Adaptive Backstepping Control of Practical Uncertain Systems

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Summary

This thesis develops the powerful and popular adaptive feedback control technology called "backstepping", with applications to dynamic uncertain systems with nonsmooth nonlinearities, such as backlash, dead-zone, hysteresis and saturation, or time-varying parameters, or with multi-inputs and multi-outputs.

In this thesis, it has been presented research results including theoretical success and practical development, such as the proof of stability and the improvement of system tracking and transient performance. With "backstepping" methodology the construction of both feedback control laws and associated Lyapunov functions is systematic. The stability and system tracking and transient performance can be obtained.

The objective of this thesis includes the following aspects:

• This work aims at designing, analyzing and implementing adaptive backstepping controls which are able to accommodate uncertain nonlinearities in industrial control systems. We will consider four types of nonsmooth nonlinear characteristics, such as backlash, dead-zone, hysteresis and saturation. Such nonlinearities often limit or deteriorate both static and dynamic performance of systems. For each nonsmooth nonlinearity we will introduce a new adaptive control scheme to compensate the effect of such nonlinearity in control systems and achieve the best possible control performance. In this thesis it will be shown how these four nonsmooth nonlinear characteristics can be adaptively compensated and how desired system performance is achieved.In particular, we are interested in designing adaptive controllers which ensure the overall control system stability. A further target is then to precisely characterize the corresponding performance and to study a suitable selection of parameters to the adaptive control scheme so as to obtain both stability and optimality results.

• Usually parameters of practical systems are changing with time. It becomes even more difficult to deal with when these time-varying parameters are unknown and there exists unknown disturbance. In this thesis, we devise a new strategy for a class of single-input single-output uncertain time-varying nonlinear systems with unknown sign of high-frequency gains in the presence of disturbances. The target is that the proposed controller can guarantee all signals bounded and obtain good transient and tracking performance.

• Most practical systems are also multi-input multi-output (MIMO) systems. For such systems, the control problem is very complicated due to the coupling among various inputs and outputs. It becomes even more difficult to deal with when there exist unknown parameters in the input or output coupling matrix. In this thesis, we design a new control law for MIMO nonlinear systems. The target is that the proposed controllers can guarantee all signals bounded. In the control of a large scale system, one usually faces poor knowledge on the interactions between subsystems. If some subsystems distribute distantly, it is difficult for a centralized controller to gather feedback signals from these subsystems. Also the design and implementation of the centralized controller are complicated. Therefore a new decentralized controller, designed independently for local subsystems and using local available signals for feedback, is proposed to overcome such problems.

Chapter 1

Introduction

Control theory attempts to improve the behavior or performance of physical systems by gathering and exploiting knowledge about the systems operation. Usually this knowledge is encoded as a descriptive mathematical model of the physical plant from which the controller design is derived. Given a mathematical representation, we are interested in designing a controller to achieve some objectives such as stability (convergence of state/output to some equilibria) or output tracking of some reference signal. Typically for robustness reasons, the controller is realized as a dynamic feedback of measured variables. From a system dynamics point of view, control theory can be divided into two main categories, linear and nonlinear systems, the dynamic features of which can be either depended on time or time invariant. In this dissertation we focus on dynamic systems (linear or nonlinear) with unknown non-smooth nonlinearities, or time-varying parameters, or with multi-inputs and multi-outputs.

1.1 Motivation

Adaptive control has been the important area of active research for over five decades now. It has seen significant development including theoretical success and practical development, such as the proof of stability and the improvement of system tracking and transient performance. One of the reasons for the rapid growth of adaptive control is its ability to control plants with uncertainties during its operation. Adaptive control is a technique of applying some method to obtain a model of the process and using this model to design a controller. An adaptive controller is formed by combining an parameter estimator, which provides estimates of unknown parameters with a control law. The parameters of the controller are adjusted during the operation of the plant. In order to obtain desired performance, it also provides adaptation methods to dealt with some uncertainties, such as flow and speed variations, external disturbance and structural uncertainties. One important approach in adaptive control is certainty equivalence based design. Such an approach has been studied extensively and a number of results have been established [1, 2, 3, 4, 5, 6, 7]. Certain schemes have also been proposed to study the robustness issues in the context of both single loop control [8, 9, 10, 11, 12, 13, 14, 15] and decentralized control of multi-loop systems [16, 17, 18, 19, 20, 21, 22, 23, 24]. Problems related to nonsmooth nonlinearities have also been well addressed in [25, 26, 27, 28]. However, transient performance is difficult to be ensured with this approach.

In the beginning of 1990s, a new approach called "backstepping" was proposed for the design of adaptive controllers. Backstepping is a recursive Lyapunov-based scheme for the class of "strict feedback" systems. In fact, when the controlled plant belongs to the class of systems transformable into the parametric-strict feedback form, this approach guarantees global regulation and tracking properties. A very appealing aspect of the backstepping design method is that it provides a systematic procedure to design stabilizing controllers, following a step-by-step algorithm. With this methodology the construction of both feedback control laws and associated Lyapunov functions is systematic. A major advantage of backstepping is that it has the flexibility to avoid cancellations of useful nonlinearities and pursue the objectives of stabilization and tracking, rather than that of linearization. Along with their advantages, they have certain drawbacks. One is that they do not offer freedom of choice of parameter update laws. For systems with many unknown parameters a drawback of adaptive backstepping is that the dynamic order of its overparametrized controller is high. On the other hand, the order of the tuning functions controller is minimal, but for high-order systems its nonlinear expressions become increasing complex. Backstepping method may be more complicated than traditional methods because of the problem of "explosion of terms" in the recursive design. A number of results using this approach has been obtained [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. However, some important practical problems such as the handling of nonsmooth nonlinearity have not been addressed using this approach.

Nonsmooth nonlinearities such as dead-zone [41, 42], backlash [27, 43], hysteresis [44, 45] and saturation [46, 47] are common in industrial control systems. Backlash, a dynamic characteristic, exists in mechanical systems, such as in hydraulic actuators. Dead-zone, a static input-output characteristic, often appears in motors, valves and biomedical actuation systems. Hysteresis, another dynamic characteristic, exists in a wide range of physical systems and devices. Saturation is always a potential problem for actuators of control systems do saturate at some level. Actuator saturation affects the transient performance and even leads to system instability.

Such nonlinearities are usually poorly known and may vary with time, and they often limit system performance. Control of systems with nonsmooth nonlinearities is an important area of control system research. A desirable control design approach for such systems should be able to accommodate system uncertainties. The need for effective control methods to deal with nonsmooth systems has motivated growing research activities in adaptive control of systems with such common practical nonsmooth nonlinearities [48, 49]. Various design methods based on different control objectives and system conditions have been developed and verified in theory and practice. Adaptive control schemes have been used to cope with actuator dead zone [50, 51, 52, 53, 54], backlash [27, 44, 55], hysteresis [45, 56, 57, 58] and saturation [46, 59, 60, 61, 62, 63]. Other schemes to handle such nonlinearities have included neural networks control in [42, 64, 65, 66, 67, 68], fuzzy logic control in [41, 69, 70, 71, 72], variable structure control in [43, 52, 73, 74, 75, 76, 77, 78], pole placement control in [46, 47, 60] and recursive least square algorithm in [79].

From the above discussion, it follows that the presence of nonsmooth nonlinearities in actuators should be accounted for. Hence, a noticeable uncertainty should be taken into account in nonlinearities models parameters, in order to match a sufficiently wide set of real situations. In this thesis, we will develop adaptive backstepping control schemes for general classes of systems to get both perfect tracking and transient performance.

In this thesis, the major methodology is backstepping. Adaptive control methods based on backstepping technique incorporated with other methodologies, such as inverse technique, are proposed to handle uncertain dynimic systems containing backlash, hysteresis, dead zone or saturation in the actuator. Although backlash, dead-zone, hysteresis and saturation characteristics are different, they are all nonsmooth in nature. Therefore, the existing backstepping adaptive control methods may not be applicable. Thus backstepping adaptive control of dynamic systems with each of these nonsmooth characteristics is a control problem that needs a systematic consideration. In this thesis it will be shown how nonsmooth nonlinear industrial characteristics can adaptively compensated and how desired system performance is achieved in the presence of such nonlinearities. The controller designed by using backstepping technique consists of new robust control laws and new estimators to estimate the unknown parameters. Besides showing global stability of the system, the transient performance of the tracking error is improved and derived to be an explicit function of design parameters. In our approaches we have chosen linear or nonlinear model with a sufficient number of adjustable parameters which provide significant flexibility in matching real situations. This flexibility will be exploited for our backstepping control schemes with such nonsmooth nonlinearities.

Most practical systems are multi-input multi-output (MIMO) systems. And usually parameters of practical systems are changing with time. For such systems, the control problem is very complicated due to the coupling among various inputs and outputs. It becomes even more difficult to deal with when there exist unknown parameters in the input or output coupling matrix. In this thesis, we will address a class of nonlinear MIMO systems with unknown disturbance. In the control of a large scale system, one usually faces poor knowledge on the plant parameters and interactions between subsystems. If some subsystems distribute distantly, it is difficult for a centralized controller to gather feedback signals from these subsystems. Also the design and implementation of the centralized controller are complicated. Therefore decentralized controllers, designed independently for local subsystems and using local available signals for feedback, are proposed to overcome such problems. The resulting decentralized controllers are also reliable in the sense that when some local controllers are out of order, the rest can still be in operation. Such decentralized controllers should be robust against the ignored interactions. Due to difficulties to consider the effects of interconnections, extension of single loop results to multi-loop interconnecting systems is challenging, which is why the number of available results is still limited. In this thesis, we develop new output feedback decentralized stabilizers for a class of interconnected systems with subsystem having arbitrary relative degrees.

1.2 Objectives

Imperfections of system components, especially those of actuators and sensors, are among the factors that severely limit the performance of feedback control loops, the vital parts of industrial automation, consumer electronics, and defense and transportation systems. Most often, a critical imperfection is a nonlinearity, which is poorly known, increases with wear and tear, and varies from component to component.

It is appealing to think of more intelligent approaches to increase the accuracy achievable with imperfect, sturdy and inexpensive, components. Can the control system, after a period of learning or adaptation, recognize the imperfection and compensate for its harmful effects? With such adaptive controllers, the component specifications could be greatly relaxed, their cost reduced, and their reliability increased.

The main objectives of this thesis are as follows:

• The main body of thesis is pointed to a direction in which the best control performance can be achieved for four most common component imperfections:
(1) dead-zone
(2) backlash
(3) hysteresis
(4) saturation.
These nonsmooth nonlinearities have different characteristics, so we will analyze each of them individually and take them into account in each of controller designs for different practical control systems. We aim at designing, analyzing and implementing adaptive backstepping control which are able to accommodate such uncertain nonsmooth nonlinearities in industrial control systems by introducing new adaptive control schemes to overcome or compensate the effect of these nonlinearities in control system. In particular, we are interested in designing adaptive controllers which ensure the overall control system stability. A further target is then to precisely characterize the corresponding performance and to study a suitable selection of parameters to the adaptive control scheme so as to obtain both stability and optimality results.

• Another feature of practical system is that their parameters change with time. This thesis also points to design new strategy for a class of single-input single-output uncertain time-varying nonlinear systems with unknown sign of high frequency gains in the presence of disturbances. The developed controller can deal with time variation and external disturbance by using backstepping and tuning functions. The objective of this is to make system BIBO stable. We develop a new backstepping control scheme for a class of multi-input multi-output nonlinear systems with respect to parameter uncertainty. The main target is that the proposed controller can guarantee system boundedness. We also address an output feedback control problem: decentralized adaptive stabilization of a class of interconnected subsystems with subsystem having arbitrary relative degrees and with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity.

1.3 Major Contribution of the Thesis

The research work done during my Ph.D studies using backstepping methodology has resulted in several basic and theoretical results. A few new control schemes have been developed for systems with nonsmooth nonlinearities (dead-zone, backlash, hysteresis and saturation). We have developed new backstepping control laws that achieve the desired objectives either by interacting with parameter update laws or by attenuating the effect of parameters estimation errors. In this way we achieve not only stronger stability properties, but also quantifiable improvements of transient performance.
The main contributions of this thesis are summarized as follows:

(1) An adaptive backstepping control algorithm is proposed for a class of uncertain dynamic nonlinear systems preceded by unknown backlash-like nonlinearity, where the backlash is modelled by a differential equation, in the presence of bounded external disturbances. By using backstepping technique, two robust adaptive control schemes are developed. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, an alternative smooth control law is proposed and the tracking error is still ensured to approach a prescribed bound in this case. The developed controllers do not require the uncertain parameters within known intervals. Also no knowledge is assumed on the bound of the 'disturbance-like' term, a combination of the external disturbance and a term separated from the backlash model. It is shown that the proposed controllers not only can guarantee global stability, but also transient performance.

(2) An output feedback control problem is considered in the control of a class of uncertain linear systems preceded by unknown backlash-like nonlinearities to achieve tracking. The controller designed consists of a new robust control law and a new estimator to estimate the unknown parameters. It is shown that all the

signals are bounded.

(3) The control of a class of uncertain dynamic nonlinear systems with unknown dead-zone nonlinearities, in the presence of bounded external disturbances, is also studied. By using backstepping technique, robust adaptive control schemes are developed. In the application of controllers, a new adaptive inverse is used to cancel the effects of the unknown dead-zone nonlinearity to avoid possible chattering caused by the sign function and the nonsmooth inverse that exists in present schemes. The tracking error is ensured to approach a prescribed bound.

(4) The development of adaptive control schemes for systems with input saturation has been a task of major practical interest as well as theoretical significance. We address the adaptive backstepping tracking of a class of nonlinear uncertain stable systems in the presence of input saturation. To compensate the effect of the saturation, we construct a new system with the same order as that of the plant in the backtsepping control design. With the error between the control input and saturated input as the input of the constructed system, a number of signals are generated to compensate the effect of saturation. It is shown that the proposed controllers can guarantee global stability.

(5) Two adaptive backstepping control algorithms are proposed for a second-order uncertain hysteretic structural system found in base isolation scheme for seismic active protection of building structures. This system exhibits a hysteretic nonlinear behavior, which is described by the so-called Bouc-Wen model. Numerical results show that the adaptive control law is working satisfactorily in the sense that the response induced by seismic action is significant reduced.

(6) A new scheme is designed for single-input single-output uncertain time-varying systems in the presence of unknown bounded disturbances. No knowledge is assumed on the sign of the term multiplying the control. The control design is

achieved by introducing certain well defined functions, estimating variation rates of parameters and incorporating a Nussbaum gain. To overcome the problem of overparametrization, tuning functions, which are different from the standard ones due to the use of projection operations, are employed. It is shown that the proposed controller can guarantee global uniform ultimate boundedness.

(7) A new scheme is proposed to design an adaptive output feedback controller for a class of multiple-input multiple-output systems in the presence of unknown disturbances. In order to reject disturbances generated from an unknown exosystems, new filters for state estimation are constructed and an adaptive internal model is employed. The control design is achieved by using backstepping, tuning functions, and SDU factorization and estimating parameters. It is shown that the proposed controller can ensure all the signals in the closed-loop system globally uniformly bounded and the tracking error to converge to zero.

(8) A new scheme is proposed to address an output feedback control problem: decentralized adaptive stabilization of a class of interconnected subsystems with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity, where the hysteresis is modelled by a differential equation. Each local controller is designed by using backstepping technique and consists of a new robust control law and a new estimator to estimate the unknown parameters. For the implementation of the controller, no knowledge is assumed on the bounds of unknown system parameters and the effect contributed by the hysteresis. There is no structure requirement on the model of each subsystem such as an upper triangular form, since a general transfer function is considered. Also the interactions between subsystems are allowed to satisfy a nonlinear bound. It is shown that all the signals are bounded. A root mean square type of bound is obtained for the system states as a function of design parameters. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters.

1.4 Organization of the Thesis

This thesis is divided into ten chapters in which chapters 3 - 9 contain the contributions. Chapters 3 - 6 follow similar structures as they seek similar goals: compensating or overcoming the effect of backlash, dead-zone, hysteresis or saturation. Every chapter starts with some well-known concepts required for the proof of main results.
In Chapter 2, we start with a brief review of adaptive control and introduce adaptive backstepping tools illustrated by a simple example. Then we present basic description of nonsmooth nonlinear characteristics: dead-zone, backlash, hysteresis and saturation and briefly describe some typical examples to illustrate them.

In Chapter 3, we address adaptive control of uncertain systems preceded by unknown backlash nonlinearity in either state feedback or output feedback control. Detailed design and analysis of backstepping algorithms in either state feedback control of nonlinear system or output feedback control of linear system are given, including structure, stability, and convergence of the algorithms. Two simple examples are illustrated to verify the effectiveness of our proposed schemes.

In Chapter 4, the schemes for state feedback control will be extended to state feedback control of nonlinear systems with unknown dead-zone. The main result of this chapter is to develop a new adaptive scheme for a class of uncertain dynamic nonlinear systems with unknown dead-zone nonlinearities, in the presence of bounded external disturbances. We will give the detailed design procedure and stability analysis. This chapter shows that control systems with backstepping controllers can be combined with a smooth inverse to handle unknown dead-zone.

In Chapter 5, we present a new scheme to design adaptive controllers for uncertain systems in the presence of input saturation. By using backstepping technique, a new robust adaptive control algorithm is developed. Besides showing stability, tracking error is also established.

In Chapter 6, we address a second-order uncertain structural system found in base isolation schemes for seismic active protection of building structures. This system exhibits a hysteretic nonlinear behavior, which is described by the so-called Bouc-Wen model. Numerical results show that the adaptive control law is working satisfactorily in the sense that the response induced by seismic action is significant reduced.

In Chapter 7, an adaptive output feedback controller is designed for single-input single-output uncertain time-varying systems in the presence of unknown bounded disturbances. The simulation results show that the controller obtained by the proposed design scheme can make the whole adaptive control system stable.

In Chapter 8, we design an adaptive output feedback controller for a class of multiple-input multiple-output systems in the presence of unknown disturbances. In order to reject external disturbances, new filters for state estimation are constructed and an adaptive internal model is employed. The control design is achieved by using backstepping, tuning functions, SDU factorization and estimation of parameters.

In Chapter 9, we provides a solution to the problem on the relaxation of subsystem relative degrees in direct decentralized system with each loop preceded by unknown backlash-like hysteresis nonlinearities by using backstepping technique. It is shown that adaptive control system is global stable in the sense that all the signals are bounded.

The conclusion and recommendation for future work are described in Chapter 10.

Chapter 2

Design Tools and Review of Nonsmooth Nonlinearities

Recursive design in this thesis is composed of simple basic steps. They are referred to as "backstepping designs" because they step back toward the control input starting with a scalar equation. After a brief review of adaptive control, this chapter introduces basic backstepping tools for systems with uncertainties and basic description of nonsmooth nonlinear characteristics, such as dead-zone [41, 42], backlash [27, 43], hysteresis [44, 45] and saturation [46, 47]. These nonsmooth nonlinear characteristics are illustrated by a few examples in this chapter.

2.1 Review

Adaptive control has a long and rich history. Its idea was conceived in the 1950s. In the fifty years of its existence, adaptive control theory has grown into the mainstream of research activity. Now adaptive control theory provides several solutions to this fundamental problem [80, 81, 82, 83, 84]. Each of these solutions is a breakthrough in the development of adaptive control. One of the reasons for the rapid growth of adaptive control is its clearly goal: to control plants with unknown and time-varying parameters and nonlinearities.

Adaptive has seen significant development including theoretical success and practical development, such as the proof of stability and the improvement of system tracking and transient performance. It provides adaptation mechanisms that adjust a controller for a system with parametric, structural, and uncertainties to achieve desired performance. Such uncertainties often appear in airplane and automobile engines, electronic devices, and industrial processes. Typical adaptive control applications reported in the literatures include temperature control, chemical reactor control, pulp dyer control, rolling mill control, automobile control, ship steering control, blood pressure control, artificial heart control, robot control, physiological control and biological control, see for examples [85, 86, 87, 88, 89].

An important feature of traditional adaptive control is its reliance on "certainty equivalence" controllers. This means that a controller is first designed as if all the plant parameters are known. The controller parameters are determined as functions of the plant parameters. Given the true values of the plant parameters, the controller parameters are calculated by solving design equations. When the true plant parameters are unknown, the controller parameters are either estimated directly or computed by solving the same design equations with plant parameter estimates.

Unlike other controllers using PID, pole placement, optimal control methods or variable structure control, whose designs are based on certain knowledge of the system parameters, adaptive controllers do not need such knowledge; they are adapted to parameter uncertainties by using performance error information online. It has been shown that adaptive control outperforms robust control when the actual uncertainty level is sufficiently high and the a-priori known uncertainty level is sufficiently conservative. On the other hand, adaptive controller are much more complex than traditional controllers, such as PID. Thus, the focus of this thesis is adaptive control.

Nonsmooth nonlinearities are among the key factors limiting both static and dynamic performance of feedback control systems. As a matter of fact, these nonlinearities are particularly harmful. Because they usually lead to a relevant deterioration of system performance. These nonsmooth nonlinear characteristics are often neglected in control system design, though they can remarkably affect systems performance. Various design methods based on different control objectives and system conditions have been developed and verified in theory and practice. Some of them are reviewed below.

Adaptive Control

Adaptive control is based on feedback of signals in a controlled system for control adaptation to effectively handle system uncertainties. It is a suitable choice for systems having parametric uncertainty. The role of adaptive control is to find a suitable controller which satisfies the control objectives and is applicable for any unknown parameter. This can be achieved by defining a parameter estimator function in the controller and tuning it appropriately in response to changes in the dynamics of the process.

Adaptive control schemes have been used to cope with actuator dead zone [50, 51, 56, 52, 54], backlash [44, 53, 55], hysteresis [45, 56, 57, 90, 58] and saturation [46, 59, 63, 60, 61, 62]. The essence of these schemes is to use adaptive algorithms to modify control signals to compensate the effects of such nonlinearities. In these schemes, the term multiplying the control and the uncertain parameters of the system must be within known intervals.

Fuzzy and Neural Network Control

Fuzzy controllers [91, 92, 93, 94] are the most important applications of fuzzy theory. They work rather different from conventional controllers; experienced knowledge is used instead of differential equations to describe a system. This knowledge can be expressed in a very natural way using linguistic variables, which are described by fuzzy sets. Neural network [95, 96] consists of many neurons, each of which performs two functions, namely aggregation of its inputs from other neurons or external environment and generation of an output from the aggregated inputs. The output from a neuron fans to other neurons to which it is connected via weighted links. Fuzzy logic system and neural networks have been used extensively in feedback control systems to deal with uncertainties. Many well-known results say that any sufficiently smooth function can be approximated arbitrarily closely on a compact set using fuzzy IF-THEN rules and neural networks with appropriate weights.

Intelligent control using neural networks to handle nonsmooth nonlinearities is presented in [42, 64, 65, 66, 67, 68], while fuzzy logic is used in [41, 69, 70, 71, 72]. The system states and uncertain weights must be within a known compact set. With this, the error resulted from using NN or fuzzy logic to approximate system functions will be bounded with known bounds. However, the states of the system cannot be guaranteed in a compact set before stability of the closed-loop system is established. Also the perfect tracking and transient performance cannot be ensured.

Variable Structure Control

Variable structure control (VSC) [97, 98, 99, 100, 101] is a well-known solution to the problem of the deterministic control of uncertain systems, since it yields invariance to a class of parameter variations. The characterizing feature of VSC is sliding motion, which occurs when the system state repeatedly crosses certain sunspaces, or sliding hyperplanes, in the state space. An important and distinct class of nonlinear controllers is comprised of variable structure controllers which use control switching to reject the effects of system modelling errors and disturbance on system behavior to enhance robustness of systems performance. The "sliding surface" in the state space constrains the system trajectory to lie within a neighborhood of the sliding surface by switching to the appropriate feedback law at any time instant. By this choice the closed loop response becomes insensitive to some uncertainties. The first, and perhaps most severe, drawback of VSC is chattering due to the discontinuities introduced by the switching function. Another drawback of the VSC is that, in general, it only applies to the uncertain systems which satisfy the matching condition i.e. where the uncertainty and the control appear in the same equation. VSC requires reliable knowledge of uncertainty i.e.

the design results in "high gain" controllers if the uncertainty set description is conservative.

Control schemes using variable structure control handling nonsmooth nonlinearities have been used as well in [43, 52, 73, 74, 75, 76, 77, 78], a describing function-based model is adopted for the input nonlinearities. Although the sliding motion is essential in variable structure control, it is an undesirable phenomenon from the adaptive control point of view. In practice, sliding motions cause chattering and also may lead to a theoretical loss of uniqueness of solutions. For the tracking performance, it is a slow convergence of the tracking errors to zero.

2.2 Backstepping

Backstepping is a recursive Lyapunov-based scheme. The idea of adaptive backstepping is to design a controller recursively by considering some of the state variables as "virtual controls" and designing for them intermediate control laws. Backstepping designs are more flexible and do not force the designed system to appear linear. They can avoid cancellations of useful nonlinearities and often introduce additional nonlinear terms to improve transient performance.

To give a clear idea of such development, we consider the following third order system

$$\begin{aligned}\dot{x}_1 &= x_2 + \phi_1^T(x_1)\theta \\ \dot{x}_2 &= x_3 + \phi_2^T(x_1, x_2)\theta \\ \dot{x}_3 &= u + \phi_3^T(x_1, x_2, x_3)\theta\end{aligned} \tag{2.1}$$

where the $p \times 1$ vector θ is constant and unknown, ϕ_1, ϕ_2 and ϕ_3 are known nonlinear functions. This system is so-called pure-feedback system as defined in [30, 102, 103, 104]. When the parameter θ is known, the pure-feedback restriction essentially amounts to feedback linearizability [105, 106]. Our problem is to globally stabilize the equations and also to achieve the asymptotic tracking of x_{1ref} by x_1.

In (2.1) the first virtual control is x_2. It is used to stabilize the first equation as a separate system. Since θ is unknown, this task is solved with an adaptive controller consisting of a control law and an estimator $\dot{\hat{\theta}} = \tau(x_1)$ to obtain an estimate $\hat{\theta}$ of θ, as in the Lyapunov-based design. In the next step the state x_3 is the "virtual control" which is used to stabilize the subsystem consisting of the first two equations of (2.1). This is again an adaptive control task, and a new update law is to be designed to estimate θ again. However, an update law has already been designed in the first step and this dose not seem to allow any freedom to proceed further. So it treats the parameter θ in the second equation of (2.1) as a new parameter and assigns to it a new estimate with a new update law. As a result, there are several estimates for the same parameter. In order to avoid this over-parametrization, a "tuning function" [107, 108, 109] is used in subsequent recursive steps and the discrepancy $\dot{\hat{\theta}} - \tau(x_1)$ is compensated with additional terms in the controller. Whenever the second derivative $\ddot{\hat{\theta}}$ would appear, it is replaced by the analytical expression for the first derivative of $\tau(x_1)$.
The design procedure is elaborated in the following.
Introducing the change of coordinates

$$z_1 = x_1 - x_{1ref}, \quad z_2 = x_2 - \alpha_1 - \dot{x}_{1ref}, \quad z_3 = x_3 - \alpha_2 - \ddot{x}_{1ref} \tag{2.2}$$

where α_1 and α_2 are virtual controllers, x_{1ref} is the reference signal.
Step 1. We start with the first equation of (2.1) by considering x_2 as control variable. The derivative of tracking error z_1 is given as

$$\dot{z}_1 = z_2 + \alpha_1 + \phi_1^T \theta \tag{2.3}$$

Designing the first stabilizing function α_1 as

$$\alpha_1 = -c_1 z_1 - \phi_1^T \hat{\theta} \tag{2.4}$$

where c_1 is a positive constant, $\hat{\theta}$ is an estimate of θ. Our task in this step is to stabilize (2.3) with respect to the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{2.5}$$

where Γ is a positive definite matrix, $\tilde{\theta} = \theta - \hat{\theta}$. Then the derivative of V_1 is

$$\dot{V}_1 = -c_1 z_1^2 - \tilde{\theta}^T(\Gamma^{-1}\dot{\hat{\theta}} - \tau_1) + z_1 z_2 \tag{2.6}$$

$$\tau_1 = \phi_1 z_1 \tag{2.7}$$

where τ_1 is the first tuning function to overcome the over-parametrization problem. *Step 2.* We now consider that x_3 is the control variable in the second equation of (2.1). We obtain

$$\dot{z}_2 = z_3 + \alpha_2 - \frac{\partial\alpha_1}{\partial x_1}x_2 + (\phi_2 - \frac{\partial\alpha_1}{\partial x_1}\phi_1)^T\theta - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_1}{\partial x_{1ref}}\dot{x}_{1ref} \tag{2.8}$$

Our task in this step is to stabilize the (z_1, z_2)-system (2.3,2.8) with respect to

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{2.9}$$

Now we select

$$\alpha_2 = -z_1 - c_2 z_2 + \frac{\partial\alpha_1}{\partial x_1}x_2 - \hat{\theta}^T(\phi_2 - \frac{\partial\alpha_1}{\partial x_1}\phi_1) + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2 + \frac{\partial\alpha_1}{\partial x_{1ref}}\dot{x}_{1ref} \tag{2.10}$$

$$\tau_2 = \tau_1 + (\phi_2 - \frac{\partial\alpha_1}{\partial x_1}\phi_1)z_2 \tag{2.11}$$

where c_2 is a positive constant. The resulting of derivative of V_2 is

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}(\Gamma\tau_2 - \dot{\hat{\theta}}) + \tilde{\theta}^T(\tau_2 - \Gamma^{-1}\dot{\hat{\theta}}) \tag{2.12}$$

Step 3. Proceeding to the last equation in (2.1), we obtain

$$\begin{aligned}\dot{z}_3 &= u - \frac{\partial\alpha_2}{\partial x_1}x_2 - \frac{\partial\alpha_2}{\partial x_2}x_3 + (\phi_3 - \frac{\partial\alpha_2}{\partial x_1}\phi_1 - \frac{\partial\alpha_2}{\partial x_2}\phi_2)^T\theta \\ &\quad -\frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_2}{\partial x_{1ref}}\dot{x}_{1ref} - \frac{\partial\alpha_2}{\partial\dot{x}_{1ref}}\ddot{x}_{1ref} \qquad (2.13)\end{aligned}$$

In this equation, the actual control input u appears and is at our disposal. We are finally in this position to design control u and update law $\dot{\hat{\theta}}$ as

$$\begin{aligned}u &= -z_2 - c_3 z_3 + \frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}x_3 + \frac{\partial\alpha_2}{\partial x_{1ref}}\dot{x}_{1ref} + \frac{\partial\alpha_2}{\partial\dot{x}_{1ref}}\ddot{x}_{1ref} \\ &\quad +(\frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma z_2 - \hat{\theta}^T)(\phi_3 - \frac{\partial\alpha_2}{\partial x_1}\phi_1 - \frac{\partial\alpha_2}{\partial x_2}\phi_2) + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\tau_3 + x_{1ref}^{(3)} \qquad (2.14) \\ \dot{\hat{\theta}} &= \Gamma\tau_3 \qquad (2.15) \\ \tau_3 &= \tau_2 + (\phi_3 - \frac{\partial\alpha_2}{\partial x_1}\phi_1 - \frac{\partial\alpha_2}{\partial x_2}\phi_2)z_3 \qquad (2.16)\end{aligned}$$

where c_3 is a positive constant. The derivative of Lapunov function $V_3 = V_2 + \frac{1}{2}z_3^2$ is

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 \qquad (2.17)$$

From the Lasalle's Theorem in Appendix B, it follows that $z_1, z_2, z_3 \to 0$, and this further implies that $\lim_{t\to\infty}(x_1 - x_{1ref}) = 0$.

Backstepping achieves the goals of stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates. For the state-feedback system, this Lyapunov function provides the proof of uniform stability and, if $x_1(t)$ is required to follow a trajectory $x_{1ref}(t)$, also the proof of asymptotic tracking $x_1(t) - x_{1ref}(t) \to 0$.

Adaptive backstepping and tuning functions have crossed the "extended matching" barrier which blocked the traditional Lyapunov-based design. They have achieved this by designing controller "stronger" than certainty equivalence controllers. So in this thesis, they will be employed in our controller design.

2.3 Nonsmooth Nonlinearities

When dealing with real control problems, the designer is inevitably led to face the difficulties tied to the presence of real physical components, which often contain nonsmooth nonlinearities such as dead-zone, backlash, hysteresis and saturation. In particular, actuators used in practice almost always contain static (e.g., dead-zone) or dynamic (e.g., backlash, hysteresis) nonlinearities, whose parameters are unknown and may vary with time. Dead-zone, backlash, hysteresis and saturation nonlinearities exist in mechanical, hydraulic, magnetic, and other types of system components.

This thesis presents adaptive backstepping controllers that are able to robustly stabilize uncertain plants, in the presence of dead-zone, backlash, hysteresis and saturation. In our approach we have chosen linear or nonlinear model for the plants to be controlled with a sufficient number of adjustable parameters which provide significant flexibility in matching real situations. This flexibility will be exploited for our backstepping control schemes with dead-zone, backlash, hysteresis and saturation nonlinearities.

The development of control techniques to mitigate effects of unknown nonsmooth nonlinearities has been studied for decades and has attracted a lot of attentions in engineering and science [45, 53, 86, 110, 111, 112]. A number of techniques are available in the literature to compensate for these nonlinearities present in the actuator. Starting from the pioneering work by [50], the idea of employing an adaptive inverse of the nonlinearity itself in the controller in order to cancel its effects has been widely used to cope with actuator dead zone [51, 41, 53, 73], backlash [43, 44, 55] and hysteresis [45, 48, 56] with unknown parameters. An adaptive inverse cascaded with the plant was employed to cancel the effects of such nonlinearities. These schemes assumed the system parameters must be inside known compact sets. Sometimes it is difficult to obtain its inverse. Intelligent control using neural networks is presented in [42, 64, 65, 66, 67, 68], while fuzzy logic is used in [41, 69, 70, 71]. The system states and uncertain weights must be within a known compact set. With this, the error resulted from using NN or fuzzy logic to approximate system functions will be bounded with known bounds. This assumption makes the control design and system analysis simpler. Variable structure control has been used as well in [43, 52, 74, 75, 76], a describing function-based model is adopted for the input nonlinearities. Although the sliding motion is essential in variable structure control, it is an undesirable phenomenon from the adaptive control point of view. In practice, sliding motions cause chattering and also may lead to a theoretical loss of uniqueness of solutions. Model reference approaches have recently been proposed to handle such nonlinearity, see for examples to cancel the effects of dead-zone in [41], backlash in [27], hysteresis in [56], saturation [59, 63] and actuator failure in [26]. Systematic design procedures for saturation have been developed for pole placement control [46, 47, 60], where all the poles and zeros of the model are strictly inside the unit circle or the plant has only also one pole at $z = 1$ and the others are within the unit circle. Very recently, the fusion of relay feedback control with robust nominal model following control has been used and experimentally tested to handle actuator dead-zone nonlinearities [113]. A recursive least square (RLS) algorithm avoiding nonlinearity inversion holding for dead zones in sensors is described in [79]. In [46, 59, 63, 47, 60, 113, 79], the plant linearity assumption is still required.

From the above discussion, it follows that the presence of nonsmooth nonlinearities in actuators should be accounted for. Hence, a noticeable uncertainty should be taken into account in nonlinearities models parameters, in order to match a sufficiently wide set of real situations.

In this thesis, the controller designed by using backstepping technique consists of new robust control laws and new estimators to estimate the unknown parameters. Besides showing global stability of the system, the transient performance of the tracking error is improved and derived to be an explicit function of design parameters. It will be shown how nonsmooth nonlinear industrial characteristics can adaptively compensated and how desired system performance is achieved in the presence of such nonlinearities. In our pragmatic approach, we have chosen general class of models with sufficient number of adjustable parameters which provide significant flexibility in matching real situations. This flexibility will be exploited for our backstepping control schemes with such nonsmooth nonlinearities. The considered plant is supposed to be preceded by the actuating device $u = f(v)$ as in Figure 2.1, u being the plant input not available for control.

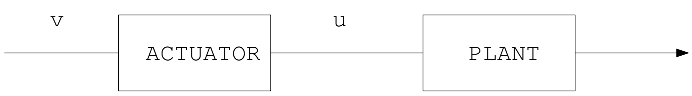


Figure 2.1: Block scheme of a plant driven by the actuator.

In the following , we consider each of nonsmooth nonlinearities in some details.

2.3.1 Dead-Zone

Dead-zone is a static input-output relationship which for a range of input values gives no output. Once the output appears, the slope between the input and the output is constant. The analytical expression of the dead-zone characteristic is

$$u(t) = \begin{cases} m_r(v(t) - b_r) & v(t) \geq b_r \\ 0 & b_l < v(t) < b_r \\ m_l(v(t) - b_l) & v(t) \leq b_l \end{cases} \tag{2.18}$$

A graphical representation of the dead-zone is shown in Figure 2.2, where v is the input and u is the output. In general, neither the break-points $b_r \geq 0, b_l \leq 0$ nor the slopes $m_r, m_l > 0$ are equal. There is no loss of generality in assuming that the zero input point is inside the dead-zone because this can always be achieved with a redefinition of the input v.

The simple dead-zone model appears in numerous studies of a wide variety of phenomena, not limited to man-made systems. We briefly describe four typical examples, starting with a bioengineering application.

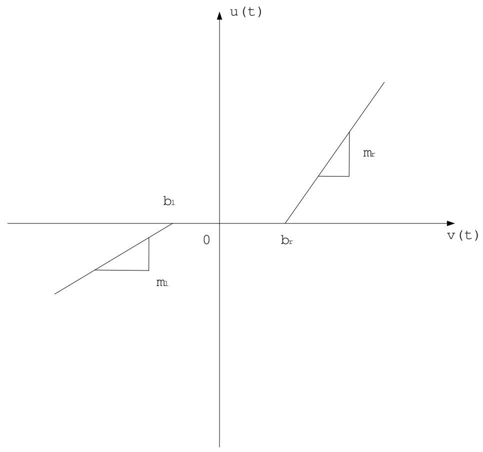


Figure 2.2: Dead-zone.

Upper-Limb Model

In functional neuromuscular stimulation a controlled electrical stimulus v is applied to inactive nerve in an attempt to replace upper motor neuron control which may be lost through cerebral stroke, brain injury, tumor, or spinal cessation. In [85] this approach has been applied to stimulation of the upper limb, concentrating on elbow flexion/extension. Two dead-zone models are employed to represent the biceps and triceps nonlinear "gains" appearing at the input of limb dynamics block in Figure 2.3. A similar model was employed in [49], [86] and [87] to adaptively control the knee joint of paraplegics.

