

Performance Analysis of Triple Correlated Selection Combining for Cooperative Diversity Systems

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Abstract—In this paper, we analyse the performance of a single-relay cooperative diversity system, which is an effective technique to combat the effects of small scale fading, in a more realistic scenario by assuming a correlation among source-to-relay (SR), relay-to-destination (RD), and source-to-destination (SD) links. A generalised closed form end-to-end symbol error probability (SEP) expression for M -ary phase-shift keying (MPSK) scheme using paired error approach has been derived over correlated Nakagami- m fading channels with decode and forward (DF) protocol being used at the relay node. Moreover, we use similar error approach to analyse the performance of a double correlated cooperative diversity system by assuming the correlation between SR and RD channels. Finally, Monte Carlo simulation has been performed to validate the theoretical results.

Index Terms—Decode and forward (DF) protocol, M -ary phase-shift keying (MPSK), Nakagami- m fading, paired error approach, and symbol error probability (SEP).

I. INTRODUCTION

Cooperative diversity is a promising technique to mitigate the effects of small scale fading and to improve system reliability and throughput in a multipath fading environment. Time, bandwidth, and size limitations preclude the use of time, frequency and spatial diversity systems respectively, compared to cooperative diversity systems. Moreover, the performance improvement of cooperative diversity techniques strictly depend on the protocols such as amplify and forward (AF), decode and forward (DF), etc. being used at the relay node, as well as the combining schemes such as maximal ratio combining (MRC), selection combining (SC), equal gain combining (EGC), etc. being used at the destination node.

In [1]-[2], AF and DF protocols have been analysed with certain implementation constraints. In [3], an end-to-end symbol error probability (SEP) expression for M -ary phase-shift keying (MPSK) scheme using paired error approach is derived by means of modified selection combining technique. All the analysis in [1]-[3] assume source-to-relay (SR), relay-to-destination (RD), and source-to-destination (SD) channels to be statistically independent. But in a more realistic scenario, out of the three channels, either of the two or all the three channels may be correlated. As mentioned in [4], if a relay is moving with certain velocity, there are possibilities that the

Doppler frequencies, multipath arrival angles, and propagation delays of both SR and RD channels may be correlated resulting in the correlation of fading factors. Similarly, if the distance between relay and SD link decreases, as mentioned in [5], SR link will partially overlap SD link leading to a spatial correlation between SR and SD links. Hence correlated fading channel model is said to be a more realistic channel model for cooperative diversity systems compared to independently distributed fading channel model.

In [4], an outage probability analysis, resulting in the degradation of receiver performance due to the correlation between SR and RD channels, with an AF relay has been carried out. But the expressions derived for outage and bit error probabilities are not simplified to give closed form solutions, resulting in computational complexity especially in the higher signal-to-noise ratio (SNR) regime. Meanwhile in [6], an effect of correlation on channel capacity for AF protocol in a single-relay system has been investigated. However, in literature there exists an inadequacy of simplified SEP analysis of cooperative diversity systems incorporating correlated Nakagami- m fading channels. On the other hand, substantial research works have been carried out over correlated fading channels in the context of receiver diversity techniques. In [7], an efficient approach to evaluate multivariate Nakagami- m probability density function (PDF) and cumulative distribution function (CDF) with an arbitrary correlation matrix is presented. Subsequently in [8], an average output SNR for triple correlated selection diversity system over correlated Nakagami- m fading channels derived from [7] is presented. In [9], a hybrid selection/MRC over correlated Nakagami- m fading channels has been analysed by deriving the SEP expression for MPSK scheme. Moreover, asymptotic error rate expressions are derived in [10] for multi-branch EGC and SC over arbitrarily correlated Rayleigh fading channels.

In this paper, an end-to-end closed form SEP analysis of a selection diversity based single-relay cooperative diversity system over correlated Nakagami- m fading channels for MPSK modulation technique is presented. DF protocol is used at the relay node and selection combining scheme is chosen for its simplicity of implementation. Most importantly, a generalised closed form SEP expression has been derived with a correla-

tion among all the links, whose correlation coefficients of the underlying Gaussian process is denoted by $\rho_{1,2}$ (correlation between SD and RD channels), $\rho_{2,3}$ (correlation between SR and RD channels), and $\rho_{1,3}$ (correlation between SD and SR channels). Moreover, the derived SEP expression can also be used for analysing double correlated cooperative diversity systems, where a special case is given in this paper with the correlation between SR and RD channels alone. Finally, Monte Carlo simulations to validate the theoretical analysis are also performed.

