

# Vibration analysis of membrane by time average in-line digital holographic interferometry

Vijay Raj Singh<sup>a</sup>, Anand Asundi<sup>b</sup>

School of Mechanical and Aerospace Engineering

Nanyang Technological University, Nanyang Avenue, Singapore, 639798

[aVI0001GH@ntu.edu.sg](mailto:VI0001GH@ntu.edu.sg), [b anand.asundi@pmail.ntu.edu.sg](mailto:anand.asundi@pmail.ntu.edu.sg)

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## Abstract

In this paper, in-line digital holography is combined with time average method and applied for vibration analysis of an aluminum membrane. The numerically reconstructed wave from time average holograms contains information of mode shape and mean deformation of the vibrating objects. At lower amplitudes of vibration only the deformation fringes are observed and as the amplitude is increases, the Bessel fringes representing the time average vibration amplitude also become visible. Experimental results for a 10mm size membrane are presented.

## 1. Introduction

Digital holography (DH) [1] is an emergent technique and an excellent tool for optical metrology. The hologram, which is the interference of the object and reference waves, is directly recorded by a CCD camera. The reconstruction process, performed numerically, provides quantitative evaluation of amplitude and phase information of the object wave. Potential application areas include bio-imaging [2], 3-D particle imaging [3], MEMS characterization [4], and interferometry [5]. The in-line DH has the advantage that it makes full and efficient use of the CCD sensor area for hologram recording. Furthermore the higher frequencies of the object wave can be recorded with shorter recording distance, which results in less speckle noise in reconstructed images. The main problem with in-line geometry is the overlapping of the twin image wave and zero-order wave with the reconstructed real image wave. Digital methods can be used for the suppression of these unwanted waves [6, 7].

Time average off-axis digital holography was demonstrated for vibration measurement by Picart et al. [8], who showed that the numerically reconstructed object is modulated by the zero order Bessel function ( $J_0$ ) similar to the conventional time average holography. The advantages of numerical reconstruction of time average holograms were recently explored for vibration measurements [9, 10]. All these methods used relatively large (>30 mm in size) objects because of either off-axis or quasi-Fourier optical configurations. More recently, we have presented time average in-line digital holographic interferometry for vibration analysis [11]. In this paper the time average in-line digital holographic interferometry is used for vibration measurement of a circular aluminium membrane with a diameter of 10mm. It is shown that by recording double exposure holograms, one with time average pattern and other without vibration, both the time average Bessel function and the mean deformation fringes are observed. Also at lower amplitudes of vibration only mean deformation are seen and as the amplitude of vibration increases it mixes with the Bessel types of fringes representing the vibration deformation. Reconstructed amplitude and phase of the real image wave used for measurement of vibration amplitude and mean deformation respectively. Also, for the proposed method the pre-processing of in-line holograms does not required for the suppression of zero-order and twin image waves.

## 2. Theory

The instantaneous displacement of a harmonically excited object, placed in  $(x, y)$  plane with a vibration frequency  $\omega$ , is  $z(x, y, t) = z(x, y) \cos \omega t$ . This object is illuminated by a coherent illumination, the scattered object wave can be written as,

$$O'(x, y, t) = O_0(x, y) \exp(i\phi) \exp\left\{i \frac{4\pi}{\lambda} z(x, y, t)\right\} \quad (1)$$

where  $O_0(x, y)$  is the amplitude of the scattered wave and the phase term  $\phi$  represents the mean state of the vibrating object and  $\lambda$  is the wavelength of illuminating beam.

For time average recording of the object wave, the exposure time should be much larger than the period of vibrations. The time average hologram is recorded by a CCD sensor placed at the plane  $(\xi, \eta)$  and can be written as,

$$H(\xi, \eta) = |O(\xi, \eta) + R(\xi, \eta)|^2 \quad (2)$$

where  $R(\xi, \eta)$  is the in-line reference beam.

In the numerical reconstruction process, the hologram is multiplied by the reference wave and the reconstructed wavefield can be obtained by using the Fresnel diffraction formula [12]. The reconstructed wavefield in the plane  $(x', y')$  at the distance  $d$  from the hologram plane is obtained as follows,

$$U(x', y', d) = \mathfrak{F}\{H(\xi, \eta)R(\xi, \eta)g(\xi, \eta)\} \quad (3)$$

Here  $g(\xi, \eta)$  is the impulse response of the coherent optical system and  $\mathfrak{F}$  represents the Fourier transform. The real image of the object is formed at the same distance as between object and CCD during recording process. The reconstructed object wave can thus be written as

$$U(x', y') = KO_0 \exp(i\phi) J_0\left\{\frac{4\pi}{\lambda} z(x', y')\right\} \quad (4)$$

where  $K$  is a constant which includes the intensity of the reference beam.