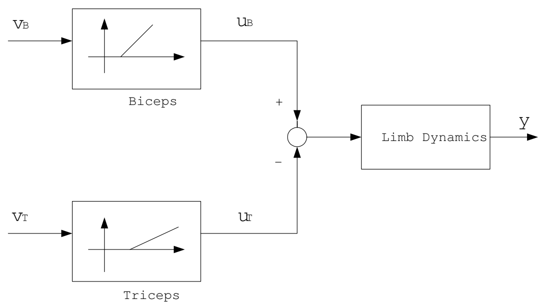


Figure 2.3: Dead-zones in upper-limb model.

Ultrasonic Motor

Ultrasonic motor(USM) is a new type motor as in [114], which is driven by the ultrasonic vibration force of piezoelectric elements. This motor has a nonlinear speed characteristics, which vary with drive conditions. In position control system, the motor shows a variable dead-zone in the control input ϕ (phase difference of applied voltages) against load torque. The drive system for the position control of USM and the block diagram of USM are shown in Figure 2.4 and Figure 2.5.

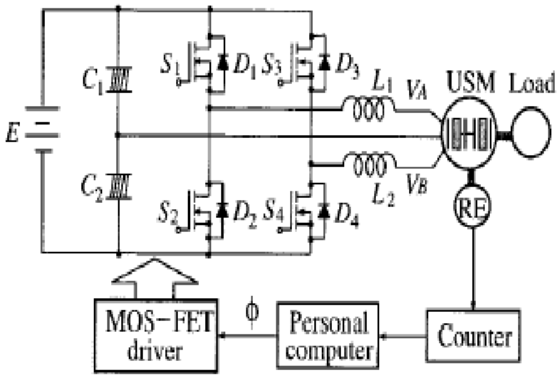


Figure 2.4: Drive system for USM.

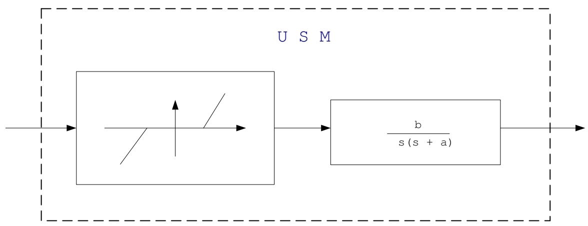


Figure 2.5: Block diagram of USM.

Servo-Valve

A common example from industrial applications is servo-valve in Figure 2.6. Its spool occludes the orifice with some overlap so that for a range of spool positions v there is no fluid flow u. This overlap prevents leakage losses which increase with wear and tear. Considering the spool position as the input v, and the load position

y as output, the hydraulic system in Figure 2.6 is represented in Figure 2.7 as a dead-zone block. It is located as the input of linear dynamics with transfer function $G(s) = \frac{K}{Ms^2+Bs}$, where $K = \frac{Ak_x}{k_p}$, $B = f + \frac{A^2}{k_p}$, $k_x = \frac{\partial g}{\partial x}$, $k_p = \frac{\partial g}{\partial P}$, $g = g(x, P) =$ flow, $A =$ area of piston, $P =$ pressure, and $f =$ viscous friction.

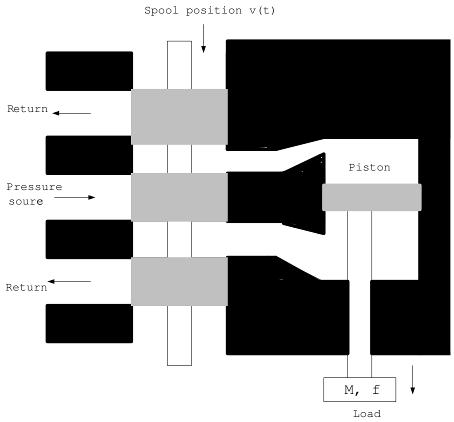


Figure 2.6: Dead-zone in servo-valve.

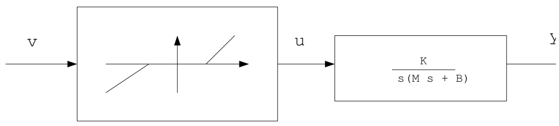


Figure 2.7: Block diagram of the servo-valve

DC Motor with Friction

A dead-zone effect is often caused by friction. In such applications the simple dead-zone model serves as aggregate static approximation of more complex microscopic dynamic phenomena. Perhaps the frequent used is a DC motor with Coulomb friction, represented in Figure 2.8. Considering motor torque T_m as the input, the transfer function in the forward path is a first-order lag with motor time constant c. When this time constant is negligible, the low-frequency approximation of the feedback loop is given by the dead-zone in Figure 2.9. The approximation can be rigorously justified as a singular perturbation [115]. It is important to observe that the friction torque characteristic is responsible for the break-points b_r and b_l, while the feed-forward gain m determines the slope.

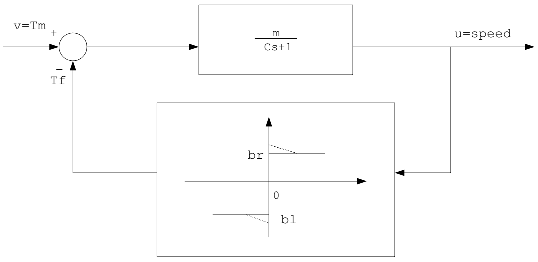


Figure 2.8: Dead-zone caused by friction.

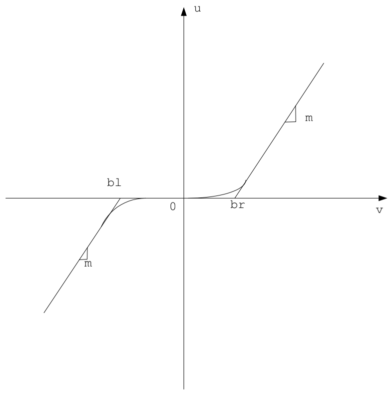


Figure 2.9: Dead-zone description.

2.3.2 Backlash

Backlash is a dynamic input-output relationship. It exists in a wide range of physical systems and devices, such as biology optics, electro-magnetism, mechanical actuators, electronic relay circuits and other areas. The analytical expression of the backlash characteristic is

$$\dot{u}(t) = \begin{cases} m\dot{v}(t) & \text{if } \dot{v}(t) \geq 0 \text{ and } u(t) = m(v(t) - c_r), \text{ or} \\ & \text{if } \dot{v}(t) \leq 0 \text{ and } u(t) = m(v(t) - c_l) \\ 0 & \text{otherwise} \end{cases} \tag{2.19}$$

where $m > 0,\ c_l < c_r$ are constant parameters. The motion on any inner segment is characterized by $\dot{u}(t) = 0$. A widely accepted characteristic of backlash is shown in Figure 2.10 where v is the input, u is the output, and c_r and c_l are the right and left "crossing".

In this thesis, another expression of the backlash characteristic is described by using a differential equation as

$$\frac{du}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - u) + B_1\frac{dv}{dt} \tag{2.20}$$

where α, c and B_1 are constants, $c > 0$ is the slope of the lines satisfying $c > B_1$. Figure 2.11 shows that the dynamic equation (2.20) can be used to model a class of backlash nonlinearities, where the parameters $\alpha = 1, c = 3.1635$, and $B_1 = 0.345$, the input signal $v(t) = 6.5sin(2.3t)$ and the initial condition $u(0) = 0$.

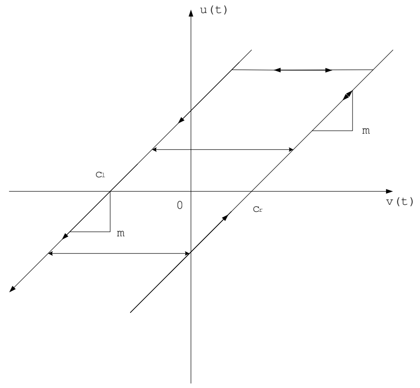


Figure 2.10: Backlash hysteresis.

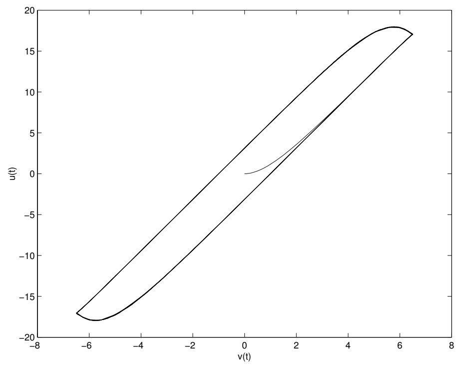


Figure 2.11: Backlash hysteresis.

The simple backlash model appears in numerous studies of a wide variety of phenomena, here we briefly describe two typical examples.

Valve Control Mechanism

An input backlash example as in [49] is shown in Figure 2.12, where the backlash is in the valve control mechanism and $G(s) = k/s$ is the transfer function relating the liquid level h with the difference between the controlled inflow u and the uncontrolled outflow d.

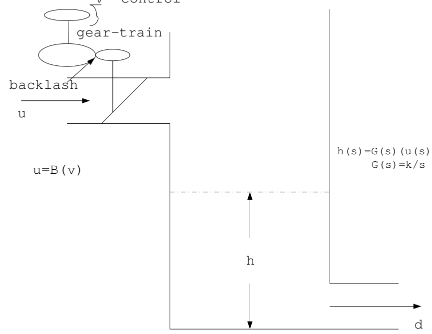


Figure 2.12: Backlash in the valve control mechanism of a liquid tank.

Positioning system

An output backlash example is a simple servo for positioning of a low inertia object in [49], as shown in Figure 2.13. In this case, $G(D)$ is the transfer function of the amplifier/motor unit. Effects of gear-train backlash in such classical servomechanism have been extensively studied.

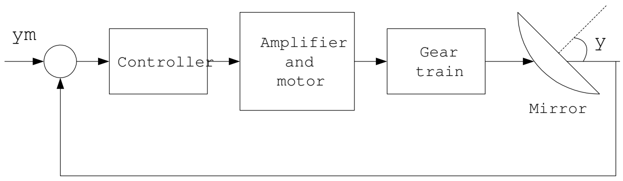


Figure 2.13: Output backlash in a positioning system.

Piezoelectric actuator

A piezoelectric actuator [116] is an electrically controllable positioning element which functions on the basis of the piezoelectric effect. A major limitation of piezoelectric actuator is the rate-independent hysteresis exhibited between voltage and displacement as shown in equation (2.20) and Figure 2.11, which severely limits system performance such as giving rise to undesirable inaccuracy or oscillations, even leading to instability.

2.3.3 Hysteresis

A hysteresis characteristics can be tuned by eight parameters: four slopes m_r, m_l, m_t, m_b and four crossing parameters c_r, c_l, c_t, c_b, where the subscripts l, r, t, b respectively indicate “left”, “right”,“top”, “bottom” sides of the hysteresis loop. The hysteresis is a dynamic nonlinearity and is described by two half-lines, two line segments, and the quadrilateral formed by those half-lines and segments.

The two half-lines and two line segments are described by:

$$\begin{aligned}
u(t) &= m_t v(t) + c_t, \;\; v(t) > v_1 = \frac{c_t + m_l c_l}{m_l - m_t} &\quad (2.21)\\
u(t) &= m_b v(t) + c_b, \;\; v(t) < v_2 = \frac{c_b + m_r c_r}{m_r - m_b} &\quad (2.22)\\
u(t) &= m_r(v(t) - c_r), \;\; v_2 < v(t) < v_3 = \frac{c_t + m_r c_r}{m_r - m_t}, \dot{v}(t) > 0, \dot{u}(t) > 0 &\quad (2.23)\\
u(t) &= m_l(v(t) - c_l), \;\; \frac{c_b + m_l c_l}{m_l - m_b} = v_4 < v(t) < v_1, \;\; \dot{v}(t) < 0, \dot{u}(t) < 0 &\quad (2.24)
\end{aligned}$$

where v_1, v_2, v_3, v_4 are the values of $v(t)$ at the upper-left, lower-right, upper-right, and lower-left corners of the quadrilateral. The motion on any inner segment is characterized by $\dot{u}(t) = 0$ even if $v(t)$ increases or deceases.

The hysteresis phenomena occur inside the loop formed by the half-lines (2.21-(2.22) and the segments (2.23-(2.24). Inside the hysteresis loop, the relationship between $u(t)$ and $v(t)$ is

$$u(t) = \begin{cases} m_t v(t) + c_d(t) & \textit{for } \dot{v}(t) < 0 \\ m_b v(t) + c_u(t) & \textit{for } \dot{v}(t) > 0 \end{cases} \quad (2.25)$$

where $c_d(t) \in (c_t, c_1), c_u(t) \in (c_2, c_b)$ are piecewise constant functions which depend on the point where $\dot{v}(t)$ changes its sign and on the past trajectories of $(v(t), u(t))$, with

$$c_1 = \begin{cases} (m_b - m_t)\frac{c_b + m_l c_l}{m_l - m_b} + c_b & \textit{for } m_t < m_b \\ (m_b - m_t)\frac{c_b + m_r c_r}{m_r - m_b} + c_b & \textit{for } m_t > m_b \\ c_b & \textit{for } m_t = m_b \end{cases} \quad (2.26)$$

$$c_2 = \begin{cases} (m_t - m_b)\frac{c_t + m_r c_r}{m_r - m_t} + c_t & \textit{for } m_t > m_b \\ (m_t - m_b)\frac{c_t + m_l c_l}{m_l - m_t} + c_t & \textit{for } m_t < m_b \\ c_t & \textit{for } m_t = m_b \end{cases} \quad (2.27)$$

The relationship (2.25) holds for a part of one of the half-lines: when $m_t > m_b$, on the half-line (2.21) with $v_1 < v(t) < v_3$, $u(t) = m_t v(t) + c_t$ for $\dot{v}(t) < 0$; when

$m_t < m_b$, on the half-line (2.22) with $v_4 < v(t) < v_2$, $u(t) = m_b v(t) + c_b$ for $\dot{v}(t) > 0$.

The signs of $\dot{u}(t)$ and $\dot{v}(t)$ are not restricted on other parts of these two half-lines: $u(t) = m_t v(t) + c_t, v(t) \geq v_3$; $u(t) = m_b v(t) + c_b, v(t) \leq v_4$; and $u(t) = m_t v(t) + c_t$, $v_1 < v(t) < v_3$ when $m_t < m_b$ or $u(t) = m_b v(t) + c_b$, $v_4 < v(t) < v_2$ when $m_t > m_b$. The model of the hysteresis and its two typical minor loops are shown in Figure 2.14.

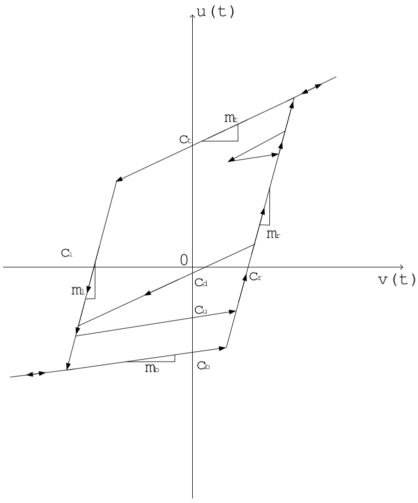


Figure 2.14: Hysteresis model.

The motion of $u(t)$ and $v(t)$ on the half-lines (2.21)-(2.22) and the segments (2.23)-

(2.24) and inside the hysteresis loop can be mathematically described as

$$\dot{u}(t) = \begin{cases} m_t\dot{v}(t) & \textit{if } v(t) \geq v_3 \textit{ and } u(t) = m_t v(t) + c_t, \\ & \textit{or if } v_4 < v(t) < v_3, \dot{v}(t) < 0, u(t) = m_t v(t) + c_d, \\ & u(t) \neq m_l(v(t) - c_l) \textit{ and } u(t) \neq m_b v(t) + c_b, \\ & \textit{or if } v_4 < v(t) < v_3,\ \dot{v}(t) < 0, \\ & u(t) = m_b v(t) + c_b \textit{ and } m_t < m_b \\ & \textit{or if } v_4 < v(t) < v_3,\ \dot{v}(t) > 0, \\ & u(t) = m_t v(t) + c_t \textit{ and } m_t < m_b \\ m_b\dot{v}(t) & \textit{if } v(t) \leq v_4 \textit{ and } u(t) = m_b v(t) + c_b, \\ & \textit{or if } v_4 < v(t) < v_3,\ \dot{v}(t) > 0, u(t) = m_b v(t) + c_u \\ & u(t) \neq m_r(v(t) - c_r) \textit{ and } u(t) \neq m_t v(t) + c_t, \\ & \textit{or if } v_4 < v(t) < v_3,\ \dot{v}(t) > 0, \\ & u(t) = m_t v(t) + c_t \textit{ and } m_t > m_b \\ & \textit{or if } v_4 < v(t) < v_3,\ \dot{v}(t) < 0, \\ & u(t) = m_b v(t) + c_b \textit{ and } m_t > m_b \\ m_r\dot{v}(t) & \textit{if } v_4 < v(t) < v_3,\ \dot{v}(t) > 0 \textit{ and } u(t) = m_r(v(t) - c_r) \\ m_l\dot{v}(t) & \textit{if } v_4 < v(t) < v_3,\ \dot{v}(t) < 0 \textit{ and } u(t) = m_l(v(t) - c_l) \\ 0 & \textit{if } \dot{v}(t) = 0 \end{cases} \tag{2.28}$$

Magnetic Suspension with Hysteresis

Typical examples of control systems with input hysteresis are magnetic suspensions and bearings. An oversimplified schematic representation of a magnetic suspension systems is shown in Figure 2.15. The position of an iron ball is detected by a light source L and a photocell P and compared with a desired reference r. The error signal $y - r$ is sent to a controller which generates the control signal - the electromagnet current I.

The magnetic force acting upon the iron ball is a nonlinear function of the ball position y and the magnetic flux ϕ. The remaining nonlinearity is the ferromagnetic hysteresis characteristic $\phi(I)$. To cast this system, we consider the current I as the signal $v(t)$ and the magnetic force F acting upon the iron ball as the signal $u(t)$. To hold the ball at some desired position $y = r$ the required force is u_s. The amount of the current v needed to generate this force depends on the operating point on the hysteresis characteristic.

The input-output model of the magnetic suspension system from the current I to the ball position y can be represented by the block diagram in Figure 2.16 where under certain simplifying assumptions the transfer function $G(s)$ has two poles: $G(s) = k/(s-p_1)(s-p_2)$. One of the poles, say p_1, is necessarily unstable, $p_1 > 0$, because the magnetic force decays with the distance and the gravitation force is constant.

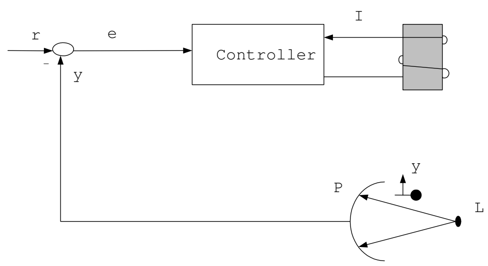


Figure 2.15: A magnetic suspension with solenoid hysteresis.

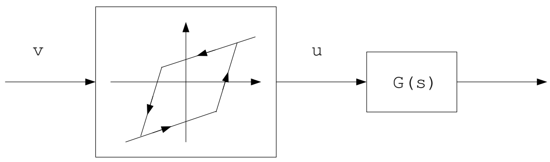


Figure 2.16: Plant with input hysteresis.

Hysteresis Motor

Hysteresis motor [117] is a self-starting synchronous motor that uses the hysteresis characteristics of the semi-hard magnetic materials. It consists of polyphase stator and rotor which contains hysteresis ring. Most of cases, semi-hard magnetic material is used for the hysteresis ring. The hysteresis ring is affected by the rotational hysteresis caused by the stator windings. It needs to determine the adequate thickness of the hysteresis ring in the hysteresis motor and the motor torque is calculated by the area of hysteresis loop determined by the field intensity in the ring. Figure 2.17 shows the basic structure of hysteresis motor. The hysteresis ring which is a part of the rotor is affected by the rotational hysteresis caused by the stator windings, and the direction of the magnetization of each element of the ring is different from that of the magnetic field or magnetic flux density. That is to say, the thicker the hysteresis ring becomes, the larger the rotational hysteresis increases and to make matters worse, the output of the thicker ring motor becomes less than that of thin rotor motor.

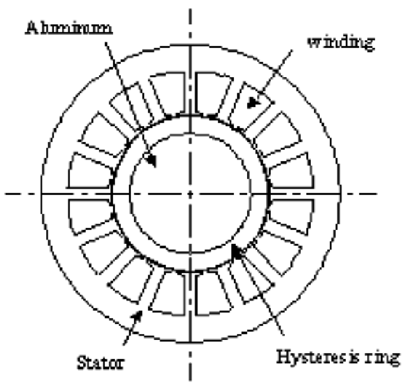


Figure 2.17: Structure of hysteresis motor.

Hysteresis in Brakes

Disk brakes are becoming important actuators in advance automotive control systems which improves safety, drivability, and the overall performance of passenger cars and trucks. A common air disk (ADB-1560) for trucks has an input-output force hysteresis characteristic. In a feedback control loop, this large hysteresis would limit the achievable dynamic performance. Therefore, for high dynamic performance the effect of hysteresis must be compensated.

2.3.4 Saturation

It is known that all real dynamic systems are subject to hard limits on input. This is due to inherent physical constraints of the dynamical system and constraints

in the controller actuators. Dynamical systems with hard limit constraints on the amplitude of control input, such as finite voltage of electrical motors and finite capacity of a pump, are the most common cases, where the hard limit constraint is modelled by a saturation nonlinearity shown in Figure 2.18. Saturation is always a potential problem for actuators of control systems as all actuators do saturate at some level. Actuator saturation affects the transient performance and even leads to system instability. Ignoring their existence may lead to sever performance deterioration and even instability in some cases.
Saturation nonlinearity is defined as

$$u = sat(v(t)) = \begin{cases} \text{sign}(v(t))u_M & |v(t)| \geq u_M \\ v(t) & |v(t)| < u_M \end{cases} \quad (2.29)$$

where u_M is a known bound of $u(t)$. If we use the *sign* function to limit the control position limits, the relationship between the applied control $u(t)$ and the control input $v(t)$ has a sharp corner when $|v(t)| = u_M$.

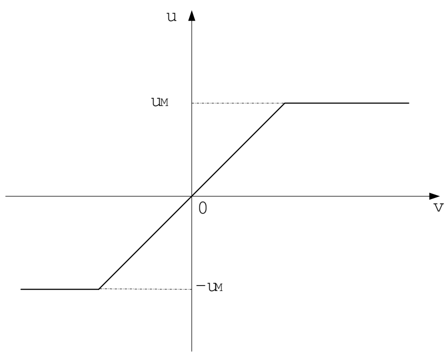


Figure 2.18: Saturation.

Input saturation constitutes a class of most encountered nonlinearities in control design. The control variables of all real-world systems are constrained or limited due to the physical nature of the actuator.

Active Micro-gravity Isolation system

An active micro-gravity isolation system [88] has saturation which limits the actuator force. A schematic of system is shown in Figure 2.19. The goal of the control design is to achieve a level of isolation between the base acceleration x_{off} and the inertial acceleration x_{on} of the isolated platform. The isolated platform must operate in a limited rattle space; hence, an additional design constraint is that the relative displacement $x_{on} - x_{off}$ does not exceed the 0.5 inch rattle space limit in order to prevent the platform from bumping into its hard stops.

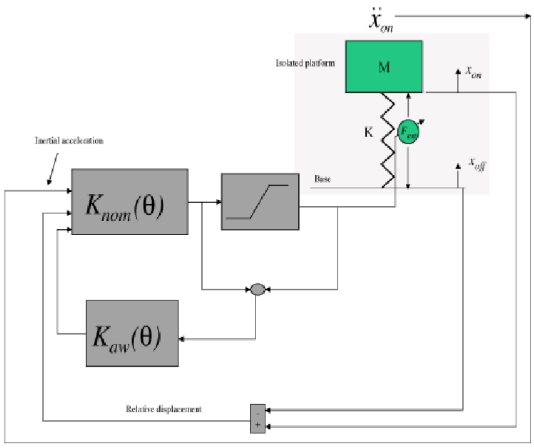


Figure 2.19: Schematic Isolation system.

Power Supply

The control of the current, position and shape of an elongated cross-section tokamak plasma [89] is complicated by the instability of the plasma vertical position. Due to the size and therefore the cost of ITER, there will naturally be smaller margins in the Poloidal Field coil power supplies implying that the feedback will experience actuator saturation during large transients due to a variety of plasma disturbances. Current saturation is relatively good due to the integrating nature of the tokamak, resulting in a reasonable time horizon for strategically handling this problem. On the other hand, voltage saturation is produced by the feedback controller itself, with no intrinsic delay.

2.4 Summary

In this Chapter, we have presented physical examples of dead-zone, backlash, hysteresis and saturation appearing indivigually. It is claer that the presence of these nonsmooth nonlinearities adversely affects the static or dynamic accuracy of feedback control systems. Moreover, it may deteriorate system performance. From the examples in this chapter we deduce that the following three classes of nonlinear plants are common in applications: plants with input nonlinearities, plants with output nonlinearities and plants with both input and output nonlinearities. There are shown in Figures 2.20(a)-(c), where $N(.)$ represent a nonsmooth nonlinearity. In this thesis, we consider the plant with input nonlinearities as in Figure 2.20(a). For this plant, we will develop backstepping schemes to cancel the effects of these nonlinearities, so that the backstepping controller can achieve improved tracking and transient performance. Note that our proposed schemes in this thesis can also be applied to other two types of plants.

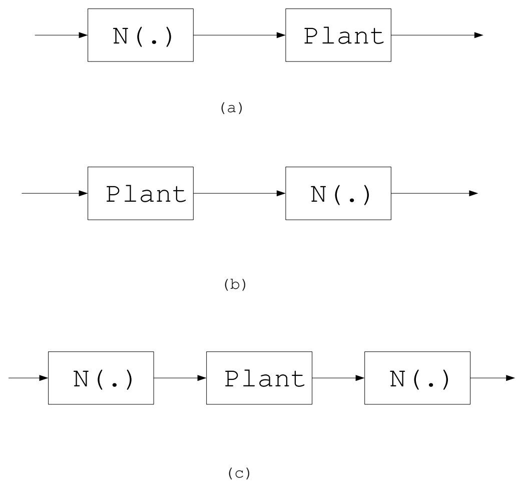


Figure 2.20: Plant models (a) with input nonlinearities; (b) with output nonlinearities; (3) with input and output nonlinearities.

Chapter 3

Adaptive Control of Uncertain Systems with Backlash Nonlinearity

In this chapter, we consider uncertain dynamic systems preceded by unknown backlash nonlinearity. By using backstepping technique, two types of robust adaptive backstepping control algorithms are developed. One is the state feedback adaptive control scheme for tracking, and another is the output feedback adaptive control scheme for tracking.

For state feedback control we develop two simple backstepping adaptive control schemes for the same class of nonlinear systems as in [43], with bounded external disturbances included in our case. Besides showing global stability of the system, the transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters.

For output feedback control we develop a new scheme for a class of uncertain linear systems preceded by unknown backlash nonlinearities. The controller designed by using backstepping technique consists of a new robust control law and a new estimator to estimate the unknown parameters. It is shown that all signals are bounded.

This chapter is organized as follows: The review of the adaptive control of backlash hysteresis is in Section 3.1. Section 3.2 states the adaptive state feedback control design based on the backstepping technique and analyzes the stability and performance. Sections 3.3 presents the adaptive output feedback control. Simulation results are presented in Section 3.4. Finally, Section 3.5 concludes the chapter.

3.1 Introduction

The development of control techniques to mitigate effects of unknown hysteresis has been studied for decades and has attracted attention [43], [44], [45], [48], [56] and [55]. Much of this interest is a consequence of its importance in present application. Interest in studying dynamic systems with hysteresis is motivated by their role as nonlinearities for which traditional control methods are insufficient and so requiring development of new approaches. For backlash hysteresis, several adaptive control schemes have recently been proposed, see for examples [43], [44], [56], [55]. In [44], [56] and [55] an inverse hysteresis nonlinearity was constructed. An adaptive hysteresis inverse cascaded with the plant was employed to cancel the effects of hysteresis. In [43] a dynamic backlash model is defined to pattern a backlash rather than constructing an inverse model to mitigate the effects of the backlash. However in [43], the term multiplying the control and the uncertain parameters of the system must be within known intervals and the 'disturbance-like' term must be bounded with known bound. Projection was used to handle the 'disturbance-like' term and unknown parameters. System stability was established and the tracking error was shown to converge to a residual. Control of such systems with unknown backlash is typically challenging.

In this chapter, we consider state feedback control of a class of special nonlinear systems and output feedback control of a class of general linear systems.

3.2 State Feedback Control

In this section, we develop two simple backstepping adaptive control schemes for the same class of nonlinear systems as in [43], with bounded external disturbances included in our case. It is shown that the proposed controllers not only can guarantee globally stability, but also transient performance. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. In our design, the term multiplying the control and the system parameters are not assumed to be within known intervals. The bound of the 'disturbance-like' term is not required. To handle such a term, an estimator is used to estimate its bound.

3.2.1 Problem Formulation

We consider the same class of systems as in [43]. For completeness, the system model is given as follows:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\big(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\big) = bu(w) + \bar{d}(t) \tag{3.1}$$

where Y_i are known continuous linear or nonlinear functions, $\bar{d}(t)$ denotes bounded external disturbances, parameters a_i are unknown constants and control gain b is unknown bounded constant, w is the control input, $u(w)$ denotes backlash type of nonlinearities described as in chapter 2.

$$\frac{du}{dt} = \alpha\Big|\frac{dw}{dt}\Big|(cw - u) + B_1\frac{dw}{dt} \tag{3.2}$$

where α, c and B_1 are constants, $c > 0$ is the slope of the lines satisfying $c > B_1$. This equation can be solved explicitly for v piecewise monotone

$$u(t) = cw(t) + d_1(w) \tag{3.3}$$

$$d_1(w) = [u_0 - cw_0]e^{-\alpha(w-w_0)\text{sign}\dot{w}} + e^{-\alpha w \text{sign}\dot{w}} \int_{w_0}^{w} [B_1 - c]e^{\alpha\xi(\text{sign}\dot{w})} d\xi \tag{3.4}$$

for $\dot{w}$ constant and $u(w_0) = w_0$. The solution indicates that dynamic equation (3.2) can be used to model a class of backlash nonlinearities. Analyzing the solution (3.3), we see that it is composed of a line with the slop c, together with a term $d_1(w)$. For $d_1(w)$, it is bounded clearly. It can be shown that if $u(w; w_0, u_0)$ is the solution of (3.4) with initial values (w_0, u_0), then, if $\dot{w} > 0(\dot{w} < 0)$ and $w \to +\infty(-\infty)$, one has

$$lim_{w\to\infty} d_1(w) = lim_{w\to\infty}[u(w; w_0, u_0) - f(w)] = -\frac{c - B_1}{\alpha} \tag{3.5}$$

$$lim_{w\to-\infty} d_1(w) = lim_{w\to-\infty}[u(w; w_0, u_0) - f(w)] = \frac{c - B_1}{\alpha} \tag{3.6}$$

It should be noted that the above convergence is exponential at the rate of α. And we get $d_1(w)$ is bounded.

From the solution structure (3.3) of model (3.2), (3.1) becomes

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\big(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\big) = \theta w(t) + d(t) \tag{3.7}$$

where $\theta = bc$ and $d(t) = bd_1(w(t)) + \bar{d}(t)$. The effect of $d(t)$ is due to both external disturbances and $bd_1(w(t))$. We call $d(t)$ a 'disturbance-like' term for simplicity of presentation.

Now equation (3.7) is rewritten in the following form

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
&\vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= -\sum_{i=1}^{r} a_i Y_i\big(x_1(t), x_2(t), \ldots, x_{(n-1}(t)\big) + \theta w(t) + d(t) \\
&= a^T Y + \theta w(t) + d(t) \qquad (3.8)
\end{aligned}$$

where $x_1 = x, x_2 = \dot{x}, \ldots, x_n = x^{(n-1)}$, $a = [-a_1, -a_2, \ldots, -a_r]^T$ and $Y = [Y_1, Y_2, \ldots, Y_r]^T$.

For the development of control laws, the following assumptions are made.

Assumption 1. The uncertain parameters b and c are such that $\theta > 0$.

Assumption 2. The desired trajectory $y_r(t)$ and its $(n-1)$th order derivatives are known and bounded.

The control objectives are to design backstepping adaptive control laws such that

- The closed loop is globally stable in sense that all the signals in the loop are uniformly ultimately bounded;
- The tracking error $x(t) - y_r(t)$ is adjustable during the transient period by an explicit choice of design parameters and $\lim_{t\to\infty} x(t) - y_r(t) = 0$ or $\lim_{t\to\infty} |x(t) - y_r(t)| - \delta_1 = 0$ for an arbitrary specified bound δ_1.

Remark 3.1 *Compared with [43], the uncertain parameters θ and a_i are not assumed inside known intervals. The bound D for $d(t)$ is not assumed to be known and it will be estimated by our adaptive controllers. Also the control objectives are not only to ensure global stability, but also transient performance.*

3.2.2 Backstepping Design and Stability Analysis

Before presenting the adaptive control design using the backstepping technique to achieve the desired control objectives, the following change of coordinates is made.

$$z_1 = x_1 - y_r \tag{3.9}$$

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \qquad i = 2, 3, \ldots, n \tag{3.10}$$

where α_{i-1} is the virtual control at the *ith step* and will be determined in later discussion. In the following, two control schemes are proposed.

Control Scheme I

To illustrate the backstepping procedures, only the last step of the design, i.e. *step* n below, is elaborated in details.

- *Step* 1: For $i = 2$, it follows from (3.8) to (3.10) that

$$\dot{z}_1 = z_2 + \alpha_1 \tag{3.11}$$

We design the virtual control law α_1 as

$$\alpha_1 = -c_1 z_1 \tag{3.12}$$

where c_1 is a positive design parameter. From (3.11) and (3.12) we have

$$z_1 \dot{z}_1 = -c_1 z_1^2 + z_1 z_2 \tag{3.13}$$

- *Step* i ($i = 2, \ldots, n-1$): Choose

$$\alpha_i = -c_i z_i - z_{i-1} + \dot{\alpha}_{i-1}(x_1, \ldots, x_{i-1}, y_r, \ldots, y_r^{(i-1)}) \tag{3.14}$$

where $c_i, i = 2, \ldots, n-1$ are positive design parameters. From (3.10) and (3.14) we obtain

$$z_i \dot{z}_i = -z_{i-1} z_i - c_i z_i^2 + z_i z_{i+1} \tag{3.15}$$

- *Step* n: From (3.8) and (3.10) we obtain

$$\dot{z}_n = \theta w(t) + a^T Y + d(t) - y_r^{(n)} - \dot{\alpha}_{n-1} \tag{3.16}$$

Then the adaptive control law is designed as follows

$$w = \hat{\vartheta} \bar{w} \tag{3.17}$$

$$\bar{w} = -c_n z_n - z_{n-1} - \hat{a}^T Y - \text{sign}(z_n) \hat{D} + y_r^{(n)} + \dot{\alpha}_{n-1} \tag{3.18}$$

$$\dot{\hat{\vartheta}} = -\gamma \bar{w} z_n \tag{3.19}$$

$$\dot{\hat{a}} = \Gamma Y z_n \tag{3.20}$$

$$\dot{\hat{D}} = \eta |z_n| \tag{3.21}$$

where c_n, γ and η are three positive design parameters, Γ is a positive definite matrix, $\hat{\vartheta}$, $\hat{a}$ and $\hat{D}$ are estimates of $\vartheta = 1/\theta$, a and D. Let $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$, $\tilde{a} = a - \hat{a}$ and $\tilde{D} = D - \hat{D}$. Note that $\theta w(t)$ in (3.16) can be expressed as

$$\theta w = \theta \hat{\vartheta} \bar{w} = \bar{w} - \theta \tilde{\vartheta} \bar{w} \tag{3.22}$$

From (3.16),(3.18) and (3.22) we obtain

$$\dot{z}_n = -c_n z_n - z_{n-1} + \tilde{a}^T Y - \text{sign}(z_n) \hat{D} + d(t) - \theta \tilde{\vartheta} \bar{w} \tag{3.23}$$

We define Lyapunov function as

$$V = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2}\tilde{a}^T \Gamma^{-1} \tilde{a} + \frac{\theta}{2\gamma}\tilde{\vartheta}^2 + \frac{1}{2\eta}\tilde{D}^2 \tag{3.24}$$

Then the derivative of V along with (3.8) and (3.17) to (3.21) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} z_i \dot{z}_i + \tilde{a}^T \Gamma^{-1} \dot{\tilde{a}} + \frac{\theta}{\gamma}\tilde{\vartheta}\dot{\tilde{\vartheta}} + \frac{1}{\eta}\tilde{D}\dot{\tilde{D}} \\ &= -\sum_{i=1}^{n} c_i z_i^2 + \tilde{a}^T Y z_n - |z_n|\hat{D} + d(t) z_n - \theta\tilde{\vartheta}\bar{w} z_n - \tilde{a}\Gamma^{-1}\dot{\hat{a}} - \frac{\theta}{\gamma}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} \\ &\leq -\sum_{i=1}^{n} c_i z_i^2 + \tilde{a}^T \Gamma^{-1}(\Gamma Y z_n - \dot{\hat{a}}) - \frac{\theta}{\gamma}\tilde{\vartheta}(\gamma \bar{w} z_n + \dot{\hat{\vartheta}}) + \frac{1}{\eta}\tilde{D}(\eta |z_n| - \dot{\hat{D}}) \\ &= -\sum_{i=1}^{n} c_i z_i^2 \end{aligned} \tag{3.25}$$

where we have used (3.13),(3.15),(3.23) and $z_n d(t) \leq |z_n| D$ to obtain (3.25).

We have the following stability and performance results based on this scheme.

