (4.3). This is motivated by the observation from Figures 4.1 and 4.2. It is easy to conclude the discrete shape of $k \rightarrow R_{\text{tot}}(k)$ is a approximate convex

function. Later on we will prove $R_{\text{tot}}(k)$ is convex in terms of k when y is relaxed.

Relaxed Formulation

In this section, we propose a convex relaxation method to approximately solve this problem. Problem (4.4) is first relaxed by replacing the nonconvex constraints $y_e \in \{0, 1\}$ with the convex constraints $y_e \in [0, 1]$, we have the relaxed formulation as follows,

$$\min_{y,w} R_{\text{tot}}(L(y,w)) \quad \text{s.t.} : \begin{cases} \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e \in [0, 1] \end{cases} \quad (4.5)$$

To take advantage of convexity of related formulation, we introduce an additional variable, the number of selected edge k , into above formulation. For each k , the problem we need to solve is

$$\min_{y,w} R_{\text{tot}}(L(y,w,k)) \quad \text{s.t.} : \begin{cases} \sum_e y_e = k \\ \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e \in [0, 1] \end{cases} \quad (4.6)$$

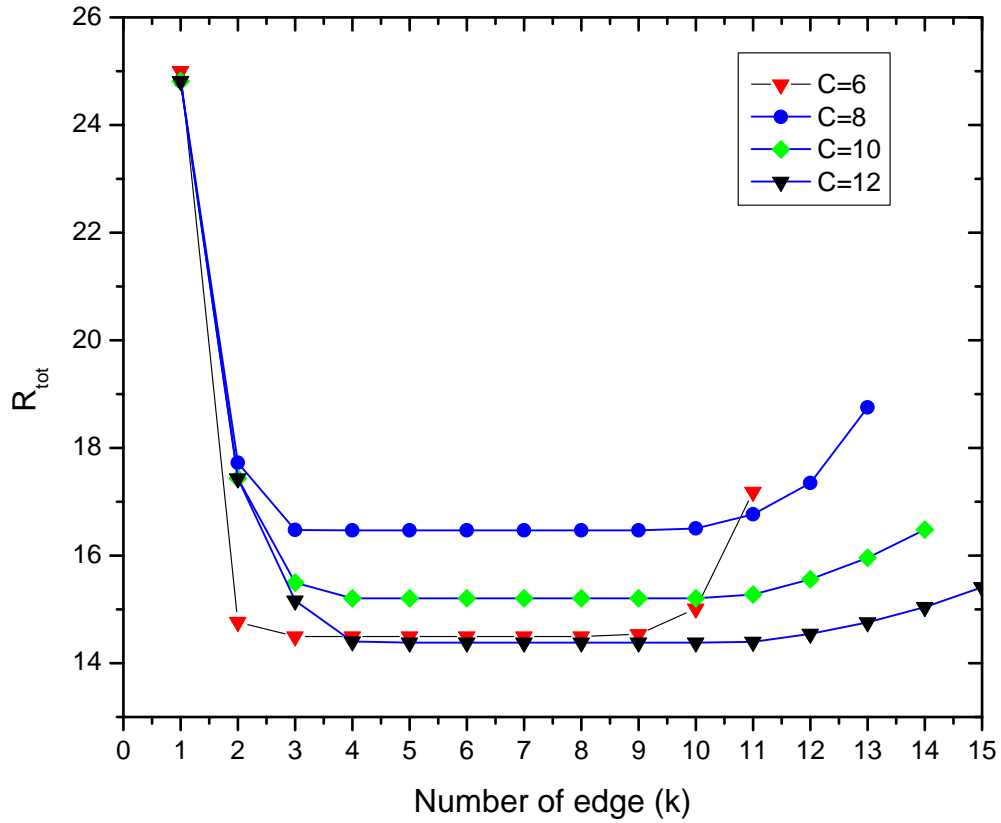


Figure 4.3: Lower bound of $R_{tot}(k)$ with $n = 6$

Lemma 1. [28] *The total effective resistance R_{tot} is a convex function of w : for $w_1, w_2 \geq 0$ and any $\lambda \in [0, 1]$, R_{tot} satisfies*

$$R_{tot}(\lambda w_1 + (1 - \lambda)w_2) \leq \lambda R_{tot}(w_1) + (1 - \lambda)R_{tot}(w_2)$$

Theorem 8. *Let $\Phi(k)$ be the objective value of problem (4.6) in terms of k , $\Phi(k)$ is a convex function.*

Relaxed SDP Formulation

Unlike the original problem, the relaxed formulation (4.6) is convex, because its objective (to be minimized) is convex in k (refer to Theorem 8), and the equality on k is linear. To solve the problem easily and efficiently, we reformulate it into the following SDP formulation,

$$\begin{aligned}
& \min_{y,w} \quad n \mathbf{tr} \mathbf{M} - n \\
& \text{s.t.} \quad \sum_e y_e = k \\
& \quad \quad c^T w \leq C \\
& \quad \quad \begin{bmatrix} \sum_e w_e h_e h_e^T + \mathbf{1}\mathbf{1}^T/n & I \\ & I & M \end{bmatrix} \succeq 0 \\
& \quad \quad \alpha y_e \leq w_e \leq \beta y_e \\
& \quad \quad y_e \in [0, 1]
\end{aligned} \tag{4.7}$$

The optimal objective value of above relaxed SDP provides a lower bound for the original problem. Figure 4.3 shows the lower bound of different values of k . The relaxed SDP reaches its minimum when k equals to certain value within interval $[k_{\min}, k_{\max}]$. In practical simulation, k_{\min} is set to be 1, k_{\max} is determined by edge cost coefficient c_i and total budget C . Here we need to find the maximum number of edges k_{\max} . No matter how robust the network is, edges are selected and added till total budget C is reached. Consider the case of

minimal weight $w = \alpha y_e$, we can solve the following knapsack problem to find k_{\max} .

$$\begin{aligned} \max_y \quad & \sum_e y_e \\ \text{s.t.} \quad & \sum_e y_e c_e \leq \frac{C}{\alpha} \\ & y_e \in \{0, 1\} \end{aligned} \tag{4.8}$$

In fact, the value of k_{\max} can be easily obtained by sorting the cost c_i in increasing order, and its value satisfies $\sum_{j=1}^{k_{\max}} c_j \leq C/\alpha$. In the case study of Figure 4.3, $k_{\max} = 12$ when $C = 6$, $k_{\max} = 15$ when $C = 12$.

In summary, the overall procedures solving the problem begin with finding an optimal k within $[k_{\min}, k_{\max}]$, and then use it to compute a feasible non-integer solution of y and w by solving the SDP formulation (4.7).

Rounding Techniques

In this section, it is assumed that the best solution \tilde{y} to the problem has been found. k edges will be chosen from \tilde{y} to form a reasonable solution \hat{y} , in which $\hat{y}_e = 1$ for k values and $\hat{y}_e = 0$ for others. Rounding techniques are often used to do so. The techniques adopted and implemented in this chapter are listed and compared to choose the one with the best performance.