The amplitude and phase of the reconstructed real image wave can be extracted as,

$$I(x', y') = |U| = |KO_0| \times \left| J_0\left\{\frac{4\pi}{\lambda} z(x', y')\right\} \right| \quad (5)$$

$$\phi(x', y') = \arctan \frac{\text{Im}(U)}{\text{Re}(U)} \quad (6)$$

Thus amplitude gives the vibration amplitude of the object and phase gives its mean deformation. However, because of the in-line geometry, the out-of-focus twin image and the zero-order term is also simultaneously reconstructed with the real image wave, which degrades the reconstructed image quality. For removal of these terms, two holograms are recorded; one without vibration and other with time average method then their subtraction will suppress the background noise caused by these unwanted waves. The amplitude subtraction will show the vibration amplitude and phase subtraction gives the mean deformation of the object.

### 3. Vibration analysis of an aluminum membrane

The vibrations of a 10-mm thin aluminum membrane excited by frequency generator are studied by using the experimental set-up shown in Fig. 1. A frequency doubled Nd-YAG (532nm) laser beam is split into two parts by a variable beam splitter. One beam is used to illuminate the object (vibrating membrane) and other is the collimated reference beam. The object beam interferes with the plane reference beam and the time average hologram is recorded by an 8 bit digital CCD with array size 1024x1024 and pixel size  $9\ \mu\text{m}$ .

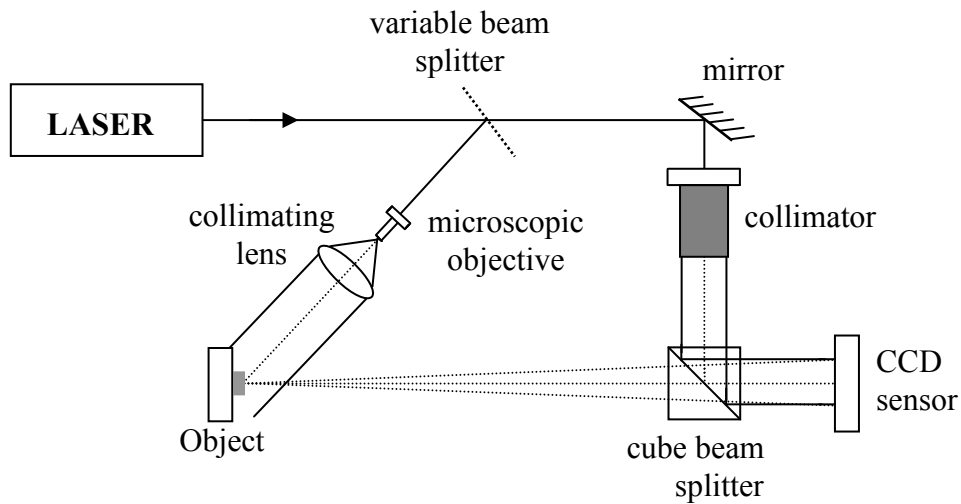


Figure 1. Experimental set-up of time average in-line digital holography

One hologram is recorded without vibration and subtracted with time average holograms recorded at different frequencies and amplitudes. Figures 2 (a), (b) and (c) shows the subtraction of phases of the three holograms recorded at frequency 7.5 KHz with increasing amplitudes (excitation voltages are 1, 2 and 3 volts). The mean deformation fringes are clearly observed which shows the change in the mean position of the diaphragm during the vibration cycle at corresponding amplitudes.