Theorem 3.1 *Consider the uncertain nonlinear system (3.1) satisfying Assumptions 1-2. With the application of controller (3.17) and the parameter update laws (3.19) to (3.21), the following statements hold:*

- *The resulting closed loop system is globally stable.*
- *The asymptotic tracking is achieved, i.e.,*

$$\lim_{t\to\infty} [x(t) - y_r(t)] = 0 \tag{3.26}$$

- *The transient tracking error performance is given by*

$$\| x(t) - y_r(t) \|_2 \leq \frac{1}{\sqrt{c_1}} \Big(\frac{1}{2}\tilde{a}(0)^T \Gamma^{-1} \tilde{a}(0) + \frac{\theta}{2\gamma}\tilde{\vartheta}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2 \Big)^{1/2} \tag{3.27}$$

Proof: From (3.25) we established that V is non increasing. Hence, $z_i, i = 1, \ldots, n$, $\hat{\vartheta}, \hat{a}, \hat{D}$ are bounded. By applying the LaSalle-Yoshizawa theorem in Appendix B

to (3.25), it further follows that $z_i(t) \to 0, i = 1, \ldots, n$ as $t \to \infty$, which implies that $\lim_{t\to\infty}[x(t) - y_r(t)] = 0$.
Then we have

$$\| z_1 \|_2^2 = \int_0^\infty |z_1(\tau)|^2 d\tau \leq \frac{1}{c_1}(V(0) - V(\infty)) \leq \frac{1}{c_1}V(0) \tag{3.28}$$

Thus, by setting $z_i(0) = 0, i = 1, \ldots, n$, we obtain

$$V(0) = \frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\theta}{2\gamma}\tilde{\vartheta}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2, \tag{3.29}$$

a decreasing function of γ, η and Γ, independent of c_1. This means that the bound resulting from (3.28) and (3.29) is

$$\| z_1 \|_2 \leq \frac{1}{\sqrt{c_1}}\left(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\theta}{2\gamma}\tilde{\vartheta}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\right)^{1/2} \tag{3.30}$$

△△△

Remark 3.2 *From Theorem 3.1 the following conclusions can be obtained:*

- *The transient performance depends on the initial estimate errors $\tilde{\vartheta}(0)$, $\tilde{a}(0)$, $\tilde{D}(0)$ and the explicit design parameters. The closer the initial estimates $\hat{\vartheta}(0), \hat{a}(0)$ and $\hat{D}(0)$ to the true values e, a and D, the better the transient performance.*
- *The bound for $\| x(t) - y_r(t) \|_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains γ, η and Γ.*
- *To improve the tracking error performance we can also increase the gain c_1. However, increasing c_1 will influence other performance such as $\| \dot{x} - \dot{y}_r \|_2$ as shown below.*

Since $\dot{V} \leq 0$, immediately from (5.19) we know

$$V(t) = \sum_{i=1}^{n}\frac{1}{2}z_i^2 + \frac{1}{2}\tilde{a}^T\Gamma^{-1}\tilde{a} + \frac{\theta}{2\gamma}\tilde{\vartheta}^2 + \frac{1}{2\eta}\tilde{D}^2 \leq V(0) \tag{3.31}$$

Then

$$\begin{aligned} \| z_i \|_\infty &\leq \sqrt{2V(0)}, \quad i = 1, \ldots, n & (3.32) \\ \| \tilde{a} \|_\infty &\leq \sqrt{\bar{\lambda}(\Gamma)}\sqrt{2V(0)} & (3.33) \end{aligned}$$

From equations (3.9),(3.10) for $i = 2$ and (3.12), we get

$$\begin{aligned} \| \dot{x} - \dot{y}_r \|_2 &= \| z_2 - c_1 z_1 \|_2 \\ &\leq \| z_2 \|_2 + c_1 \| z_1 \|_2 & (3.34) \end{aligned}$$

Similar to the proof of (3.30), we can get $\| z_2 \|_2 \leq \frac{1}{\sqrt{c_2}}\sqrt{V(0)}$ and thus

$$\| \dot{x} - \dot{y}_r \|_2 \leq (\frac{1}{\sqrt{c_2}} + \sqrt{c_1})\sqrt{V(0)} \tag{3.35}$$

From equation (3.35) we can see that increasing c_1 also increase the error $\| \dot{x} - \dot{y}_r \|_2$. This suggests to fix the gain c_1 to some acceptable value and adjust the other gains such as γ, η and Γ.

Control Scheme II

In the previous scheme, a discontinuous function $sgn(z_n)$ is involved in the control and this may cause chattering. To avoid this, we now propose an alternative smooth control scheme.

Firstly we define a function $sg_i(z_i)$ as follows

$$sg_i(z_i) = \begin{cases} \dfrac{z_i}{|z_i|} & |z_i| \geq \delta_i \\ \dfrac{z_i^{(2q+1)}}{(\delta_i^2 - z_i^2)^{n-i+2} + |z_i|^{(2q+1)}} & |z_i| < \delta_i \end{cases} \tag{3.36}$$

where $\delta_i (i = 1, \ldots, n)$ is a positive design parameter and $q = round\{(n-i+2)/2\}$, where $round\{x\}$ means the element of x to the nearest integer. Clearly $2q + 1 \geq$

$(n - i + 2)$.

Remark 3.3 *Note that $sg_i(z_i)$ is $(n - i + 2)th$ order differentiable, so that this function can be used in the recursive backstepping control design, which required the function continuous differentiable. The function is used in the control scheme to remove the effect of disturbance and avoid chattering problem caused by discontinuous function.*

We also design a function $f_i(z_i)$ as

$$f_i(z_i) = \begin{cases} 1 & |z_i| \geq \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \tag{3.37}$$

Then we can get

$$sg_i(z_i)f_i(z_i) = \begin{cases} 1 & z_i \geq \delta_i \\ 0 & |z_i| < \delta_i \\ -1 & z_i \leq \delta_i \end{cases} \tag{3.38}$$

To ensure the resultant functions are differentiable, we replace z_i^2 by $(|z_i| - \delta_i)^{n-i+2} sg_i(z_i)$ in the Lyapunov functions for $i = 1, \ldots, n$ in Scheme I and we also replace z_i by $(|z_i| - \delta_i)^{n-i+1} sg_i$ in the design procedure as detailed below.

- *Step 1*: we design virtual control law α_1 as

$$\alpha_1 = -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) - (\delta_2 + 1)sg_1(z_1) \tag{3.39}$$

where c_1 is a positive design parameter. We choose Lyapunov function V_1 as

$$V_1 = \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} f_1 \tag{3.40}$$

Then the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= (|z_1| - \delta_1)^n f_1 s g_1(z_1) \dot{z}_1 \\ &\leq -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} f_1 + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 \end{aligned} \tag{3.41}$$

where (3.39) has been used.

- *Step 2*: we design virtual control law α_2 as

$$\alpha_2 = -(c_2 + \frac{5}{4})(|z_2| - \delta_2)^{n-1} s g_2(z_2) + \dot{\alpha}_1 - (\delta_3 + 1) s g_2(z_2) \tag{3.42}$$

where c_2 is positive design parameter.

We design Lyapunov function V_2 as

$$V_2 = \frac{1}{n}(|z_2| - \delta_2)^n f_2 + V_1 \tag{3.43}$$

Then the derivative of V_2 is

$$\dot{V}_2 \leq -\sum_{i=1}^{2} c_i (|z_i| - \delta_i)^{2(n-i+1)} f_i + M_2 + (|z_2| - \delta_2)^{n-1} (|z_3| - \delta_3 - 1) f_2 \tag{3.44}$$

where $M_2 = -\frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 - (|z_2| - \delta_2)^{2(n-1)} f_2$. Now we show that $M_2 < 0$. It is clear that $M_2 \leq 0$ for $|z_2| < \delta_2 + 1$. For $|z_2| \geq \delta_2 + 1$

$$\begin{aligned} M_2 &\leq -\frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + \frac{1}{4}(|z_1| - \delta_1)^{2n} f_1^2 \\ &\quad + (|z_2| - \delta_2 - 1)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &< (|z_2| - \delta_2)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &= (|z_2| - \delta_2)^2 (1 - (|z_2| - \delta_2)^{2(n-2)}) \\ &\leq 0 \end{aligned} \tag{3.45}$$

Then (3.44) is written as

$$\dot{V}_2 \leq -\sum_{i=1}^{2} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i + (|z_2| - \delta_2)^{n-1}(|z_3| - \delta_3 - 1) f_2 \tag{3.46}$$

- *Step i ($i = 3, \dots, n-1$)*: Choose

$$\alpha_i = -(c_i + \frac{5}{4})(|z_i| - \delta_i)^{n-i+1} sg_i(z_i) + \dot{\alpha}_{i-1} - (\delta_{i+1} + 1) sg_i(z_i) \tag{3.47}$$

where c_i is positive design parameter.

- *Step n*: The control law and parameter update laws are designed as follows

$$w = \hat{\vartheta}\bar{w} \tag{3.48}$$

$$\bar{w} = -(c_n + 1)(|z_n| - \delta_n) sg_n(z_n) - \hat{a}^T Y - sg_n \hat{D} + y_r^{(n)} + \dot{\alpha}_{n-1} \tag{3.49}$$

$$\dot{\hat{\vartheta}} = -\gamma \bar{w}(|z_n| - \delta_n) f_n sg_n(z_n) \tag{3.50}$$

$$\dot{\hat{a}} = \Gamma Y(|z_n| - \delta_n) f_n sg_n(z_n) \tag{3.51}$$

$$\dot{\hat{D}} = \eta(|z_n| - \delta_n) f_n \tag{3.52}$$

where c_n, γ and η are three positive design parameters, Γ is a positive definite matrix, $\hat{\vartheta}$, $\hat{a}$ and $\hat{D}$ are estimates of $\vartheta = 1/\theta$, a and D. We define Lyapunov function as

$$V = \sum_{i=1}^{n} \frac{1}{n-i+2}(|z_i| - \delta_i)^{n-i+2} f_i + \frac{1}{2}\tilde{a}^T \Gamma^{-1} \tilde{a} + \frac{\theta}{2\gamma}\tilde{\vartheta}^2 + \frac{1}{2\eta}\tilde{D}^2 \tag{3.53}$$

Then the derivative of V is given by

$$\begin{aligned}\dot{V} &= \dot{V}_i + (|z_n| - \delta_n)^2 f_n sg_n(z_n)\dot{z}_n + \tilde{a}^T\Gamma^{-1}\dot{\tilde{a}} + \frac{\theta}{\gamma}\tilde{\vartheta}\dot{\tilde{\vartheta}} + \frac{1}{\eta}\tilde{D}\dot{\tilde{D}} \\ &\leq -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i + \tilde{a}^T\Gamma^{-1}\big(\Gamma Y(|z_n| - \delta_n) f_n sg_n(z_n) - \dot{\hat{a}}\big) \\ &\quad -\frac{\theta}{\gamma}\tilde{\vartheta}\big(\gamma\bar{w}(|z_n| - \delta_n) f_n sg_n(z_n) + \dot{\hat{\vartheta}}\big) + \frac{1}{\eta}\tilde{D}\big(\eta(|z_n| - \delta_n) f_n - \dot{\hat{D}}\big) \\ &= -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i \end{aligned} \tag{3.54}$$

where (3.8),(3.39),(3.42) and (3.48) to (3.52) have been used.

Theorem 3.2 *Consider the uncertain nonlinear system (3.1) satisfying Assumptions 1-2. With the application of controller (3.48) and the parameter update laws (3.50) to (3.52), the following statements hold:*

- *The resulting closed loop system is globally stable.*
- *The tracking error converges to δ_1 asymptotically, i.e.,*

$$\lim_{t\to\infty} |x(t) - y_r(t)| = \delta_1, \quad |z_1| \geq \delta_1 \tag{3.55}$$

- *The transient tracking error performance is given by*

$$\|\, |x(t) - y_r(t)| - \delta_1 \,\|_2 \leq c_1^{\frac{-1}{2n}} \big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\theta}{2\gamma}\tilde{\vartheta}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\big)^{\frac{1}{2n}} \tag{3.56}$$

with $z_i(0) = \delta_i, i = 1, \ldots, n$,

Proof: Based (3.54), we established that V is non increasing. Hence, $|z_i| - \delta_i$ $(i = 1, \ldots, n)$, $\hat{\vartheta}, \hat{a}, \hat{D}$ are bounded. By applying the LaSalle-Yoshizawa theorem in Appendix B to (3.54), it further follows that $|z_i| - \delta_i \to 0, i = 1, \ldots, n$ as $t \to \infty$, which implies that $\lim_{t\to\infty} |x(t) - y_r(t)| = \delta_1$.

From (3.54) we establish that V is non increasing. Then we have

$$\| z_1 - \delta_1 \|_2^{2n} = \int_0^\infty |z_1(\tau) - \delta_1|^{2n} d\tau \leq \frac{1}{c_1}(V(0) - V(\infty)) \leq \frac{1}{c_1} V(0) \tag{3.57}$$

Thus, by setting $z_i(0) = \delta_i, i = 1, \ldots, n$, we obtain

$$V(0) = \frac{1}{2}\tilde{a}(0)^T \Gamma^{-1} \tilde{a}(0) + \frac{\theta}{2\gamma}\tilde{\vartheta}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2, \tag{3.58}$$

a decreasing function of γ, η and Γ, independent of c_1. This means that the bound resulting from (3.57) and (3.58) is

$$\| |x(t) - y_r(t)| - \delta_1 \|_2 \leq c_1^{\frac{-1}{2n}} \left(\frac{1}{2}\tilde{a}(0)^T \Gamma^{-1} \tilde{a}(0) + \frac{\theta}{2\gamma}\tilde{\vartheta}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\right)^{\frac{1}{2n}} \tag{3.59}$$

$\triangle\triangle\triangle$

Remark 3.4 *From Theorem 3.2 the following conclusions can also be obtained:*

- *The transient performance depends on the initial estimate errors $\tilde{\vartheta}(0)$, $\tilde{a}(0)$, $\tilde{D}(0)$ and the explicit design parameters.*
- *The bound for $\| x(t) - y_r(t) \|_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains c_1, γ, η and Γ.*

Remark 3.5 *To further improve system performance such as the tracking error, especially in the case without using sign functions, it is worthy to take the system hysteresis into account in the controller design, instead of only considering its effect like bounded disturbances. The first step of achieving this is perhaps to obtain an efficient adaptive hysteresis inverse which is still unclear and currently under investigation.*

3.3 Output Feedback Control

In this section, a new scheme is proposed to address an output feedback control problem: control of a class of uncertain linear systems preceded by unknown backlash nonlinearity, where the backlash is modelled by a differential equation. The controller designed by using backstepping technique consists of a new robust control law and a new estimator to estimate the unknown parameters. For the implementation of the controller, no knowledge is assumed on the bounds of unknown system parameters and the effect contributed by the backlash. It is shown that all the signals are bounded. A bound for the truncated L_2 norm of the tracking error is obtained as a function of design parameters. Simulation studies also verify the effectiveness of the proposed scheme.

3.3.1 Plant Model

The class of single-input single-output linear systems is given by

$$y(s) = \frac{B(s)}{A(s)}u(s) = \frac{\bar{b}_m s^m + \cdots + \bar{b}_1 s + \bar{b}_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}u(s) \tag{3.60}$$

$$\frac{du}{dt} = \alpha\left|\frac{dw}{dt}\right|(cv - u) + B_1\frac{dw}{dt} \tag{3.61}$$

where the coefficients a_i and $\bar{b}_i$ are constant but unknown, u denotes a backlash nonlinearity, w is the design controller. (3.61) is treated as in previous section. The control objective is for the system output y to asymptotically track a reference signal $y_r(t)$. Regarding the system and the reference signal, the following assumptions are made:

Assumption 1. The plant is minimum phase, i.e., the polynomial $B(s) = \bar{b}_m s^m + \cdots + \bar{b}_1 s + \bar{b}_0$ is Hurwitz.

Assumption 2. The relative degree ($\rho = n - m$) and an upper bound for the plant order (n) are known.

Assumption 3. The reference signal $y_r(t)$ and its first ρ derivatives are known and bounded, and, in addition, $y_r^{(\rho)}(t)$ is piecewise continuous.

Assumption 4. The sign of high-frequency gain ($\text{sign}(b_m)$), where $b_m = \bar{b}_m c$, is known.

3.3.2 State Estimation Filters

We start by representing the plant (3.60) as in the observer canonical form

$$\begin{aligned} \dot{x} &= Ax - ya + \begin{bmatrix} 0_{(\rho-1)\times 1} \\ b \end{bmatrix} w + D(t) \\ y &= e_1^T x \end{aligned} \tag{3.62}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & \\ \vdots & I_{n-1} \\ 0 & \ldots 0 \end{bmatrix}, \; b = \begin{bmatrix} \bar{b}_m c \\ \vdots \\ \bar{b}_0 c \end{bmatrix} = \begin{bmatrix} b_m \\ \vdots \\ b_0 \end{bmatrix}, \; D(t) = \begin{bmatrix} 0_{(\rho-1)\times 1} \\ \bar{b}_m d(t) \\ \vdots \\ \bar{b}_0 d(t) \end{bmatrix} \\ a &= [a_{n-1}, \; \ldots \; a_0]^T \end{aligned}$$

In order to proceed, we rewrite (3.62) as

$$\begin{aligned} \dot{x} &= Ax + F(y,w)^T\theta + D(t) \\ y &= e_1^T x \end{aligned} \tag{3.63}$$

where

$$F(y,w)^T = \left[\begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} w, \; -I_n y \right], \tag{3.64}$$

and the $p = n + m + 1-$dimensional parameter vector θ is defined by

$$\theta = \begin{bmatrix} b \\ a \end{bmatrix}. \tag{3.65}$$

For state estimation, by following the standard procedures and similar filters as in [30], we can obtain

$$\begin{aligned}
\dot{\lambda} &= A_0\lambda + e_n w && (3.66)\\
\dot{\eta} &= A_0\eta + e_n y && (3.67)\\
\Omega^T &= [v_m, \ldots, v_1, v_0, \Xi] && (3.68)\\
v_j &= A_0^j\lambda, \qquad j = 0, \ldots, m && (3.69)\\
\Xi &= -[A_0^{n-1}\eta, \ldots, A_0\eta, \eta] && (3.70)\\
\xi &= -A_0^n\eta && (3.71)
\end{aligned}$$

where the vector $k = [k_1, \ldots, k_n]^T$ is chosen so that the matrix $A_o = A - ke_1^T$ is Hurwitz. Hence there exists a P such that $PA_0 + A_0P^T = -2I,\ P = P^T > 0$. With these designed filters our state estimate is

$$\hat{x} = \xi + \Omega^T\theta \tag{3.72}$$

and the state estimation error $\epsilon = x - \hat{x}$ satisfies

$$\dot{\epsilon} = A_0\epsilon + D(t) \tag{3.73}$$

Let $V_\epsilon = \epsilon^T P\epsilon$. It can be shown that

$$\begin{aligned}
\dot{V}_\epsilon &= \epsilon^T(PA_0 + A_0^T P)\epsilon + 2\epsilon^T PD(t)\\
&\leq -\epsilon^T\epsilon + \| PD(t) \|^2 && (3.74)
\end{aligned}$$

Then system (3.63) can be expressed as

$$\dot{y} = b_m v_{m,2} + \xi_2 + \bar{\delta}^T \theta + \epsilon_2 \tag{3.75}$$

$$\dot{v}_{m,i} = v_{m,i+1} - k_i v_{m,1}, \qquad i = 2, \ldots, \rho - 1 \tag{3.76}$$

$$\dot{v}_{m,\rho} = v_{m,\rho+1} - k_\rho v_{m,1} + w \tag{3.77}$$

where

$$\delta = [v_{m,2}, v_{m-1,2}, \ldots, v_{0,2}, \Xi_{(2)} - y e_1^T]^T \tag{3.78}$$

$$\bar{\delta} = [0, v_{m-1,2}, \ldots, v_{0,2}, \Xi_{(2)} - y e_1^T]^T \tag{3.79}$$

and $v_{i,2}, \epsilon_2, \xi_2$ denote the second entries of v_i, ϵ, ξ respectively. All of its states are available for feedback.

3.3.3 Design of Adaptive Controllers

(a) Design Procedure

As usual in backstepping approach, the following change of coordinates is made.

$$z_1 = y - y_r \tag{3.80}$$

$$z_i = v_{m,i} - \hat{\vartheta} y_r^{(i-1)} - \alpha_{i-1}, \ i = 2, 3, \ldots, \rho \tag{3.81}$$

where $\hat{\vartheta}$ is an estimate of $\eta = 1/b_m$ and α_{i-1} is the virtual control at the *ith step* and will be determined in later discussion.

To illustrate the backstepping procedures, only the first and the last steps of the design, i.e. *steps 1 and n* below, are elaborated in details.

- *Step* 1: We start with the equation for the tracking error z_1 obtained from (3.63) and (3.80) that

$$\dot{z}_1 = b_m v_{m,2} + \xi_2 + \bar{\delta}^T\theta + \epsilon_2 - \dot{y}_r \tag{3.82}$$

By substituting (3.81) for $i = 2$ into (3.82) and using $\tilde{\vartheta} = \frac{1}{b_m} - \frac{1}{\hat{b}_m}$, we get

$$\dot{z}_1 = b_m\alpha_1 + \xi_2 + \bar{\delta}^T\theta + \epsilon_2 - b_m\tilde{\vartheta}\dot{y}_r + b_m z_2 \tag{3.83}$$

We design the virtual control law α_1 as

$$\alpha_1 = \hat{\vartheta}\bar{\alpha}_1 \tag{3.84}$$

$$\bar{\alpha}_1 = -c_1 z_1 - d_1 z_1 - \xi_2 - \bar{\delta}^T\hat{\theta} \tag{3.85}$$

where c_1 and d_1 are positive design parameters, $\hat{\theta}$ is the estimate of θ. From (3.83) and (3.84) we have

$$\begin{aligned}\dot{z}_1 &= -c_1 z_1 - d_1 z_1 + \epsilon_2 + \bar{\delta}^T\tilde{\theta} - b_m(\dot{y}_r + \bar{\alpha}_1)\tilde{\vartheta} + b_m z_2 \\ &= -(c_1 + d_1)z_1 + \epsilon_2 + (\delta - \hat{\vartheta}(\dot{y}_r + \bar{\alpha}_1)e_1)^T\tilde{\theta} - b_m(\dot{y}_r + \bar{\alpha}_1)\tilde{\vartheta} + \hat{b}_m z_2\end{aligned} \tag{3.86}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, we have

$$b_m\alpha_1 = b_m\hat{\vartheta}\bar{\alpha}_1 = \bar{\alpha}_1 - b_m\tilde{\vartheta}\bar{\alpha}_1 \tag{3.87}$$

$$\begin{aligned}\bar{\delta}^T\tilde{\theta} + b_m z_2 &= \bar{\delta}^T\tilde{\theta} + \tilde{b}_m z_2 + \hat{b}_m z_2 \\ &= \bar{\delta}^T\tilde{\theta} + (v_{m,2} - \hat{\vartheta}\dot{y}_r - \alpha_1)e_1^T\tilde{\theta} + \hat{b}_m z_2 \\ &= (\delta - \hat{\vartheta}(\dot{y}_r + \bar{\alpha}_1)e_1)^T\tilde{\theta} + \hat{b}_m z_2\end{aligned} \tag{3.88}$$

We consider the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{|b_m|}{2\gamma}\tilde{\vartheta}^2 + \frac{1}{2d_1}V_\epsilon \tag{3.89}$$

where Γ is a positive definite design matrix and γ is a positive design parameter. We examine the derivative of V_1

$$\begin{aligned}\dot{V}_1 &\leq z_1\dot{z}_1 - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \frac{|b_m|}{\gamma}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{1}{2d_1}\epsilon^T\epsilon + \frac{1}{2d_1} \parallel PD(t) \parallel^2 \\ &\leq -c_1z_1^2 + \hat{b}_m z_1 z_2 - d_1 z_1^2 + z_1\epsilon_2 - |b_m|\tilde{\vartheta}\frac{1}{\gamma}[\gamma\text{sign}(b_m)(\dot{y}_r + \bar{\alpha}_1)z_1 + \dot{\hat{\vartheta}}] \\ &\quad +\tilde{\theta}^T\Gamma^{-1}[\Gamma(\delta - \hat{\vartheta}(\dot{y}_r + \bar{\alpha}_1)e_1)z_1 - \dot{\hat{\theta}}] - \frac{1}{2d_1}\epsilon^T\epsilon + \frac{1}{2d_1} \parallel PD(t) \parallel^2\end{aligned} \tag{3.90}$$

Now we choose

$$\dot{\hat{\vartheta}} = -\gamma\text{sign}(b_m)(\dot{y}_r + \bar{\alpha}_1)z_1 - \gamma l_e(\hat{\vartheta} - e_0) \tag{3.91}$$

$$\tau_1 = (\delta - \hat{\vartheta}(\dot{y}_r + \bar{\alpha}_1)e_1)z_1 \tag{3.92}$$

where l_e, e_0 are positive design constants.

From the choice, the following useful property can be obtained:

$$\begin{aligned}l_e\tilde{\vartheta}(\hat{\vartheta} - e_0) &= -l_e(\hat{\vartheta} - e)(\frac{1}{2}(\hat{\vartheta} - e) + \frac{1}{2}(\hat{\vartheta} + e) - e_0) \\ &\quad -\frac{1}{2}l_e\tilde{\vartheta}^2 + \frac{1}{2}l_e(e - e_0)^2\end{aligned} \tag{3.93}$$

Then the following derivation for the derivative of V_1 can be carried out by using (3.91)-(3.93)

$$\begin{aligned}\dot{V}_1 &\leq -c_1z_1^2 + \hat{b}_m z_1 z_2 - \frac{|b_m|}{2}l_e\tilde{\vartheta}^2 - \frac{1}{4d_1}\epsilon^T\epsilon + \frac{|b_m|}{2}l_e(e - e_0)^2 \\ &\quad +\tilde{\theta}^T(\tau_1 - \Gamma^{-1}\dot{\hat{\theta}}) + \frac{1}{2d_1} \parallel PD(t) \parallel^2\end{aligned} \tag{3.94}$$

Remark 3.6 *Note that a new term $\gamma l_e(\hat{\vartheta} - e_0)$ is introduced in the parameter update law (3.91) compared with the traditional estimator using backstepping. This term is used to mitigate the backlash effect for system stability as shown in later discussion.*

- *Step i ($i = 2, \ldots, \rho$)*: Choose virtual control laws

$$\alpha_2 = -\hat{b}_m z_1 - \left[c_2 + d_2\left(\frac{\partial \alpha_1}{\partial y}\right)^2\right] z_2 + \beta_2 + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2 + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma l_\theta (\hat{\theta} - \theta_0) \quad (3.95)$$

$$\alpha_i = -z_{i-1} - \left[c_i + d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2\right] z_i + \beta_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma l_\theta (\hat{\theta} - \theta_0) - \left(\sum_{k=2}^{i-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\right) \Gamma \frac{\partial \alpha_{i-1}}{\partial y} \delta \quad (3.96)$$

where $c_i, i = 3, \ldots, \rho$ are positive design parameters, and

$$\tau_i = \tau_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y} \delta z_i \quad (3.97)$$

$$\beta_i = \frac{\partial \alpha_{i-1}}{\partial y}(\xi_2 + \delta^T \hat{\theta}) + \frac{\partial \alpha_{i-1}}{\partial \eta}(A_0 \eta + e_n y) + k_i v_{m,1} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} + \left(y_r^{(i-1)} + \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}}\right) \dot{\hat{\vartheta}} + \sum_{j=1}^{m+i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j}(-k_j \lambda_1 + \lambda_{j+1}) \quad (3.98)$$

Then the adaptive controller and parameter update law are finally given by

$$w = \alpha_\rho - v_{m,\rho+1} + \hat{\vartheta} y_r^{(\rho)} \quad (3.99)$$

$$\dot{\hat{\theta}} = \Gamma \tau_\rho + \Gamma l_\theta (\hat{\theta} - \theta_0) \quad (3.100)$$

where l_θ and θ_0 are positive design constants.

Remark 3.7 *Again note that $\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma l_\theta (\hat{\theta} - \theta_0)$ and $\Gamma l_\theta (\hat{\theta} - \theta_0)$ are added in the virtual control (3.96) and in the parameter update law $\dot{\hat{\theta}}$, respectively. These terms are employed to ensure system stability and its performance in the presence of backlash effects as shown below.*

(b) Stability Analysis

We define the final Lyapunov function V_ρ as

$$V_\rho = \sum_{i=1}^{\rho} \frac{1}{2} z_i^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{|b_m|}{2\gamma}\tilde{\vartheta}^2 + \sum_{i=1}^{\rho} \frac{1}{2d_i} V_\epsilon \tag{3.101}$$

Note that

$$\Gamma\tau_{i-1} - \dot{\hat{\theta}} = \Gamma\tau_{i-1} - \Gamma\tau_i + \Gamma\tau_i - \dot{\hat{\theta}} = \Gamma\frac{\partial \alpha_{i-1}}{\partial y}\delta z_i + (\Gamma\tau_i - \dot{\hat{\theta}}) \tag{3.102}$$

$$l_\theta \tilde{\theta}^T(\hat{\theta} - \theta_0) \leq -\frac{1}{2} l_\theta \parallel \tilde{\theta} \parallel^2 + \frac{1}{2} l_\theta \parallel \theta - \theta_0 \parallel^2 \tag{3.103}$$

From (3.96) - (3.100), the derivative of the last Lyapunov function satisfies

$$\begin{aligned}
\dot{V}_\rho &= \sum_{i=1}^{\rho} z_i \dot{z}_i - \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} - \frac{|b_m|}{\gamma}\tilde{\vartheta}\dot{\hat{\vartheta}} + \sum_{i=1}^{\rho}\frac{1}{2d_i}\dot{V}_\epsilon \\
&\leq -\sum_{i=1}^{\rho} c_i z_i^2 - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma\tau_\rho) - \sum_{i=1}^{\rho}\frac{1}{4d_i}\epsilon^T\epsilon - \frac{|b_m|}{2} l_e \tilde{\vartheta}^2 + \frac{|b_m|}{2} l_e (e - e_0)^2 \\
&\quad + (\sum_{k=2}^{\rho} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}})[\Gamma\tau_\rho + \Gamma l_\theta(\hat{\theta} - \theta_0) - \dot{\hat{\theta}}] + \sum_{i=1}^{\rho}\frac{1}{2d_i} \parallel PD(t) \parallel^2 \\
&\leq -\sum_{i=1}^{\rho} c_i z_i^2 - \frac{1}{2} l_\theta \parallel \tilde{\theta} \parallel^2 + \frac{1}{2} l_\theta \parallel \theta - \theta_0 \parallel^2 + \sum_{i=1}^{\rho}\frac{1}{2d_i} \parallel PD(t) \parallel^2 \\
&\quad - \frac{|b_m|}{2} l_e \tilde{\vartheta}^2 + \frac{|b_m|}{2} l_e (e - e_0)^2 - \sum_{i=1}^{\rho}\frac{1}{4d_i}\epsilon^T\epsilon \\
&\leq -\sum_{i=1}^{\rho} c_i z_i^2 - \frac{|b_m|}{2} l_e \tilde{\vartheta}^2 - \frac{1}{2} l_\theta \parallel \tilde{\theta} \parallel^2 - \sum_{i=1}^{\rho}\frac{1}{4d_i}\epsilon^T\epsilon + M^*
\end{aligned} \tag{3.104}$$

where

$$M^* = M + \sum_{i=1}^{\rho}\frac{1}{2d_i} \parallel P \parallel^2 D_{max}^2 \tag{3.105}$$

$$M = \frac{|b_m|}{2} l_e (e - e_0)^2 + \frac{1}{2} l_\theta \parallel \theta - \theta_0 \parallel^2 \tag{3.106}$$

In (3.105), D_{max} denotes the bound of $D(t)$ which may not be available. Notice that

$$-\sum_{i=1}^{\rho} c_i z_i^2 - \frac{|b_m|}{2} l_e \tilde{\vartheta}^2 - \frac{1}{2} l_\theta \parallel \tilde{\theta} \parallel^2 \quad - \quad \sum_{i=1}^{\rho} \frac{1}{4d_i} \epsilon^T \epsilon \leq -f_- \bar{V}_\rho \tag{3.107}$$

and

$$V_\rho \;=\; \sum_{i=1}^{\rho} \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{|b_m|}{2\gamma} \tilde{\vartheta}^2 + \sum_{i=1}^{\rho} \frac{1}{2d_i} V_\epsilon \leq f_+ \bar{V}_\rho \tag{3.108}$$

where

$$\bar{V}_\rho \;=\; \sum_{i=1}^{\rho} z_i^2 + \tilde{\theta}^T \tilde{\theta} + \tilde{\vartheta}^2 + \sum_{i=1}^{\rho} \epsilon^T \epsilon \tag{3.109}$$

$$f_- \;=\; min\{c_i, \frac{|b_m|}{2} l_e, \frac{1}{2} l_\theta, \frac{1}{4d_i}\} \tag{3.110}$$

$$f_+ \;=\; max\{\frac{1}{2}, \frac{1}{2}\lambda_{max}(\Gamma), \frac{|b_m|}{2\gamma}, \frac{1}{2d_i}\lambda_{max}(P)\} \tag{3.111}$$

where $\lambda_{max}(P)$ and $\lambda_{max}(\Gamma)$ are the maximum eigenvalues of P and Γ, respectively. Therefore, from (3.104) we obtain

$$\dot{V}_\rho \;\leq -f^* V_\rho + M^* \tag{3.112}$$

By direct integrations of the differential inequality (3.112), we have

$$V_\rho \;\leq\; V_\rho(0) e^{-f^* t} + \frac{M^*}{f^*}(1 - e^{-f^* t}) \leq V_\rho(0) + \frac{M^*}{f^*} \tag{3.113}$$

where $f^* = f^- / f^+$. This shows that V_ρ is uniformly bounded. Thus $z_i, \hat{\vartheta}, \hat{\theta}$ and ϵ are bounded. Since z_1 and y_r are bounded, y is also bounded. Then from (3.66) and (3.67) we can show that λ, η and x are bounded as in [30]. Therefore boundedness of all signals in the system is ensured as formally stated in the following Theorem.

Theorem 3.3 *Consider the closed-loop adaptive system consisting of the plant (3.60) under Assumptions 1-4, the controller (3.99), the estimator (3.91), (3.100), and the filters (3.66) and (3.67). All the signals in the system are globally uniformly bounded.*

We now derive a bound for the vector $z(t)$ where $z(t) = [z_1, z_2, \ldots, z_\rho]^T$. Firstly, the following definitions are made.

$$c_0 = min_{1\le i\le \rho} c_i, \quad d_0 = \sum_{i=1}^{\rho} \frac{1}{2d_i} \tag{3.114}$$

$$\| z \|_{[0,T]} = \sqrt{\frac{1}{T}\int_0^T z(t)^2 dt} \tag{3.115}$$

Then from (3.104), we have

$$\dot{V}_\rho \le -c_0 \| z \|^2 + M^* \tag{3.116}$$

Integrating both sides, we obtain

$$\| z \|_{[0,T]} \le \frac{1}{c_0}[\frac{|V_\rho(0) - V_\rho(T)|}{T} + M + \frac{1}{T} d_0 \| P \|^2 \int_0^T D(t)^2 dt] \tag{3.117}$$

On the other hand, from (3.112), we have

$$\begin{aligned} \frac{|V_\rho(0) - V_\rho(T)|}{T} &\le \frac{1 - e^{-f^* T}}{T}(\frac{M}{f^*} + V_\rho(0)) + \frac{1}{T} d_0 \| P \|^2 \int_0^T e^{-f^*(T-t)} D(t)^2 dt \\ &\le M + f^* V_\rho(0) + \frac{\| P \|^2}{T} d_0 \int_0^T D(t)^2 dt, \quad \forall T \ge 0, \end{aligned} \tag{3.118}$$

where we have used the fact that $e^{-f^*(T-t)} \le 1$ and $\frac{1-e^{-f^*T}}{T} \le f^*$.
By setting $z_i(0) = 0$, the initial value of the Lyapunov function is

$$V_\rho(0) = \frac{1}{2} \| \tilde{\theta}(0) \|^2_{\Gamma^{-1}} + \frac{|b_m|}{2\gamma} |\tilde{\vartheta}(0)|^2 + d_0 |\epsilon(0)|^2_P \tag{3.119}$$

Using (3.110) and (3.111), the fact that $f^*/c_0 \leq 2$, the a bound resulting from (3.117) - (3.119) is given by

$$\begin{aligned} \| z \|_{[0,T]} \leq & \| \tilde{\theta}(0) \|^2_{\Gamma^{-1}} + \frac{|b_m|}{\gamma} |\tilde{\vartheta}(0)|^2 + 2d_0 |\epsilon(0)|^2_P + \frac{2}{Tc_0} d_0 \| P \|^2 \int_0^T D(t)^2 dt \\ & + \frac{1}{c_0} (|b_m| l_e (e - e_0)^2 + l_\theta \| \theta - \theta_0 \|^2) \end{aligned} \tag{3.120}$$

Remark 3.8 *Regarding the above bound, the following conclusions can be drawn:*

- *The transient performance in the sense of truncated norm given in (3.120) depends on the initial estimate errors* $\tilde{\theta}(0), \tilde{\vartheta}(0)$ *and* $\epsilon(0)$*. The closer the initial estimates to the true values, the better the transient performance.*
- *This bound can also be systematically reduced by increasing* Γ, γ, c_0 *and decreasing* d_0, l_e, l_θ*.*

3.4 Simulation Studies

3.4.1 Design Example 1: State Feedback Control

In this section, we illustrate the state feedback methodologies on the same example system in [43] which is described as:

$$\dot{x} = a \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + bu(t), \quad u(t) = B(\omega(t)) \tag{3.121}$$

where $u(t)$ represents the output of the the backlash described by (3.2). The actual parameter values are $b = 1$ and $a = 1$. Without control, i.e. $u(t) = 0$, (3.121) is unstable, because $\dot{x} = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} > 0$ for $x > 0$, and $\dot{x} < 0$ for $x < 0$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 12.5\text{sin}(2.3t)$ as in [43].

The two adaptive backstepping schemes are used.

In the simulation of Scheme I, the robust adaptive control law (3.17)-(3.21) was used, taking $c_1 = 2, \gamma = \Gamma = \eta = 0.4$. The initial values are chosen to $\hat{\vartheta}(0) =$

$0.8/3, \hat{a}(0) = 1.5, \hat{D}(0) = 2, x(0) = 1.05$ and $w(0) = 0$ which are the same as in [43]. The simulation results presented in the Figure 3.3 and Figure 3.4 are system tracking error and input. The effectiveness of adaptive Scheme I is demonstrated by the fact that the tracking error is reduced to zero after a few periods of the reference input as shown in Figure 3.3. Figure 3.5 and 3.6 show the tracking error with different design parameters c_1 and γ, Γ, η. Clearly, the transient and tracking performance of tracking error are improved by increasing the parameters c_1 and γ, Γ, η respectively.