- *Greedy*: Choosing k biggest elements \hat{y}_e from the relaxed optimal solution \tilde{y} to form \hat{y} .

- *Random*: Random rounding is a powerful method in design and analysis of approximation algorithms, and determines a feasible solution which is provably not too far away from the best-possible solution. During implementation, \tilde{y} is normalized first, and then normalized values are set as probability value P_e . For each $e \in \{1, \dots, m\}$, $\hat{y}_e = 1$ with probability P_e and $\hat{y}_e = 0$ otherwise. Then k elements are randomly chosen according to the probability distribution. Multiple iterations are required, and the best values are kept.
- *Step by Step*: In each step, we select the maximal element from \tilde{y} and update the graph by adding this edge. Then we solve this problem with the updated graph again and iterates till k edges are added. This rounding method needs to solve the relaxed SDP (4.7) for k times. When n or k are big, the computation time might be extremely long.

The performances of these three rounding techniques are compared via a case study, in which the graph is set up with 6 nodes. Figure 4.4 presents the results with R_{tot} as function of k . The lower bound obtained by relaxation is also shown in the figure. It is important to note that when some techniques fail to find a feasible solution, the corresponding values are deleted from the figure. We can also see the algorithms do not achieve the minimal value at the maximum value k . Each method shown has some advantages and disadvantages. Step by step rounding method gives the best result in term of performance. However, its computation time needed to solve k SDP is the worst. Greedy rounding technique provides

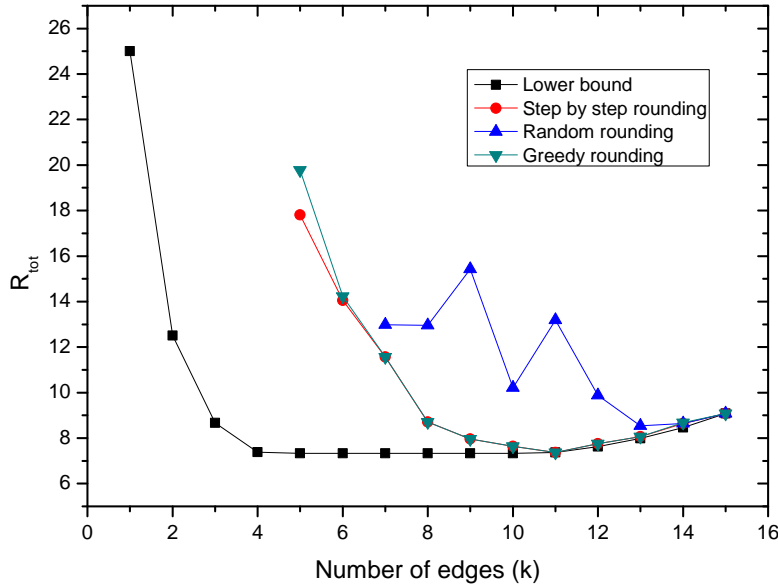


Figure 4.4: The performance of three rounding techniques as function of k . The results are represented for $n = 6$ and $C = 12$.

the shortest computation time, while its performance is not good as the step by step method when k is relatively small.

Relaxed SDP with Step by step Rounding

For a given k , the optimal topology of network can be found via iterating each k and then solve the corresponding weight assignment problem. When k_{\max} is large, it's impossible to test all the possible case of k . Motivated by convexity property of total effective resistance on k , one-dimensional search algorithm can used to speed up the whole search process. As the edges number of k is limited to be an integer value, continuous optimization algorithms

cannot be used. In this chapter, we use the golden section search method to sequentially reduce the interval range where the extremum exists. The technique maintains the function values for triples points whose distances form a golden ratio $\phi \approx 0.618$. Assume in t th iteration, $[a_t, b_t]$ is the current bracket interval, $f(a_t)$ and $f(b_t)$ have already been calculated previous. Let $c_t = \lfloor b_t + \phi(a_t - b_t) \rfloor$, $d_t = \lceil a_t + \phi(b_t - a_t) \rceil$, here the value is floored and ceiled to get an integer. The updating rules used are : if $f(c) < f(d)$ then $[a_{t+1}, b_{t+1}] = [a_t, d_t]$; otherwise, $[a_{t+1}, b_{t+1}] = [c_t, b_t]$. We only need to compute one new value at each step. On average this method shortens the interval with a factor ϕ .

Algorithm 5 Relaxed SDP with step by step rounding technique

Require: k_{max}

Initialize $a \leftarrow 1$ and $b \leftarrow k_{max}$

Solve relaxed SDP problem (4.7) to get $\Phi(a)$ and $\Phi(b)$.

while $a \leq b$ **do**

 Compute c, d via golden section search method

 Compute $\Phi(c), \Phi(d)$ via solving relaxed SDP problem

if $\Phi(c) \leq \Phi(d)$ **then**

$b \leftarrow d$

else

$a \leftarrow c$

end if

end while

$k_{opt} \leftarrow \text{Round}((a + b)/2)$

for $i = 1$ to k **do**

$j \leftarrow \arg \max_e \{\tilde{y}_e | \tilde{y}_e \leq 1\}$.

$\hat{y}_j \leftarrow 1, w_e \leftarrow w_{opt_e}$

 Solve relaxed SDP problem (4.7) with $k - i$.

end for

return R_{tot}, \hat{y}, w_e .

The details of the algorithm, relaxed SDP with step by step rounding technique, are shown in Algorithm (5). Two subroutines are included into this algorithm. In the first

subroutine, the algorithm tries to find an optimal k_{opt} for the relaxed SDP problem. Assume complexity T needs to solve the problem. There are total $S = -\frac{\lg k_{max}}{\lg \phi}$ steps, therefore, the complexity of the golden section search part can be approximated by $O(ST)$. After obtaining k_{opt} , as the step by step rounding needs k_{opt} iterations to select edge, the complexity associated with this part equals to $O(Tk_{opt})$, thus the total complexity of this algorithm is approximated by $O(T(S + k_{opt}))$.

To test the full algorithm, a Watts-Strogatz graph [84] with 30 nodes, 40 edges, mean node degree 8, and rewiring probability 0.5 is generated. The optimization results are shown in Figure 4.5. The selected edges are denoted by red lines, and line width implies each edge's weight. Thicker line indicates a higher value of edge weight.