The rest of the paper is organised as follows. Section II discusses on the system model for cooperative diversity system considering DF protocol. It also discusses on various assumptions being carried out in the computation and simulation analysis. In section III, the end-to-end closed form SEP expression for MPSK signalling has been derived. Numerical results and inferences are discussed in section IV. Finally, concluding remarks are given in section V.

II. SYSTEM MODEL

Let us consider a three node cooperative diversity system model which comprises source, relay, and destination nodes. The transmission phase in this system can be divided in to two orthogonal time slots and a symbol by symbol transmission method is assumed. In phase 1, the source node will broadcast its complex information-bearing signal (s), which is one of the M constellation points in the signal space ($s \in S_1, \dots, S_M$), to the relay and destination nodes where

$$S_t = \sqrt{2E_s} \exp\left(\frac{-j 2\pi(t-1)}{M}\right), \quad (1)$$

$t = 1, 2, \dots, M$, $j = \sqrt{-1}$, and E_s - Energy of the symbol S_t . In phase 1, the complex baseband signal (s) received at the relay and destination is modelled respectively as

$$r_{sr} = h_{sr}s + n_{sr}, \quad r_{sd} = h_{sd}s + n_{sd}, \quad (2)$$

where h_{sd} and h_{sr} are the fading channel gains of SD and SR links respectively and n_{sd} and n_{sr} are the additive white Gaussian noises of SD and SR links respectively. Moreover, n_{sd} and n_{sr} are modelled as complex Gaussian random variables with zero mean and variance $2N_0$. Since DF protocol is used at the relay node, it will detect the complex baseband signal following the decision rule which is given by

$$\hat{s} = \arg \left\{ \max_{s \in S} \operatorname{Re}(s^* h_{sr}^* r_{sr}) \right\}, \quad (3)$$

where $(.)^*$ denotes the complex conjugate operation.

In the second transmission phase, relay will alone forward the detected signal \hat{s} to the destination node. Now the received complex baseband signal at the destination is modelled as

$$r_{rd} = h_{rd}\hat{s} + n_{rd}, \quad (4)$$

where \hat{s} is the detected MPSK signal at the relay node, h_{rd} is the fading gain of RD link and n_{rd} is the additive white Gaussian noise which is modelled as zero mean complex Gaussian random variable with variance $2N_0$. Moreover, h_{sd} ,

h_{rd} , and h_{sr} can be modelled as $2m$ dimensional column vectors which follow Gaussian distribution with zero mean and $r_1 = |h_{sd}|$, $r_2 = |h_{rd}|$, and $r_3 = |h_{sr}|$ follow Nakagami- m distribution, where $|h_{ij}|$ means norm of h_{ij} . The PDF of Nakagami- m distribution is given by [7]

$$f(r) = \begin{cases} \frac{2 r^{2m-1}}{\Gamma(m) \Omega^m} \exp\left(-\frac{r^2}{\Omega}\right), & r \geq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

where $\Omega = \frac{\mathbf{E}(r^2)}{m} = 2\sigma^2$, σ^2 being the variance of the Gaussian distribution, $\Gamma(\cdot)$ indicates gamma integral function, m represents the fading severity which should satisfy $m \geq 1/2$, and (5) reduces to Rayleigh distribution for $m=1$.

Note that in two orthogonal time frames, two copies of the same message signal are received at the destination over correlated SD, SR, and RD links. Since selection combining scheme is used at the destination node, the decision rule can be framed as

$$\hat{s} = \begin{cases} \arg \{ \max_{s \in S} \operatorname{Re}(s^* h_{rd}^* r_{rd}) \} & \text{if } \gamma_{rd} > \gamma_{sd}, \\ \arg \{ \max_{s \in S} \operatorname{Re}(s^* h_{sd}^* r_{sd}) \} & \text{if } \gamma_{sd} > \gamma_{rd}, \end{cases} \quad (6)$$

where γ_{rd} , γ_{sd} , and γ_{sr} are the instantaneous SNR values of RD, SD, and SR links respectively, which are given by

$$\gamma_{rd} = \frac{E_s}{N_0} |h_{rd}|^2, \quad \gamma_{sd} = \frac{E_s}{N_0} |h_{sd}|^2, \quad \gamma_{sr} = \frac{E_s}{N_0} |h_{sr}|^2, \quad (7)$$