In the simulation of Scheme II by using the robust adaptive control law (3.48)-(3.52), we choose $c_1, \gamma, \eta, \Gamma$ and the initial values to be same as above and $\delta_1 = 0.1$. The simulation results presented in the Figure 3.7 and Figure 3.8 are system tracking error and input. The effectiveness of adaptive Scheme II is also demonstrated by the fact that the tracking error is reduced to $\delta_1 = 0.1$ after a few periods of the reference input as shown in Figure 3.7.

As a conclusion, all the results verify our theoretical findings and show the effectiveness of the control schemes.

3.4.2 Design Example 2: State Feedback Control

In this section, we illustrate the above methodology for state feedback control of the system described as:

$$\ddot{x} = a_1 \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} - a_2(\dot{x}^2 + 2x)\sin(\dot{x}) - 0.5a_3 x \sin(3t) + bu(t) \tag{3.122}$$

where $u(t)$ represents the output of the backlash nonlinearity (3.2). The actual parameter values are $b = 1$ and $a_1 = a_2 = a_3 = 1$. The parameters of the backlash at $\alpha = 1, c = 3, B_1 = 0.3$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 5\sin(t)$.

In the simulation of Scheme I, we take $c_1 = 2, c_2 = 2, \gamma = 0.8, \Gamma = 0.8I_3, \eta = 0.8$.

The initial values are chosen as follows: $\hat{\eta}(0) = 0.25, \hat{a}(0) = [1.5\ 1\ 1]^T, \hat{D}(0) = 2$,$x(0) = [1.05,\ 1]^T$ and $v(0) = 0$. The simulation results presented in the Figure 3.9 and Figure 3.10 are system tracking error and input. The effectiveness of adaptive Scheme I is demonstrated by the fact that the tracking error is reduced to zero after a few periods of the reference input as shown in Figure 3.9. The chattering phenomena in Figure 3.10 is caused by the *sign* function used in the controller. It can be avoided by adaptive Scheme II. Figure 3.11 and Figure 3.12 show the tracking error with different design parameters c_1, c_2 and γ, Γ, η. Clearly, the transient and tracking performance of tracking error are improved by increasing the parameters c_1, c_2 and γ, Γ, η respectively.

In the simulation of Scheme II, we choose $c_1, \gamma, \eta, \Gamma$ and the initial values to be same as above and $\delta_1 = 0.06$. The simulation results presented in the Figure 3.13 and Figure 3.14 are system tracking error and input. In Figure 3.13 the tracking error is reduced to $\delta_1 = 0.06$ after a few periods of the reference input. The control input w is bounded and has no chattering problem as shown in Figure 3.14.

As a conclusion, all the results verify our theoretical findings and show the effectiveness of the control schemes.

3.4.3 Design Example 3: Output Feedback Control

We illustrate the output feedback method on a simple linear systems

$$y(s) = \frac{\bar{b}}{s^2 + as} u(s) \tag{3.123}$$

$$u(t) = B(w(t)) \tag{3.124}$$

where $\bar{b} = 1, a = 1.2$, $B(w)$ is the backlash described by (3.2) with parameters $\alpha = 1, c = 3.1635, B_1 = 0.345$. These parameters are not need to be known in the controller design. The objective is to control the system output y to follow a desired trajectory $y_r(t) = 2\sin(2t)$. The filters from (3.66) and (3.67) are implemented as

$$\dot{\eta} = A_0\eta + e_2 y \tag{3.125}$$

$$\dot{\lambda} = A_0\lambda + e_2 w \tag{3.126}$$

$$\Xi = -[A_0\eta, \eta], \quad \xi = -A_0^2\eta \quad v = \lambda \tag{3.127}$$

$$A_0 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \tag{3.128}$$

The adaptive control laws α_1 (3.84), $w(t)$ (3.99), and parameter update laws $\hat{\vartheta}$ (3.91), $\hat{\theta}$ (3.100) were used, where $\hat{\vartheta}$ and $\hat{\theta}$ are estimates of $e = 1/\bar{b}c$ and $\theta = [\bar{b}c, a]^T$, respectively. The design parameters are chosen as $c_1 = c_2 = 5, d_1 = d_2 = 0.1, \gamma = 2, \Gamma = I_2, l_e = 0.1, l_\theta = 0.1, k_1 = 6, k_2 = 8$. The initials are set $y(0) = 1, \hat{\vartheta}(0) = 0.2, \hat{\theta}(0) = [2, 0.5]^T$. The simulation results presented in the Figure 3.15 shows the system output y and the desired trajectory signal. Figure 3.16 shows the control signal $w(t)$. Figure 3.17 and Figure 3.18 show the tracking error with different design parameters c_1, c_2 and γ, Γ. Clearly, the transient and tracking performance of tracking error are improved by increasing the parameters c_1, c_2 and γ, Γ respectively. These simulation results verify that our proposed scheme is effective to cope with backlash nonlinearity.

3.4.4 Design Example 4: Valve control Mechanism

In this section, we illustrate our proposed scheme on valve control mechanism shown in Figure 3.1 as in chapter 2, where the backlash is in the control inflow $u = B(v)$ and the output is the liquid level h. The transfer function $h(p)$ can be expressed as

$$h(p) = G(p)(u(p) - d(p)) = \frac{k}{p}(u(p) - d(p)) \tag{3.129}$$

where d is the uncontrolled outflow which is treated as a disturbance and $p = \frac{d}{dt}$. The backlash is expressed as follows

$$\frac{du}{dt} = \alpha|\frac{dv}{dt}|(cv - u) + B_1\frac{dv}{dt} \tag{3.130}$$

where α, c and B_1 are constants. The true parameters are set as $k = 2, \alpha = 1, c = 3, B_1 = 0.2$. The objective is to control the liquid level to $2m$.

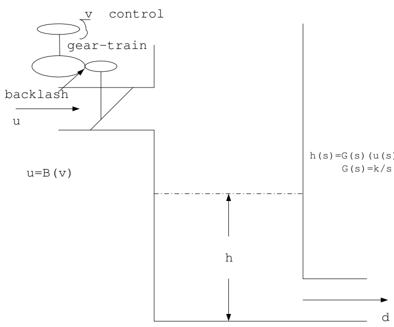


Figure 3.1: Backlash in the valve control mechanism of a liquid tank.

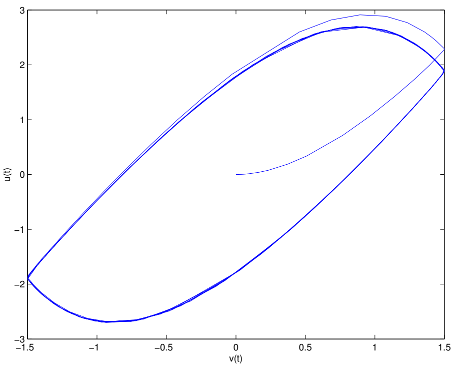


Figure 3.2: Input backlash.

The adaptive control law is designed by Scheme II as follows

$$v = \hat{\vartheta}\bar{w} \tag{3.131}$$
$$\bar{w} = -c_1(|z_1| - \delta_1)sg_1 - sg_1\hat{D} + y_r \tag{3.132}$$
$$\dot{\hat{\vartheta}} = -\gamma\bar{w}(|z_1| - \delta_1)f_1 sg_1 \tag{3.133}$$
$$\dot{\hat{D}} = \eta(|z_1| - \delta_1)f_1 \tag{3.134}$$

where $z_1 = h - y_r, y_r = 2$, $\hat{D}$ is an estimate of D which is the bound of $kd_1(t) - kd(t)$. In the simulation, the design parameters are chosen as $c_1 = 2, \gamma = 1, \eta = 2, \delta_1 = 0.01$ and the initial value is chosen as $h(0) = 3, \hat{\vartheta}(0) = 0.15$. The disturbance is selected as $\sin(2t)$. Figure 3.2 is the response of input backlash. The simulation results with the controller designed by proposed Scheme II and the scheme in [43] presented in Figure 3.19 to Figure 3.22 show that the tracking error of controlled liquid level and the input control using the controller with scheme in [43] and with our proposed scheme II. It is remarkable that the tracking performance of our proposed scheme is better than that of the scheme in [43].

3.5 Summary

In this chapter, we present two types of robust adaptive backstepping control algorithms: state feedback control of a class of nonlinear system with unknown backlash and output feedback control of a class of linear system with unknown backlash.

For state feedback control, two backstepping adaptive controller design schemes are developed. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. The developed backstepping controls do not require the model parameters within known intervals and the knowledge on the bound of 'disturbance-like' term is not required. Besides showing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters.

For output feedback control, an adaptive control scheme with certain modifications to the existing backstepping control design is proposed to achieve tracking. For the implementation of the controller, no a priori knowledge on the bounds of all the unknown parameters and the backlash effect is required. It is shown that adaptive control system is global stable in the sense that all the signals are bounded. Also a truncated L_2 bound is derived for the tracking error as a function of the design parameters. Simulation results verify the effectiveness of the proposed schemes.

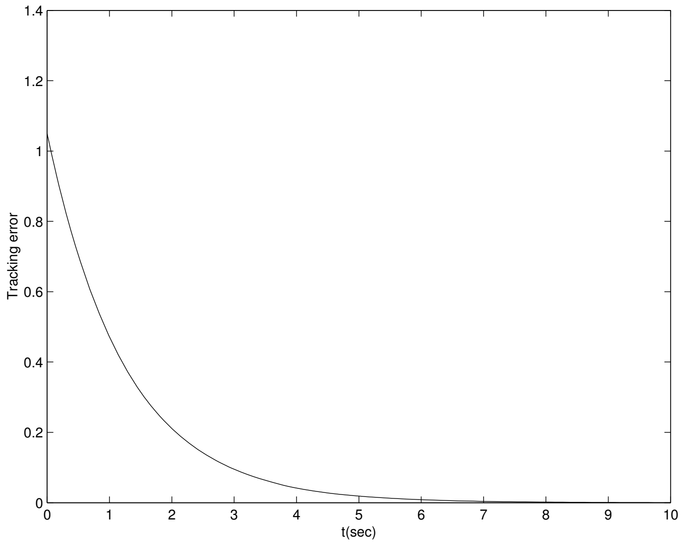


Figure 3.3: Tracking error-Control Scheme I.

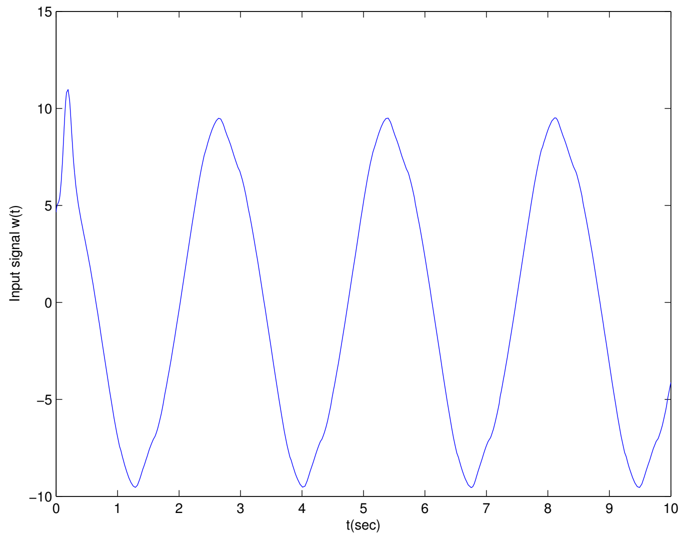


Figure 3.4: Control signal $w(t)$-Control Scheme I.

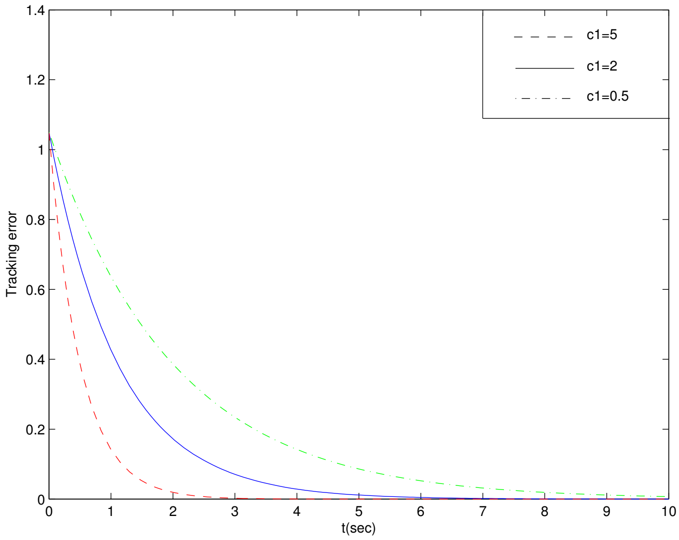


Figure 3.5: Comparison of tracking error with different parameter c_1.

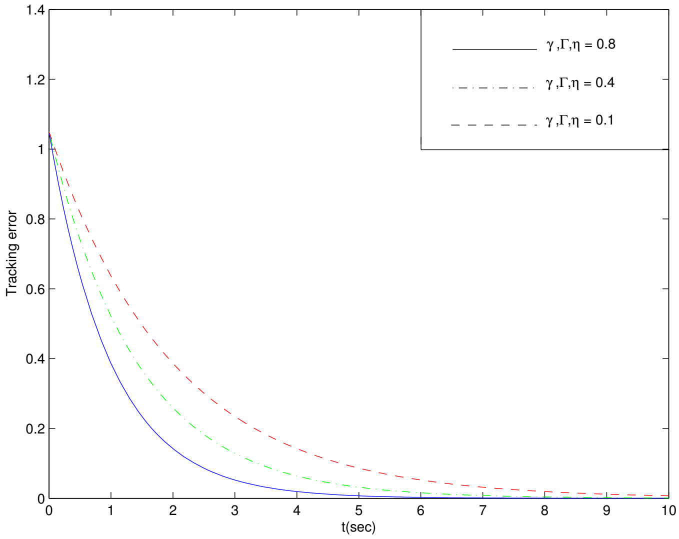


Figure 3.6: Comparison of tracking error with different parameters γ, Γ, η.

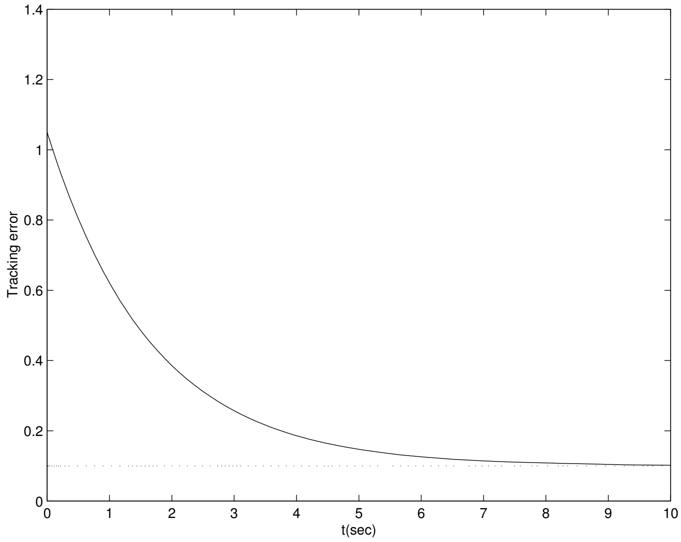


Figure 3.7: Tracking error-Control Scheme II.

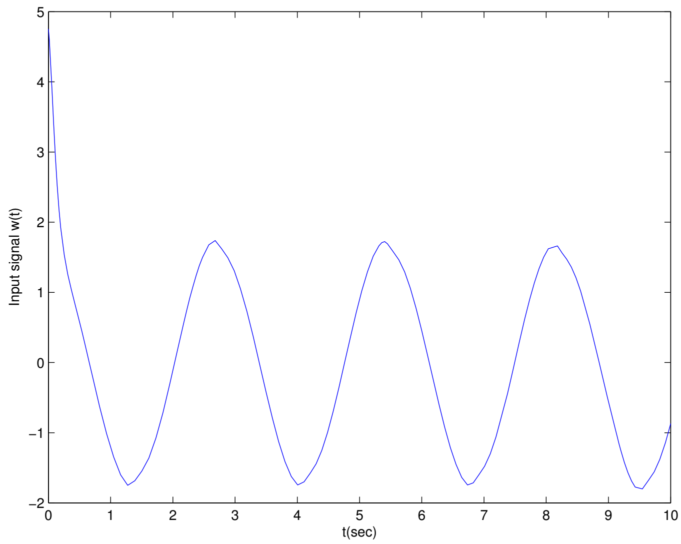


Figure 3.8: Control signal $w(t)$-Control Scheme II.

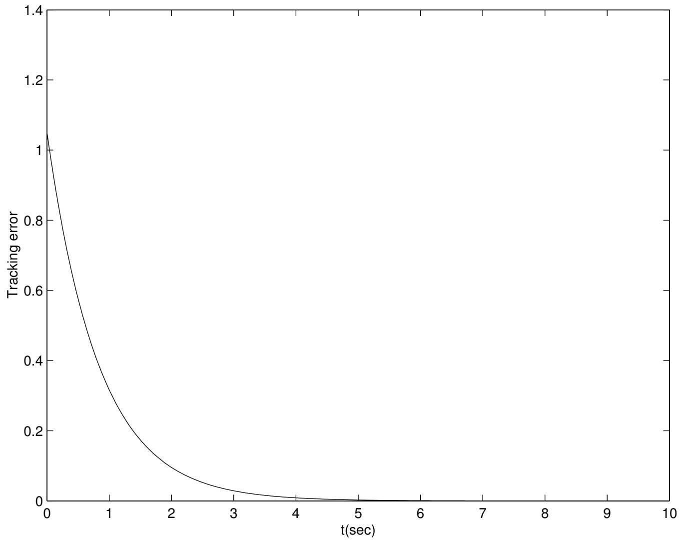


Figure 3.9: Tracking error-Scheme I

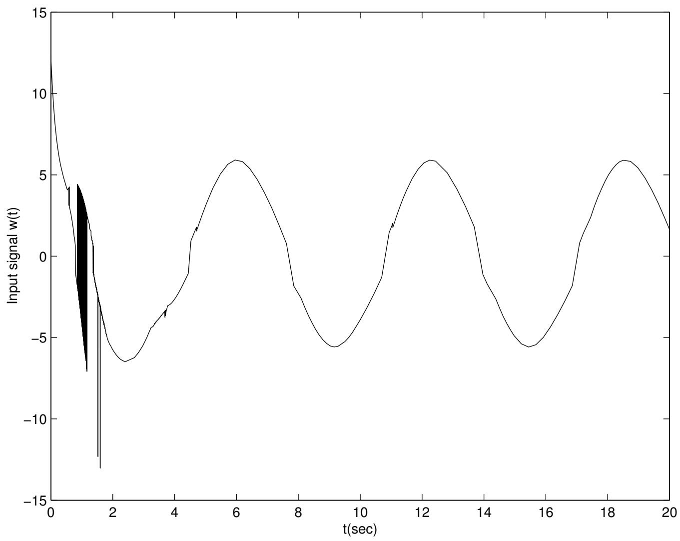


Figure 3.10: Control signal $w(t)$-Scheme I.

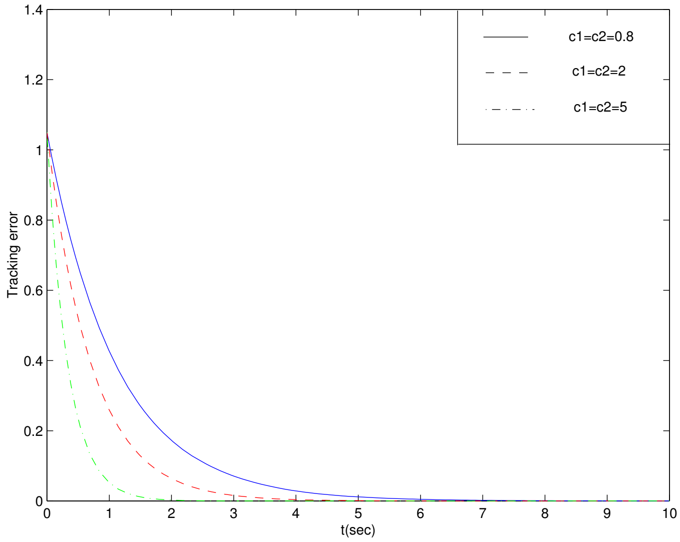


Figure 3.11: Comparison of tracking error with different parameter c_1, c_2.

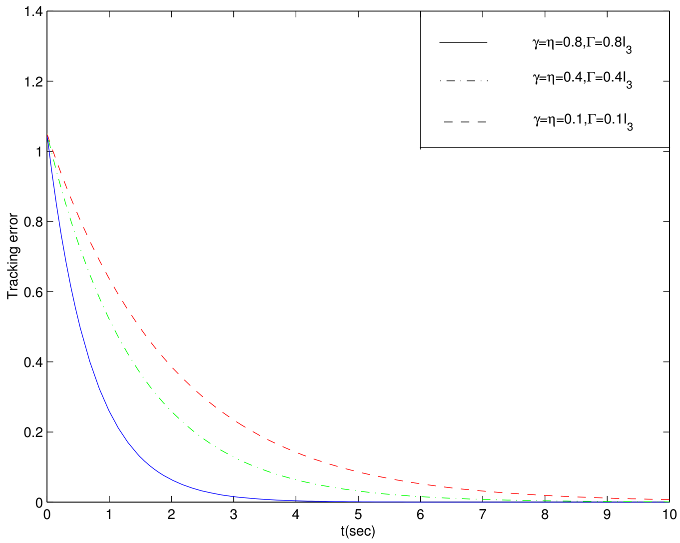


Figure 3.12: Comparison of tracking error with different parameters γ, Γ, η

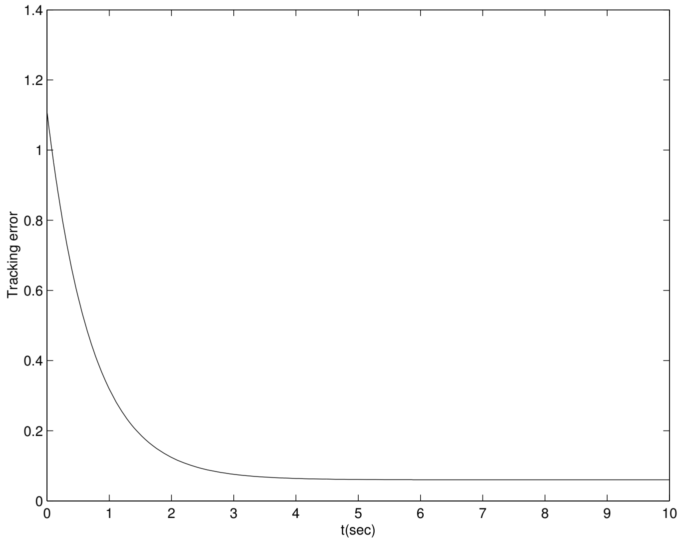


Figure 3.13: Tracking error-Scheme II.

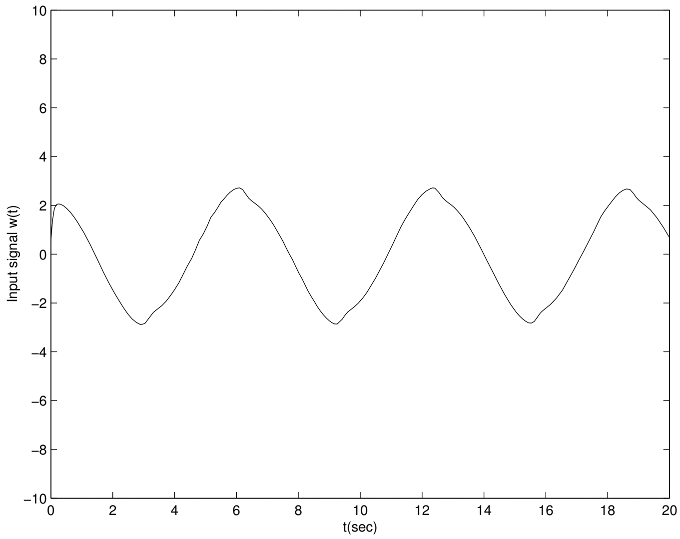


Figure 3.14: Control signal $v(t)$-Scheme II.

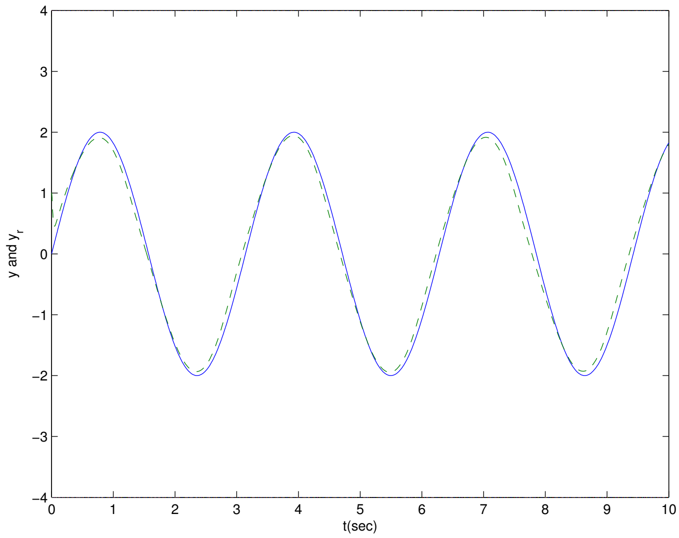


Figure 3.15: Output y(dashed) and trajectory y_r(solid line).

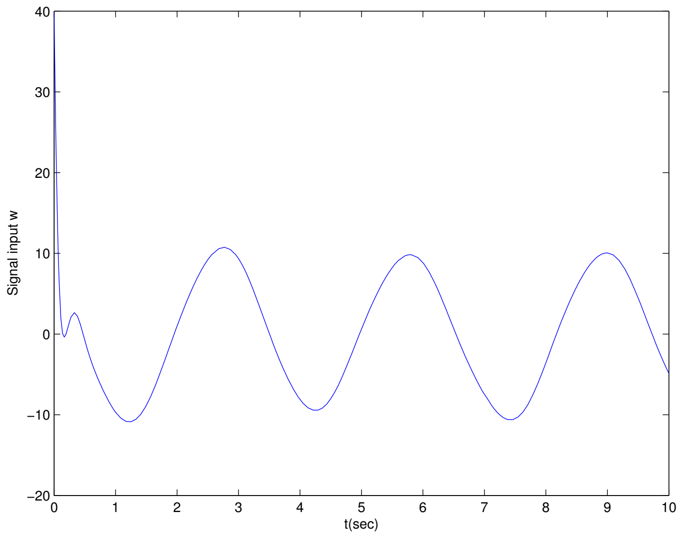


Figure 3.16: Control signal $w(t)$.

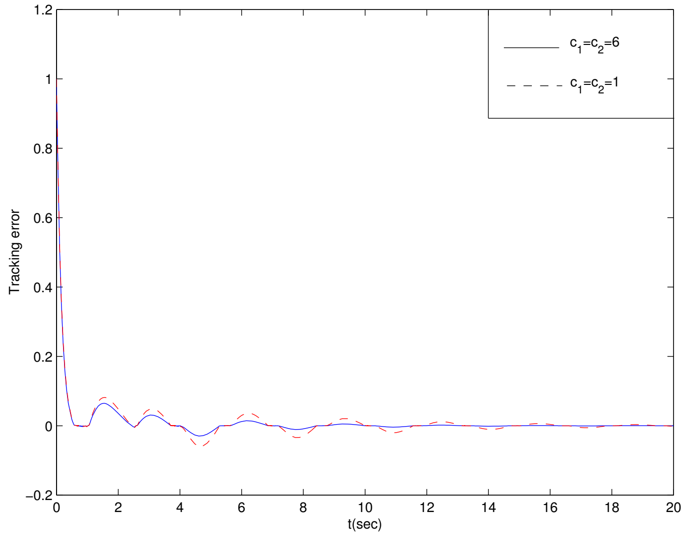


Figure 3.17: Comparison of tracking error with different parameter c_1, c_2.

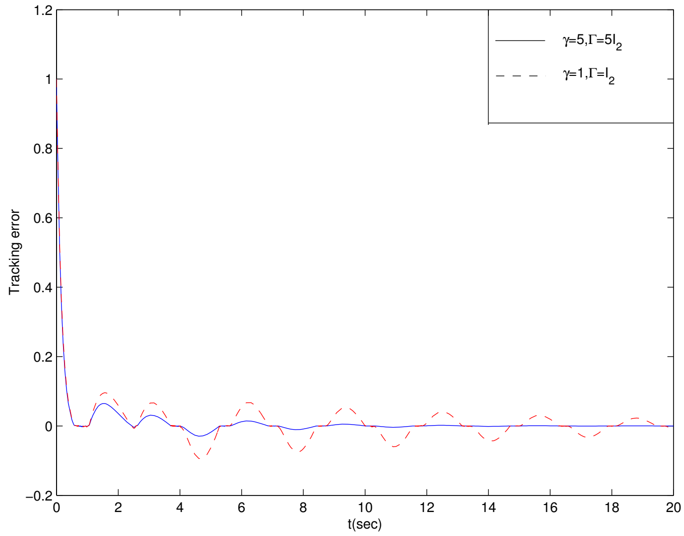


Figure 3.18: Comparison of tracking error with different parameters γ, Γ.

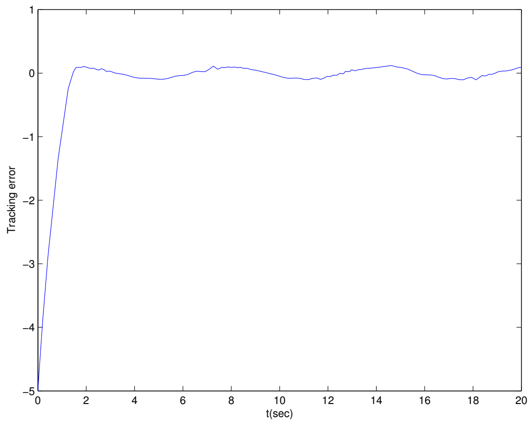


Figure 3.19: Tracking error.

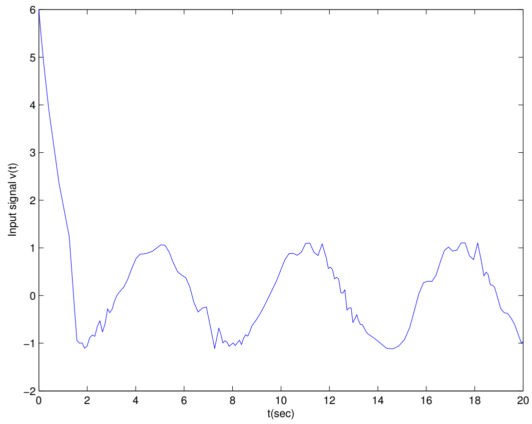


Figure 3.20: Control signal.

Controller designed by Scheme Scheme in [43]

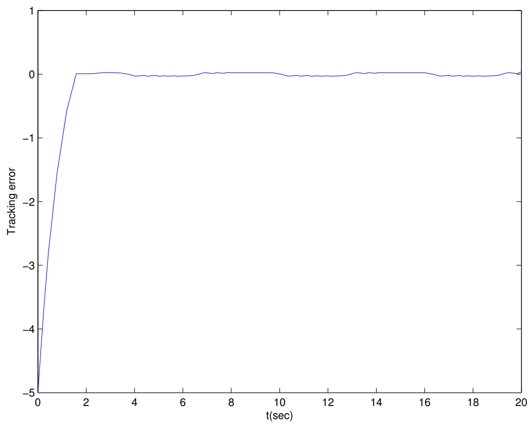


Figure 3.21: Tracking error.

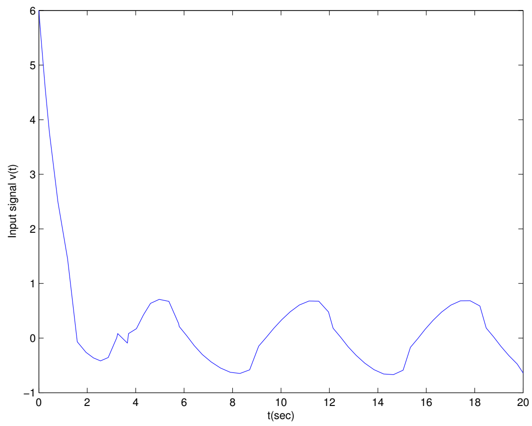


Figure 3.22: Control signal.

Controller designed by our proposed Scheme II

Chapter 4

Adaptive Control of Uncertain Nonlinear Systems with Dead-zone Nonlinearity

In this chapter, we present new adaptive schemes for uncertain nonlinear systems preceded by unknown dead-zone nonlinearity. Robust adaptive backstepping control algorithms are developed for state feedback tracking of a class of uncertain dynamic nonlinear systems preceded by unknown dead-zone nonlinearities, in the presence of bounded external disturbances. The new design for output feedback tracking is achieved by introducing a new smooth inverse function of the dead-zone and using it in the controller design with backstepping technique. For the design and implementation of the controllers, no knowledge is assumed on the unknown system parameters and also the dead-zone. It is shown that the proposed controllers not only can guarantee global stability, but also transient performance.

4.1 Introduction

Dead-zone, which can severely limit system performances, is one of the most important nonsmooth nonlinearities arisen in actuators, such as valves and DC servo motors and other devices. Therefore the effect of dead-zone should be taken into consideration in the design and analysis of control systems. In most practical motion systems, the dead-zone parameters are poorly known, and thus robust adaptive control techniques may be applied to design controllers. The study of adaptive control for systems with unknown dead-zone at the input was initiated in [50], where an adaptive scheme was proposed with full state measurement. An immediate method for the control of dead-zone is to construct an adaptive dead-zone inverse. This approach was used in [41, 42, 53, 73, 74, 52, 51], where the output of a dead-zone is measurable. In [41] a fuzzy pre-compensator was proposed in nonlinear industrial motion system. Such approaches promise to improve the tracking performance of motion system in presence of unknown dead-zones. An alternative approach based on sliding mode control is proposed in [74, 52]. In [51] and [57], an adaptive state feedback controller employees an adaptive inverse for a class of nonlinear systems. Dead-zone pre-compensation using neural network have also been used in feedback control systems [42], where the uncertain NN weights must be within a known compact set. With this, the error resulted from using NN to approximate system functions will be bounded with known bounds. This assumption makes the control design and system analysis simpler. In [118], a robust adaptive control scheme is developed for a class of nonlinear systems without using the dead-zone inverse, where the dead-zone slopes in the positive and negative region are the same and the unknown system parameters are inside a known compact set. For all approaches mentioned above, strictly speaking, only local stability is ensured in the sense that the initial values of the parameter estimates, which can be considered as part of system state variables, must be chosen from the compact set. If the true parameters are outside the set, system stability cannot be ensured. Also the developed schemes cannot ensure the transient performance due to their design methods.

In this chapter, we consider state feedback control and output feedback control of a class of nonlinear systems.

4.2 State Feedback Backstepping Control

In this section, we consider the same class of systems as in [118], i.e., a class of uncertain dynamic nonlinear systems preceded by unknown dead-zone nonlinearities, in the presence of bounded external disturbances. By using backstepping technique, robust adaptive backstepping control algorithms are developed. Unlike some existing control schemes for systems with dead-zone, the developed backstepping controllers do not require the uncertain parameters within known intervals. Also no knowledge is assumed on the bound of the 'disturbance-like' term, a combination of the external disturbances and a term separated from the dead-zone model. It is shown that the proposed controllers not only can guarantee global stability, but also transient performance.

4.2.1 Problem Statement

We consider the class of nonlinear systems with unknown dead-zone given as follows:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\big(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\big) = bu(v) + \bar{d}(t) \tag{4.1}$$

where Y_i are known continuous linear or nonlinear functions, $\bar{d}(t)$ denotes bounded external disturbances, parameters a_i are unknown constants and control gain b is unknown bounded constant, v is the control input, $u(v)$ denotes dead-zone nonlinearity described by

$$u = \begin{cases} m(v(t) - b_r) & v(t) \geq b_r \\ 0 & b_l < v(t) < b_r \\ m(v(t) - b_l) & v(t) \leq b_l \end{cases} \tag{4.2}$$

where $b_r \geq 0, b_l \leq 0$ and $m > 0$ are constants, v is the input and u is the output.For plant (4.1) with dead-zone nonlinearity, the $u(t)$ can be expressed as

$$u(t) = mv(t) - d_1(v(t)) \tag{4.3}$$

where

$$d_1(v(t)) = \begin{cases} -mb_r & v(t) \geq b_r \\ -mv(t) & b_l < v(t) < b_r \\ -mb_l & v(t) \leq b_l \end{cases} \tag{4.4}$$

It is clear that $d_1(v(t))$ is bounded.
From the structure (4.3) of model (4.2), (4.1) becomes

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\big(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\big) = \beta v(t) + d(t) \tag{4.5}$$

where $\beta = bm$ and $d(t) = bd_1(v(t)) + \bar{d}(t)$. The effect of $d(t)$ is due to both external disturbances and $bd_1(v(t))$. We call $d(t)$ a 'disturbance-like' term for simplicity of presentation and use D to denote its bound.
Now equation (4.5) is rewritten in the following form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= -\sum_{i=1}^{r} a_i Y_i\big(x_1(t), x_2(t), \ldots, x_{(n-1)}(t)\big) + \beta v(t) + d(t) \\ &= a^T Y + \beta v(t) + d(t) \end{aligned} \tag{4.6}$$

where $x_1 = x, x_2 = \dot{x}, \ldots, x_n = x^{(n-1)}$, $a = [-a_1, -a_2, \ldots, -a_r]^T$ and $Y = [Y_1, Y_2, \ldots, Y_r]^T$.

For the development of control laws, the following assumptions are made.

Assumption 1. The uncertain parameters b and m are such that $\beta > 0$.

Assumption 2. The desired trajectory $y_r(t)$ and its $(n-1)$th order derivatives are known and bounded.