4.3 Clustering Based Convex Relaxation

Air transportation network is often described as a complex adaptive system or further characterized as a System of Systems [23]. The underlying network structure reveals multiple, and heterogeneous characteristics. The overall transportation network is composed of individual airline subnetwork and so on. The convex relaxation method proposed in Section 4.3 is going to be used to optimize large-scale network. However, it is infeasible for the network size which is large than 90 in the author's workstation. The reason is that the most time-consuming part of the convex relaxation method is solving the SDP, a polynomial in the size of the variables, approximated to n^2 . The value of n of large-scale network is

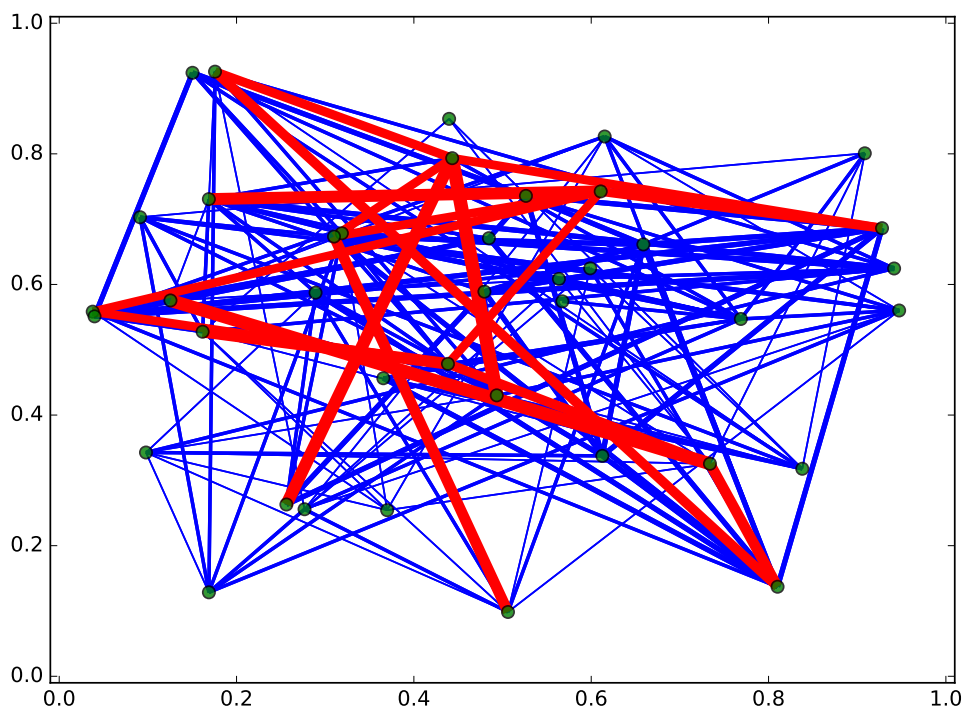


Figure 4.5: Optimization results for Watts-Strogatz model graph with 30 nodes. $\alpha = 2$, $\beta = 5$, $C = 40$.

thousands and thus making it impossible to get a reasonable value. For example, the worldwide air transportation network might include over thousands and millions of flight routes. Therefore, it is necessary to propose a method to get some feasible solutions for large-scale network. We design a clustering based convex relaxation method, the clustering algorithm first hierarchically clusters the airport, and then airports in critical levels are selected as the candidate graph nodes. Finally, convex relaxation method developed in the previous section is used to optimize the graph formed by critical airports.

Measuring Hub Hierarchy

The connectivity between hub airports plays a more important role than other airports. For example, in weather-related delay case, the flights between hubs have more priorities to be alleviated. In contrast, those flights between non-hub might be significantly postponed or even cancelled. The idea in this section is first to identify the hubs associated with the objective network, and then optimize the network only formed by hubs. So the core difficulty lies in how to successfully determine the hubs. Hub identification of air transportation network is not straightforward as there are no consistent definitions of hubs [66]. The simplest idea is to predefine a simple dichotomy of large hubs, medium hubs, small hub, nonhub primary and nonprimary commercial service. This dichotomy is based on the percentage of annual boarding passengers, it only reflects the level of a hub and its hierarchical property, the dichotomy does not take into consideration other properties of air transportation networks, including quality of service during the operation and topology of flight routes.

The metric we utilized is based on the work in [66], where a metric including hub connectivity \mathbb{C}_i , hub service level regarding flight operations per day \mathbb{F}_i , and the volume of the passengers moving per month \mathbb{V}_i , was proposed for hub definition. These three indicators successfully capture the hierarchical structure between the infrastructure, the operating frequency, and the spatial requirements of the hubs.

The connectivity \mathbb{C}_i measures the number of available routes to airport i , it implies the operation strategy of an airline carrier or alliance. The flight frequency per day \mathbb{F}_i is

the number of the daily flights from the airport to other airports, it's a complementary to connectivity \mathbb{C}_i . Two airports with same connectivity and different flight frequency are of the different operational level. Similar to \mathbb{F}_i , passenger volume \mathbb{V}_i defines the enplaned passenger at airport i . These three measures are associated with each airport; then clustering algorithms can be employed on the three measures that define the groupings of the similar hubs and the number of each grouping.

Gaussian Mixture Model

The idea of this section is to divide the airport into κ clusters according to the property related to each airport. The algorithm we employ here is Gaussian mixture model (GMM) that allows the data in 3-n space be divided into several clusters; each cluster can be mathematically represented by a parametric distribution from a model based perspective. Thus the entire dataset is therefore modeled by a linear superposition of this distribution. In practice, the mixture of Gaussian distribution is the most widely used model:

$$p(\mathbf{x}|\Theta) = \sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x}|\mu_k, \Sigma_k) \quad (4.9)$$

where the parameters are $\Theta = (\pi_1, \dots, \pi_K, \mu_1, \dots, \mu_K, \Sigma_1, \dots, \Sigma_K)$ such that $\sum_{k=1}^K \pi_k = 1$ and $\mathcal{N}(\cdot)$ is a Gaussian density function parameterized by its mean and variance (μ_k, Σ_k) . In other words, let's assume the K Gaussian density components are mixed with K mixing coefficients π_k .

Suppose $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_N)$ is a set of observations, and we wish to model the dataset using a mixture of Gaussians, and therefore we need to find Θ such that $p(\mathbf{X}|\Theta)$ is maximal. Assume the data points are sampled independently, and we can use the log likelihood function to represent the Gaussian mixture model of the i.i.d. data set:

$$\mathcal{L}(\Theta) = \ln p(\mathbf{X}|\Theta) = \sum_{n=1}^N \ln \left\{ \sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x}_n | \mu_k, \Sigma_k) \right\} \quad (4.10)$$

The log of the sum makes above formulation be difficult to optimize. To simplify the log likelihood expressions, we introduce a K -dimensional binary random variable \mathbf{z} . The variable is 1-of- K representation with a particular element \mathbf{z}_k is equal to 1 and all other elements are equal to 0. In addition to the observed dataset \mathbf{X} , suppose we are also given the values of the corresponding discrete variable \mathbf{Z} , this likelihood function takes the following form:

$$L(\Theta) = \ln p(\mathbf{X}, \mathbf{Z}|\Theta) = \sum_{n=1}^N \sum_{k=1}^K \mathbf{z}_{nk} \{ \ln \pi_k + \ln N(\mathbf{x}_n | \mu_k, \Sigma_k) \} \quad (4.11)$$

where \mathbf{z}_{nk} denotes the k th component of \mathbf{z}_n .