and the corresponding average SNR values of RD, SD, and SR links are denoted as $\Gamma_{rd} = \mathbf{E}[\gamma_{rd}] = \frac{E_s \Omega_{rd} m}{N_0}$, $\Gamma_{sd} = \mathbf{E}[\gamma_{sd}] = \frac{E_s \Omega_{sd} m}{N_0}$, and $\Gamma_{sr} = \mathbf{E}[\gamma_{sr}] = \frac{E_s \Omega_{sr} m}{N_0}$ respectively. $\mathbf{E}[\cdot]$ denotes the expectation operator. Since $|h_{rd}|^2$, $|h_{sd}|^2$, and $|h_{sr}|^2$ are all random in nature, the corresponding instantaneous SNR values (γ_{rd} , γ_{sd} , and γ_{sr}) are also random in nature which follow gamma distribution with mean values equal to their average SNR values (i.e $\Gamma = \Gamma_{sd} = \Gamma_{rd} = \Gamma_{sr} = 2m \frac{E_s}{N_0}$) by assuming σ^2 as unity. We also assume that all the links are identical in nature (i.e $m = m_{sd} = m_{rd} = m_{sr}$).

III. PERFORMANCE ANALYSIS

In this section, we derive the SEP for MPSK signalling over correlated Nakagami- m fading environment. Since we assume all the links are correlated, we need to use trivariate Nakagami- m PDF as follows [8]

$$f_{r_1, r_2, r_3}(R_1, R_2, R_3) = \frac{|W|^m |p_{1,2} p_{2,3}|^{-(m-1)} R_1^m R_2^m R_3^m}{2^{m-1} \Gamma(m)} \times \exp\left[-\frac{1}{2} (p_{1,1} R_1^2 + p_{2,2} R_2^2 + p_{3,3} R_3^2)\right] \times I_{m-1}(|p_{1,2}| R_1 R_2) \times I_{m-1}(|p_{2,3}| R_2 R_3), \quad (8)$$

where $I_v(\cdot)$ represents the v^{th} order modified Bessel function [11, eq.(8.445)], $\Omega = \Omega_{sr} = \Omega_{rd} = \Omega_{sd} = \frac{\mathbf{E}(r_i^2)}{m} = 2$, and W is a tridiagonal matrix which is given by [8]

$$W = C^{-1} = \begin{bmatrix} p_{1,1} & p_{1,2} & 0 \\ p_{1,2} & p_{2,2} & p_{2,3} \\ 0 & p_{2,3} & p_{3,3} \end{bmatrix}. \quad (9)$$

In (9), C denotes a Green's matrix which is given by

$$C = \begin{bmatrix} \rho'_{1,1} & \rho'_{1,2} & \rho'_{1,3} \\ \rho'_{1,2} & \rho'_{2,2} & \rho'_{2,3} \\ \rho'_{1,3} & \rho'_{2,3} & \rho'_{3,3} \end{bmatrix}. \quad (10)$$

(10) is a Green's matrix approximation [7] of an arbitrary correlation matrix of the underlying Gaussian process which is given by

$$\Sigma = \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \rho_{1,3} \\ \rho_{1,2} & \rho_{2,2} & \rho_{2,3} \\ \rho_{1,3} & \rho_{2,3} & \rho_{3,3} \end{bmatrix}. \quad (11)$$

Since all the instantaneous SNR values as given in (7) follow gamma distribution, to derive the SEP expression for MPSK scheme, we need to use trivariate gamma distribution, whose PDF can be derived from the CDF of the correlated SD, RD, and SR links as follows

$$F_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) = F_{r_1, r_2, r_3}(\sqrt{n_1}, \sqrt{n_2}, \sqrt{n_3}), \quad (12)$$

where $n_1 = \frac{N_0 t_1}{E_s}$, $n_2 = \frac{N_0 t_2}{E_s}$, and $n_3 = \frac{N_0 t_3}{E_s}$.

Differentiating (12) and substituting (8), we get the PDF of trivariate gamma distribution as

$$f_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{|W|^m |p_{1,2}|^{2k} |p_{2,3}|^{2l}}{\Gamma(m) k! l!} \times \frac{m^B t_1^{v'-1} t_2^{v-1} t_3^{v''-1} \exp\left(-\frac{m}{\Gamma}(p_{1,1}t_1 + p_{2,2}t_2 + p_{3,3}t_3)\right)}{\Gamma^B \Gamma(v') \Gamma(v'')}, \quad (13)$$

where $B = 3m + 2k + 2l$, $v' = m + k$, $v'' = m + l$, and $v = m + l + k$.