The control objectives are to design backstepping adaptive control laws such that

• The closed loop is globally stable in sense that all the signals in the loop are uniformly ultimately bounded;

• The tracking error $x(t) - y_r(t)$ is adjustable during the transient period by an explicit choice of design parameters and $\lim_{t\to\infty} x(t) - y_r(t) = 0$ or $\lim_{t\to\infty} |x(t) - y_r(t)| - \delta_1 = 0$ for an arbitrary specified bound δ_1.

4.2.2 Controller Design

We develop two adaptive backstepping designs, where the controller designs are similar to the state feedback backstepping control of backlash hysteresis in Chapter 3. These schemes are now concisely summarized in the tables 4.2 and 4.2.

Table 4.1: Adaptive Backstepping Controller-Scheme I

Adaptive Control Laws:

$$\begin{aligned} \alpha_1 &= -c_1 z_1 && (4.7)\\ \alpha_i &= -c_i z_i - z_{i-1} + \dot{\alpha}_{i-1}(x_1, \ldots, x_{i-1}, y_r, \ldots, y_r^{(i-1)}), i = 2, \ldots, n && (4.8)\\ \bar{v} &= -c_n z_n - z_{n-1} - \hat{a}^T Y - \text{sign}(z_n)\hat{D} + y_r^{(n)} + \dot{\alpha}_{n-1} &&\\ v &= \hat{e}\bar{v} && (4.9) \end{aligned}$$

Parameter Update Laws:

$$\begin{aligned} \dot{\hat{e}} &= -\gamma \bar{v} z_n && (4.10)\\ \dot{\hat{a}} &= \Gamma Y z_n && (4.11)\\ \dot{\hat{D}} &= \eta |z_n| && (4.12) \end{aligned}$$

where $c_i, i = 1, \ldots, n$, are positive design parameters, γ and η are two positive design parameters, Γ is a positive definite matrix, $\hat{e}$, $\hat{a}$ and $\hat{D}$ are estimates of $e = 1/\beta$, a and D.

Table 4.2: Adaptive Backstepping Controller-Scheme II

Functions:

$$sg_i(z_i) = \begin{cases} \dfrac{z_i}{|z_i|} & |z_i| \geq \delta_i \\ \dfrac{z_i^{(2q+1)}}{(\delta_i^2 - z_i^2)^{n-i+2} + |z_i|^{(2q+1)}} & |z_i| < \delta_i \end{cases} \quad i = 1, ..., n \quad (4.13)$$

$$f_i(z_i) = \begin{cases} 1 & |z_i| \geq \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \quad i = 1, \ldots, n \quad (4.14)$$

Adaptive Control Laws:

$$\alpha_1 = -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) - (\delta_2 + 1)sg_1(z_1) \quad (4.15)$$

$$\alpha_2 = -(c_2 + \frac{5}{4})(|z_2| - \delta_2)^{n-1} sg_2(z_2) + \dot{\alpha}_1 - (\delta_3 + 1)sg_2(z_2) \quad (4.16)$$

$$\alpha_i = -(c_i + \frac{5}{4})(|z_i| - \delta_i)^{n-i+1} sg_i(z_i) + \dot{\alpha}_{i-1} - (\delta_{i+1} + 1)sg_i(z_i) \quad (4.17)$$

$$(i = 3, \ldots, n)$$

$$\bar{v} = -(c_n + 1)(|z_n| - \delta_n)sg_n(z_n) - \hat{a}^T Y - sg_n \hat{D} + y_r^{(n)} + \dot{\alpha}_{n-1}$$

$$v = \hat{e}\bar{v} \quad (4.18)$$

Parameter Update Laws:

$$\dot{\hat{e}} = -\gamma\bar{v}(|z_n| - \delta_n)f_n sg_n(z_n) \quad (4.19)$$

$$\dot{\hat{a}} = \Gamma Y(|z_n| - \delta_n)f_n sg_n(z_n) \quad (4.20)$$

$$\dot{\hat{D}} = \eta(|z_n| - \delta_n)f_n \quad (4.21)$$

where $\delta_i (i = 1, \ldots, n)$ are positive design parameters and $q = round\{(n-i+2)/2\}$, $round\{x\}$ means the element of x to the nearest integer.

Theorem 4.1 *Consider the uncertain nonlinear system (4.1) satisfying Assumptions 1-2. With the application of controller (4.9) and the parameter update laws*

(4.10) to (4.12), the following statements hold:

- *The resulting closed loop system is globally stable.*
- *The asymptotic tracking is achieved, i.e.,*

$$\lim_{t\to\infty}[x(t) - y_r(t)] = 0$$

- *The transient tracking error performance is given by*

$$\| x(t) - y_r(t) \|_2 \;\leq\; \frac{1}{\sqrt{c_1}}\Big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\Big)^{1/2}$$

Theorem 4.2 *Consider the uncertain nonlinear system (4.1) satisfying Assumptions 1-2. With the application of controller (4.18) and the parameter update laws (4.19) to (4.21), the following statements hold:*

- *The resulting closed loop system is globally stable.*
- *The tracking error converges to δ_1 asymptotically, i.e.,*

$$\lim_{t\to\infty}|x(t) - y_r(t)| = \delta_1, \quad |z_1| \geq \delta_1$$

- *The transient tracking error performance is given by*

$$\| |x(t) - y_r(t)| - \delta_1 \|_2 \;\leq\; c_1^{\frac{-1}{2n}}\Big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\Big)^{\frac{1}{2n}}$$

with $z_i(0) = \delta_i, i = 1, \ldots, n,$

Remark 4.1 *From the above two Theorems the following conclusions can be obtained:*

- *The transient performance depends on the initial estimate errors $\tilde{e}(0)$, $\tilde{a}(0)$, $\tilde{D}(0)$ and the explicit design parameters. The closer the initial estimates $\hat{e}(0), \hat{a}(0)$ and $\hat{D}(0)$ to the true values e, a and D, the better the transient performance.*
- *The bound for $\| x(t) - y_r(t) \|_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the*

transient performance by increasing the adaptation gains γ, η and Γ.

- *To improve the tracking error performance we can also increase the gain c_1. However, increasing c_1 will influence other performance such as $\| \dot{x} - \dot{y}_r \|_2$.*

4.3 Output Feedback Control Using Backstepping and Inverse Technique

In this section, we will address the output feedback control of similar class of nonlinear systems as in [43], [74] and [118], in the presence of unknown dead-zone actuator nonlinearity. We take the dead-zone into account in our controller design unlike in [74] and [118]. A new smooth inverse of the dead-zone will be introduced to compensate the effect of the dead-zone in controller design with backstepping approach. Such a smooth inverse can avoid chattering problems that may occur in the nonsmooth inverse approach proposed in [41], [49], [119] and [120]. The specific treatment of the dead-zone may bring performance improvement. As system output feedback is employed, a state observer is required. To obtain such an observer, a new parametrization of the state observer for the plant is proposed to include two sets of parameters: one from the dead-zone nonlinearity and the other from the plant. With our approach, a priori knowledge on system parameters and dead-zone nonlinearity is no longer needed. Besides showing global stability of the system, the transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters and thus our scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way.

4.3.1 System Model

We consider the class of uncertain nonlinear systems with unknown dead-zone nonlinearity. For completeness, the system model is given as follows:

$$x^{(n)}(t) + a_1 Y_1(x(t)) + a_2 Y_2(x(t)) + \cdots + a_r Y_r(x(t)) = bu \tag{4.22}$$

$$y = x_1, \quad u = DZ(v) \tag{4.23}$$

where Y_i are known continuous linear or nonlinear functions, parameters a_i and control gain b are unknown constants, $v(t)$ is the output from the controller, $u(t)$ is the input to the system and $y(t)$ is the system output. The actuator nonlinearity $DZ(v)$ is described as a dead-zone characteristic.

The control objective is to design an output feedback control law for $v(t)$ to ensure that all closed-loop signals are bounded and the plant output $y(t)$ tracks a given reference signal $y_r(t)$ under the following assumptions:

Assumption 1: The sign of b is known and the reference signal $y_r(t)$ and its first nth derivatives are known and bounded.

Assumption 2: The dead-zone parameters m_r and m_l satisfy $m_r \geq m_{r0}$ and $m_l \geq m_{l0}$, where m_{r0} and m_{l0} are two small positive constants.

4.3.2 Dead-zone Characteristic

Dead-zone, backlash, hysteresis, and piecewise-linear characteristics are typical examples of actuator nonlinearity. These nonlinear characteristics have break-points so that they are non-differentiable (nonsmooth) but can be parameterized in [49] and [57].

The parameterized model of the dead-zone characteristic $DZ(.)$ can be unified as in chapter 2.

$$u(t) = DZ(v(t)) = \begin{cases} m_r(v(t) - b_r) & v(t) \geq b_r \\ 0 & b_l < v(t) < b_r \\ m_l(v(t) - b_l) & v(t) \leq b_l \end{cases} \tag{4.24}$$

where $b_r \geq 0, b_l \leq 0$ and $m_r > 0, m_l > 0$ are constants. In general, the break-points $|b_r| \neq |b_l|$ and the slopes $m_r \neq m_l$.

The essence of compensating dead-zone effect is to employ a dead-zone inverse as shown in [41, 49, 119, 120]. In this paper, we propose a smooth inverse for the dead-zone as follows:

$$v(t) = DI(u(t)) = \frac{u(t) + m_r b_r}{m_r}\phi_r(u) + \frac{u(t) + m_l b_l}{m_l}\phi_l(u) \tag{4.25}$$

where $\phi_r(u)$ and $\phi_l(u)$ are smooth continuous indicator functions defined as

$$\phi_r(u) = \frac{e^{u/e_0}}{e^{u/e_0} + e^{-u/e_0}} \tag{4.26}$$

$$\phi_l(u) = \frac{e^{-u/e_0}}{e^{u/e_0} + e^{-u/e_0}} \tag{4.27}$$

Such an inverse is shown in Figure 4.1.

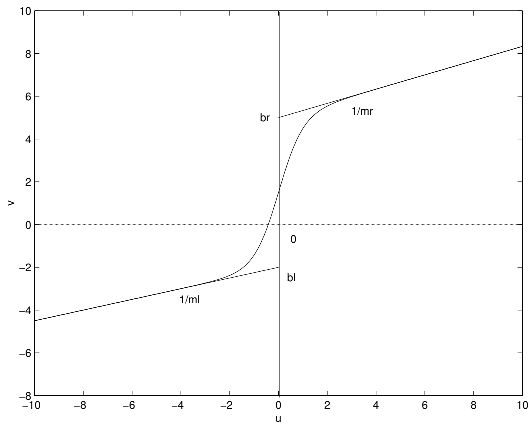


Figure 4.1: Dead-zone inverse.

As $e_0 \to 0$, $\phi_r(u)$ and $\phi_l(u)$ approaches the indicator functions defined in [49].

Remark 4.2 *Note that the functions $\phi_r(u)$ and $\phi_l(u)$ are continuous and differentiable. This is different from the inverse in [41], [119] and [49], where the inverse indicator functions are nonsmooth. The latter case may cause chattering phenomenon in the recursive backstepping control.*

To design adaptive controller for the system , we parameterize the dead-zone as

$$u(t) = -\theta^T \omega \tag{4.28}$$

where

$$\theta = [m_r, m_r b_r, m_l, m_l b_l]^T \tag{4.29}$$

$$\omega(t) = [-\sigma_r(t)v(t), \sigma_r(t), -\sigma_l(t)v(t), \sigma_l(t)]^T \tag{4.30}$$

$$\sigma_r(t) = \begin{cases} 1 & if\ u(t) > 0 \\ 0 & otherwise \end{cases} \tag{4.31}$$

$$\sigma_l(t) = \begin{cases} 1 & if\ u(t) < 0 \\ 0 & otherwise \end{cases} \tag{4.32}$$

As θ is unknown and ω is unavailable, the actual control input to the plant $u_d(t)$ is designed as

$$u_d(t) = -\hat{\theta}^T \hat{\omega}(t) \tag{4.33}$$

$$\hat{\theta} = [\widehat{m_r}, \widehat{m_r b_r}, \widehat{m_l}, \widehat{m_l b_l}]^T \tag{4.34}$$

$$\hat{\omega}(t) = [-\phi_r(v)v(t), \phi_l(v), -\phi_l(v)v(t), \phi_l(v)]^T \tag{4.35}$$

where $\hat{\theta}$ is an estimate of θ. Then corresponding control output $v(t)$ is given by

$$v(t) = \widehat{DI}(u_d(t)) = \frac{u_d(t) + \widehat{m_r b_r}}{\widehat{m_r}} \phi_r(u_d) + \frac{u_d(t) + \widehat{m_l b_l}}{\widehat{m_l}} \phi_l(u_d(t)) \tag{4.36}$$

The resulting error between u and u_d is

$$u(t) - u_d(t) = (\hat{\theta} - \theta)^T \hat{\omega}(t) + d_N(t) \tag{4.37}$$

where $d_N(t) = \theta^T \hat{\omega}(t) - u(t)$. The bound of $d_N(t)$ can be obtained as

$$\begin{aligned} |d_N(t)| &= |\theta^T \hat{\omega}(t) - u(t)| \\ &\leq \begin{cases} \frac{1}{2} e^{-1} |m_r - m_l| e_0 + \frac{|m_r b_r - m_l b_l|}{e^{2br/e_0} + 1} & v(t) \geq b_r \\ max\{|m_r b_r|, |m_l b_l|\} & b_l < v(t) < b_r \\ \frac{1}{2} e^{-1} |m_r - m_l| e_0 + \frac{|m_r b_r - m_l b_l|}{e^{-2b_l/e_0} + 1} & v(t) \leq b_l \end{cases} \end{aligned} \tag{4.38}$$

where we have used that $|v|e^{-|v|} \leq e^{-1}$. Note that when $b_l \leq v \leq b_r$ the bound of $d_N(t)$ decreases as e_0 increases, while outside this range the bound decreases as e_0 decreases. It has the desired properties that $d_N(t)$ is bounded for all $t \geq 0$ and $d_N(t)$ approaches to 0 as $\hat{\theta} \rightarrow \theta$ and $e_0 \rightarrow 0$. Figure 4.2 shows the characteristics of $d_N(t)$.

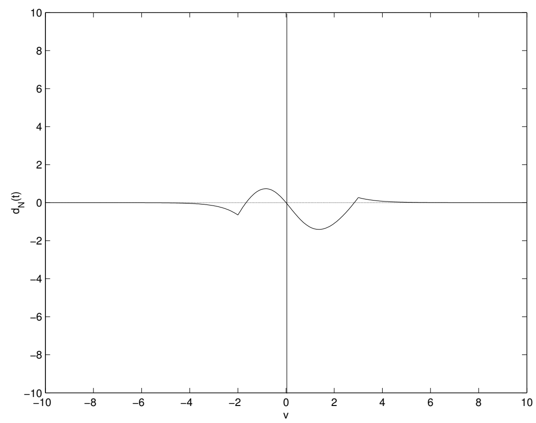


Figure 4.2: Approximation error $d_N(t)$.

4.3.3 State Observer

As we consider output feedback, a state observer is required. To design such an observer, we re-write plant equation (4.22) as

$$\dot{x} = Ax + aYe_n + bue_n \tag{4.39}$$

$$y = cx, \quad u = DZ(v) \tag{4.40}$$

where

$$A = \begin{bmatrix} 0 & \\ \vdots & I_{n-1} \\ 0 & \dots 0 \end{bmatrix}, \; a = \begin{bmatrix} -a_1 \\ \vdots \\ -a_r \end{bmatrix}, \; Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_r \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}^T \quad e_n = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}^T$$

To construct an observer for (4.39) and (4.40), we choose $k = [k_1, \dots, k_n]^T$ such that all eigenvalues of $A_0 = A - kc$ are at some desired stable locations. If the signal $u(t)$ were available then we would implement the following filters

$$\hat{x}(t) = \xi_0 - \sum_{i=1}^{r} a_i \xi_i + b\eta \tag{4.41}$$

$$\dot{\xi}_0 = A_0 \xi_0 + ky + \chi \tag{4.42}$$

$$\dot{\xi}_i = A_0 \xi_i + Y_i e_n, \; i = 1, \dots, r \tag{4.43}$$

$$\dot{\eta} = A_0 \eta + e_n u \tag{4.44}$$

where χ is a design signal specified later. It can be shown that the state estimation error $\epsilon = x(t) - \hat{x}(t)$ satisfies

$$\dot{\epsilon} = A_0 \epsilon - \chi \tag{4.45}$$

Hence, $lim_{t \to \infty} \epsilon(t) = 0$ exponentially if $\chi = 0$.

In our control problem, the signal $u(t)$ is not available. Thus the signal η in (4.44) needs to be re-parameterized. From (4.33) and (4.37), we know that

$$u(t) = -\theta^T \hat{\omega}(t) + d_N(t) \tag{4.46}$$

Let $p = \frac{d}{dt}$. With $\Delta(p) = det(pI - A_0)$, we express (4.44)

$$\eta(t) = [\eta_1(t), \eta_2(t), \ldots, \eta_n(t)]^T = [q_1(p), q_2(p), \ldots, q_n(p)]^T \frac{1}{\Delta(p)} u(t) \tag{4.47}$$

for some known polynomials $q_i(p), i = 1, \ldots, n$. Using (4.46) and (4.47), we obtain

$$\eta_i(t) = -\theta^T \hat{\omega}_i(t) + d_i(t) \tag{4.48}$$

$$\hat{\omega}_i(t) = \frac{q_i(p) I_4}{\Delta(p)} \hat{\omega}(t) \tag{4.49}$$

$$d_i(t) = \frac{q_i(p)}{\Delta(p)} d_N(t) \tag{4.50}$$

Based on (4.48), $\hat{\omega}_i$ is available for controller design in place of u. Denoting the second component of ξ_i as $\xi_{i2}, i = 0, \ldots, r$, we have

$$\hat{x}_2 = \xi_{02} - \sum_{i=1}^{r} a_i \xi_{i2} - b\theta^T \hat{\omega}_2(t) + bd_2(t) \tag{4.51}$$

$$\hat{\omega}_2(t) = \frac{(p + k_1) I_4}{p^n + k_1 p^{n-1} + \cdots + k_{n-1} p + k_n} \hat{\omega}(t) \tag{4.52}$$

The term $\theta^T \hat{\omega}_2^{(n-1)}$ renders a place to the signal u_d at the last step of the backstepping design.

4.3.4 Backstepping Design with Dead-zone Inverse

Design Procedure

As usual in backstepping approach, the following change of coordinates is made.

$$z_1 = y - y_r \tag{4.53}$$

$$z_i = -\hat{\theta}^T \hat{\omega}_2^{(i-2)} - \hat{e} y_r^{(i-1)} - \alpha_{i-1}, \ i = 2, 3, \dots, \rho \tag{4.54}$$

where $\hat{e}$ is an estimate of $e = 1/b$ and α_{i-1} is the virtual control at the *ith step* and will be determined in later discussion.

Firstly, we define functions $sg_i(z_i)$ and $f_i(z_i)$ as follows

$$sg_i(z_i) = \begin{cases} \frac{z_i}{|z_i|} & |z_i| \geq \delta_i \\ \frac{z_i^{(2q+1)}}{(\delta_i^2 - z_i^2)^{n-i+2} + |z_i|^{(2q+1)}} & |z_i| < \delta_i \end{cases} \tag{4.55}$$

$$f_i(z_i) = \begin{cases} 1 & |z_i| \geq \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \tag{4.56}$$

where $\delta_i (i = 1, \dots, n)$ is a positive design parameter and $q = round\{(n-i+2)/2\}$, where $round\{x\}$ means the element of x to the nearest integer. Clearly $2q + 1 \geq (n - i + 2)$. It can be shown that $sg_i(z_i)$ is $(n - i + 2)th$ order differentiable.

As the backstepping design procedures are similar to [121], only the first and the last steps of the design, i.e. *steps 1 and n* below, are elaborated in details.

• *Step* 1: We start with the equation for the tracking error z_1 obtained from (4.39), (4.51) and (4.53) to obtain

$$\dot{z}_1 = \xi_{02} + a^T \xi_2 + b z_2 + b\alpha_1 - b\tilde{\theta}^T \hat{\omega}_2(t) + d(t) + \epsilon_2 - b\tilde{e}\dot{y}_r \tag{4.57}$$

where $d(t) = bd_2(t), \tilde{\theta} = \theta - \hat{\theta}$. From (4.38) and (4.50), there exists a positive constant D such that

$$|d(t)| \leq D.$$

Remark 4.3 *The unknown bound D of $d(t)$ will be estimated online and thus it is not assumed to be known in contrast with [41], [119] and [49]. In fact, bounded external disturbance can also be treated in the same way, even though disturbance is not considered explicitly in this paper.*

Now select the first virtual control law α_1 as

$$\alpha_1 = \hat{e}\bar{\alpha}_1 \tag{4.58}$$

$$\begin{aligned} \bar{\alpha}_1 = & -(c_1 + \frac{\hat{b}^2}{4})(|z_1| - \delta_1)^n sg_1 - \xi_{02} - \hat{a}^T \xi_2 \\ & -\hat{D}_1 sg_1 - (\delta_2 + 1)\sqrt{\hat{b}^2 + \delta_0} \cdot sg_1 \end{aligned} \tag{4.59}$$

where δ_0 is a small positive real number, $\hat{e}, \hat{a}$ and $\hat{b}$ are estimates of e, a and b, $\hat{D}_1$ is an estimate of D. We define a positive definite function V_1 as

$$\begin{aligned} V_1 = & \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} f_1 + \frac{1}{2}|b|\tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta} + \frac{1}{2}\tilde{a}^T \Gamma_a \tilde{a} \\ & + \frac{|b|}{2\gamma_1}\tilde{e}^2 + \frac{1}{2\gamma_{d1}}\tilde{D}_1^2 + \frac{1}{2l_1}\epsilon^T P \epsilon \end{aligned} \tag{4.60}$$

where $\tilde{a} = a - \hat{a}, \tilde{e} = e - \hat{e}$, Γ_θ, Γ_a are positive definite matrices, γ_1, γ_{d1} are positive constants, and $P = P^T > 0$ satisfies the equation $PA_0 + A_0^T P = -2I$. We select the adaptive update laws as

$$\dot{\hat{\theta}} = Proj\{-\text{sign}(b)\Gamma_\theta \hat{\omega}_2(t)(|z_1| - \delta_1)^{2n} f_1 sg_1\} \tag{4.61}$$

$$\dot{\hat{e}} = -\text{sign}(b)\gamma_1(\bar{\alpha}_1 + \dot{y}_r)(|z_1| - \delta_1)^{2n} f_1 sg_1 \tag{4.62}$$

$$\dot{\hat{D}}_1 = \gamma_{d1}(|z_1| - \delta_1)^n f_1 \tag{4.63}$$

where $Proj(.)$ is a smooth projection operation to ensure the estimates $\hat{m}_r(t) \geq m_{r0}$ and $\hat{m}_l(t) \geq m_{l0}$. Such an operation can be found in [30]. Then form (4.60) to (4.62), we obtain the time derivative of V_1 as

$$\begin{aligned} \dot{V}_1 &\leq -(c_1 + \frac{\hat{b}^2}{4})(|z_1| - \delta_1)^{2n} f_1 + \tilde{a}^T(\tau_{a1} - \Gamma_a^{-1}\dot{\hat{a}}) + \epsilon^T(\tau_{\chi 1} - \frac{1}{l_1}P\chi) \\ &\quad +(|z_1| - \delta_1)^n f_1 sg_1\big(bz_2 - (\delta_2 + 1)\sqrt{\hat{b}^2 + \delta_0}\big) - \frac{1}{l_1}\epsilon^T\epsilon \qquad (4.64) \\ \tau_{a1} &= \xi_2(|z_1| - \delta_1)^n f_1 sg_1 \qquad (4.65) \\ \tau_{\chi 1} &= e_2(|z_1| - \delta_1)^n f_1 sg_1 \qquad (4.66) \end{aligned}$$

where $e_2 = [0, 1, 0, \ldots, 0]^T$.

- *Step* 2: Using (4.41), (4.42), (4.58) and (4.54), we write

$$\begin{aligned} \dot{z}_2 &= -\frac{d}{dt}\big(\hat{\theta}^T\hat{\omega}_2 - \hat{e}\dot{y}_r\big) - \dot{\alpha}_1 \\ &= -\dot{\hat{\theta}}^T\hat{\omega}_2 + z_3 - \dot{\hat{e}}\dot{y}_r + \alpha_2 - \frac{\partial\alpha_1}{\partial y}\big(\xi_{02} + a^T\xi_2 - b\theta^T\hat{\omega}_2(t) + d(t) + \epsilon_2\big) \\ &\quad -\frac{\partial\alpha_1}{\partial\hat{e}}\dot{\hat{e}} - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r - \frac{\partial\alpha_1}{\partial\xi_0}(A_0\xi_0 + ky + \chi) \\ &\quad -\sum_{i=1}^{r}\frac{\partial\alpha_1}{\partial\xi_i}\dot{\xi}_i - \frac{\partial\alpha_1}{\partial\hat{a}}\dot{\hat{a}} - \frac{\partial\alpha_1}{\partial\hat{D}_1}\dot{\hat{D}}_1 - \frac{\partial\alpha_1}{\partial\hat{\omega}_2}\dot{\hat{\omega}}_2 \\ &= z_3 + \alpha_2 + \beta_2 - \frac{\partial\alpha_1}{\partial y}a^T\xi_2 + \frac{\partial\alpha_1}{\partial y}\Theta^T\hat{\omega}_2(t) - \frac{\partial\alpha_1}{\partial\hat{a}}\dot{\hat{a}} \\ &\quad -\frac{\partial\alpha_1}{\partial\xi_0}\chi - \frac{\partial\alpha_1}{\partial y}d(t) - \frac{\partial\alpha_1}{\partial y}\epsilon_2 \qquad (4.67) \\ \beta_2 &= -\dot{\hat{e}}\dot{y}_r - \dot{\hat{\theta}}^T\hat{\omega}_2 - \frac{\partial\alpha_1}{\partial y}\xi_{02} - \frac{\partial\alpha_1}{\partial\hat{e}}\dot{\hat{e}} - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r - \frac{\partial\alpha_1}{\partial\xi_0}(A_0\xi_0 + ky) \\ &\quad -\sum_{i=1}^{r}\frac{\partial\alpha_1}{\partial\xi_i}\dot{\xi}_i - \frac{\partial\alpha_1}{\partial\hat{\omega}_2}\dot{\hat{\omega}}_2 - \frac{\partial\alpha_1}{\partial\hat{D}_1}\dot{\hat{D}}_1 \qquad (4.68) \end{aligned}$$

where $\Theta = b\theta$.

Remark 4.4 *Noted that in (4.68), the term $\hat{\omega}_2$ is continuous and differentiable because $\hat{\omega}$ is continuous function from the definitions of the dead-zone (4.26), (4.27), (4.35) and (4.52).*

We choose the virtual control law for α_2 and the adaptive updated law for $\hat{b}$, the estimate of b as

$$\begin{aligned}\alpha_2 &= -(c_2+1)(|z_2|-\delta_2)^{n-1}sg_2 - \beta_2 - (\delta_3+1)sg_2 + \frac{\partial\alpha_1}{\partial y}\hat{a}^T\xi_2 - \frac{\partial\alpha_1}{\partial y}\hat{\Theta}^T\hat{\omega}_2(t) \\ &+\sqrt{\| \frac{\partial\alpha_1}{\partial y} \|^2 + \delta_0}\cdot\hat{D}_2 sg_2 + \frac{\partial\alpha_1}{\partial\hat{a}}\Gamma_a\tau_{a2} + \frac{\partial\alpha_1}{\partial\xi_0}l_1P^{-1}\tau_{\chi2} \qquad (4.69)\end{aligned}$$

$$\dot{\hat{b}} = \gamma_2(|z_1|-\delta_1)^n f_1 sg_1 z_2 \qquad (4.70)$$

$$\dot{\hat{D}}_2 = \gamma_{d2}\sqrt{\| \frac{\partial\alpha_1}{\partial y} \|^2 + \delta_0}\cdot(|z_2|-\delta_2)^{n-1}f_2 \qquad (4.71)$$

$$\tau_{a2} = \tau_{a1} - \frac{\partial\alpha_1}{\partial y}\xi_2(|z_2|-\delta_2)^{n-1}f_2 sg_2 \qquad (4.72)$$

$$\tau_{\chi2} = \tau_{\chi1} - \frac{\partial\alpha_1}{\partial y}(|z_2|-\delta_2)^{n-1}f_2 sg_2 e_2 \qquad (4.73)$$

where γ_2 and γ_{d2} are positive constants. Defining a positive Lyapunov function V_2 as

$$V_2 = V_1 + \frac{1}{n}(|z_2|-\delta_2)^n f_2 + \frac{1}{2}\tilde{\Theta}^T\Gamma_\Theta^{-1}\tilde{\Theta} + \frac{1}{2\gamma_2}\tilde{b}^2 + \frac{1}{2\gamma_{d2}}\tilde{D}_2^2 \qquad (4.74)$$

Then the derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &\le -\sum_{i=1}^{2}c_i(|z_i|-\delta_i)^{2(n-i+1)}f_i + \tilde{a}^T(\tau_{a2}-\Gamma_a^{-1}\dot{\hat{a}}) + \epsilon^T(\tau_{\chi2} - \frac{1}{l_1}P\chi) \\ &+\Big[\frac{\partial\alpha_1}{\partial\hat{a}}(\Gamma_a\tau_{a2}-\dot{\hat{a}}) + \frac{\partial\alpha_1}{\partial\xi_0}(l_1P^{-1}\tau_{\chi2}-\chi)\Big](|z_2|-\delta_2)^{n-1}f_2 sg_2 - \frac{1}{l_1}\epsilon^T\epsilon \\ &+\tilde{\Theta}^T(\tau_{\Theta2}+\Gamma_\Theta^{-1}\dot{\hat{\Theta}}) + (|z_2|-\delta_2)^{n-1}(|z_3|-\delta_3-1)f_2 + M_2 \qquad (4.75)\end{aligned}$$

$$\tau_{\Theta2} = -\frac{\partial\alpha_1}{\partial y}\hat{\omega}_2(|z_2|-\delta_2)^{n-1}f_2 sg_2 \qquad (4.76)$$

where

$$M_2 = -\frac{\hat{b}^2}{4}(|z_1| - \delta_1)^{2n} f_1 + |\hat{b}|(|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 - (|z_2| - \delta_2)^{2(n-1)} f_2$$

Now we show that $M_2 < 0$. This is quite clear for $|z_2| < \delta_2 + 1$. For $|z_2| \geq \delta_2 + 1$

$$\begin{aligned} M_2 &\leq -\frac{\hat{b}^2}{4}(|z_1| - \delta_1)^{2n} f_1 + \frac{\hat{b}^2}{4}(|z_1| - \delta_1)^{2n} f_1^2 \\ &\quad +(|z_2| - \delta_2 - 1)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &< (|z_2| - \delta_2)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &= (|z_2| - \delta_2)^2 (1 - (|z_2| - \delta_2)^{2(n-2)}) \\ &\leq 0 \end{aligned} \tag{4.77}$$

- *Step i*, $i = 3, \ \ldots, \ n$: We choose

$$\begin{aligned} \alpha_i &= -(c_i + 1)(|z_i| - \delta_i)^{n-i+1} sg_2 - \beta_i - (\delta_{i+1} + 1) sg_i + \frac{\partial \alpha_{i-1}}{\partial y} \hat{a}^T \xi_2 \\ &+ \sqrt{\| \frac{\partial \alpha_{i-1}}{\partial y} \|^2 + \delta_0} \cdot \hat{D}_i sg_i + \frac{\partial \alpha_{i-1}}{\partial \hat{a}} \Gamma_a \tau_{ai} + \frac{\partial \alpha_{i-1}}{\partial \xi_0} l_1 P^{-1} \tau_{\chi i} + \frac{\partial \alpha_{i-1}}{\partial \hat{\Theta}} \Gamma_\Theta \tau_{\Theta i} \\ &+ \sum_{k=2}^{i-1} (|z_k| - \delta_k)^{n-k+1} f_k sg_k \big[- \frac{\partial \alpha_{k-1}}{\partial \hat{a}} \frac{\partial \alpha_{i-1}}{\partial y} \xi_2 - \frac{\partial \alpha_{k-1}}{\partial \xi_0} \frac{\partial \alpha_{i-1}}{\partial y} l_1 P^{-1} e_2 \big] \\ &- \sum_{k=3}^{i-1} (|z_k| - \delta_k)^{n-k+1} f_k sg_k \frac{\partial \alpha_{k-1}}{\partial \hat{\Theta}} \frac{\partial \alpha_{i-1}}{\partial y} \hat{\omega}_2 - \frac{\partial \alpha_{i-1}}{\partial y} \hat{\Theta}^T \hat{\omega}_2(t) \end{aligned} \tag{4.78}$$

$$\dot{\hat{b}} = \gamma_2 (|z_1| - \delta_1)^n f_1 sg_1 z_2 \tag{4.79}$$

$$\dot{\hat{D}}_i = \gamma_{di} \sqrt{\| \frac{\partial \alpha_{i-1}}{\partial y} \|^2 + \delta_0} \cdot (|z_i| - \delta_i)^{n-i+1} f_i \tag{4.80}$$

$$\tau_{ai} = \tau_{ai-1} - \frac{\partial \alpha_{i-1}}{\partial y} \xi_2 (|z_i| - \delta_i)^{n-i+1} f_i sg_i \tag{4.81}$$

$$\tau_{\chi i} = \tau_{\chi i-1} - \frac{\partial \alpha_{i-1}}{\partial y} (|z_i| - \delta_i)^{n-i+1} f_i sg_i e_2 \tag{4.82}$$

$$\tau_{\Theta i} = \tau_{\Theta i-1} - \frac{\partial \alpha_{i-1}}{\partial y} (|z_i| - \delta_i)^{n-i+1} f_i sg_i \hat{\omega}_2 \tag{4.83}$$

and

$$V_i = \sum_{k=1}^{i}[\frac{1}{n-k+2}(|z_k|-\delta_k)^{n-k+2}f_k + \frac{1}{2\gamma_{dk}}\tilde{D}_k^2] + \frac{1}{2}|b|\tilde{\theta}^T\Gamma_\theta^{-1}\tilde{\theta} + \frac{1}{2}\tilde{a}^T\Gamma_a\tilde{a} + \frac{|b|}{2\gamma_1}\tilde{e}^2 + \frac{1}{2}\tilde{\Theta}^T\Gamma_\Theta^{-1}\tilde{\Theta} + \frac{1}{2\gamma_2}\tilde{b}^2 + \frac{1}{2l_1}\epsilon^T P\epsilon \quad (4.84)$$

where $\hat{\Theta}$, $\hat{D}_k$ are estimates of $\Theta = b\theta$ and D, $\tilde{\Theta} = \Theta - \hat{\Theta}$, $\tilde{b} = b - \hat{b}$, $\tilde{D}_k = D - \hat{D}_k$, β_i contains all known terms, $\gamma_2, \gamma_{di}, i = 1, \ldots, n$ are positive constants, Γ_Θ is a positive definite matrix.

Step n: Using (4.36), (4.46) and (4.52), we have

$$\hat{\theta}^T\hat{\omega}_2^{(n-1)} = \hat{\theta}^T\frac{(p^n + k_1p^{n-1})I_4}{p^n + k_1p^{n-1} + \cdots + k_{n-1}p + k_n}\hat{\omega}(t) = u_d(t) + \omega_0 \quad (4.85)$$

where ω_0 is given by

$$\omega_0 = -\frac{(k_2p^{n-2} + \cdots + k_{n-1}p + k_n)I_4}{p^n + k_1p^{n-1} + \cdots + k_{n-1}p + k_n}\hat{\omega}(t) \quad (4.86)$$

With this equation, the derivative of $z_n = -\hat{\theta}^T\hat{\omega}_2^{(n-2)} - \hat{e}y_r^{(n-1)} - \alpha_{n-1}$ is

$$\dot{z}_n = u_d + \beta_n - \frac{\partial\alpha_{n-1}}{\partial y}a^T\xi_2 + \frac{\partial\alpha_{n-1}}{\partial y}\Theta^T\hat{\omega}_2(t) - \frac{\partial\alpha_{n-1}}{\partial\hat{a}}\dot{\hat{a}} - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{D}_j}\dot{\hat{D}}_j - \frac{\partial\alpha_{n-1}}{\partial\hat{\Theta}}\dot{\hat{\Theta}} - \frac{\partial\alpha_{n-1}}{\partial\xi_0}\chi - \frac{\partial\alpha_{n-1}}{\partial y}d(t) - \frac{\partial\alpha_{n-1}}{\partial y}\epsilon_2 \quad (4.87)$$

where β_n contains all known terms.

We choose the update laws for $\hat{a}$, $\hat{D}$, $\hat{\Theta}$

$$\dot{\hat{a}} = \Gamma_a\tau_{an} \quad (4.88)$$

$$\dot{\hat{D}}_n = \gamma_{dn}\sqrt{\| \frac{\partial\alpha_{n-1}}{\partial y} \|^2 + \delta_0} \cdot (|z_n| - \delta_n)f_n \quad (4.89)$$

$$\dot{\hat{\Theta}} = -\Gamma_\Theta\tau_{\Theta n} \quad (4.90)$$

and the design signal as

$$\chi \ = \ l_1 P^{-1} \tau_{\chi n} \tag{4.91}$$

Finally the control law is given by

$$v(t) \ = \ \frac{u_d(t) + \widehat{m_r b_r}}{\widehat{m_r}} \phi_r(u_d) + \frac{u_d(t) + \widehat{m_l b_l}}{\widehat{m_l}} \phi_l(u_d) \tag{4.92}$$

$$u_d \ = \ \alpha_n \tag{4.93}$$

4.3.5 Stability Analysis

Define a positive definite Lyapunov function V_n as

$$V_n = V_{n-1} + \frac{1}{2}(|z_n| - \delta_n)^2 f_n + \frac{1}{2\gamma_{dn}} \tilde{D}_n^2 \tag{4.94}$$

With this choice of adaptive controller and parameter update laws, the derivative of V_n becomes

$$\begin{aligned}
\dot{V}_n \ \leq \ & -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i + \tilde{a}^T(\tau_{an} - \Gamma_a^{-1}\dot{\hat{a}}) + \epsilon^T(\tau_{\chi n} - \frac{1}{l_1} P\chi) \\
+ \ & \tilde{\Theta}^T(\tau_{\Theta n} + \Gamma_\Theta^{-1}\dot{\hat{\Theta}}) - \frac{1}{l_1}\epsilon^T\epsilon + \sum_{k=3}^{n}(|z_k| - \delta_k)^{n-k+1} f_k \frac{\partial \alpha_{k-1}}{\partial \hat{\Theta}}(\Gamma_\Theta \tau_{d\Theta} + \dot{\hat{\Theta}}) \\
+ \ & \sum_{k=2}^{n}(|z_k| - \delta_k)^{n-k+1} f_k \big[\frac{\partial \alpha_{k-1}}{\partial \hat{a}}(\Gamma_a \tau_{an} - \dot{\hat{a}}) + \frac{\partial \alpha_{k-1}}{\partial \xi_0}(l_1 P^{-1}\tau_{\chi n} - \chi)\big] \\
\leq \ & -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i - \frac{1}{l_1}\epsilon^T\epsilon
\end{aligned} \tag{4.95}$$

From (4.95), we get the following Lemma.