The likelihood function can be optimized using Expectation-Maximization (EM) algorithm. The updating rule for the parameter $\Theta = (\pi, \mu, \Sigma)$ is

$$\mu_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^N \gamma(z_{nk}) \mathbf{x}_n \quad (4.12)$$

$$\Sigma_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^N \gamma(z_{nk}) (\mathbf{x}_n - \mu_k^{\text{new}})(\mathbf{x}_n - \mu_k^{\text{new}})^T \quad (4.13)$$

$$\pi_k^{\text{new}} = N_k/N \quad (4.14)$$

where $N_k = \sum_{n=1}^N \gamma(z_{nk})$, $\gamma(z_{nk}) = \frac{\pi_k N(\mathbf{x}_n | \mu_k, \Sigma_k)}{\sum_{j=1}^K \pi_j N(\mathbf{x}_n | \mu_j, \Sigma_j)}$

There is a close similarity between the k -means algorithm with the EM algorithm for Gaussian mixture [5], whereas the k -means performs a *hard* assignment of the data points to the cluster, and each data point is assigned to one unique group. The EM algorithm makes use of the posterior probabilities to make a *soft* assignment for each point. The Gaussian mixture model generalizes the k -mean clustering, and includes the covariance structure of the data and the centers of the latent Gaussians, it helps to express the data uncertainty.

GMM Based Convex Relaxation Algorithm

Algorithm 6 GMM Based Convex Relaxation Algorithm

Require: $\mathbf{X} = [\mathbb{C}, \mathbb{F}, \mathbb{V}]_{N \times 3}$

Initialize lowest_bic $\leftarrow +\infty$

for κ in [2, 3, 4, 5] **do**

for cv_type in [*spherical*, *tied*, *diag*, *full*] **do**

 gmm \leftarrow *GaussianMixture*(κ , cv_type)

 gmm.*fit*(\mathbf{X})

if gmm.*bic*(\mathbf{X}) < lowest_bic **then**

 best_gmm \leftarrow gmm

end if

end for

end for

clusters = best_gmm.*predict*(\mathbf{X})

Select airports A from clusters _{i} based on C

Optimize the graph formed by A using **Algorithm** (5)

return R_{tot} , \hat{y} , w_e .

To apply the GMM to successfully determinate the hub hierarchy grouping, we need to choose the number of the cluster κ . Similar to the work in [66], κ is set to be a value within 2 to 5.

The optimal number of clusters is chosen by its Bayesian information criterion (BIC) [68]. BIC is a criterion to select an optimal model from a set of models; the model with the lowest BIC is preferred. Moreover, model selection also concerns with the covariance type of the model. We grid search four types of covariance, including **spherical**, **tied**, **diagonal**, and **full**, and using BIC to decide the optimal one.

To sum up, the full algorithm is listed in Algorithm 2. The best parameters κ and covariance type is selected in the first part of the Algorithm 2. The remaining uses the optimal model to cluster the airports. The results include several clusters, and each includes several airports. Finally, the airports in certain cluster are selected based on the total cost C , the selected airports and routes within them are formed as a candidate graph for optimization.

4.4 Case Study

In this section, two real air transportation networks including Jetstar Asia Airway and domestic American Airline are studied. The first case study demonstrates the feasibility of and efficiency of convex relaxation for a network with node size less than 80; the second case study provides the results of clustering based convex relaxation for large-scale air

transportation network.

Jetstar Asia Airway

Table 4.2: The total 68 routes and their cancellation rates.

<i>Route Name</i>	<i>Cancellation Rate (%)</i>	<i>Route Name</i>	<i>Cancellation Rate (%)</i>	<i>Route Name</i>	<i>Cancellation Rate (%)</i>
ADL-AKL	0	ADL-BNE	0	ADL-CNS	0
ADL-DPS	9	ADL-DRW	0	ADL-MEL	1
ADL-OOL	1	ADL-PER	1	ADL-SYD	1
AKL-CHC	1	AKL-DUD	0	AKL-MEL	1
AKL-OOL	0	AKL-SYD	1	AKL-WLG	0
AKL-ZQN	3	AVV-SYD	0	AYQ-SYD	0
BNE-CNS	0	BNE-DPS	9	BNE-DRW	0
BNE-HBA	0	BNE-HTI	0	BNE-LST	0
BNE-MEL	2	BNE-MKY	1	BNE-NTL	0
BNE-PER	0	BNE-PPP	0	BNE-SYD	2
BNE-TSV	1	BNK-MEL	0	BNK-SYD	0
CHC-MEL	0	CHC-OOL	0	CHC-SYD	0
CHC-WLG	0	CNS-DRW	0	CNS-MEL	0
CNS-OOL	0	CNS-PER	3	CNS-SYD	0
DPS-DRW	6	DPS-MEL	9	DPS-PER	4
DPS-SYD	9	DRW-MEL	0	DRW-SYD	0
HBA-MEL	1	HBA-SYD	0	HTI-MEL	0
HTI-SYD	0	LOP-PER	0	LST-MEL	4
LST-SYD	0	MCY-MEL	0	MCY-SYD	0
MEL-NTL	0	MEL-OOL	1	MEL-PER	1
MEL-SYD	4	MEL-TSV	0	MEL-ZQN	2
NTL-OOL	0	OOL-SYD	1	PER-SYD	0
SYD-TSV	0	SYD-ZQN	2		

According to the current route map of Jetstar Asia Airway, there are 26 airports and 68 routes. These 26 airports include Adelaide (ADL), Auckland (AKL), Avalon (AVV), Ayer

Rock (AYQ), Brisbane (BNE), Ballina (BNK), Christchurch (CHC), Cairns (CNS), Denpasar (DPS), Darwin (DRW), Dunedin (DUD), Hobart (HBA), Hamilton Island (HTI), Praya (LOP), Launceston (LST), Maroochydore (MCY), Melbourne (MEL), Mackay (MKY), Newcastle (NTL), Coolangatta (OOL), Perth (PER), Proserpine (PPP), Sydney (SYD), Townsville (TSV), Wellington (WLG), and Queenstown (ZQN).

The assignment of flight route weights are based on historical data of flight cancellations [27]. The website is a leader in providing flight data services and applications to customers serving the global travel industry. It also provides public data related global airline cancellation and delays, airline on-time performance, and airport performance. Its value rates associated with each flight route are listed in Table 4.2. The cancellation rate ranges from 0% to 9%. A linear interpolation mechanism is designed that maps the failure rate [0%, 9%] to route weight [0, 3]. The design of mechanisms to map cancellation rates to different weights is driven by the actual operation of the airline industry. A high weighted flight route indicates that aircraft or aircraft are less affected by unforeseen accidents, such as aircraft machinery problems and bad weather, etc. The cost coefficient of potential flight route is assumed to be 1 in the case study. The optimization results with two different parameters setting have been shown in Table 4.3. Each selected flight route, represented by <ORIGIN-DESTINATION> pair, is associated an optimized weight satisfying the weights constraint.