Now the conditional error probability conditioned on γ , which is the general notation for instantaneous SNR of the given link, for MPSK scheme is given by

$$P_e(\gamma) = \frac{1}{\pi} \int_0^\phi \exp\left(-\frac{\gamma \sin^2\left(\frac{\pi}{M}\right)}{\sin^2 \phi}\right) d\phi, \quad (14)$$

where $\phi = \frac{\pi(M-1)}{M}$. The average end-to-end SEP expression can be represented as

$$P_e = P_{eSD} + P_{eSRD}. \quad (15)$$

The average SEP expression for SD link under the condition $\gamma_{sd} > \gamma_{rd}$ is given by

$$P_{eSD} = \int_0^\infty \int_0^{t_1} \int_0^\infty \frac{1}{\pi} \int_0^\phi \exp\left(-\frac{t_1 \sin^2\left(\frac{\pi}{M}\right)}{\sin^2 \phi}\right) d\phi \times f_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) dt_3 dt_2 dt_1. \quad (16)$$

Substituting (13) in (16) and further simplification using [11, eq.(6.455)], the average SEP expression can be written in the form of a single integral with Gauss hypergeometric

function [11, eq.(9.100)]. Now the Gauss hypergeometric function can be expanded and written in the form of an infinite series expression, which can be simplified further using [9, appendix-C] to get the closed form solution as follows

$$P_{eSD} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{|W|^m |p_{1,2}|^{2k} |p_{2,3}|^{2l} \Gamma(A)}{\Gamma(m) k! l! \Gamma(v')(v) (p_{3,3})^{v''} D^A} \times \left(\sum_{H=0}^{\infty} \vartheta'(H) \zeta(F, \theta, C_n) \right), \quad (17)$$

where $A = 2m + 2k + l$, $F = H + A$, $\theta = \frac{\pi}{M}(M-1)$, $D = p_{2,2} + p_{1,1}$, $C_n = \frac{mD}{\Gamma \sin^2(\pi/M)}$, $\vartheta'(H) = \frac{(a)_H (b)_H (p_{2,2})^H}{(c)_H H! D^H}$, $a = 1$, $b = A$, $c = v + 1$,

$$\zeta(F, \theta, C_n) = \left(\frac{\theta}{\pi} + \sum_{i=1}^F (-1)^i \binom{F}{i} \right) \tau_{n,i}(C_n, \theta),$$

$$\tau_{n,i}(C_n, \theta) = \frac{1}{\pi(1+C_n)^{i-\frac{1}{2}}} \sum_{L=0}^{i-1} \binom{i-1}{L} \binom{2L}{L} \times \left(\frac{C_n}{4} \right)^L \left[\tan^{-1}(\sqrt{1+C_n} \tan \theta) + \sqrt{1+C_n} \frac{\tan \theta}{2} \right] \times \sum_{p=1}^L \frac{4^p}{\binom{2p}{p}} \frac{1}{p(1+(1+C_n)\tan^2 \theta)^p}, \quad (18)$$

and $(x)_H$ is defined as follows

$$(x)_H = \begin{cases} 1, & H = 0, \\ x(x+1)(x+2)\dots(x+H-1), & H > 0. \end{cases} \quad (19)$$

We next proceed to derive the average SEP expression for cooperative (SRD) link by assuming $\gamma_{rd} > \gamma_{sd}$. According to paired error approach [3, eq.(9)], the probability of making a correct decision at the destination node is given by

$$P_{cSRD}(\gamma_{sr}, \gamma_{rd}) = (1 - P_e(\gamma_{sr}))(1 - P_e(\gamma_{rd})) + \sum_{\substack{L=1 \\ L \neq n}}^M P_{n,L}(\gamma_{sr}) P_{L,n}(\gamma_{rd}), \quad (20)$$

where P_{cSRD} represents the probability of making a correct decision at the destination node conditioned on SR and RD links, $P_e(\gamma_{sr})$ and $P_e(\gamma_{rd})$ denote the probabilities of making a wrong decision at the relay and destination nodes respectively, $P_{n,L}(\gamma_{sr})$ and $P_{L,n}(\gamma_{rd})$ represent the conditional paired error probabilities conditioned on SR and RD links respectively, where the former represents the probability that the transmitted symbol S_n is being detected as S_L at the relay node (where $L \neq n$), and the latter represents the probability that the transmitted symbol S_L from the relay node is being detected as S_n at the destination node. From [3], the conditional paired error probability is given by

$$P_{n,L}(\gamma) = \frac{1}{2\pi} \int_0^{\pi-\phi_1} \exp\left(-\frac{\gamma \sin^2(\phi_1)}{\sin^2(\phi)}\right) d\phi - \frac{1}{2\pi} \int_0^{\pi-\phi_2} \exp\left(-\frac{\gamma \sin^2(\phi_2)}{\sin^2(\phi)}\right) d\phi, \quad (21)$$

where $\phi_1 = \left(\frac{2\pi(L-n)}{M} - \frac{\pi}{M}\right)$ and $\phi_2 = \left(\frac{2\pi(L-n)}{M} + \frac{\pi}{M}\right)$.