Lemma 4.1 *The adaptive controller designed above ensures that $z_1, \ldots, z_n$, $\hat{\theta}$, $\hat{e}$, $\hat{b}, \hat{a}, \hat{\Theta}, \hat{D}_i, \epsilon$ are all bounded.*

With Lemma 1, all the signals in the closed-loop can be shown to be bounded and a bound can be established for the tracking error, as stated in the following theorem.

Theorem 4.3 *Consider the system consisting of the parameter estimators given by (4.61), (4.62), (4.79), (4.88)-(4.90), adaptive controllers designed using (4.92) and (4.93) with virtual control laws (4.58) and (4.78), and plant (4.22) with a dead-zone nonlinearity (4.24). The system is globally stable in the sense that all signals in the closed loop are bounded. Furthermore*

- *The tracking error converges to* $[-\delta_1, \delta_1]$ *asymptotically, i.e.,*

$$\lim_{t\to\infty} |y(t) - y_r(t)| = \delta_1, \quad |z_1| \geq \delta_1 \tag{4.96}$$

- *The transient tracking error performance is given by*

$$\begin{aligned} \| \, |y(t) \quad - \quad y_r(t)| - \delta_1 \, \|_2 \leq & \frac{1}{c_1^{2n}}\Big(\frac{1}{2}\tilde{a}(0)^T\Gamma_a^{-1}\tilde{a}(0) + \frac{|b|}{2\Gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2\Gamma_\Theta}\tilde{\Theta}(0)^2 \\ & + \frac{|b|}{2\gamma_1}\tilde{e}(0)^2 + \sum_{i=1}^{n}\frac{1}{2\gamma_{di}}\tilde{D}_i(0)^2 + \frac{1}{2\gamma_2}\tilde{b}(0)^2 + \frac{1}{2l_1}\epsilon(0)^2\Big)^{1/2n} \end{aligned} \tag{4.97}$$

with $z_i(0) = 0, i = 1, \ldots, n,$

Proof: From Lemma 4.1, we have that $z_1, \ldots, z_n, \hat{\theta}, \hat{e}, \hat{b}, \hat{a}, \hat{\Theta}, \hat{D}_i, \epsilon$ are bounded. The tracking error performance can be obtained from (4.95) following similar approaches to those in [121]. What we need to prove is the boundedness of state x, controller output v and plant input u. From state observers (4.42) and (4.43), and (4.45), we have that $\xi_0, \ldots, \xi_r$ are bounded. Re-writing plant (4.22) as

$$p^n y + \sum_{i=1}^{r} a_i Y_i\big(y, py, \ldots, p^{n-1}y\big) = bu \tag{4.98}$$

and using (4.47), we have

$$\eta_2 = \frac{q_2(p)}{\Delta(p)}u = \frac{p^n q_2(p)}{b\Delta(p)}y + \frac{q_2(p)}{b\Delta(p)}\sum_{i=1}^{r} a_i Y_i(y, py, \ldots, p^{n-1}y) \tag{4.99}$$

Since $\Delta(p) = p^n + k_1 p^{n-1} + \cdots + k_n$ is Hurwitz, so $\frac{q_2(p)}{b\Delta(p)}$ is stable. We have that η_2 is bounded because y is bounded. From (4.46) and (4.48), we have $\eta_2 = -\theta^T\hat{\omega}_2(t) + d_2(t)$. As $d_2(t) \in L^\infty$, then $\theta^T\hat{\omega}_2 \in L^\infty$.

Express (4.49) as

$$\hat{\omega}_2(t) = [-\frac{q_2(p)}{\Delta(p)}\phi_r(v)v, \frac{q_2(p)}{\Delta(p)}\phi_r(v), -\frac{q_2(p)}{\Delta(p)}\phi_l(v)v, \frac{q_2(p)}{\Delta(p)}\phi_l(v)]^T \tag{4.100}$$

$$\begin{aligned}\theta^T\hat{\omega}_2(t) &= -m_r\frac{q_2(p)}{\Delta(p)}\phi_r(v)v + m_r b_r\frac{q_2(p)}{\Delta(p)}\phi_r(v) \\ &\quad -m_l\frac{q_2(p)}{\Delta(p)}\phi_l(v)v + m_l b_l\frac{q_2(p)}{\Delta(p)}\phi_l(v)\end{aligned} \tag{4.101}$$

Because $\phi_r(v) \in L^\infty$, $\phi_l(v) \in L^\infty$ and $\frac{q_2(p)}{\Delta(p)}$ is stable, the terms $\frac{q_2(p)}{\Delta(p)}\phi_r(v)$ and $\frac{q_2(p)}{\Delta(p)}\phi_l(v)$ in (4.100) are bounded.

We now show that $\hat{\omega}_2$ is bounded in two cases:

Case 1. If $v(t)$ is bounded, $\hat{\omega}_2$ is bounded directly from (4.100).

Case 2. In case that $v(t)$ is unbounded, we divide $R^+ = [0, \infty)$ into two subsequences $R^+ = R_1 \cup R_2$, where $R_1 = \{t \,|v(t) \geq 0\}$ and $R_2 = \{t \,|v(t) < 0\}$. Then the following two situations are considered.

(i). $t \in R_1$. From (4.27) we get

$$\begin{aligned}\phi_l(v)\cdot v &= \frac{e^{-v/e_0}}{e^{v/e_0} + e^{-v/e_0}}\cdot v = \frac{v}{1 + e^{2v/e_0}} \\ \text{Thus} \quad \phi_l(v)\cdot v &\to 0, \text{ when } v \to +\infty \; for\ t \in R_1.\end{aligned} \tag{4.102}$$

So in equation (4.101), the third term $m_l\frac{q_2(p)}{\Delta(p)}\phi_l(v)v(t) \to 0$, with the boundedness of second term and fourth term and $\theta^T\hat{\omega}_2 \in L^\infty$, we see that the first term $m_r\frac{q_2(p)}{\Delta(p)}\phi_r(v)v$ is bounded for $t \in R_1$.

(ii). $t \in R_2$. Similarly from (4.26) we can show that

$$\begin{aligned} \phi_r(v) \cdot v &= \frac{e^{v/e_0}}{e^{v/e_0} + e^{-v/e_0}} \cdot v = \frac{v}{1 + e^{-2v/e_0}} \\ \text{Thus} \quad \phi_r(v) \cdot v &\to 0, \text{ when } v \to -\infty \ for\ t \in R_2. \end{aligned} \tag{4.103}$$

and the third term $m_l \frac{q_2(p)}{\Delta(p)} \phi_l(v) v$ is bounded for $t \in R_2$.

Combining (i) and (ii), we get that for all $t \in R^+$, $\frac{q_2(p)}{\Delta(p)} \phi_r(v) v$ and $\frac{q_2(p)}{\Delta(p)} \phi_l(v) v$ are bounded. Then $\hat{\omega}_2$ is bounded from (4.100). In summary, from the two cases we obtain the boundedness of $\hat{\omega}_2$.

Since $\hat{\theta}^T \hat{\omega}_2$ and z_2 are bounded, from $z_2 = -\hat{\theta}^T \hat{\omega}_2 - \hat{e} \dot{y}_r - \alpha_1$ we can obtain the boundedness of α_1. From (4.58), we have $\bar{\alpha}_1$ is bounded. From (4.78), $\alpha_2, \ldots, \alpha_n$ are bounded, and so is χ. From (4.93) we have that $u_d(t)$ is bounded, and so are $v = \widehat{D}I(u_d)$ and $u = DI(v)$. It following from (4.49) that $\hat{\omega}_i \in L^\infty, i = 1, \ldots, n$. From (4.44), we have that η is bounded. Then $\hat{x}$ is bounded from (4.41) and finally $x(t) = \hat{x}(t) + \epsilon(t)$ is bounded from (4.41-4.44).

△△△

Remark 4.5 *The transient performance depends on the initial estimate errors and the explicit design parameters. The closer the initial estimates to the true values, the better the transient performance. The bound for* $\| y(t) - y_r(t) \|_2$ *is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains* $\gamma_1, \gamma_{di}, \gamma_2$ *and* $\Gamma_a, \Gamma_\theta, \Gamma_\Theta$.

4.4 Illustrative Examples

4.4.1 Example 1: State Feedback Backstepping Control

In this section, we illustrate the above methodology for state feedback control of the same example system in [118] which is described as:

$$\ddot{x} = a_1 \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} - a_2(\dot{x}^2 + 2x)\sin(\dot{x}) - 0.5a_3 x \sin(3t) + bu(t) \quad (4.104)$$

where $u(t)$ represents the output of the dead-zone nonlinearity. The actual parameter values are $b = 1$ and $a_1 = a_2 = a_3 = 1$. The parameters of the dead-zone at $b_r = 0.5, b_l = -0.6, m = 1$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 2.5\sin(t)$.

In the simulation of Scheme I, the robust adaptive control law (4.9)-(4.12) was used, taking $c_1 = 2, c_2 = 2, \gamma = 0.2, \Gamma = 0.2I_3, \eta = 0.2$. The initial values are chosen as follows: $\hat{e}(0) = 0.25, \hat{a}(0) = [1.5\ 1\ 1]^T, \hat{D}(0) = 2, x(0) = [1\ 1.05]^T$ and $v(0) = 0$. The simulation results presented in the Figure 4.5 and Figure 4.6 are system tracking error and input. The effectiveness of adaptive Scheme I is demonstrated by the fact that the tracking error is reduced to zero after a few periods of the reference input as shown in Figure 4.5. The chattering phenomena in Figure 4.6 is caused by the *sign* function used in the controller. It can be avoided by adaptive Scheme II.

In the simulation of Scheme II by using the robust adaptive control law (4.18)-(4.21), we choose $c_1, \gamma, \eta, \Gamma$ and the initial values to be same as above and $\delta_1 = 0.05$. The simulation results presented in the Figure 4.7 and Figure 4.8 are system tracking error and input. In Figure 4.7 the tracking error is reduced to $\delta_1 = 0.05$ after a few periods of the reference input. The control input v is bounded and has no chattering problem as shown in Figure 4.8.

As a conclusion, all the results verify our theoretical findings and show the effectiveness of the control schemes.

4.4.2 Example 2: Output Feedback Inverse Control

In this section, we illustrate the above methodology for output feedback control of the same system as in [43] and [74], which is described as:

$$\begin{aligned} \dot{x} &= a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} + bu(t) \\ u &= DZ(v) \end{aligned} \tag{4.105}$$

where u represents the output of the dead-zone nonlinearity. The actual parameter values are $b = 1$ and $a = 1$, and the dead-zone parameter values are $m_r = 1.05; m_l = 1.05; b_r = 0.3; b_l = -0.5$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 8.5\sin(2.5t)$.

In the simulations, taking $c_1 = 4, \Gamma_a = 0.1$, $\gamma_1 = 0.3$, $\gamma_2 = 0.2$, $\Gamma_\theta = [0.1, 0.1, 0.1, 0.1]^T$, $e_0 = 1, \delta_1 = 0.02$ and the initial parameters $\hat{e}(0) = 0.3, \hat{a}(0) = 1.5, \hat{D}(0) = 0.4, \hat{\theta}(0) = [1, 1, 0.2, -0.3]^T$. The initial state is chosen as $x(0) = -0.5$. The parameters and the initial states are the same as in [74]. For comparison, the scheme in [74] and our proposed scheme are both applied to the system. The simulation results presented in the Figure 4.9 and Figure 4.10 are the tracking error and the controller output $v(t)$.

Note that the tracking error with the proposed scheme is reduced to zero after a few periods of the reference input as shown in Figure 4.9. However, for the scheme in [74], the tracking error converges to a residual. It is remarkable that the tracking performance of our scheme is better than that of other scheme, while the control effort is about same.

As a conclusion, the simulation results verify our theoretical findings and show the effectiveness of our control scheme. Also system performance is improved by our scheme.

4.4.3 Example 3: Application to Servo-Valve

In this section, we apply our proposed scheme to an industrial application as in chapter 2, servo-valve in Figure 4.3. Its spool occludes the orifice with some overlap so that for a range of spool positions v there is no fluid flow u. This overlap prevents leakage losses which increase with wear and tear. Considering the spool position as the input v, and the load position y as output, the hydraulic system in Figure 4.3 is represented in Figure 4.4 as a dead-zone block. It is located as the input of linear dynamics with transfer function $G(s) = \frac{K}{Ms^2+Bs}$, where $K = \frac{Ak_x}{k_p}$, $B = f + \frac{A^2}{k_p}$, $k_x = \frac{\partial g}{\partial x}$, $k_p = \frac{\partial g}{\partial P}$, $g = g(x, P) =$ flow, $A =$ area of position, $P =$ pressure, and $f =$ viscous friction.

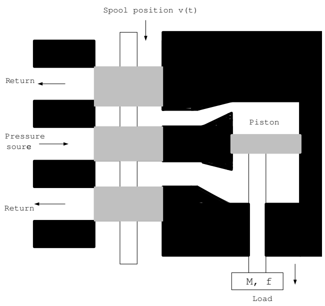


Figure 4.3: Dead-zone in servo-valve.

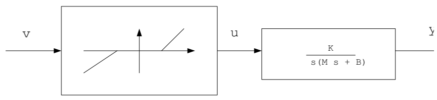


Figure 4.4: Block diagram.

The system is modelled as

$$M\ddot{y} + B\dot{y} = Ku(v) \tag{4.106}$$

where $u(v)$ represents the dead-zone nonlinearity. The adaptive control law is designed as follows

$$v(t) = \frac{u_d(t) + \widehat{m_r b_r}}{\widehat{m_r}}\phi_r(u_d) + \frac{u_d(t) + \widehat{m_l b_l}}{\widehat{m_l}}\phi_l(u_d(t)) \tag{4.107}$$

$$u_d = \hat{e}\bar{u} \tag{4.108}$$

$$\bar{u} = -(c_2 + 1)(|z_2| - \delta_2)sg_2 + \dot{\alpha}_1 + \hat{a}\dot{y} - sg_2\hat{D} \tag{4.109}$$

$$\alpha_1 = -(c_1+1)(|z_1|-\delta_1)^2 sg_1 \tag{4.110}$$
$$\dot{\hat{e}} = -\gamma\bar{u}(|z_2|-\delta_2)f_2 sg_2 \tag{4.111}$$
$$\dot{\hat{a}} = -\eta\dot{y}(|z_2|-\delta_2)f_2 sg_2 \tag{4.112}$$
$$\dot{\hat{\theta}} = Proj\{-\text{sign}(\frac{K}{M})\Gamma_\theta\hat{\omega}(|z_2|-\delta_2)f_2\} \tag{4.113}$$

where $z_1 = y - y_r, z_2\dot{y} - \dot{y}_r, y_r = 2 + 0.5\sin(t)$, $\hat{a}$ is an estimate of $a = B/M$ and $\hat{e}$ is an estimate of $e = M/K$. In the simulation, the design parameters are chosen as $c_1 = c_2 = 1, \gamma = 0.5, \eta = 0.6, \Gamma_\theta = 0.2I_4$ and the initial value is chosen as $y(0) = 2.2, \hat{e}(0) = \hat{a}(0) = 1, \hat{\theta}(0) = [1, 1, 0.4, -0.4]^T$. The simulation results presented in Figures 4.11-4.16 show that the load position y and y_r and the spool position v using the controller without considering dead-zone, with hard inverse and with our proposed smooth inverse. Clearly, our proposed scheme improves the system performance greatly.

4.5 Summary

In this chapter, we present two types of robust adaptive backstepping control algorithms: state feedback control and output feedback control of nonlinear system with unknown dead-zone.

For state feedback control two backstepping adaptive controller design schemes are developed. For output feedback control, a backstepping adaptive controller is design for a class of uncertain nonlinear SISO systems preceded by uncertain dead-zone actuator nonlinearity. We propose a new smooth adaptive inverse to compensate the effect of the unknown dead-zone. Such an inverse can avoid possible chattering phenomenon which may be caused by nonsmooth inverse. The inverse function is employed in the backstepping controller design. For the design and implementation of the controller, no knowledge is assumed on the unknown system parameters and nonlinearity. Besides showing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters. Simulation results illustrates the effectiveness of proposed schemes.

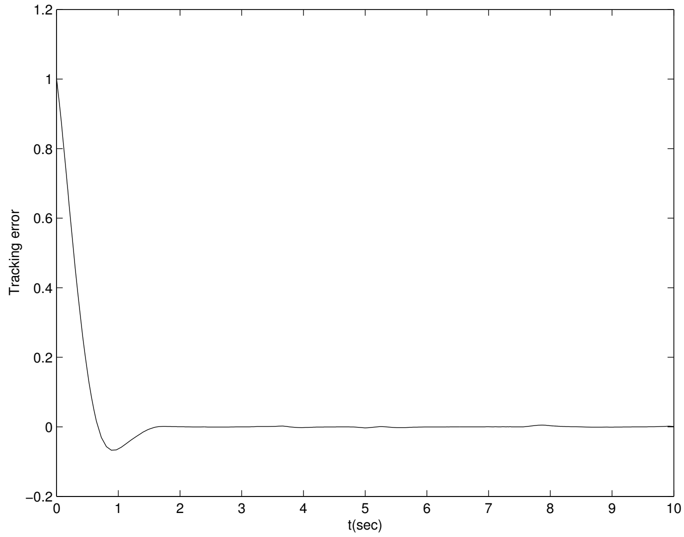


Figure 4.5: Tracking error-Scheme I.

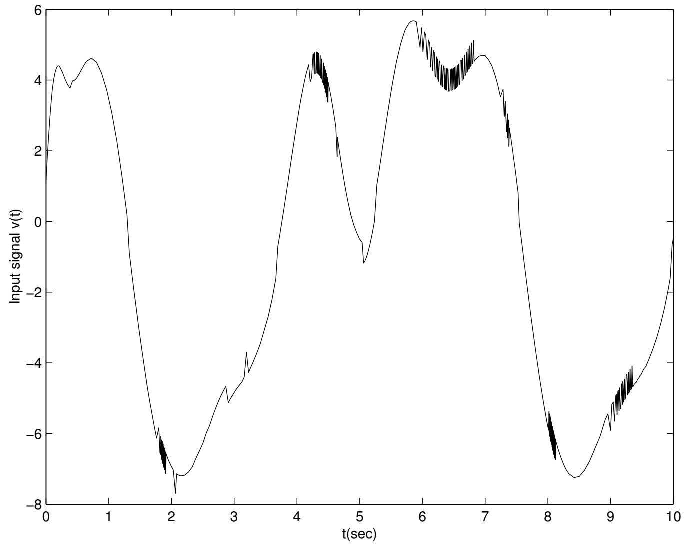


Figure 4.6: Control signal $v(t)$-Scheme I.

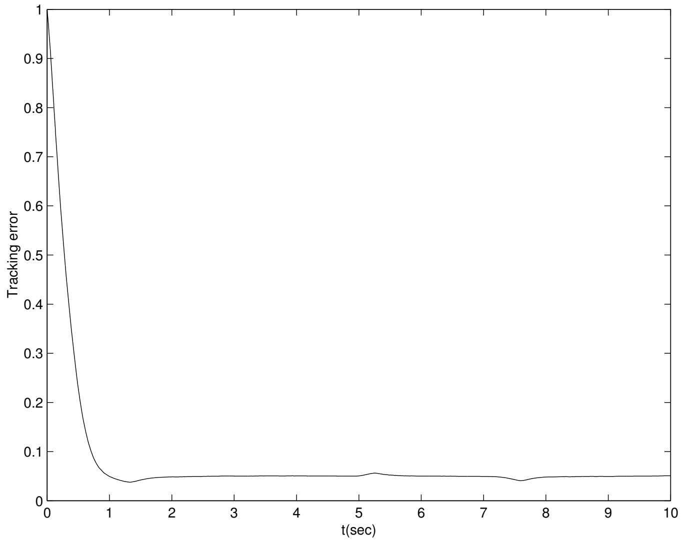


Figure 4.7: Tracking error-Scheme II.

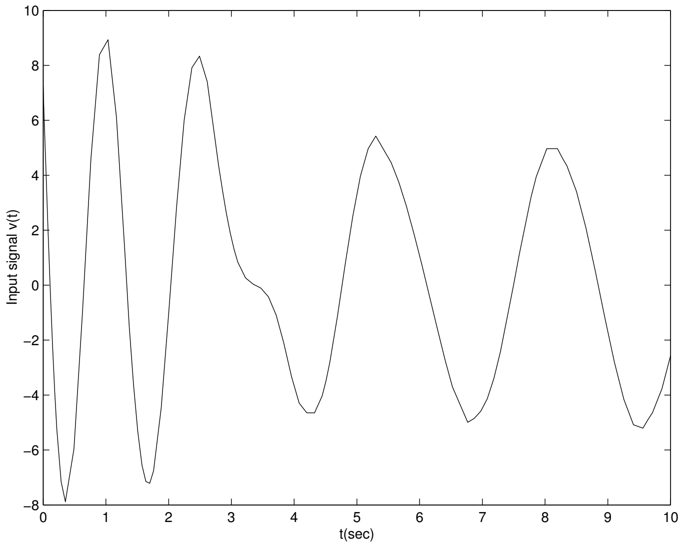
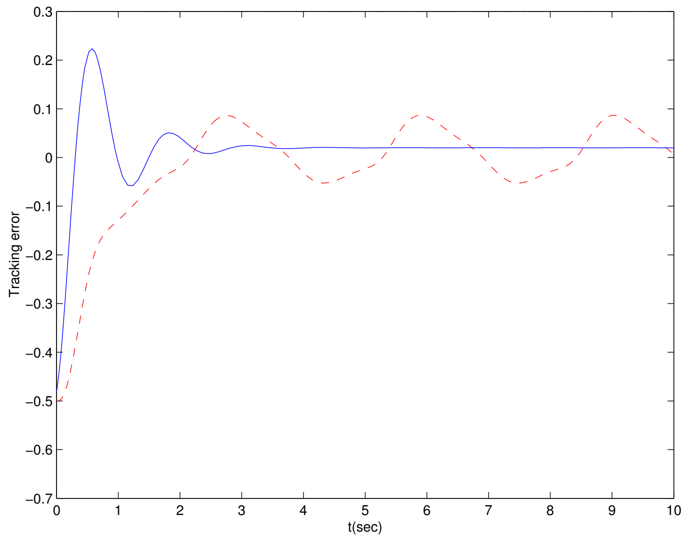
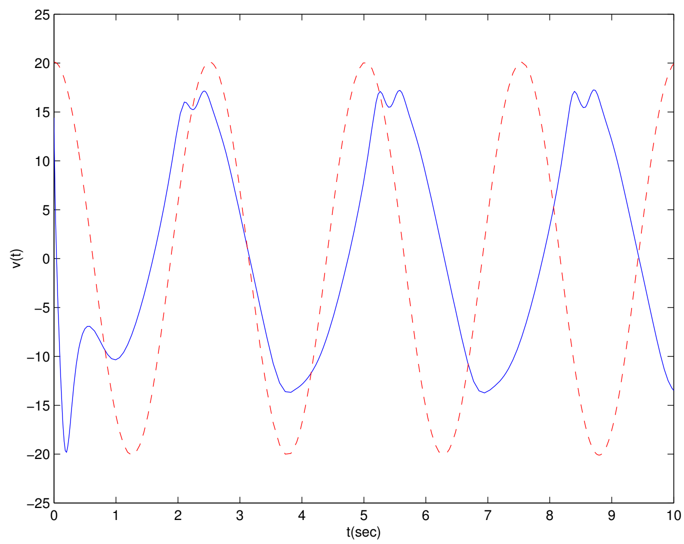


Figure 4.8: Control signal $v(t)$-Scheme II.



(dashed: scheme in [74], solid: proposed scheme)

Figure 4.9: Tracking error.



(dashed: scheme in [74], solid: proposed scheme)

Figure 4.10: Control signal $v(t)$.

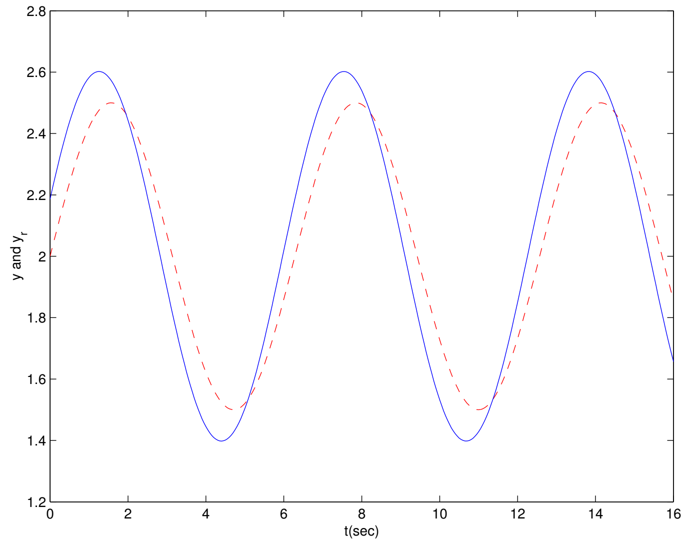


Figure 4.11: Without considering dead-zone: Load position (y: solid line; y_r: dashed line).

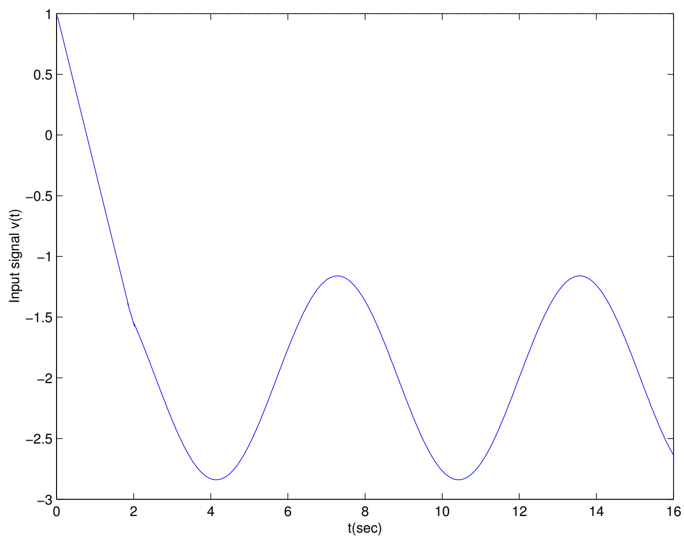


Figure 4.12: Without considering dead-zone: Spool position v.

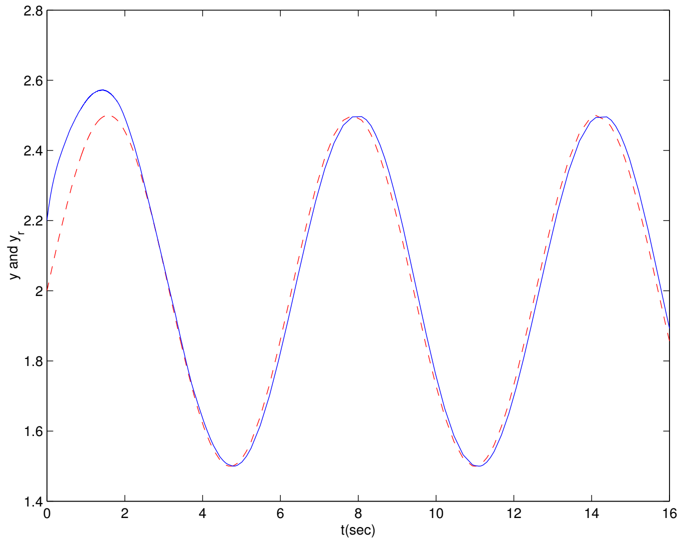


Figure 4.13: With hard inverse: Load position (y: solid line; y_r: dashed line).

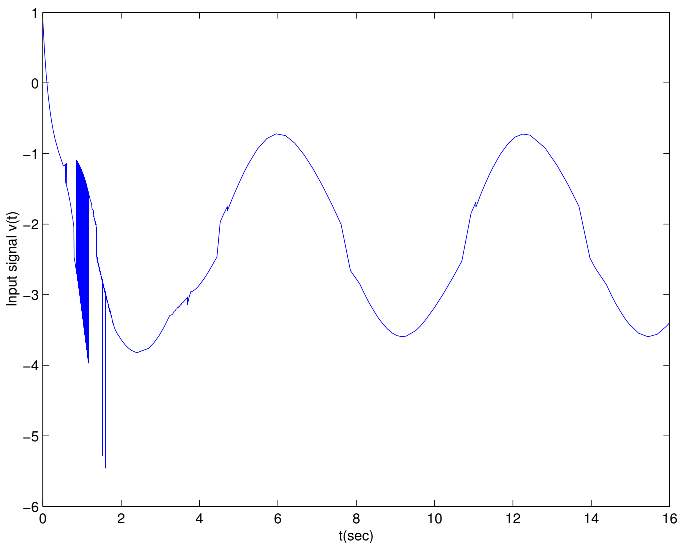


Figure 4.14: With hard inverse: Spool position v.

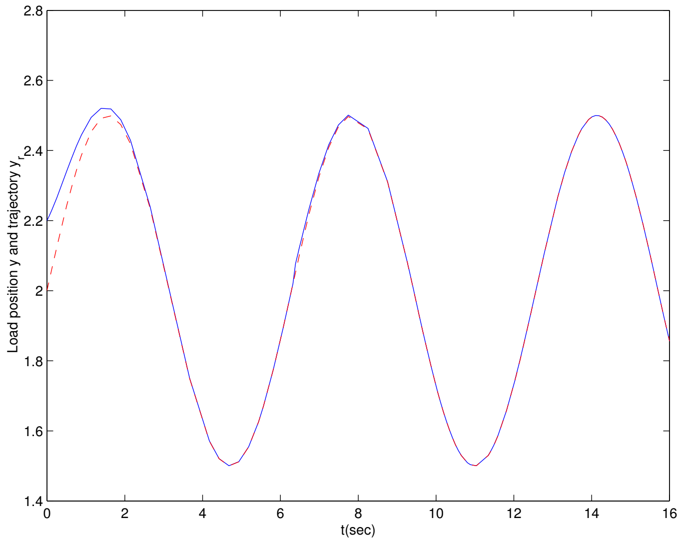


Figure 4.15: Our proposed scheme: Load position (y: solid line; y_r: dashed line).

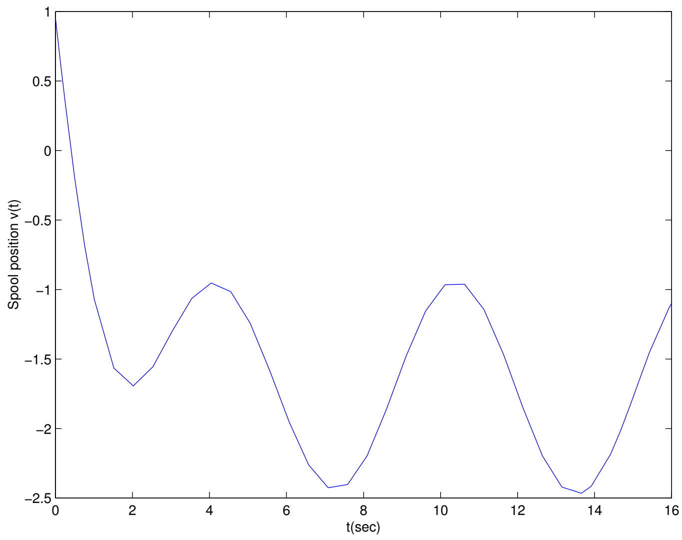


Figure 4.16: Our proposed scheme: Spool position v.

Chapter 5

Robust Adaptive Control of Uncertain Systems in the Presence of Input Saturation

In this chapter, we present a new scheme to design adaptive controllers for uncertain nonlinear systems in the presence of input saturation. By using backstepping technique, a new robust adaptive control algorithm is developed. The developed controllers do not require uncertain parameters within a known compact set. Besides showing global stability, transient performance is also established and can be adjusted by tuning certain design parameters.

5.1 Introduction

In many practical dynamic systems, physical input saturation on hardware dictates that the magnitude of the control signal is always constrained. Saturation is a potential problem for actuators of control systems. It often severely limits system performance, giving rise to undesirable inaccuracy or leading instability. The development of adaptive control schemes for systems with input saturation has been a task of major practical interest as well as theoretical significance.

However, the number of available results by taking saturation into account in the design and analysis is still limited due to the difficulty of the problem. For linear stable systems with known parameters and input saturation, a few control schemes have been proposed, for examples, anti-windup schemes in [122, 123], low-gain control in [124, 125] and linear feedback regulation in [126]. When the system parameters are unknown, adaptive control schemes have been proposed, for examples, model reference adaptive control in [59, 63], predictive control in [61], discrete-time control approaches in [46], indirect adaptive regulator in [60, 62], where uncertain parameters must be inside a known compact set. An adaptive force-balancing control scheme with actuator limits for a MEMS gyroscope was also presented in [127], where the plant is a stable second-order uncertain linear system. The system tracking error is shown to approach a signal generated by an artificially constructed system.

Backstepping approach is a Lyapunov-based recursive design procedure. With this technique, transient performance can be established and improved with explicit tuning of design parameters. A great deal of attention has been paid to tackle both linear and nonlinear systems with unknown parameters. A number of results have been obtained as summarized in [30]. Some robustness issues have also been addressed, see for examples, [34, 121]. However, the effect of saturation nonlinearity has not been addressed with this approach, especially in the absence of a priori knowledge of system parameters. To solve such a problem, certain modifications of standard backstepping controllers are required.

In this chapter, we will address the problem of controlling a class of uncertain nonlinear systems in the presence of saturation. To deal with saturation, we construct a new system with the same order as that of the plant similar to [127]. With the error between the control input and saturated input as the input of the constructed system, a number of signals are generated to compensate the effect of saturation. With the proposed adaptive backstepping controller, the system tracking error is shown to approach a signal generated by the constructed system. The tracking error is also adjustable by an explicit choice of design parameters. Thus our designed backstepping scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way.

5.2 System Description and Problem Statement

We consider the same class of systems given in [121, 43]. For completeness, the system model is given as follows:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i\big(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\big) = u(v) \tag{5.1}$$

where $Y_i(x, \dot{x}, \ldots, x^{(n-1)})$ are known continuous linear or nonlinear functions, parameters a_i are unknown constants, v is the control input, and $u(v(t)) \in R$ denotes the plant input subject to saturation described by

$$u(v(t)) \quad = \quad sat(v(t)) = \begin{cases} sign(v(t))u_M & |v(t)| \geq u_M \\ v(t) & |v(t)| < u_M \end{cases} \tag{5.2}$$

where u_M is the saturation bound of $u(t)$.

For the development of control laws, the following assumption is made.

Assumption 1. The plant is bounded input bounded output stable.

The control objectives are to design backstepping adaptive control law $v(t)$ such that

• The closed loop system is globally stable in sense that all the signals in the system are uniformly ultimately bounded;

• The tracking error $y(t) - y_r(t)$ is adjustable by an explicit choice of design parameters.

5.3 Design of Adaptive Controllers

Now equation (5.1) is rewritten in the following form

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
&\vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= -\sum_{i=1}^{r} a_i Y_i\big(x_1(t), x_2(t), \ldots, x_{(n-1)}(t)\big) + bu(v) \\
&= a^T Y + u(v) \qquad (5.3) \\
y &= x_1 \qquad (5.4)
\end{aligned}$$

where $x_1 = x, x_2 = \dot{x}, \ldots, x_n = x^{(n-1)}$, $a = [-a_1, -a_2, \ldots, -a_r]^T$ and $Y = [Y_1, Y_2, \ldots, Y_r]^T$.

In order to compensate the effect of the saturation, the following system is constructed to generate signals $\lambda(t) = [\lambda_1, \ldots, \lambda_n]^T$

$$\begin{aligned}
\dot{\lambda}_1 &= \lambda_2 - c_1\lambda_1 \\
\dot{\lambda}_i &= \lambda_{i+1} - c_i\lambda_i, \qquad i = 2, 3, \ldots, n \\
\dot{\lambda}_n &= -c_n\lambda_n + \Delta u \qquad (5.5)
\end{aligned}$$

where c_i are positive constants and $\Delta u = u(v) - v$.

The following change of coordinates is made.

$$z_1 = y - y_r - \lambda_1 \qquad (5.6)$$

$$z_i = x_i - \alpha_{i-1} - y_r^{(i-1)} - \lambda_i, \qquad i = 2, 3, \ldots, n \qquad (5.7)$$

where α_{i-1} is the virtual control at the *ith step* to be determined.

Remark 5.1 *With the error Δu as the input of the constructed system, it has no effect on z_i. Thus it will not affect the design of controllers. Then by following* *the standard backstepping approach, the adaptive law will ensure the boundedness of parameter estimates regardless of Δu. On the other hand, such estimates will depend on Δu when standard backstepping is used without using the transformed systems.*

In the following, backstepping control scheme is proposed. To illustrate the design procedures, only the first and the last step are elaborated in details.

• *Step 1*: Starting from the equations for the tracking error obtained from (5.3) to (5.7), we get

$$\begin{aligned} \dot{z}_1 &= x_2 - \lambda_2 + c_1\lambda_1 - \dot{y}_r \\ &= z_2 + \alpha_1 + c_1\lambda_1 \end{aligned} \tag{5.8}$$

We design the virtual control law α_1 as

$$\alpha_1 = -c_1(x_1 - y_r) \tag{5.9}$$

where $c_1 > 1/2$ is a positive design parameter. A positive Lyapunov function V_1 is defined as

$$V_1 = \frac{1}{2}z_1^2 \tag{5.10}$$

Then the derivative of V_1 along with (5.8) and (5.9) is given as

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 + z_1 z_2 \\ &\leq -c_1 z_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \\ &= -\bar{c}_1 z_1^2 + \frac{1}{2}z_2^2 \end{aligned} \tag{5.11}$$

where $\bar{c}_1 = c_1 - \frac{1}{2} > 0$.