Table 4.3: Edge selection and weight assignment for the Jetstar Asia Airway

$\langle C, \alpha, \beta \rangle$	<i>Flight Route</i>	<i>Edge Weight</i>
$\langle 4, 1, 3 \rangle$	DUD-LOP	1.44
	DUD-MKY	1.28
	DUD-PPP	1.28
$\langle 12, 2, 3 \rangle$	DUD-MKY	2.15
	DUD-PPP	2.15
	PPP-WLG	2.00
	MKY-WLG	2.00
	DUD-LOP	3.00
	AVV-MKY	2.17
	AVV-PPP	2.17
	AYQ-PPP	2.17
	AYQ-MKY	2.17

Domestic American Airline

We used the clustering based convex relaxation to optimize the air transportation network of domestic American Airline, which operates 412 airports and 6719 direct flight routes. The data we used was collected in the T100 database of the US transportation plan in the first quarter of 2016. This database contains domestic and international airline and segment data. We get \mathbb{C}_i from the flight route flows, \mathbb{F} from the number of operations on the route, and \mathbb{V} from the number of passengers traveling on each segment of per airline per month.

We employ GMM on the data defined by three metrics to group similar hubs and to determine the number of airport within each tier of hierarchy. There are total 16 different models with the combination of clusters number and covariance type. The optimal number of clusters is determined by finding the minimal BIC score associated with the model. The

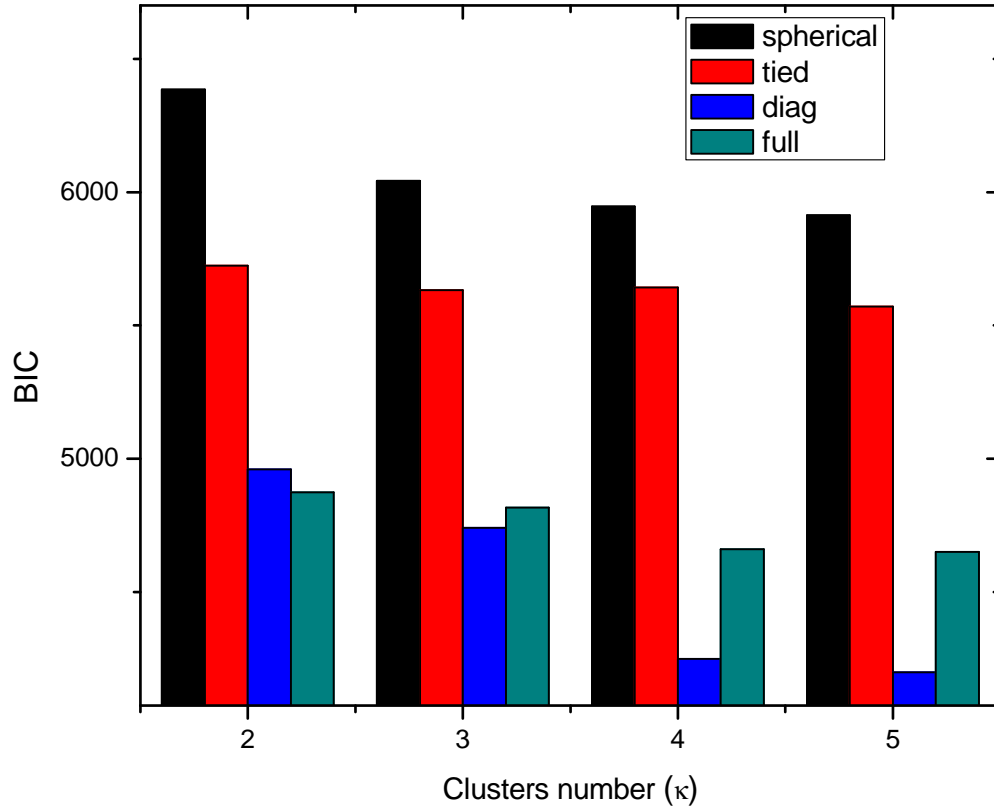


Figure 4.6: BIC score per model

BIC score of each model is illustrated in Figure 4.8. The optimal model in terms of minimal BIC score has parameters $\kappa = 5$ and *diagonal* covariance. We use this optimal model to group the airports into different clusters and then label each group with corresponding tier level. The mean value of each cluster is used to determine the tier level. The larger of the mean value, the higher tier of the cluster belongs to.

The airports are classified into 5 clusters. Table 4.4 shows the first three tier airports. From tier 1 to tier 5, the criticality of airport monotonically decreases. For example, airports in tier 3 have a significantly lower mean of connectivity and flight operations compared to

those in tier 1 and 2. The clustering results are meaningful and explainable. Within all the airports, 2.18% airports are within tier 1, 15.1% airports are within tier 2, and 26.9% are within tier 3.

In this case study, instead of using the whole network, we select the network formed by airports in top two tiers for optimization. The lower bound is $\alpha = 1$, the upper bound $\beta = 5$, and the total budget is $C = 50$. The total running time is 261 seconds on our workstation. The total effective resistance we achieve is $R_{\text{tot}} = 46.7$. The optimal results are shown in Figure 4.7. Blue lines denote the existing routes, red lines are the 22 flight routes which are selected to be added to the current graph. Edges with higher weight are represented by a thicker line. In fact, the network formed by airports in tier 1 is a fully connected network, these 22 routes are mostly added within tier 2 airports or between tier 1 and tier 2 airports. In summary, Figure 4.7 shows that the computation for large-scale network optimization is very efficient. In a real worldwide network, a large κ can be adopted to classify the airport into more clusters, and then same optimization procedures can be applied.

4.5 Summary

This chapter deals with the problem of optimizing the robustness of air transportation network under the budget constraint. The robustness optimization problem is mathematically reformulated into minimizing the total effective resistance under several constraints, namely route budget allocation problem. To demonstrate the difficulty to solve it exactly,

Table 4.4: Clustering result for domestic American Airline, 2016

<i>Tier Type</i>	<i>Airports</i>	$[\bar{C}, \bar{F}, \bar{V}]$
Tier 1	BOS, FLL, LAS, LAX, LGA, MCO, PHX, SAN, SFO	$[87, 2.4 \times 10^4, 1.5 \times 10^5]$
Tier 2	ABQ, ANC, ATL, AUS, AZA, BDL, BHM, BNA, BUF, BWI, CHS, CLE, CLT, CMH, CVG, DAL, DCA, DEN, DFW, DTW, EWR, HNL, HOU, IAD, IAH, IND, JAX, JFK, MCI, MDW, MEM, MIA, MKE, MSP, MSY, OAK, OKC, OMA, ONT, ORD, ORF, PBI, PDX, PHL, PIE, PIT, RDU, RIC, RNO, RSW, SAT, SDF, SEA, SFB, SJC, SJU, SLC, SMF, SNA, STL, TPA, TUS	$[63, 1.1 \times 10^4, 9.6 \times 10^4]$
Tier 3	ABE, AGS, ALB, AMA, ASE, AVL, AVP, BFL, BGR, BIL, BLI, BMI, BOI, BTR, BTV, BUR, BZN, CAE, CAK, CHA, CHO, CID, COS, CRP, CRW, DAB, DAY, DSM, ECP, EGE, ELP, EUG, EVV, EYW, FAR, FAT, FAY, FCA, FNT, FSD, FWA, GEG, GJT, GNV, GPT, GRB, GRK, GRR, GSO, GSP, GTF, HDN, HPN, HRL, HSV, ICT, ILM, ISP, JAC, JAN, JNU, KOA, LBB, LEX, LFT, LGB, LIH, LIT, MAF, MDT, MFE, MFR, MGM, MHT, MLB, MLI, MOB, MSN, MSO, MTJ, MYR, OAJ, OGG, PGD, PHF, PIA, PNS, PSC, PSP, PVD, PWM, RAP, ROA, ROC, SAV, SBA, SBN, SCE, SGF, SHV, SRQ, STT, SWF, SYR, TLH, TRI, TTN, TUL, TYS, VPS, XNA	$[14, 1.6 \times 10^3, 9.5 \times 10^4]$