From (20), we can write the conditional SEP at the destination node for SRD link as follows

$$P_{eSRD}(\gamma_{sr}, \gamma_{rd}) = P_e(\gamma_{sr}) + P_e(\gamma_{rd}) - P_e(\gamma_{sr})P_e(\gamma_{rd}) - \sum_{\substack{L=1 \\ L \neq n}}^M P_{n,L}(\gamma_{sr})P_{L,n}(\gamma_{rd}). \quad (22)$$

Now the average SEP of SRD link can be written by averaging (22) over the statistics of trivariate gamma distribution given in (13) as

$$P_{eSRD} = P_{eSRD}(\Gamma_{sr}, \Gamma_{rd}) = P_{e1SRD} + P_{e2SRD} - P_{e3SRD} - P_{e4SRD}, \quad (23)$$

where P_{e1SRD} in (23) is obtained as follows

$$P_{e1SRD} = \int_0^\infty \int_0^\infty \int_0^{t_2} P_e(t_3) f_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) \times dt_1 dt_2 dt_3. \quad (24)$$

Substituting (14) and (13) in (24) and further simplification using [11, eq.(6.455)] and [9, appendix-C], we get the closed form expression as follows

$$P_{e1SRD} = \sum_{k=0}^\infty \sum_{l=0}^\infty \frac{|W|^m |p_{1,2}|^{2k} |p_{2,3}|^{2l} \Gamma(A) m^A}{\Gamma(m) k! l! v! \Gamma^A} \times \left(\frac{{}_2F_1(1, A; v' + 1; \frac{\alpha_1}{\alpha_1 + \beta_1}) \zeta(G, \theta, D_n)}{(\alpha_1 + \beta_1)^A (p_{3,3})^{v''}} \right), \quad (25)$$

where $G = m+l$, $\alpha_1 = \frac{p_{1,1}m}{\Gamma}$, $\beta_1 = \frac{p_{2,2}m}{\Gamma}$, $D_n = \frac{p_{3,3}m}{\Gamma \sin^2(\pi/M)}$.

Now the second term in (23), P_{e2SRD} can be written as

$$P_{e2SRD} = \int_0^\infty \int_0^\infty \int_0^{t_2} P_e(t_2) f_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) \times dt_1 dt_2 dt_3. \quad (26)$$

Substituting (14) and (13) in (26) and further simplification using [11, eq.(6.455)], the average SEP expression can be written in the form of a single integral with Gauss hypergeometric function. As mentioned earlier, the single integral with Gauss hypergeometric function can be simplified using [9, appendix-C] to get the closed form solution as follows

$$P_{e2SRD} = \sum_{k=0}^\infty \sum_{l=0}^\infty \frac{|W|^m |p_{1,2}|^{2k} |p_{2,3}|^{2l} \Gamma(A)}{\Gamma(m) k! l! v! (p_{3,3})^{v''} D^A} \times \sum_{H=0}^\infty \vartheta(H) \zeta(F, \theta, C_n) \quad (27)$$

where $\vartheta(H) = \frac{(a)_H (b)_H (p_{1,1})^H}{(v'+1)_H H! D^H}$.

We now derive the third term P_{e3SRD} in (23) as follows

$$P_{e3SRD} = \int_0^\infty \int_0^\infty \int_0^{t_2} P_e(t_3) P_e(t_2) f_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) \times dt_1 dt_2 dt_3. \quad (28)$$

TABLE I
LIST OF DIFFERENT SUMMATION LIMITS

k	l	H	SEP	Term	SNR (dB)
5	5	2	0.0033	P_{eSD}	15
10	10	5	0.0033	P_{eSD}	15
5	5	-	0.0090	P_{e1SRD}	15
10	10	-	0.0091	P_{e1SRD}	15
15	15	-	0.0091	P_{e1SRD}	15
5	5	2	0.0011	P_{e4SRD}	10
10	10	5	0.0011	P_{e4SRD}	10