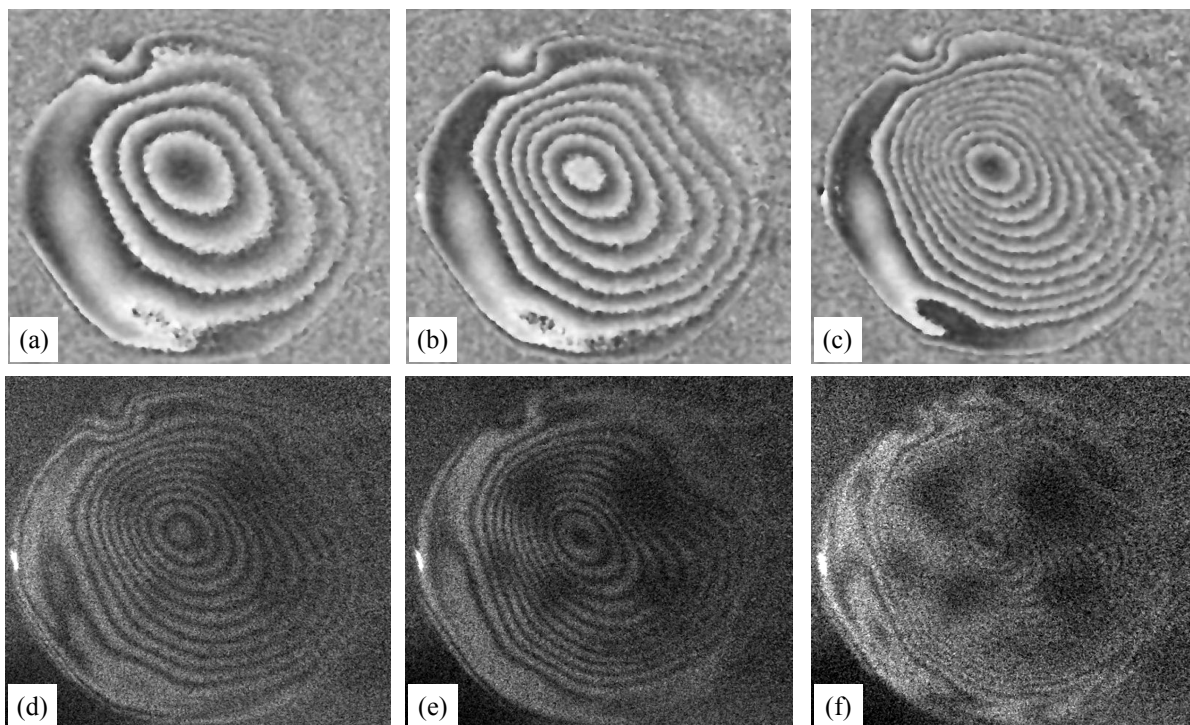


Figure 2 (a)-(c) Mean deformation fringes, (d)-(f) Combine time average and deformation fringes

As the amplitudes of vibration increases, the time average fringes also becomes visible, this can be shown by the wavefield subtraction as shown in Fig. 2(d), (e) and (f). To separate the vibration fringes from the deformation fringe, the amplitudes as defined by eqn. (5) of the holograms are first calculated and then subtracted. Figure 3(a) shows the subtraction of wavefields reconstructed by time average hologram recorded at a resonant vibration frequency of 8.5 KHz and excitation voltage 5V with the hologram recorded without vibration. The mixing of fringes as discussed above is observed. The time average vibration fringes can be obtained by amplitude subtraction as shown in Fig. 3(b), the same pattern can also be obtained by reconstruction of amplitude of a single time average hologram, but for reconstruction of single in-line hologram the pre-processing is required to suppress the background noise caused by twin image and zero-order waves. In the present case no such pre-processing is required. The corresponding deformation fringes can be obtained by phase subtraction of the reconstructed wavefields as shown in Fig. 3(c). The vibration amplitude can be calculated using eqn(5) by putting first, second... zeros of Bessel fringes equal to 2.4048, 5.5201... respectively. For the mean static deformation fringes, the sensitivity is  $\lambda/2$  as for any double exposure interferometry.

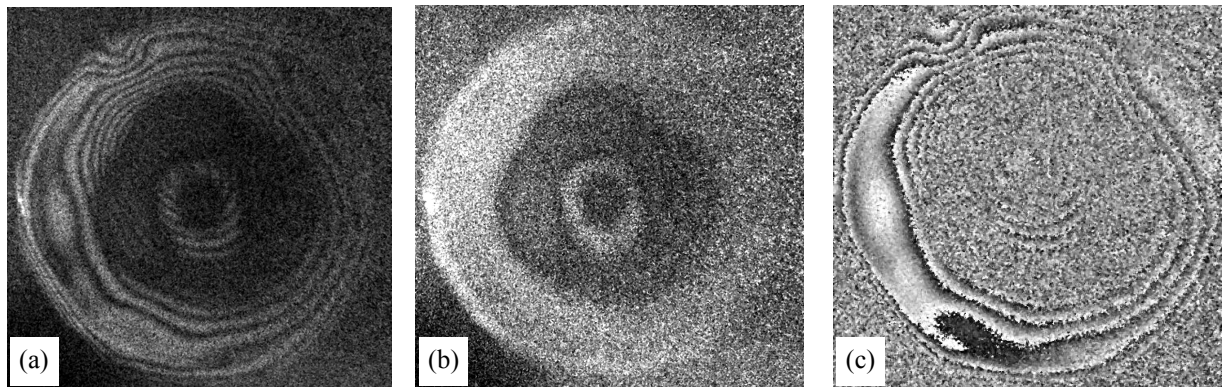


Figure 3 (a) Wavefield subtraction, (b) Amplitude Subtraction shows the vibration fringes. and (c) Phase subtraction shows the mean deformation fringes

#### 4. Conclusion

Time average in-line digital holography is presented for vibration measurement of membranes. For the suppression of background noise because of in-line geometry, two holograms are recorded, one without vibration and other with vibration and then subtracted. It is shown that at the lower amplitudes of vibration only the mean deformation fringes are observed and as the amplitude of vibration increases the time average fringes also becomes visible and mixes with the deformation fringes. The numerical reconstructions of amplitudes and phase are used to get the time average and deformation fringes respectively.

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