a small network is first exactly solved using brute-force method, although it's infeasible for large-scale network, it gives us some ideas of the property of the problem. Then a convex relaxation method is proposed to solve the problem with approximate performance and acceptable computational efficiency. This method lays the foundation for the algorithm to solve large-scale networks. We finally propose a clustering based convex relaxation method

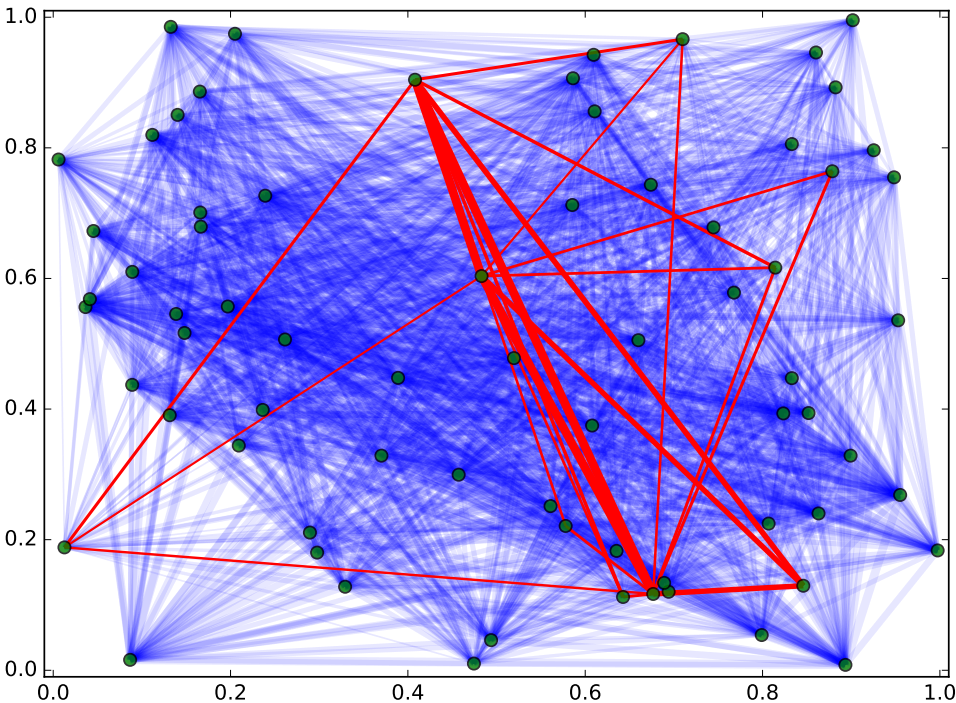


Figure 4.7: Optimization result in the case of $\alpha = 1$, $\beta = 5$, and $C = 50$

for the large-scale network problem. The Gaussian mixture model is used to cluster the airports into several tiers, this will greatly decrease the scale of the network formed by top tier airports, which makes it be possible to get a reasonable solution for the large-scale network. Two real networks including Jetstar Asia Airway and domestic American Airline are used to demonstrate the practical usability of the method developed.

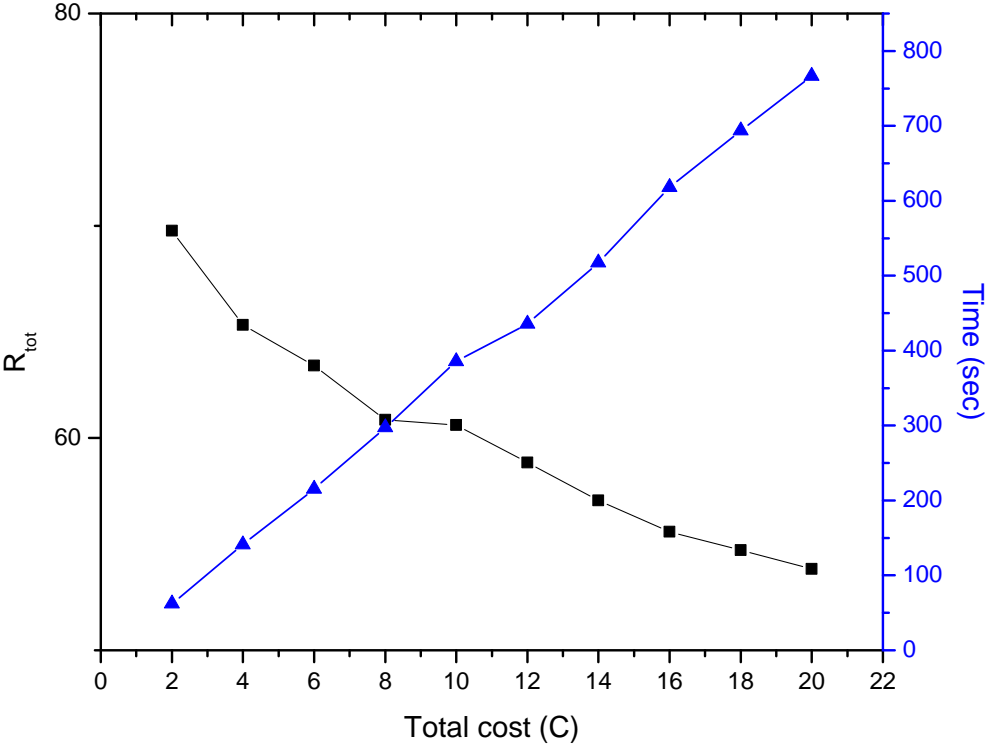


Figure 4.8: Total effective resistance and computation time vs budget

4.6 Appendix

Proof of Theorem 8

Proof. Consider k_1 and k_2 in \mathbf{R} , $\lambda \in [0, 1]$, such that $\Phi(k_i) > 0$ for $i = 1, 2$, we have

$$\Phi(k_\lambda) = \min_{y,w} R_{\text{tot}}(L(y,w)) \text{ s.t. : } \left\{ \begin{array}{l} \sum_e y_e = \lambda k_1 + (1-\lambda)k_2 \\ \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e \in [0,1] \end{array} \right.$$