Substituting (14) and (13) in (28) and further simplification using [11, eq.(6.455)] and [9, appendix-C], we get the closed form solution as follows

$$P_{e3SRD} = \sum_{k=0}^\infty \sum_{l=0}^\infty \frac{|W|^m |p_{1,2}|^{2k} |p_{2,3}|^{2l} \Gamma(A)}{\Gamma(m) k! l! v! (p_{3,3})^{v''}} \times \left(\frac{\zeta(G, \theta, D_n)}{D^A} \times \sum_{H=0}^\infty \vartheta(H) \zeta(F, \theta, C_n) \right). \quad (29)$$

Finally, the last term in (23) is derived as follows

$$P_{e4SRD} = \int_0^\infty \int_0^\infty \int_0^{t_2} \sum_{\substack{L=1 \\ L \neq n}}^M P_{n,L}(t_3) P_{L,n}(t_2) \times f_{\gamma_{sd}, \gamma_{rd}, \gamma_{sr}}(t_1, t_2, t_3) dt_1 dt_2 dt_3. \quad (30)$$

By substituting (21),(13), and by using [11, eq.(6.455)] and [9, appendix-c], the closed form solution for (30) after further simplification can be written as

$$P_{e4SRD} = \sum_{\substack{L=1 \\ L \neq n}}^M \sum_{k=0}^\infty \sum_{l=0}^\infty \frac{|W|^m |p_{1,2}|^{2k} |p_{2,3}|^{2l} \Gamma(A)}{4 \Gamma(m) k! l! v! (p_{3,3})^{v''} D^A} \times \left[\zeta(G, \theta_1, A_n) - \zeta(G, \theta_2, B_n) \right] \sum_{H=0}^\infty \vartheta(H) \times \left[\zeta(F, \theta_3, A_{n1}) - \zeta(F, \theta_4, B_{n1}) \right], \quad (31)$$

where $\theta_1 = \pi - \phi_1$, $\theta_2 = \pi - \phi_2$, $\theta_3 = \pi - \frac{2\pi(n-L)}{M} + \frac{\pi}{M}$, $\theta_4 = \pi - \frac{2\pi(n-L)}{M} - \frac{\pi}{M}$, $A_n = \frac{p_{3,3}m}{\Gamma \sin^2(\phi_1)}$, $B_n = \frac{p_{3,3}m}{\Gamma \sin^2(\phi_2)}$, $A_{n1} = \frac{Dm}{\Gamma \sin^2(\frac{2\pi(n-L)}{M} - \frac{\pi}{M})}$, and $B_{n1} = \frac{Dm}{\Gamma \sin^2(\frac{2\pi(n-L)}{M} + \frac{\pi}{M})}$.

Now, the closed form average end-to-end SEP expression, which requires very less computation time, for binary phase-shift keying (BPSK) scheme can be obtained from (17),(25),(27), and (29) and is given by

$$P_e = P_{eSD} + P_{e1SRD} + P_{e2SRD} - 2 P_{e3SRD}. \quad (32)$$

Similarly from (17),(25),(27),(29), and (31), we obtain the closed form SEP expression for MPSK scheme and is given by

$$P_e = P_{eSD} + P_{e1SRD} + P_{e2SRD} - P_{e3SRD} - P_{e4SRD}. \quad (33)$$

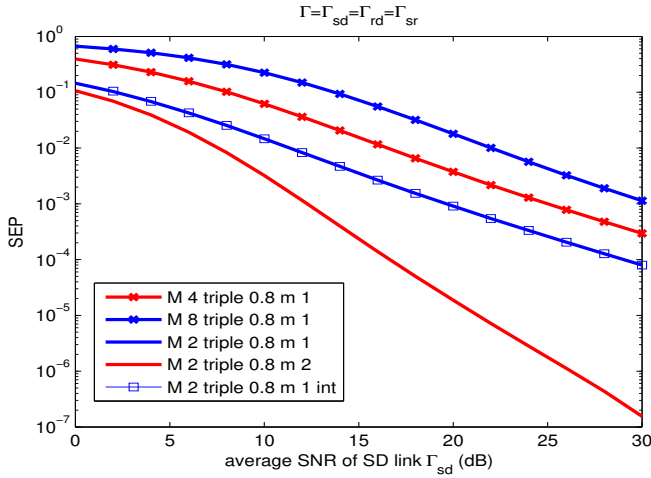


Fig. 1. Performance comparison of triple correlated selection combining technique for various modulation schemes and fading severity conditions over Nakagami- m fading channel (assuming $\rho'_{1,1} = 1.0, \rho'_{2,2} = 1.0, \rho'_{3,3} = 1.0, \rho'_{1,2} = \rho'_{2,1} = 0.8522, \rho'_{1,3} = \rho'_{3,1} = 0.7425, \rho'_{2,3} = \rho'_{3,2} = 0.8712$)