• *Step i* $(i = 2, \ldots, n-1)$: For $z_i = x_i - \alpha_{i-1} - y_r^{(i-1)} - \lambda_i$, we choose virtual control law α_i as

$$\alpha_i = -c_i(x_i - \alpha_{i-1} - y_r^{(i-1)}) + \dot{\alpha}_{i-1}(x_1, \ldots, x_{i-1}) \tag{5.12}$$

where $c_i, i = 2, \ldots, n-1$ are positive design parameters satisfying $c_i > 1$. From (5.7) and (7.66) we obtain

$$z_i \dot{z}_i = -c_i z_i^2 + z_i z_{i+1} \tag{5.13}$$

We choose Lyapunov function as

$$V_i = \sum_{k=1}^{i} \frac{1}{2} z_k^2 \tag{5.14}$$

Then the derivative of V_i along with (7.66) and (5.13) is given by

$$\begin{aligned} \dot{V}_i &\leq -c_i z_i^2 + z_i z_{i+1} + \frac{1}{2} z_i^2 \\ &\leq -\sum_{i=1}^{i} \bar{c}_i z_i^2 + \frac{1}{2} z_{i+1}^2 \end{aligned} \tag{5.15}$$

where $\bar{c}_i = c_i - 1 > 0$.

- *Step* n: From (5.3) and (5.7) for $i = n$, we obtain

$$\dot{z}_n = v + a^T Y - \dot{\alpha}_{n-1} + c_n \lambda_n - y_r^{(n)} \tag{5.16}$$

We design the adaptive control law $v(t)$ as follows

$$v = -c_n(x_n - \alpha_{n-1} - y_r^{(n-1)}) - \hat{a}^T Y + \dot{\alpha}_{n-1}(x_1, \ldots, x_{n-1}) + y_r^{(n)} \tag{5.17}$$

where c_n is a positive design parameter satisfying $c_n > \frac{1}{2}$, $\hat{a}$ is an estimate of a. The parameter update law is designed as

$$\dot{\hat{a}} = \Gamma Y z_n \tag{5.18}$$

where Γ is a positive definite matrix. We define a positive Lyapunov function V_n as

$$V = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} \tag{5.19}$$

where $\tilde{a} = a - \hat{a}$. Then the derivative of V along with (5.16) to (5.18) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} z_i \dot{z}_i + \tilde{a}^T \Gamma^{-1} \dot{\tilde{a}} \\ &\leq -\sum_{i=1}^{n} \bar{c}_i z_i^2 + \tilde{a}^T \Gamma^{-1} (\Gamma Y z_n - \dot{\hat{a}}) \qquad (5.20) \\ &= -\sum_{i=1}^{n} \bar{c}_i z_i^2 \qquad (5.21) \end{aligned}$$

where $\bar{c}_n = c_n - \frac{1}{2}$.

This shows that V is uniformly bounded. Thus $z_i, i = 1, \ldots, n$ and $\hat{a}$ are bounded. From Assumption 1, we have that $x_i, i = 1, \ldots, n$ are bounded as the plant is stable and its input is bounded. So that the boundedness of $\alpha_1, \ldots, \alpha_{n-1}$ and control signal $v(t)$ can be obtained from (5.9), (7.66) and (5.17). Thus $\Delta u = u(v) - v$ is also bounded. Therefore boundedness of all signals in the closed loop system is ensured as stated in the following theorem.

Theorem 5.1 *Consider the uncertain nonlinear system (5.1) in the presence of input saturation satisfying Assumption 1. With the application of controller (5.17) and the parameter update law (5.18), the following statements hold:*

- *The steady state tracking error satisfies*

$$\lim_{t\to\infty}[y(t) - y_r(t) - \lambda_1(t)] = 0 \tag{5.22}$$

- *A bound of the transient tracking error will be given by*

$$\| y(t) - y_r(t) \|_2 \;\; \leq \;\; \frac{1}{\sqrt{\bar{c}_1}}(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0))^{1/2} + \frac{1}{\sqrt{c_0}} \| \Delta u \|_2 \tag{5.23}$$

Proof: From (5.21) we established that V is non increasing. Hence, $z_i, i = 1, \dots, n$, $\hat{a}$ are bounded. By applying the LaSalle-Yoshizawa theorem in [30] to (5.21), it further follows that $z_i(t) \to 0, i = 1, \dots, n$ as $t \to \infty$, which implies that $\lim_{t\to\infty}[y(t) - y_r(t) - \lambda_1] = 0$.

From (5.21) we also have that

$$\begin{aligned} \| z_1 \|_2^2 &= \| y - y_r - \lambda_1 \|_2^2 = \int_0^\infty |z_1(\tau)|^2 d\tau \\ &\leq \frac{1}{\bar{c}_1}(V(0) - V(\infty)) \leq \frac{1}{\bar{c}_1}V(0) \end{aligned} \tag{5.24}$$

Thus, by setting $z_i(0) = 0, i = 1, \dots, n$, we obtain

$$V(0) \;\; = \;\; \frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) \tag{5.25}$$

a decreasing function of Γ, independent of $\bar{c}_1$. This means that the bound resulting from (5.24) and (5.25) is

$$\| y(t) - y_r(t) - \lambda_1(t) \|_2 \;\; \leq \;\; \frac{1}{\sqrt{\bar{c}_1}}\left(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0)\right)^{1/2} \tag{5.26}$$

Now we derive the bound of λ_1.

We construct the positive Lyapunov function $V_\lambda = \sum_{i=1}^n \frac{1}{2}\lambda_i^2$. Then the derivative of V_λ is given as

$$\begin{aligned} \dot{V}_\lambda &= -c_1\lambda_1^2 + \lambda_1\lambda_2 - c_2\lambda_2^2 + \lambda_2\lambda_3 + \cdots + \lambda_{n-1}\lambda_n - c_n\lambda_n^2 + \lambda_n\Delta u \\ &\leq \sum_{i=1}^{n} -\bar{c}_i\lambda_i^2 + \Delta u^2 \\ &\leq -c_0 \parallel \lambda \parallel^2 + \Delta u^2 \end{aligned} \tag{5.27}$$

where $\bar{c}_1 = c_1 - \frac{1}{2}$, $\bar{c}_i = c_i - 1 (i = 2, \ldots, n-1)$, $\bar{c}_n = c_n - \frac{3}{4}$, $c_0 = min_{1\leq i\leq n}\bar{c}_i$. Integrating both sides of (5.27), we have

$$\begin{aligned} \parallel \lambda \parallel_2^2 &= \int_0^\infty \parallel \lambda \parallel^2 d\tau \\ &\leq \frac{1}{c_0}\big[(V_\lambda(0) - V_\lambda(\infty)) + \int_0^\infty (\Delta u)^2 d\tau\big] \end{aligned} \tag{5.28}$$

By setting $\lambda_i(0) = 0$, the initial value of the Lyapunov function is $V_\lambda(0) = 0$. Then a bound on the state $\parallel \lambda \parallel_2$ is established as follows

$$\parallel \lambda \parallel_2 \leq \frac{1}{\sqrt{c_0}} \parallel \Delta u \parallel_2 \tag{5.29}$$

Thus from (5.26) and (5.29), it is obtained

$$\parallel y - y_r \parallel_2 \leq \frac{1}{\sqrt{\bar{c}_1}}\big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0)\big)^{1/2} + \frac{1}{\sqrt{c_0}} \parallel \Delta u \parallel_2 \tag{5.30}$$

△△△

From Theorem 1 the following conclusions can be obtained.

Remark 5.2 *The transient performance depends on the initial estimate error* $\tilde{a}(0)$ *and the explicit design parameters. The closer the initial estimate* $\hat{a}(0)$ *to the true value* a*, the better the transient performance.*

Remark 5.3 *The bound for* $\parallel y(t) - y_r(t) \parallel_2$ *is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error* *estimate on the transient performance by increasing the adaptation gain* Γ *and parameter* c_1.

Remark 5.4 *The bound of* $\| y(t) - y_r(t) \|_2$ *depends on the bound of* Δu*, which effects on the performance can be decreased by increasing parameter* c_0*. If* $\Delta u \to 0$ *as* $t \to \infty$*, we have* $\lambda_1 \to 0$*. Then* $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$*. This implies that if the system has no saturation or the control signal is not saturated as* $t \to \infty$*, then perfect tracking is ensured.*

5.4 Simulation Study

In this section, we illustrate the above methodology on the following example. We consider a second-order system depicted in Figure 5.1 which is modelled by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{m}x_1 - \frac{c}{m}x_2 + \frac{1}{m}sat(v) \\ y &= x_1 \end{aligned} \tag{5.31}$$

where x_1 and x_2 are the position and velocity, m is the mass of the object, k is the stiffness constant of the spring and c is the damping. The input saturation limit is $13N$. The true parameters are set as $m = 1.25kg$, $c = 2N \cdot s/m$, $k = 8N/m$, which are not needed to be known in our controller design. The design parameters are chosen as $c_1 = c_2 = 5$ and $\Gamma = 0.5I_2$. The adaptive control law and parameter update laws are followed by (5.17) and (5.18), where $a = [\frac{-k}{m}, \frac{-c}{m}]^T, b = \frac{1}{m}$. The initial parameter values are selected as $\hat{a}(0) = [-6, -1]^T$ and $\hat{e}(0) = 1$. The desired trajectory is given as

$$r(t) = -0.2\cos(2\pi \times 1.5t) + 0.2 \ [m]$$

and the initial conditions are $x_1(0) = 0.6m, x_2(0) = 0.8$.

Two controllers, designed using the standard backstepping approach [30] and the

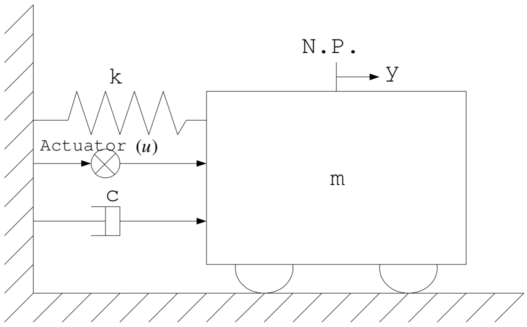


Figure 5.1: Spring, mass and damper system.

proposed scheme respectively, are applied to system (5.31). Simulation results on system tracking error and control signal are presented in Figures 5.2 to 5.5. Significantly improved performance is clearly seen with the proposed scheme. It is also observed that the input signal is not saturated for $t > 8$ second and the perfect tracking is obtained. These results indicate that the proposed backstepping adaptive controller is effective and practically useful.

5.5 Conclusion

This chapter presents a new scheme to design adaptive backstepping controller for a class of uncertain nonlinear systems in the presence of input saturation. We propose a new control law to compensate the effect of the saturation nonlinearity using backstepping technique. The developed backstepping controls do not require the model parameters within known intervals. Besides showing global stability, we also give an explicit bound on the performance of the tracking error in terms of design parameters. Simulation results illustrate the effectiveness of our proposed scheme. Also improvement of system performance over a backstepping adaptive controller designed without considering saturation is observed.

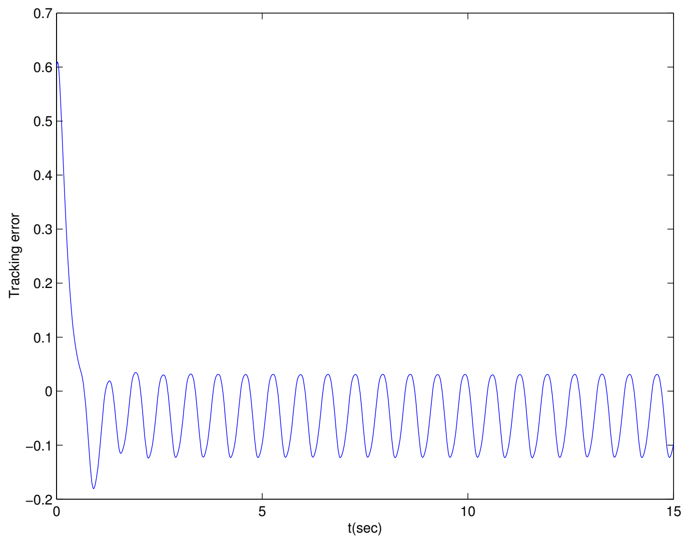


Figure 5.2: Tracking error.

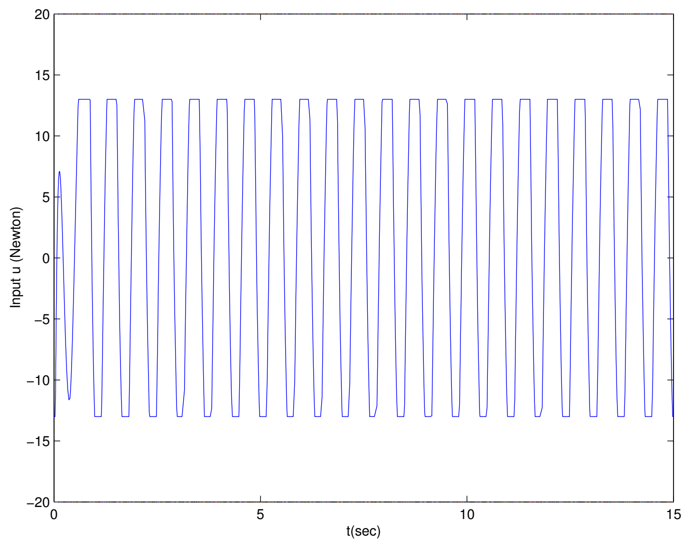


Figure 5.3: Control signal.

System performance with the controller designed using standard backstepping approach.

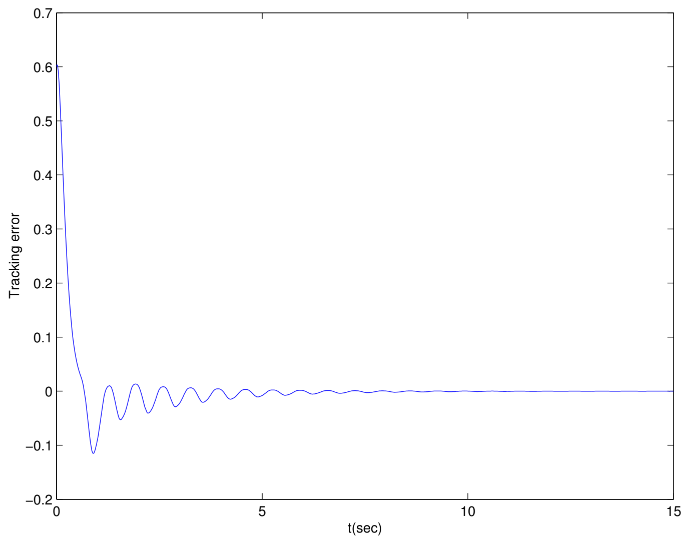


Figure 5.4: Tracking error.

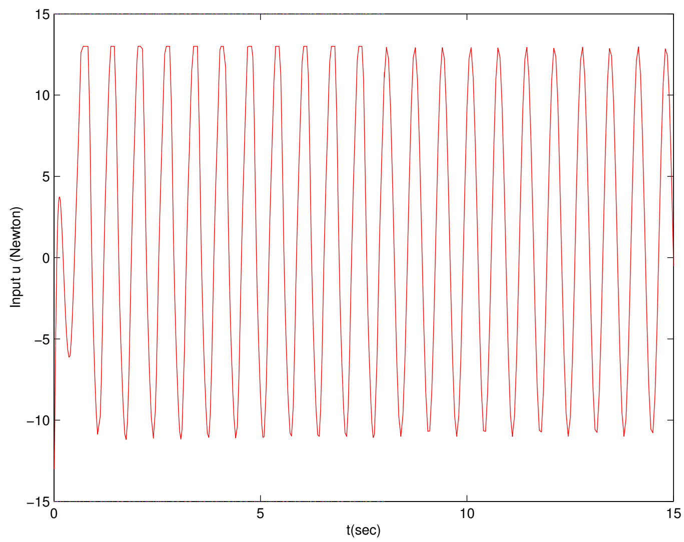


Figure 5.5: Control signal.

System performance with the controller designed using proposed scheme.

Chapter 6

Adaptive Control of a Hysteretic Structural System in Base Isolation Scheme

In this chapter, we present two adaptive backstepping control algorithms for a second-order uncertain hysteretic structural system found in base isolation scheme for seismic active protection of building structures. The hysteretic nonlinear behavior is described by a Bouc-Wen model. It is shown that not only global stability is guaranteed by the proposed controller, but also both transient and asymptotic performances are quantified as explicit functions of the design parameters so that designers can tune the design parameters in an explicit way to obtain the required closed loop behavior.

6.1 Introduction

The modelling and identification of nonlinear hysteretic systems is a problem widely encountered in the structural dynamics field. Nonlinear hysteretic behavior is seen commonly in structures experiencing strong ground earthquake excitation. Because of the hysteretic nature of the restoring force in such situation, the nonlinear force cannot be expressed in the form of an algebraic function involving the instantaneous values of the state variables of the system. Studies of this problem have been reported in the works of [128], [129], [130], [131], [132] and [133]. In [128], an adaptive controller was designed for a class of state-feedback nonlinear systems with unknown hysteresis to counteract the effect of an earthquake excitation. To represent the behavior of a seismic base isolation scheme which has a nonlinear hysteretic behavior, the Bouc-Wen model associated to a second-order structural system is used. This behavior is described in ([134, 135, 136, 137, 138]). The system considered arises from a class of nonlinear oscillators, which are common in structural engineering models [128, 139]. The proposed controller is designed to counteract the effect of an earthquake excitation and mitigate the seismic displacement response of the system. In the controller design, the true hysteretic behavior is not required to be known. However, the system uncertain parameters must be within some known intervals and the effect of the hysteresis is treated in a similar way of handling a bounded disturbance. The bound of the effect is also required for the design. Certain structural information in the model is not exploited.

In this chapter, we develop two backstepping adaptive control design schemes for the same class of system as in [128]. In the design, no knowledge is assumed on the term multiplying the control and other uncertain parameters. In the first scheme, we use some available structure information in the design and the residual effect of the hysteresis is treated as a bounded disturbance. An update law is used to estimate the bound involving this partial hysteresis effect and external disturbance. In the second scheme, we further take the structure of the Bouc-Wen model describing the hysteresis into account in the controller design, if apriori knowledge on some parameters of the model is available. It is shown that the proposed controller can guarantee global stability and achieve tracking performance. Also with the proposed scheme, both transient and asymptotic performances are quantified as explicit functions of the design parameters so that designers can tune the design parameters in an explicit way to obtain the required closed loop behavior. Comparing with the scheme in [128], system performance with the first scheme applied is still improved even though we need much less knowledge from the system. When the second scheme is applied, the performance has been significantly improved comparing with the first scheme and the scheme in [128]. Simulation results verify the effectiveness of our adaptive controllers.

6.2 Problem Formulation

Consider the following second-order uncertain nonlinear system illustrated in Figure 6.1 modelled as

$$m\ddot{x} + \bar{c}\dot{x} + \Phi(x,t) = f(t) + u(t) \tag{6.1}$$

where m is an unknown positive parameter, $\bar{c}$ is an uncertain parameter and Φ represents a nonlinear component, $f(t)$ is an external disturbance with unknown bound, and $u(t)$ is control input. In the structural system, m and $\bar{c}$ are the mass and the damping coefficients, respectively, and restoring force Φ characterizes a hysteretic behavior of isolator material, which is usually made with inelastic rubber bearings, x is the position, $u(t)$ is an active control force supplied by appropriate actuators, $f(t)$ is an exciting unknown force, which is described as $f(t) = -ma(t)$, where $a(t)$ is earthquake ground acceleration. The hysteresis force Φ is described by the so-called Bouc-Wen model ([134],[135],[136],[138],[140]) in the following form:

$$\Phi(x,t) = \alpha kx(t) + (1-\alpha)Dkz(t) \tag{6.2}$$

$$\dot{z} = D^{-1}[A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \lambda\dot{x}|z|^n] \tag{6.3}$$

where A, β, λ are nondimensional parameters which control the shape and the size of the hysteresis loop, n is an integer that governs the smoothness of the transition from elastic to plastic response, z is a dependent variable, which is the solution of the nonlinear first order differential equation (6.3). This model represents the restoring force $\Phi(x,t)$ by superposition of elastic component αkx and a hysteretic component $(1-\alpha)Dkz$, in which $D > 0$ is the yield constant displacement and α

is the post to pre-yielding stiffness ratio. The hysteretic part involves an auxiliary variable z which is the solution of the nonlinear first order differential equation (6.3). The Bouc-Wen model is able to capture, in an analytic form, a range of shapes of hysteretic cycles which match the behavior of a wide class of nonlinear structures. It is widely used in structure dynamics, particularly to describe rubber bearing isolation schemes.

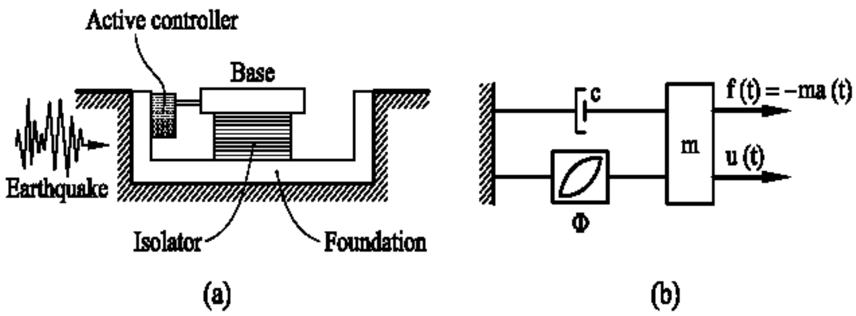


Figure 6.1: Base isolation system (a) and physical model (b).

Now we prove the boundedness of $z(t)$. From dynamic system (6.3), we have

$$\begin{aligned} \dot{z} &= -D^{-1}|z|^{n-1}[\ \beta|\dot{x}|z + \lambda\dot{x}|z|] + D^{-1}A\dot{x} \\ &= -D^{-1}|z|^{n-1}|\dot{x}|\left[\beta + \lambda \text{sign}(z)\text{sign}(\dot{x})\right] z + D^{-1}A\dot{x} \end{aligned} \tag{6.4}$$

We construct a positive Lyapunov function $V_z = z(t)^2/2$. Its derivative takes different forms depending on the signs of $\dot{x}$ and z. The following analysis determines the condition on the Bouc-Wen model parameters, such that $z(t)$ is globally bounded. Consider the case $A > 0$. There are three possibilities.

- $\ast$ $P1 : \beta + \lambda > 0$ *and* $\beta - \lambda \geq 0$;
- $\ast$ $P2 : \beta + \lambda > 0$ *and* $\beta - \lambda < 0$;
- $\ast$ $P3 : \beta + \lambda \leq 0$.

Let us now focus on the case $P1$. Indeed, setting $Q1 := \{\dot{x} \geq 0 \ and \ z \geq 0\}$, and denoting $\dot{V}_{Q_1}$ as the expression of the derivative of the Lyapunov function V_z over the set $Q1$, we have $\dot{V}_{Q1} = z\dot{x}D^{-1}(A - (\beta + \lambda)z^n)$. Thus $\dot{V}_{Q1} \leq 0$ for

$$z \geq \sqrt[n]{A/(\beta + \lambda)} \triangleq z^0$$

Similarly, $\dot{V}_{Q2} \leq 0$ for $|z| \geq z^0$, where $Q2 = \{\dot{x} \leq 0, \ and \ z \leq 0\}$.
Also setting $Q3 = \{\dot{x} \geq 0, \ and \ z \leq 0\}$, we have $\dot{V}_{Q3} = z\dot{x}D^{-1}(A + (\beta - \lambda)|z|^n)$. In this case, $\dot{V}_{Q3} \leq 0$ for all values of z. The same conclusion is drawn in the case that $Q4 = \{\dot{x} \leq 0, \ and \ z \geq 0\}$.
We then conclude that for all possibilities of the signs of $\dot{x}$ and z, we have $\dot{V}_z \leq 0$ for all $|z| \geq z^0$. By theorem 4.10 in [141] we conclude that $z(t)$ is bounded for every piecewise function $\dot{x}$ and every initial condition $z(0)$. The bounds on $z(t)$ can be derived from [141] as follows: If the initial condition of z is such that $|z(0)| \leq z^0$, then $|z| \leq z^0$ for all $t \geq 0$. If the initial condition of z is such that $z(0) \geq z^0$, then $|z| \leq z(0)$ for all $t \geq 0$.
We now turn to the case $P2$ by considering $\dot{V}_z$ in the regions $\{\dot{x} \geq 0, \ and \ z \geq z^0\}$, $\{\dot{x} \geq 0, \ and \ -z^1 \leq z \leq 0\}$, $\{\dot{x} \leq 0, \ and \ 0 \leq z \leq z^1\}$, and $\{\dot{x} \leq 0, \ and \ -z^0 \leq z \leq 0\}$, where

$$z^1 \triangleq \sqrt[n]{A/(\lambda - \beta)}$$

By following the similar argument earlier, we can show that $\dot{V}_z \leq 0$ for the initial state $z(0)$ such that $|z(0)| \leq z^1$.
Following the same analysis for the case $P3$, we can see that z may be unbounded for some functions $\dot{x}$. This implies that the region of ultimate boundedness is empty in this case.
A similar analysis can be carried out for the case $A < 0$ and $A = 0$ and a conclusion draw from the analysis is summarized in the following lemma.

Lemma 6.1 *Consider the nonlinear dynamic system (6.3). Then for any piece-*

wise continuous signal x and $\dot{x}$, the output $z(t)$ is globally bounded if and only if the parameters of system (6.3) satisfies the inequality $\beta > |\lambda|$.

The control objective is to design a backstepping adaptive control law such that

- The closed loop is global uniform ultimate bounded.
- The tracking error $x(t) - y_r(t)$ is made arbitrarily small both in the transient period and steady state by an explicit choice of the design parameters, where $y_r(t)$ is a known bounded reference signal.

6.3 Control Design and Main Results

In this section, we develop two adaptive backstepping design schemes. In Scheme I, we use some available structure information in the design and the residual effect of the hysteresis is treated as a bounded disturbance with unknown bound. An update law is used to estimate the bound involving the effect of the hysteresis and the external disturbance. In Scheme II, we assume certain apriori information of the parameters in the Bouc-Wen model (6.3) is available and construct a variable $\bar{z}(t)$ to approximate $z(t)$ in (6.3) in our controller design. To illustrate the backstepping procedures, only the first scheme is elaborated in details.

6.3.1 Control Scheme I

The Bouc-Wen nonlinear restoring force $\Phi(x,t)$ in (6.2) can be parameterized as follows.

$$\Phi(t) = \theta_1 x(t) + R(t) \tag{6.5}$$

where $\theta_1 = \alpha k$ is uncertain parameter and $R(t) = (1-\alpha)Dkz(t)$. Note that $x(t)$ is an available signal.

For the residual term R we have the following inequality:

$$|R(t)| \leq (1-\alpha_{min})D_{max}k_{max}max_{t\geq 0}|z(t)| \tag{6.6}$$

Then we rewrite equations (6.1) and (6.5) in the following form

$$\dot{x}_1 = x_2 \tag{6.7}$$

$$\begin{aligned} \dot{x}_2 &= \frac{1}{m}\big(u(t) - \bar{c}x_2 - \theta_1 x(t) - R(t) + f(t)\big) \\ &= \theta^T \varphi(t) + \frac{1}{m}\big(u(t) + d(t)\big) \end{aligned} \tag{6.8}$$

where $x_1 = x, x_2 = \dot{x}$, $\theta = [\frac{\bar{c}}{m}, \frac{\theta_1}{m}]^T$ is a constant vector of uncertain parameters, $\varphi = [-x_2, -x(t)]^T$ and $d(t) = f(t) - R(t)$. Note that $R(t)$ is bounded as $z(t)$ has been shown bounded in Lemma 6.1. So $d(t)$ is bounded with unknown bound F. Before presenting the adaptive control design using the backstepping technique to achieve the desired control objectives, the following change of coordinates is made.

$$z_1 = x_1 - y_r \tag{6.9}$$

$$z_2 = x_2 - \dot{y}_r - \alpha_1 \tag{6.10}$$

where α_1 is the virtual control and will be determined in later discussion.

- *Step* 1: We design the virtual control law α_1 as

$$\alpha_1 = -c_1 z_1 \tag{6.11}$$

where c_1 is a positive design parameter. From (6.7) and (6.11) we have

$$z_1\dot{z}_1 = -c_1 z_1^2 + z_1 z_2 \tag{6.12}$$

- *Step* 2: From (6.8) and (6.10), we have

$$\dot{z}_2 = \theta^T\varphi + \frac{1}{m}\big(d(t) + u(t)\big) - \ddot{y}_r - \dot{\alpha}_1 \tag{6.13}$$

Then the control law and parameter update laws are given below.

$$u = -\hat{F}\text{sign}(z_2) + \hat{m}\bar{u} \tag{6.14}$$

$$\bar{u} = -c_2 z_2 - z_1 - \hat{\theta}\varphi + \ddot{y}_r + \dot{\alpha}_1 \tag{6.15}$$

$$\dot{\hat{\theta}} = \Gamma\varphi z_2 \tag{6.16}$$

$$\dot{\hat{m}} = -\gamma\bar{u}z_2 \tag{6.17}$$

$$\dot{\hat{F}} = \gamma_f |z_2| \tag{6.18}$$

where c_2, γ and γ_f are designed positive parameters, Γ is a positive definite design matrix. $\hat{\theta}$, $\hat{m}$ and $\hat{F}$ are estimates of θ, m and F.

Remark 6.1 *Note that a parameter update law is used to estimate the bound F of the disturbance $d(t)$, so there is no need to know this bound.*

We define a positive Lyapunov function as

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma m}\tilde{m}^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{1}{2m\gamma_f}\tilde{F}^2 \tag{6.19}$$

where $\tilde{m} = m - \hat{m}$, $\tilde{\theta} = \theta - \hat{\theta}$ and $\tilde{F} = F - \hat{F}$.

Note that $\frac{1}{m}u$ in (6.13) can be expressed as

$$\begin{aligned} \frac{1}{m}u &= \frac{1}{m}\hat{m}\bar{u} - \frac{1}{m}\hat{F}\text{sign}(z_2) \\ &= \bar{u} - \frac{1}{m}\tilde{m}\bar{u} - \frac{1}{m}\hat{F}\text{sign}(z_2) \end{aligned} \tag{6.20}$$

Then the derivative of V along with (6.14-6.17) is given by

$$\begin{aligned}
\dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + \frac{1}{\gamma m}\tilde{m}\dot{\tilde{m}} + \frac{1}{\gamma_f m}\tilde{F}\dot{\tilde{F}} \\
&\le -c_1 z_1^2 - c_2 z_2^2 + \tilde{\theta}^T\Gamma^{-1}\big(\Gamma\varphi z_2 - \dot{\hat{\theta}}\big) \\
&\quad -\frac{1}{\gamma m}\tilde{m}\big(\gamma\bar{u}z_2 + \dot{\hat{m}}\big) + \frac{1}{m\gamma_f}\tilde{F}\big(\gamma_f|z_2| - \dot{\hat{F}}\big) \\
&= -c_1 z_1^2 - c_2 z_2^2 \qquad (6.21)
\end{aligned}$$

Based on (6.21), we can obtain the result on system stability and performance as stated below.

Theorem 6.1 *Consider the uncertain nonlinear system (6.1). With the application of the controller (6.14) and the parameter update laws (6.16), (6.17) and (6.18), the following statements hold:*

- *The resulting closed loop system is global uniform ultimate bounded.*
- *The asymptotic tracking is achieved, i.e.,*

$$\lim_{t\to\infty}[x(t) - y_r(t)] = 0 \qquad (6.22)$$

- *The transient displacement tracking error performance is given by*

$$\| x(t) - y_r(t) \|_2 \le \frac{1}{\sqrt{c_1}} \times \Big(\frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2\Big)^{1/2} \qquad (6.23)$$

- *The transient velocity tracking error performance is given by*

$$\| \dot{x} - \dot{y}_r \|_2 \le \Big(\frac{1}{\sqrt{c_2}} + \sqrt{c_1}\Big) \times \Big(\frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2\Big)^{1/2} \qquad (6.24)$$

Proof: Equation (6.21) shows that $V(t)$ is globally uniformly bounded. This implies that $z_1, z_2, \tilde{\theta}, \tilde{m}, \tilde{F}$ are bounded. The state variables x_1, x_2 and the parameter estimates $\hat{\theta}, \hat{m}, \hat{F}$ are also bounded. Thus u is bounded from (6.14) because of the boundedness of $z_1, z_2, \hat{\theta}, \hat{m}, \hat{F}$.

Since V is non increasing from (6.21), we have

$$\| z_1 \|_2^2 = \int_0^\infty |z_1(\tau)|^2 d\tau \leq \frac{1}{c_1}(V(0) - V(\infty)) \leq \frac{1}{c_1} V(0) \tag{6.25}$$

Thus, by setting $z_1(0) = z_2(0) = 0$, we obtain

$$V(0) = \frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2 \tag{6.26}$$

a decreasing function of γ, γ_f and Γ, independent of c_1. This means that the bounds resulting from (6.25) and (6.26)

$$\| z_1 \|_2 \leq \frac{1}{\sqrt{c_1}}\Big(\frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2\Big)^{1/2} \tag{6.27}$$

can be asymptotically reduced either by increasing c_1 or by simultaneously increasing γ, γ_f and Γ. The bound for $\| z_1 \|_2$ is explicit.

From equations (6.8) to (6.11), we get

$$\| \dot{x} - \dot{y}_r \|_2 = \| z_2 - c_1 z_1 \|_2 \leq \| z_2 \|_2 + c_1 \| z_1 \|_2 \tag{6.28}$$

Similarly, we can get $\| z_2 \|_2 \leq \frac{1}{\sqrt{c_2}}\sqrt{V(0)}$. Along with (6.27) we get

$$\| \dot{x} - \dot{y}_r \|_2 \leq \Big(\frac{1}{\sqrt{c_2}} + \sqrt{c_1}\Big)\Big(\frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2\Big)^{1/2} \tag{6.29}$$

Remark 6.2 *From Theorem 6.1 the following conclusions can be obtained:*

- *Boundedness of the adaptive system is guaranteed to be global, uniform and ul-*

timate for any positive values of the design parameters $c_1, c_2, \gamma, \gamma_f$ and Γ. No a priori information is required about the parameter uncertainty.

- *We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains γ, γ_f and Γ. And thus the bound for $\| x - y_r \|$ is an explicit function of desired parameters.*
- *The transient performance depends on the initial estimate errors $\tilde{\theta}(0)$ and $\tilde{m}(0)$. The closer the initial estimates $\hat{\theta}(0), \hat{F}(0)$ and $\hat{m}(0)$ to the true values θ, F and m, the better the transient performance. The asymptotic behavior is not affected by the initial estimate errors.*
- *To improve the displacement tracking error performance we can also increase the gain c_1. However, increasing the gain c_1 will also increase the velocity tracking error as shown above. Improving the closed loop displacement behavior may be done at the expense of the increase in the control signal amplitude. This suggests to fix the gain c_1 to some acceptable value and adjust the other gains. By fixing the gain c_1, increasing the gain c_2 or by simultaneously increasing γ, γ_f and Γ, we can achieve a velocity tracking error as small as desired.*

6.3.2 Control Scheme II:

In this section, we assume that certain apriori information of the Bouc-Wen model parameters is available. Thus we further exploit the structure of the model in our controller design to improve system performance.

The Bouc-Wen nonlinear restoring force $\Phi(x,t)$ in (6.2) can be parameterized as follows.

$$\Phi(t) = \theta_1 x(t) + \theta_2 z(t) \tag{6.30}$$

where $\theta_1 = \alpha k$ and $\theta_2 = (1-\alpha)Dk$ are uncertain parameters.

Assumption: Parameters A, β, D, λ are inside some known intervals.

With the above assumption, a signal $\bar{z}(t)$ can be generated using an equation

$$\dot{\bar{z}} = D_0^{-1}[A_0\dot{x} - \beta_0|\dot{x}||\bar{z}|^{n-1}\bar{z} - \lambda_0\dot{x}|\bar{z}|^n] \tag{6.31}$$

where $A_0, \beta_0, D_0, \lambda_0$ are inside the known intervals. With this $\bar{z}(t)$, we approximate $\Phi(x,t)$ by $\bar{\Phi}(x,t)$ as $\bar{\Phi}(x,t) = \theta_1 x(t) + \theta_2\bar{z}(t)$.

Remark 6.3 *From Lemma 6.1, $\bar{z}(t) - z(t)$ is bounded. Note that $\Phi - \bar{\Phi} = \theta_2(z - \bar{z})$. So $\Phi - \bar{\Phi}$ is also bounded. This bounded error can then be combined with external disturbance to get f with its combined bound F estimated as in Scheme I. It is expected that $\| \Phi - \bar{\Phi} \| \leq \| \Phi \|$.*

Then we rewrite equations (6.1) and (6.5) in the following form

$$\dot{x}_1 = x_2 \tag{6.32}$$

$$\dot{x}_2 = \theta^T\varphi(t) + \frac{1}{m}\big(u(t) + f(t)\big) \tag{6.33}$$

where $x_1 = x, x_2 = \dot{x}$, $\theta = [\frac{\bar{c}}{m}, \frac{\theta_1}{m}, \frac{\theta_2}{m}]^T$ is a constant vector of uncertain parameters, and $\varphi = [-x_2, -x(t), -\bar{z}(t)]^T$.

The controller design is similar to the Scheme I. We only give the resulting control laws.

$$u = -\hat{F}\text{sign}(z_2) + \hat{m}\bar{u} \tag{6.34}$$

$$\bar{u} = -c_2 z_2 - z_1 - \hat{\theta}\varphi + \ddot{y}_r + \dot{\alpha}_1 \tag{6.35}$$

$$\alpha_1 = -c_1 z_1 \tag{6.36}$$

$$\dot{\hat{\theta}} = \Gamma\varphi z_2 \tag{6.37}$$

$$\dot{\hat{m}} = -\gamma\bar{u}z_2 \tag{6.38}$$

$$\dot{\hat{F}} = \gamma_f|z_2| \tag{6.39}$$

where c_1, c_2, γ and γ_f are designed positive parameters, Γ is a positive definite design matrix, $\hat{\theta}$, $\hat{m}$ and $\hat{F}$ are estimates of θ, m and F.