$$\leq \min_{y,w} R_{\text{tot}}(L(y,w)) \text{ s.t. : } \left\{ \begin{array}{l} \sum_e y_e^{(1)} = k_1, \sum_e y_e^{(2)} = k_2 \\ y_e = \lambda y_e^{(1)} + (1-\lambda)y_e^{(2)} \\ \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e^{(j)} \in [0,1], \quad j = 1,2 \end{array} \right.$$

$$\leq \min_{y,w} R_{\text{tot}}(L(y,w)) \text{ s.t. : } \left\{ \begin{array}{l} \sum_e y_e^{(1)} = k_1, \sum_e y_e^{(2)} = k_2 \\ w_e = \lambda w_e^{(1)} + (1-\lambda)w_e^{(2)} \\ y_e = \lambda y_e^{(1)} + (1-\lambda)y_e^{(2)} \\ \sum_e w_e c_e \leq C \\ \alpha y_e^{(j)} \leq w_e^{(j)} \leq \beta y_e^{(j)} \\ y_e^{(j)} \in [0,1], \quad j = 1,2 \end{array} \right.$$

$$= \min_{y,w} R_{\text{tot}}(L(y,w)) \quad \text{s.t. :} \quad \left\{ \begin{array}{l} \sum_e y_e^{(1)} = k_1, \sum_e y_e^{(2)} = k_2 \\ w_e = \lambda w_e^{(1)} + (1-\lambda)w_e^{(2)} \\ \sum_e w_e c_e \leq C \\ \alpha y_e^{(j)} \leq w_e^{(j)} \leq \beta y_e^{(j)} \\ y_e^{(j)} \in [0,1], \quad j = 1,2 \end{array} \right.$$

$$\leq \lambda \min_{y,w} R_{\text{tot}}(L(y,w)) \quad \text{s.t. :} \quad \left\{ \begin{array}{l} \sum_e y_e = k_1 \\ \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e \in [0,1] \end{array} \right.$$

$$+(1-\lambda) \min_{y,w} R_{\text{tot}}(L(y,w)) \quad \text{s.t. :} \quad \left\{ \begin{array}{l} \sum_e y_e = k_2 \\ \sum_e w_e c_e \leq C \\ \alpha y_e \leq w_e \leq \beta y_e \\ y_e \in [0,1] \end{array} \right.$$

$$= \lambda \Phi(k_1) + (1-\lambda) \Phi(k_2)$$

The third inequality satisfies as $w \rightarrow R_{\text{tot}}(w)$ is convex shown in Lemma (1). The proof procedure shows Φ is convex in k . □

Chapter 5

Conclusion and Future Work

5.1 Conclusion of the Results

In this thesis, we first surveyed and then compared the robustness measures by performing an experiment on several toy graphs and a real air transportation network. The results showed that the total effective resistance was a promising measure to represent the robustness of the air transportation network. The measure captured the local characteristic as well as the global property of the network. We then introduced the graphical description of the air transportation network.

The network optimization problem is demonstrated to be equal to the minimization of the total effective resistance under several constraints. Two effective resistance minimization problems are proposed using different strategies. The first problem is *flight route selection problem*. To solve it, we designed a convex relaxation method with the step by step rounding

technique. We further proved the submodular characteristic of the objective function and proposed a submodular greedy algorithm, that allowed scaling to the network size far beyond the capabilities of the convex relaxation and guaranteed a bounded optimality gap. These two algorithms were tested on three real air transportation networks of small, medium, and large-scales. The case study on the small-scale drone cargo network indicated that the edges tend to connect the low-degree airports and transfer airports when the number of selected edges is small. When the number of selected edge is increased, edges with higher weights slowly begin to connect the airports with a high node degree. For the medium-scale network of SF Airline, the results showed that the edges with medium weight occupied a large proportion, indicating these edges are influential for the robustness improvement. For the large-scale worldwide network, the computation time linearly increases and the network robustness continuously improves as the number of selected edges grows.

The second problem was *route budget allocation optimization*. The problem was concerned with how to select the edges to be added and their edge weights when a fixed operating budget is provided. The problem was first accurately solved by the brute-force method to demonstrate the difficulty in obtaining an optimal solution. To achieve better efficiency, a convex relaxation formulation for medium-scale network was proposed. The inability of convex relaxation for the large-scale problem motivated us to design a clustering based convex relaxation, as it greatly reduces the dimensionality of the problem. Three measures were combined to define the airport hierarchy. Two real air transportation networks were studied. The first case, Jetstar Asia Airway, showed that the edge weight is small when the total

budget given is small. The values of edge weight increase when the total budget provided is large. In the second case, domestic American Airline, five tiers airports were hierarchically clustered, and the optimization results on the top two tier network showed the computation to be very efficient.

Both problems are able to maximize the robustness in an air transportation network for different levels of organization and provide quantitative results for the decision makers. The network planners (route map planners) in airline companies can make use of the results to optimize the network. The aviation authorities, such as FAA in the US and ATMB (Air Traffic Management Bureau) in China, can also use them to evaluate a regional or national air transportation network and then give their suggestions to airline companies. The method is also applicable for edge deletion when the airlines cut down their operating budgets and try to sustain its robustness to the largest degree.

However, the proposed algorithms are also limited in several aspects: 1) To solve the large-scale problem, both problems adopt a decomposition-based method to get the feasible solution. Its performance is not guaranteed. 2) Only the Gaussian mixture model is considered as the clustering algorithm, and we do not compare the performances of different unsupervised algorithms. 3) The budget constraint in the route budget allocation problem is to enforce the operation cost that is linear to the edge weight. Since the edge weight is defined to be the strength of a link, the cost does not have to be proportional to the edge weight. We do not consider the fixed cost and nonlinear cost in this research, which will limit the model to be applied to a more general cost structure. 4) Only the undirected graph is

considered, the developed method is not applicable to the directed graph. 5) The graphical model does not take into account the properties of the airport, for example, the runway configurations, through output of the airport, economic level, and facility level, into the route selection. All of these informations will influence the selection level. 6) The models do not take all the reasonable constraints into consideration, such as the number of connected legs.

5.2 Detailed Contributions

This research takes important steps towards analyzing and optimizing the air transportation network using the total effective resistance. The main contributions of the research are:

1. The research introduces the total effective resistance into the air transportation region for the first time. We systematically compare this measure with other 10 measures by checking two basic criteria on five standard graphs in Section 1.2. The results showed that the total effective resistance satisfies both the global property and local property, and also embodies the weighted aspect of the route. In particular, the total effective resistance is minimal when the network is partitioned and maximal when it is completely connected; it increases when new routes are selected and added to the network. Compared to the state-of-the-art, algebraic connectivity that only includes the network global information, the total effective resistance also reflects the local changes of the network.