IV. NUMERICAL RESULTS AND DISCUSSIONS

Table I shows the list of various summation limits and its convergence values used in the computation. The summation limits k, l , and H have been truncated to 10, 10, and 5 respectively for the terms P_{eSD}, P_{e2SRD} , and P_{e3SRD} , since adding more than the specified values did not show any improvement in the 4th decimal figure. Moreover, the term P_{e1SRD} has been truncated to $k=10$ and $l=10$ for better convergence. However, the term P_{e4SRD} has been truncated to $k=5, l=5$, and $H=2$, since this term is having very less impact on the end-to-end SEP values.

Fig.1 shows the performance comparison of MPSK scheme for different values of M (i.e $M = 2, 4$, and 8) and m (i.e $m = 1$ and 2) over correlated Nakagami- m fading channels assuming strong correlation between all possible fading statistics. From the plots of SEP vs Average SNR of SD link (with $\Gamma = \Gamma_{sd} = \Gamma_{rd} = \Gamma_{sr}$), it has been inferred that to obtain an SEP of 10^{-3} , 8-PSK and quadrature phase-shift keying (QPSK) signalling require average SNR values of 30 and 24 dB respectively, whereas BPSK system requires about 18 dB to satisfy the same error probability criteria assuming $m=1$. It shows the performance improvement of BPSK system over QPSK and 8PSK systems as expected. Fig.1 also shows the comparison of closed form average end-to-end SEP values and average end-to-end SEP values calculated using single numerical integration for BPSK scheme. It has been inferred that the performance of both the SEP expressions are most likely equal for all the SNR values, which shows the positive impact on computing the closed form SEP expression. The SEP vs Average SNR plots for two different fading severity conditions (i.e for $m=1$ and $m=2$) assuming BPSK signalling are also compared in Fig.1. From the plot, as the effect of fading severity decreases (or as the value of m increases), there is a substantial improvement in the SEP performance as

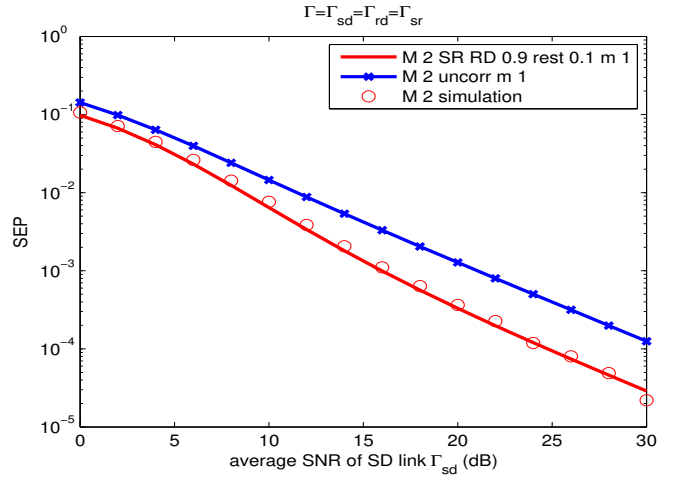


Fig. 2. Performance comparison of double correlated cooperative selection diversity system with uncorrelated system (assuming $m = 1, \rho'_{1,1} = 1.0, \rho'_{2,2} = 1.0, \rho'_{3,3} = 1.0, \rho'_{1,2} = \rho'_{2,1} = 0.1032, \rho'_{1,3} = \rho'_{3,1} = 0.0964, \rho'_{2,3} = \rho'_{3,2} = 0.9345$)

expected.

The performance comparison of double correlated cooperative diversity based BPSK system assuming strong correlation between SR and RD links and negligible correlation values between rest of the links and its uncorrelated counterpart is shown in Fig.2. It has been inferred from the plot that to obtain the SEP of 10^{-3} , SNR required for double correlated cooperative diversity based BPSK system is 4 dB less when compared to uncorrelated system. So the existence of correlation between SR and RD channels shows a positive impact on correlated cooperative diversity system. Moreover, the performance improvement is due to the improved error performance of individual SD, RD, and SR links over correlated Nakagami- m fading channels compared to its performance over uncorrelated Nakagami- m fading statistics. It has also been inferred that the computed SEP values in Fig.2 well agree with the simulated SEP values obtained by Monte Carlo simulation.

Fig.3 shows the comparison of performance analysis of correlated cooperative diversity based BPSK system for different values of correlation coefficients. There is a marginal degradation in the performance improvement of correlated BPSK system compared to uncorrelated system in the lower SNR regime. However, in the higher SNR regime, there is a substantial improvement in the performance of correlated cooperative diversity system. This is because, the SEP of the cooperative (SRD) link is having a major impact on the end-to-end SEP in the higher SNR regime, since SEP performance of SD link is negligible. So the correlation between SR and RD links is playing a significant role in the higher SNR regime leading to the improved error performance. Moreover, irrespective of the correlation existing between the channels, there is always a substantial improvement in the performance of cooperative diversity system compared to non-cooperative system as shown in Fig.3 and the computation of SEP ex-

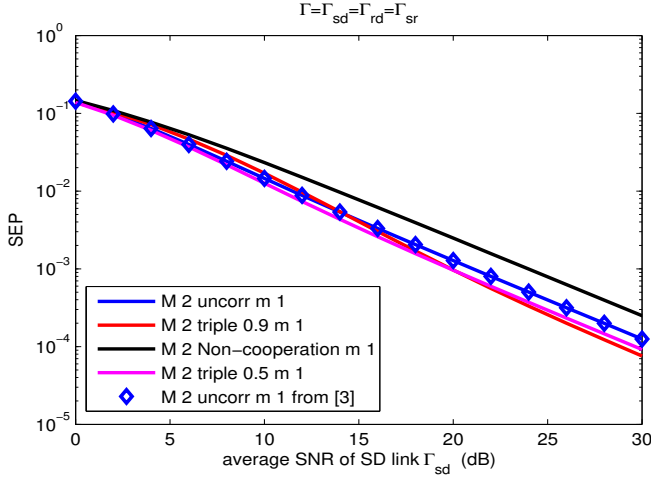


Fig. 3. Performance comparison of triple correlated cooperative diversity system for different correlation coefficients with non-cooperative direct link (assuming $m = 1, \rho'_{1,1} = 1.0, \rho'_{2,2} = 1.0, \rho'_{3,3} = 1.0, \rho'_{1,2} = \rho'_{2,1} = 0.9299, \rho'_{1,3} = \rho'_{3,1} = 0.8689, \rho'_{2,3} = \rho'_{3,2} = 0.9345$)

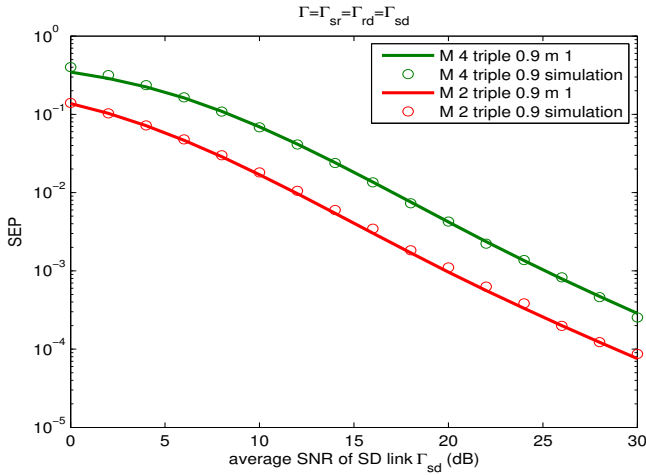


Fig. 4. Comparison of computed and simulated triple correlated selection combining technique for BPSK and QPSK modulation schemes (assuming $m=1$)

pression well agree with the expression given in [3, eq.(18)] for uncorrelated case. Finally, Fig.4 shows that the computed SEP values well agree with the SEP values obtained by Monte Carlo simulations.

V. CONCLUSION

In this paper, a generalised closed form SEP expression for triple correlated cooperative diversity system has been derived for MPSK signalling assuming DF protocol and selection diversity combining scheme being used at the relay and destination nodes respectively over correlated Nakagami- m fading channels. The correlation is assumed to exist among all possible fading links, as well as between SR and RD links. From the results, it has been inferred that double correlated cooperative diversity system in our case outperforms uncorrelated cooperative system due to the existence of correlation

between SR and RD links. On the flip side, when there is an existence of correlation among all the three links, there is a slight decrease in the performance of cooperative diversity system in the lower SNR regime ; however, in the higher SNR regime, there is a substantial improvement in the performance due to the major impact of SRD link on the end-to-end system performance.

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