2. Compared to the existing literature, we introduce the total effective resistance in the flight route selection problem for the first time in Section 3.1, providing a new idea for the robust design of the aviation network. Since many airlines have established routes to provide services locally or globally, it is not necessary to build a new air transport network from scratch. Therefore, in practice, it is usually possible to improve the robustness by selecting additional routes from a set of potentially available as well as feasible routes and adding them to the existing network. Although similar work is conducted in [85], in which the algebraic connectivity is selected to optimize the robustness of the air transportation network, we have shown in Section 1.2, the total effective resistance is a more promising measure than the algebraic connectivity for an air transportation network. The constraints of the flight route selection problem are same with the ones in [85], the only difference lies in the objective function, the total effective resistance vs algebraic connectivity, so the results in this research will be more reasonable and applicable. Meanwhile, they only solve the small-scale problem and do not consider the large-scale problem, which is significant meaningful in the real-life application. We provide two algorithms to systematically solve the small, medium, and large-scale flight route selection problem.
3. The flight route selection problem has a nonlinear objective and integer constraints, in order to solve such a combinatorial optimization problem, we have designed two kinds of algorithms for different network scales; these two algorithms make a trade-off between

the performance and computation time, (refer to Section 3.2). For small and medium-scale networks, we first prove that the flight route selection problem is a binary convex problem. Although, it can be solved by semi-definite positive programming directly, to improve the solution efficiency, we derive the dual problem of the primal problem. Then, we have the duality gap according to the optimal objective of the primal and dual problem. The duality gap can serve as a terminated condition, and we can design a revised interior-point algorithm for the convex relaxation problem. By comparing three rounding techniques, we finally choose the step-by-step rounding strategy, and thus, we have the interior-point algorithm with the step-by-step rounding technique.

4. To approach a large-scale problem that contains several hundred nodes, (refer to Section 3.3), we further develop a submodular greedy algorithm that provides a promising optimality performance within the practical time limits. The flight route selection problem can be regarded as a set function problem, and hence, we prove the problem satisfies the monotone submodular property. Thus, it enables the greedy algorithm to efficiently obtain a good solution with a guaranteed optimality gap. However, even with the high efficiency of the basic greedy algorithm, it can be still computationally intensive if the marginal function is not efficiently evaluated, especially when facing a large-scale network with a huge route set. The marginal function can be efficiently derived by performing a rank-1 update in the following iterations, which finally enables us to develop an accelerated submodular greedy algorithm.

We further compare the computation time between the basic submodular greedy algorithm and the accelerated version. Both algorithms are applied to a scale-free network, allowing us to conclude that the running time of the accelerated greedy algorithm is substantially better than the basic version. We also compare the performance of the submodular greedy algorithm and the convex relaxation method with the step-by-step rounding technique. It is observed that, although the convex relaxation with the step-by-step rounding technique always provides a relatively better optimality performance, the greedy algorithm achieves a substantial reduction in computation time.

5. We also introduce the total effective resistance into the route budget allocation problem for the first time in Section 4.1. In this problem, each edge is associated with an operating cost that we assume is a linear function of the edge weight. Given the total budget, we need to determine the number of added edges and their corresponding weights. A similar problem is investigated in [86], in which the algebraic connectivity is selected to be the objective function and the same constraints are applied. Similar to the flight route selection problem, the results of the route budget allocation with the total effective resistance will be more reasonable and applicable than with the algebraic connectivity.
6. The algorithms to solve the route budget allocation problem are specifically proposed to tackle with the high computational complexity (please refer to Section 4.2). They are designed with different performance considerations for the small and large-scale

network. To demonstrate the difficulty of obtaining an exact solution, a small network is exactly solved using the brute-force method; although it is unsolvable for the large-scale network, it gives us some insights into the problem complexity and ideas of how to solve the problem. We prove the problem is convex in terms of the number of edges added when the binary variable is relaxed. Then, an approximate algorithm is designed and to take advantage of convexity of related formulation, we introduce an additional variable, i.e. the number of selected edge, into the relaxed formulation. We reformulate it into an SDP formulation that provides a lower bound for the original problem. We solve a knapsack problem to find the maximum number of edges. Then, the best solution of the relaxed problem should be rounded to an integer solution. We compare three techniques and choose the one with the best performance, thus obtaining the algorithm named relaxed SDP with the step-by-step rounding technique. The optimal topology of the network can be found via iterating each k and then solve the corresponding weight assignment problem.

When the number of candidate edges is large, it's impossible to test all the possible cases. Motivated by convexity property of the total effective resistance on k , the one-dimensional search algorithm can be used to speed up the whole search process. As the edges number of k is limited to be an integer value, continuous optimization algorithms cannot be used. The golden-section search method is selected to sequentially reduce the interval range where the extremum exists.

7. We also propose a clustering-based approximation algorithm for the large-scale network in Section 4.3. The clustering algorithm first hierarchically groups the airport, and then airports in critical levels are selected as the candidate graph nodes. Finally, the relaxed SDP with step-by-step rounding techniques is used to obtain the integer solutions.
8. As there are no consistent agreements on the definition of the hub, we used three metrics, namely hub connectivity, hub service level regarding flight operations per day, and the volume of the passengers moving per month to define the hub. Compared with the methods proposed in the literature, this one provides a general framework to classify the airports and takes multiple factors into consideration. This general classification method also improves its exhibility, and different organizations could select the factors that greatly related to the hub to classify the airports.

5.3 Future Work

Probabilistic flight route selection

In this thesis, the two problems we solved are deterministic, which indicates if an edge is chosen then it will be added to the existing network with 100% guarantee. However, in practice, the route to be added might be affected by severe disruption events, and the route selected may not always be added successfully with 100% guarantee. In this scenario, the flight route selection problem could be mathematically expressed in the form of minimizing

the expectation of the total effective resistance. We could also consider the stochastic version of the route budget allocation problem. The adaptive submodularity concept could be used to solve the stochastic optimization problem. As we are able to prove the problem satisfies the conditions of adaptive submodularity, a simple adaptive greedy algorithm is guaranteed to be competitive with the optimal policy.

Incorporating the network functionality

This dissertation only investigates the structural robustness of the air transportation network. Nevertheless, the functional robustness of the network, which refers to its ability to maintain the traffic flow balance on individual nodes in the face of possible disruptions, is also significant to the network optimization. Improving the functional robustness of the air transportation network beforehand could considerably mitigate the impact of potential disruptions. The functional robustness can be included in the objective function or as a constraint into the optimization problem.

Directed graph networks

This thesis investigates the undirected graph networks. However, the study on the directed network might be a practical issue. The result for the directed case will provide more general results because it allows different link strengths and the operating cost for each link direction. Note that the effective resistance is also feasible to reflect the properties of directed

graphs as shown in [92]. In the future work, we plan to relax the undirected assumption and extend it to the cases with directed edges.

Exploring the application of effective resistance

Effective resistance is studied in many areas, e.g., social network, electrical network, and resistor network. How to use the developed methods in this thesis for other research problems also appears to be a promising direction.

5.4 Summary

This chapter began with a summary of the basic problems we have solved, the algorithms proposed to solve the problem, and the current limitations in the research work. Then we summarized our key contributions/findings, which are contextualized with the state-of-the-art measure in the robustness of air transportation network. Finally, we pointed out the potential research directions that might fill in the gaps that we left in this dissertation.

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