Remark 6.4 *Certain information of the hysteretic structure is used in our controller design, unlike the Scheme I and the scheme in [128], where the effect of hysteresis is treated as a bounded disturbance. This is reflected in* $\hat{\theta}$ *and* φ *of the designed controller in (6.34)-(6.39).*

Following the similar analysis to Scheme I, we can establish that $x, \dot{x}, \hat{\theta}, \hat{m}, \hat{F}$ are all bounded. Then from Lemma 6.1 and (6.34), (6.35), u is also bounded. Thus similar to Theorem 6.1, the result on system stability and performance can be established and now stated in the following theorem.

Theorem 6.2 *Consider the uncertain nonlinear system (6.1). With the application of the controller (6.34) and the parameter update laws (6.37), (6.38) and (6.39), the following statements hold:*

- *The resulting closed loop system is global uniform ultimate bounded.*
- *The asymptotic tracking is achieved, i.e.,*

$$\lim_{t\to\infty}[x(t) - y_r(t)] = 0 \tag{6.40}$$

- *The transient displacement tracking error performance is given by*

$$\| x(t) - y_r(t) \|_2 \leq \frac{1}{\sqrt{c_1}} \times \Big(\frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2\Big)^{1/2} \tag{6.41}$$

- *The transient velocity tracking error performance is given by*

$$\| \dot{x} - \dot{y}_r \|_2 \leq (\frac{1}{\sqrt{c_2}} + \sqrt{c_1}) \times \Big(\frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{2\gamma m}\tilde{m}(0)^2 + \frac{1}{2m\gamma_f}\tilde{F}(0)^2\Big)^{1/2} \tag{6.42}$$

6.4 Simulation Results

In this section we test our proposed backstepping controller. For simulation studies, the following values are selected as "true" parameters for the system and the hysteresis model: $m = 156 \times 10^3 [Kg], k = 6 \times 10^6 [N/m], c = 2 \times 10^4 [Ns/m]$, $\alpha = 0.6, D = 0.6[m], A = 1, \beta = 0.5, \lambda = 0.4, n = 3$. In fact, it is not required to know the exact values of these parameters to implement the controller.

The control objective is to mitigate the seismic displacement response of the system, so the reference trajectory $y_r(t)$ is set to 0.

When Scheme I is used, we take the following set of design parameters: $\gamma = 0.1, \Gamma = I_3, \gamma_f = 0.5, c_1 = 0.3, c_2 = 3$, $\hat{m}(0) = 300 \times 10^3$, $\hat{D}(0) = 200 * 10^3$, and $\hat{\theta}(0) = [0.15, 12, 5]^T$.

When Scheme II is used, we take the following set of design parameters: $\gamma = 0.1, \Gamma = I_2, \gamma_f = 0.5, c_1 = 0.3, c_2 = 3$, $\hat{m}(0) = 300 \times 10^3$, $\hat{F}(0) = 200 * 10^3$, $\hat{\theta}(0) = [0.15, 12]^T$, $A_0 = 1.5, \beta_0 = 0.6, \lambda_0 = 0.3, D = 0.5$. Note that the uncertainties of these parameters are $50\%, 20\%, 25\%, 16.7\%$, respectively.

For comparison, the scheme in [128] and our proposed Scheme I and Scheme II are all applied to the system (6.1) to (6.3). Figure 6.2 shows the earthquake ground acceleration. Figure 6.3 displays the results of the hysteresis behavior.

The simulation results with the proposed two schemes are presented in Figures 6.4 to 6.8. Figures 6.4 and 6.5 show the time histories of the displacement x_1 and the velocity x_2 without control and with control using Scheme I. After t = 20 seconds, the excitation stops and the uncontrolled case corresponds to free vibration response. The open loop system exhibits a low damping behavior. On the contrary, the proposed control drive the response towards zero rapidly, thus introducing a significant damping effect into the system. Figures 6.6 and 6.7 show the time histories of the displacement x_1 and the velocity x_2 with active control using proposed Scheme I and the scheme in [128]. Figure 6.8 shows the time history of the control signal with Scheme I. Clearly, system performance is improved by Scheme I, even though we need much less apriori knowledge from the system.

The simulation results with the proposed Scheme II are also presented in Figures 6.9 to 6.15. Figure 6.10 displays system hysteresis behavior $\Phi(t)$ and the approximated hysteresis behavior $\bar{\Phi}(t)$ with Scheme II. Figure 6.11 displays the behavior of $\Phi - \bar{\Phi}$. As noted, $|\Phi - \bar{\Phi}|$ is smaller than $|\Phi(t)|$. A significant reduction in the magnitude of x and $\dot{x}$ can be observed with Scheme II, compared with Scheme I and the scheme in [128].

As a conclusion, the simulation results verify our theoretical findings and show the effectiveness of our control schemes. System performance is improved by our proposed schemes. Also Scheme II is better than Scheme I in improving system performance, but requires more apriori knowledge.

6.5 Summary

This chapter has presented two backstepping adaptive controllers for a second-order uncertain building structural system involving hysteretic phenomena. The hysteretic nonlinear behavior is described by the so-called Bouc-Wen model. The control strategies have been applied to a system found in base isolation schemes for seismic active protection of building structures. The developed backstepping controls do not require the model parameters within known intervals. In the first scheme, the partial effect of the hysteresis is treated as a bounded disturbance. In the second scheme, we further take the structure of the hysteresis into account in our controller design. It is shown that the proposed controllers can guarantee global uniform ultimate bounded and achieve tracking to a desired precision. Numerical results show that the adaptive control law is working satisfactorily in that the response induced by seismic action is significant reduced.

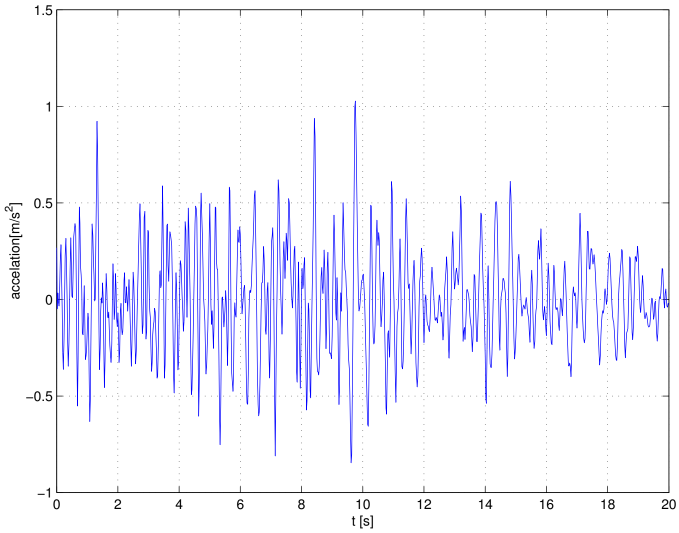


Figure 6.2: Earthquake ground acceleration.

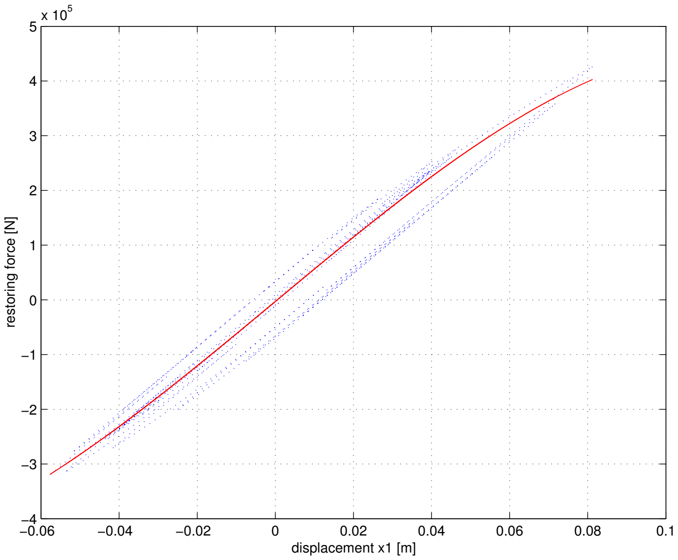


Figure 6.3: Hysteresis identification.

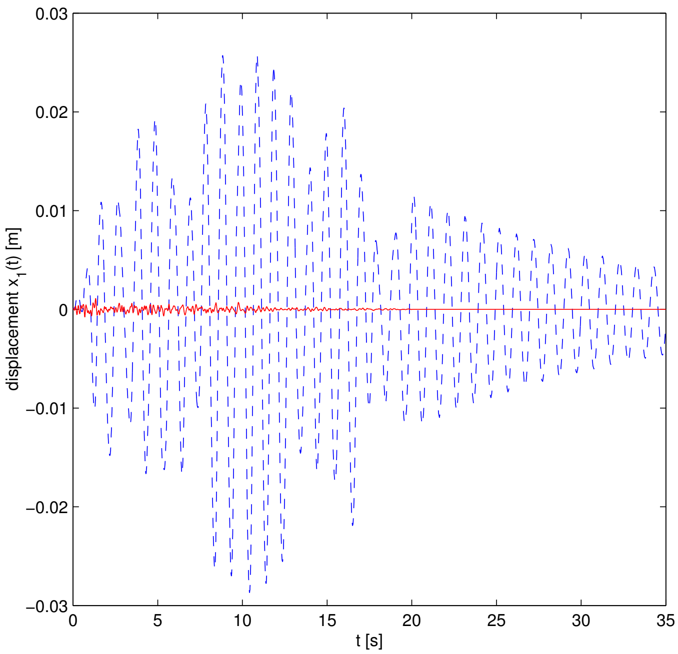


Figure 6.4: Displacement x_1.

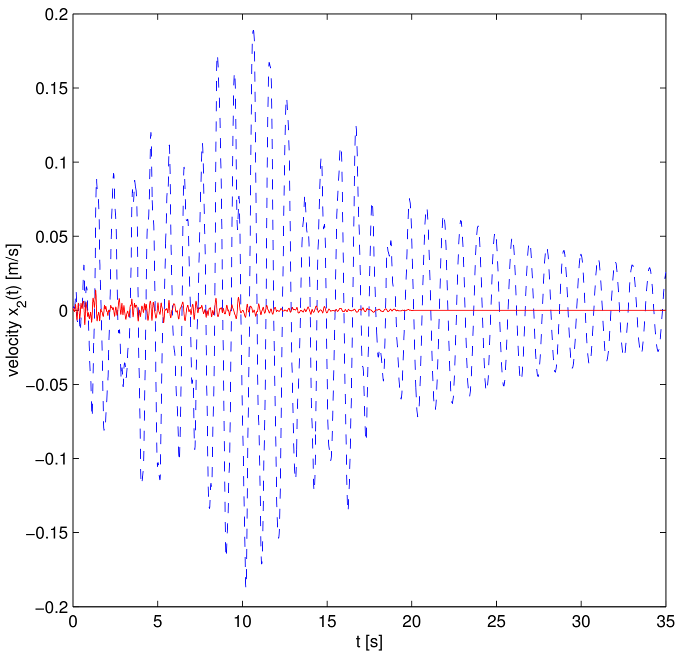


Figure 6.5: Velocity x_2.

Without Control – dashed line
With Scheme I – solid line.

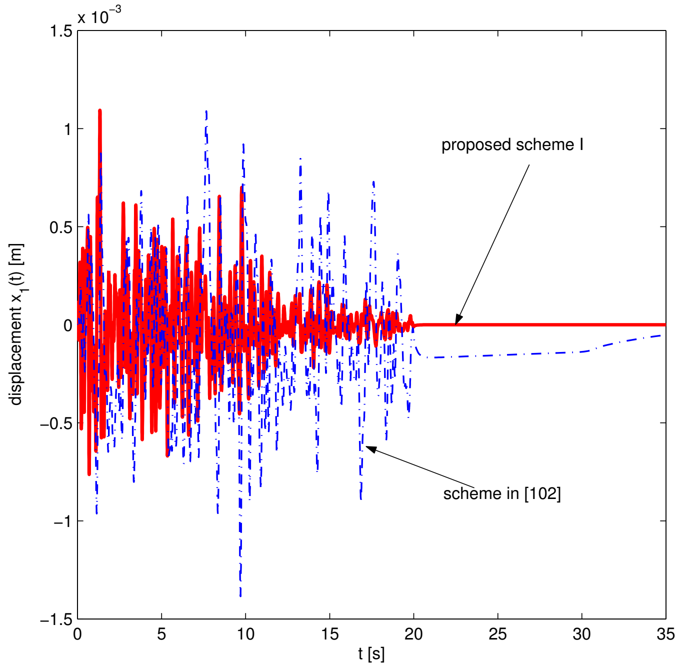


Figure 6.6: Displacement x_1.

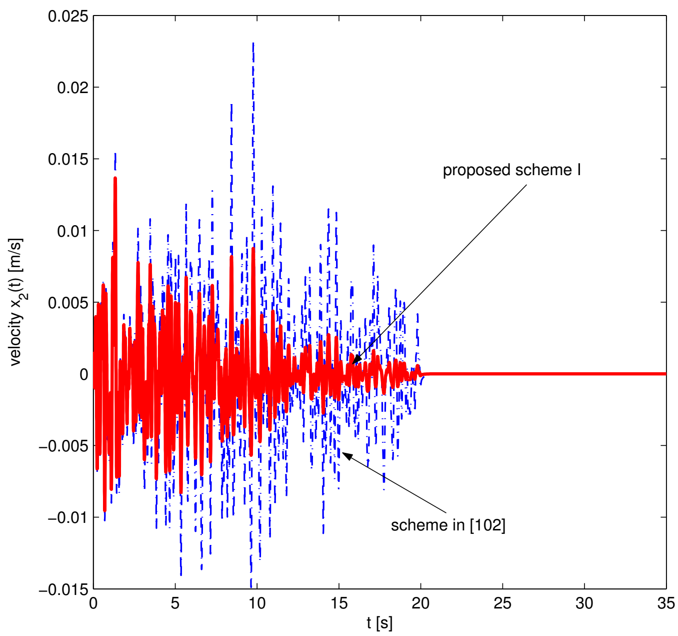


Figure 6.7: Velocity x_2.

Scheme I (solid) and the Scheme in [128] (dashed).

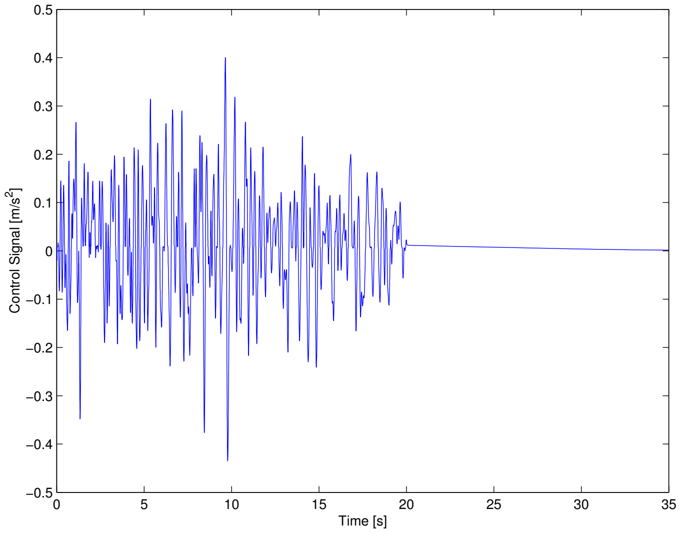


Figure 6.8: Control Signal $u(t)/m$ with Scheme I.

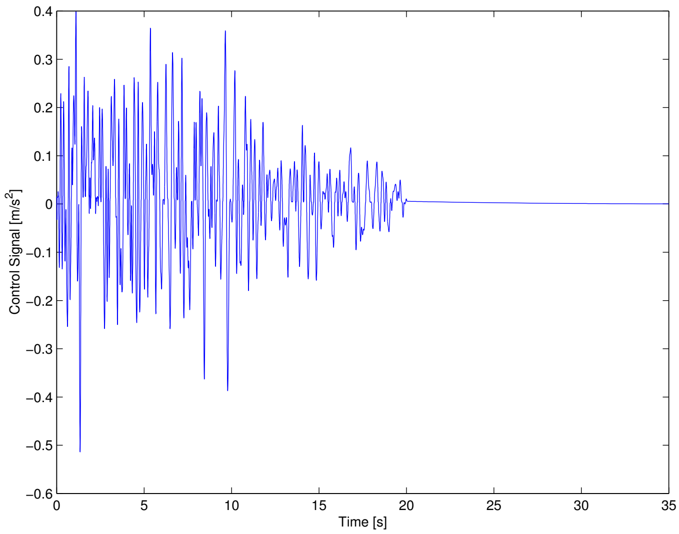


Figure 6.9: Control Signal $u(t)/m$ with Scheme II.

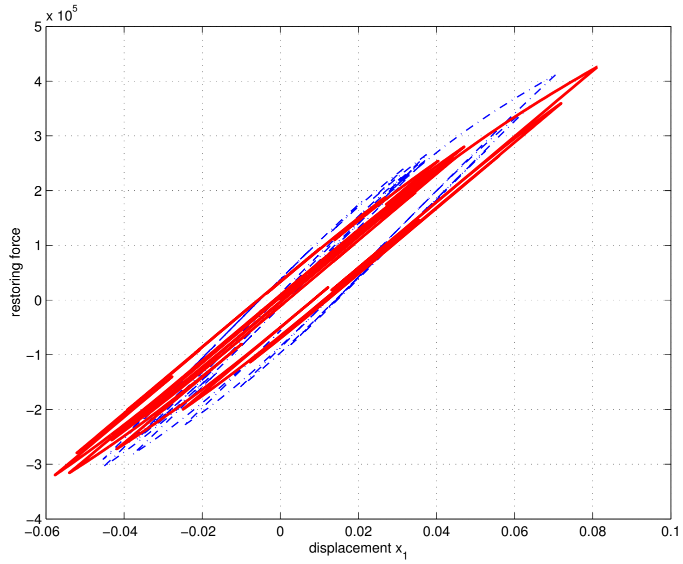


Figure 6.10: True system hysteresis Φ – solid line.

Approximated hysteresis $\bar{\Phi}$ with Scheme II – dashed line.

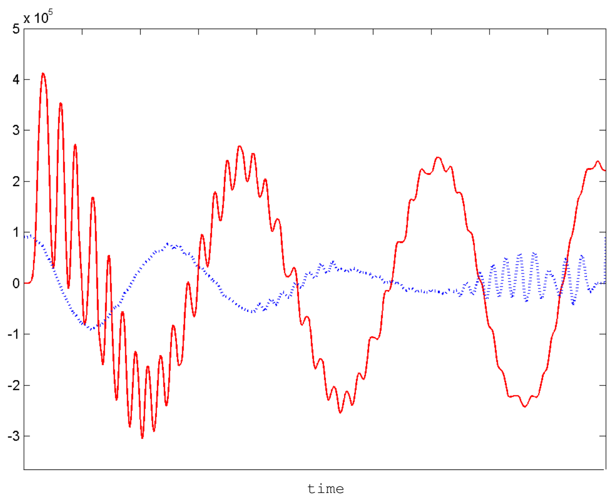


Figure 6.11: True hysteresis Φ – solid line; $\Phi - \bar{\Phi}$ – dot line.

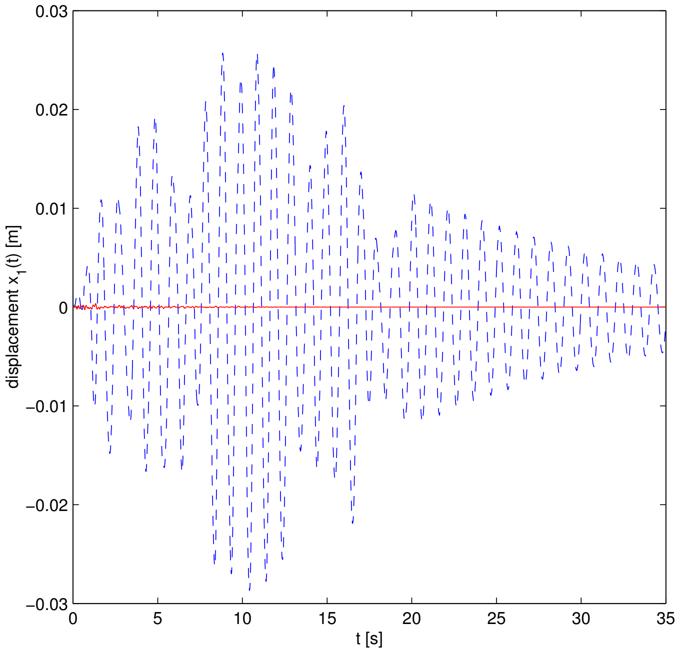


Figure 6.12: Displacement x_1.

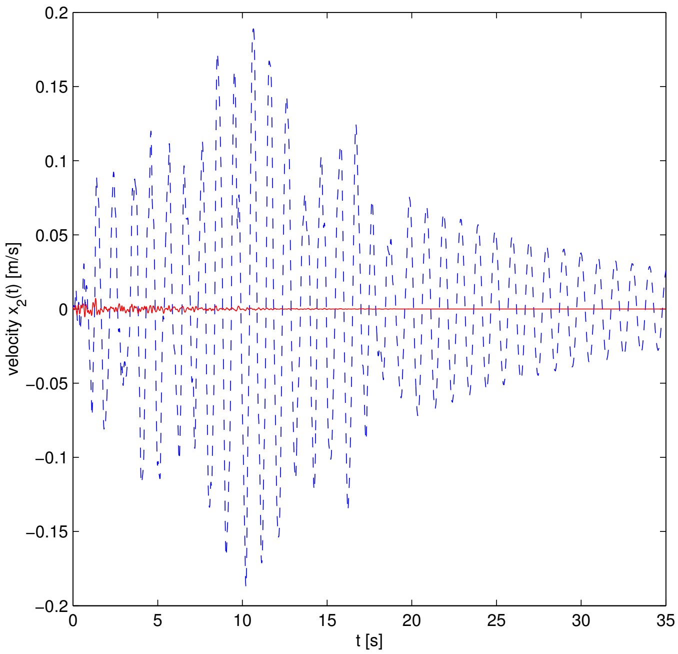


Figure 6.13: Velocity x_2 .

Without Control – dashed line
With Scheme II – solid line.

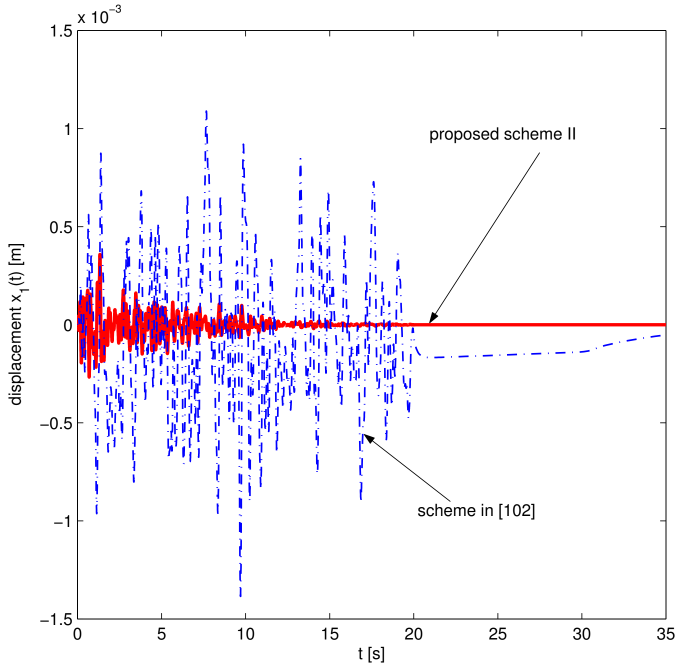


Figure 6.14: Displacement x_1.

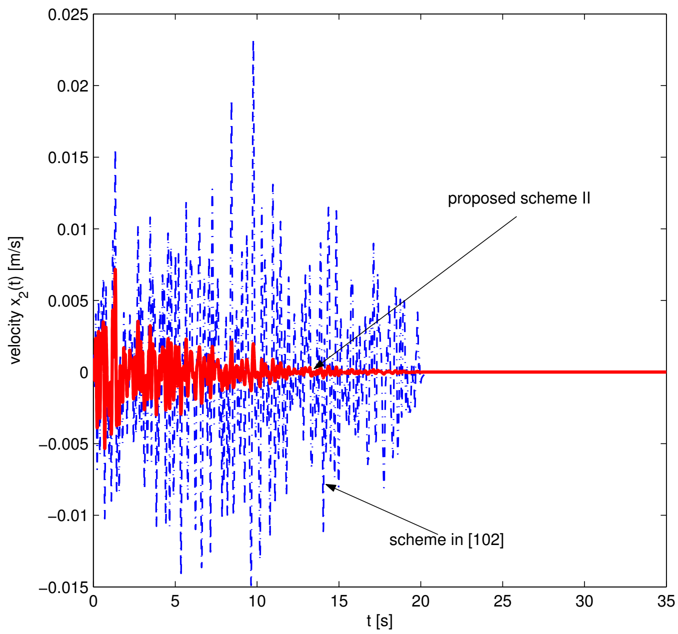


Figure 6.15: Velocity x_2.

Scheme II (solid) and the Scheme in [128] (dashed).

Chapter 7

Output Control of Time-varying Nonlinear Systems

The task of this chapter is to introduce a new scheme to design adaptive controllers for single-input single-output uncertain time-varying systems in the presence of unknown bounded disturbances. No knowledge is assumed on the sign of the term multiplying the control. The control design is achieved by introducing certain well defined functions, estimating variation rates of parameters and incorporating a Nussbaum gain. To overcome the problem of overparametrization, tuning functions, which are different from the standard ones in [30] due to the use of projection operations, are employed. It is shown that the proposed controller can guarantee global uniform ultimate boundedness.

7.1 Background

Adaptive control has seen significant development. However, only limited number of results are available for nonlinear systems with time-varying parameters and/or without the knowledge on the sign of the term multiplying the control, i.e. high frequency gain in the case of linear systems, in the presence of external disturbances. In this paper, we shall also call this term as high frequency gain for nonlinear systems for simplicity.

In [142], output feedback control was considered for linear time-varying systems when the sign of high-frequency gain is known. In [34], the problem of adaptive control with unknown sign of high-frequency gain for linear time invariant systems was studied. In [143], Nussbaum gain incorporating with the backstepping technique was used to design adaptive output stabilizer for high order uncertain time invariant nonlinear systems with unknown sign of high-frequency gain in the absence of external disturbances. The nonlinearities considered should satisfy sector conditions. In [35], disturbance decoupling was addressed for nonlinear time invariant systems with known sign of the high frequency gain. The result obtained is critically depending on a function of the system output y and the reference trajectory y_r. It should be noted that such a function is undefined at the time instants when $y = y_r$. Therefore, the control signal is undefined at these time instants. In [144], a flat zone was used to handle the problem nonlinear time invariant systems with unknown sign of high frequency gain in the presence of disturbances. The bound of the disturbance and all the unknown parameters need to be estimated at every step in the backstepping process. This results in the problem of overparametrization and makes the implementation complicated. In [145] state-feedback control was considered for a class of uncertain time-varying nonlinear systems in the presence of disturbances. Due to state feedback, no filter is required for state estimation. Thus the derivatives of the time varying parameters and the term of the disturbance need not to be considered in controller design. This also makes the stability analysis greatly simplified. Again, parameters are required to be estimated at every step, which result in overparametrization. In the case of output feedback control of nonlinear time-varying systems in the presence of disturbances, no result is available. In this case, filters similar to [30] are required to estimate system states and the state equations of the state estimation error will be used in the design and analysis. In these equations, the external disturbances and derivatives of time-varying parameters will appear and have great impact on the errors. This makes the design and analysis quite difficult, especially when the sign of high frequency gain is unknown and tuning functions are used.

In this chapter, we consider such a case and propose a new control design scheme to solve the problem. The nonlinearities considered are not required to satisfy the sector type of conditions like [143]. To handle the disturbances, well defined functions are introduced to eliminate their effects in the Lyapunov functions employed in the recursive design steps. To deal with the time variation problem, an estimator is used to estimate the bound of the variation rates. Furthermore, the overparamterization problem is also solved by using the concept of tuning functions. As projection operation is used, the design of tuning functions are different from existing schemes as in [30]. With our proposed controller, global system stability is ensured.

7.2 System Model and Problem Formulation

7.2.1 Problem Formulation

Consider the following class of single-input-single-output (SISO) nonlinear time-varying systems in the feedback form

$$\begin{aligned}
\dot{x}_1 &= x_2 + \theta_{a1}(t)\psi_{a1}(y) + d_1(t)\phi_{a1}(y) + \psi_{01}(y) \\
&\vdots \\
\dot{x}_{\rho-1} &= x_\rho + \theta_{a\rho-1}(t)\psi_{a\rho-1}(y) + d_{\rho-1}(t)\phi_{a\rho-1}(y) + \psi_{0\rho-1}(y) \\
\dot{x}_\rho &= x_{\rho+1} + \theta_{a\rho}(t)\psi_{a\rho}(y) + d_\rho(t)\phi_{a\rho}(y) + \psi_{0\rho}(y) + b_m(t)u \\
&\vdots \\
\dot{x}_n &= \theta_{an}(t)\psi_{an}(y) + d_n(t)\phi_{an}(y) + \psi_{0n}(y) + b_0(t)u \\
y &= e_1^T x
\end{aligned} \tag{7.1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n, u \in R$ and $y \in R$ are system states, input and output respectively, $b_i(t), i = m, \ldots, 0$ are bounded uncertain time-varying piecewise continuous high-frequency gains, $\theta_{ai}(t) \in R^{p_i}$ are uncertain time-varying parameters, $d_i(t), i = 1, \ldots, n$ denote unknown time-varying bounded disturbances, ψ_{ai}

and ϕ_{ai} are known smooth nonlinear functions in R^n. Similar class of systems was analyzed in [146].

In order to cope with the unknown sign of high-frequency gain, the Nussbaum gain technique is employed in this chapter. A function $N(\chi)$ is called a Nussbaum-type function if it has the following properties

$$lim_{s\to\infty} sup \frac{1}{s}\int_0^s N(\chi)d\chi = \infty \quad (7.2)$$

$$lim_{s\to\infty} inf \frac{1}{s}\int_0^s N(\chi)d\chi = -\infty \quad (7.3)$$

In this chapter, the even Nussbaum function $exp(\chi^2)\cos(\frac{\pi}{2}\chi)$ is exploited. As in Chapter 5, the following Lemma will be employed in later analysis.

Lemma 7.1 *Let $V(t)$ and $\chi(t)$ be a smooth function defined on $[0, t_f)$ with $V(t) \geq 0$, $\forall t \in [0, t_f)$, and $N(\chi) = exp(\chi^2)\cos(\frac{\pi}{2}\chi)$ be an even smooth Nussbaum-type function. If the following inequality holds:*

$$V(t) \leq f_0 + e^{-f_1 t}\int_0^t g_1 N(\chi)\dot{\chi}d\tau + e^{-f_1 t}\int_0^t \dot{\chi}(t)e^{f_1\tau}d\tau \quad (7.4)$$

where constant $f_1 > 0$, g_1 is a parameter which takes values in the unknown closed intervals $I_1 = [l_1^-, l_1^+]$ with $0 \notin I_1$, and f_0 represents some suitable constant, then $V(t), \chi(t)$ and $\int_0^t g_1 N(\chi)\dot{\chi}d\tau$ must be bounded on $[0, t_f)$.

For the considered system in (7.1), the following assumptions are imposed.

Assumption 1. The uncertain parameter vector θ is inside a compact set Ω_θ, where $\theta = [b_m(t), \ldots, b_0(t), \theta_{a1}(t), \ldots, \theta_{an}(t)]^T$. In addition, there exists an unknown bounded positive constant q so that $q \geq \| \dot{\theta} \|$. Also q is inside a compact intervals $\Omega_q = [I^-, I^+]$ and $b_m(t) \neq 0, \forall t$.

Assumption 2. The relative degree ρ is fixed and known. This is ensured by Assumption 1.

Assumption 3. The reference signal y_r and its $(\rho - 1)$th order derivatives are also assumed to be known and bounded.

Assumption 4. The system is minimum phase in the sense defined in [146].

Definition: System is said to be minimum phase if its zero dynamics, subject to appropriate initial conditions and a suitable control producing output identically zero, is stable.

7.2.2 State Estimation Filters

In order to design the desired adaptive control law with output via backstepping procedures, we now transform system (7.1) into the following form

$$\dot{x} = Ax + F(y,u)^T\theta + \Phi_a(y)d(t)^T + \psi_0(y) \tag{7.5}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \tag{7.6}$$

$$F(y,u)^T = \left[\begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} u, \quad \Psi_a(y) \right] \tag{7.7}$$

$$\Psi_a(y) = \begin{bmatrix} \psi_{a1}^T & 0 & \dots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \psi_{an}^T \end{bmatrix} = \begin{bmatrix} \Psi_{a1}(y) \\ \vdots \\ \Psi_{an}(y) \end{bmatrix} \tag{7.8}$$

$$\Phi_a(y) = \begin{bmatrix} \phi_{a1}^T & 0 & \dots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \phi_{an}^T \end{bmatrix} == \begin{bmatrix} \Phi_{a1}^T(y) \\ \vdots \\ \Phi_{an}^T(y) \end{bmatrix} \tag{7.9}$$

$$\theta = [b_m(t), \dots, b_0(t), \theta_{a1}(t), \dots, \theta_{an}(t)]^T \tag{7.10}$$

$$d(t) = [d_1(t), \ldots, d_n(t)] \tag{7.11}$$

$$\psi_0(y) = [\psi_{01}(y), \ldots, \psi_{0n}(y)]^T \tag{7.12}$$

We employ the filters similar to those in [30], i.e.

$$\dot{\xi} = A_0\xi + ky + \psi_0(y) \tag{7.13}$$

$$\dot{\Omega}^T = A_0\Omega^T + F(y, u)^T \tag{7.14}$$

where

$$k \triangleq [k_1, k_2, \ldots, k_n]^T \tag{7.15}$$

$$A_0 = A - ke_1^T \tag{7.16}$$

The vector k in (7.15) is chosen such that the matrix A_0 is strictly stable. It can be shown that Ω obtained from (7.14) satisfies the following equations.

$$\Omega^T = [v_m, \ldots, v_1, v_0, \Xi] \tag{7.17}$$

$$\dot{\Xi} = A_0\Xi + \Psi_a(y) \tag{7.18}$$

$$\dot{\lambda} = A_0\lambda + e_n u \tag{7.19}$$

$$v_j = A_0^j\lambda \tag{7.20}$$

From our designed filters, system (7.1) can be represented as

$$\dot{y} = b_m v_{m,2} + \beta + \bar{\omega}^T\theta + \epsilon_2 + d(t)\Phi_{a1}(y) \tag{7.21}$$

$$\dot{v}_{m,i} = v_{m,i+1} - k_i v_{m,1}, \qquad i = 2, 3, \ldots, \rho - 1 \tag{7.22}$$

$$\dot{v}_{m,\rho} = v_{m,\rho+1} - k_\rho v_{m,1} + u \tag{7.23}$$

where

$$\beta = \xi_2 + \psi_{01} \tag{7.24}$$

$$\omega = [v_{m,2}, v_{m-1,2}, \ldots, v_{0,2}, \Xi_2 + \Psi_{a1}]^T \tag{7.25}$$

$$\bar{\omega} = [0, v_{m-1,2}, \ldots, v_{0,2}, \Xi_2 + \Psi_{a1}]^T \tag{7.26}$$

In above equations, $\epsilon_2, v_{i,2}$ and $\xi_{i,2}$ denote the second entries of ϵ, v_i and ξ_i respectively, ϵ is the estimation error defined in (7.28).
With the above filters, a state estimate is given by

$$\hat{x} = \xi + \Omega^T \theta \tag{7.27}$$

and the estimation errors ϵ is defined as

$$\epsilon = x - \hat{x} \tag{7.28}$$

From the equations (7.5), (7.13), (7.14), (7.27) and (7.28), the estimation error satisfies

$$\dot{\epsilon} = A_0 \epsilon + \Phi_a(y) d(t)^T - \Omega^T \dot{\theta} \tag{7.29}$$

Remark 7.1 *The error ϵ will be used in our design and analysis given later. As the disturbances and derivatives of time-varying parameters appear in (7.29), their effects should be considered in controller design. However for the state-feedback control in [145], no filter is required for state estimation. Their effects may not be necessarily considered in controller design and this makes problem much simpler.*

We now divide the error ϵ into two parts, i.e. $\epsilon = \epsilon_a + \epsilon_b$, where ϵ_a satisfies

$$\dot{\epsilon}_a = A_0 \epsilon_a + \Phi_a(y) d(t)^T \tag{7.30}$$

with $\epsilon_a(0) = \epsilon(0)$, and $\epsilon_b = \int_0^t e^{A_0(t-\tau)}(-\Omega^T\dot{\theta})d\tau$. It can be shown that

$$\begin{aligned} \| \epsilon_b \| &\leq \int_0^t \| e^{A_0(t-\tau)} \| \| \Omega \| \| \dot{\theta} \| d\tau \\ &\leq q\int_0^t \| e^{A_0(t-\tau)} \| \| \Omega \| d\tau \\ &\leq q\int_0^t e^{-\lambda_\theta(t-\tau)}k_\theta \| \Omega \| d\tau \end{aligned} \tag{7.31}$$

where λ_θ and k_θ are chosen positive parameters so that

$$k_\theta e^{-\lambda_\theta t} \geq \| e^{A_0 t} \|, \quad \forall t \geq 0 \tag{7.32}$$

Thus ϵ_b satisfies that

$$|\epsilon_b| \leq h(t)q \tag{7.33}$$

where $h(t)$ is generated by

$$\dot{h} = -\lambda_\theta h + k_\theta(\| \Omega \|^2 + \frac{1}{4}). \tag{7.34}$$

Suppose $P \in R^{n\times n}$ is a positive definite matrix, satisfying $PA_0 + A_0^T P \leq -3I$ and let

$$V_\epsilon = \epsilon_a^T P \epsilon_a \tag{7.35}$$

It can be shown that

$$\begin{aligned} \dot{V}_\epsilon &= \epsilon_a^T(PA_0 + A_0^T P)\epsilon_a + 2\epsilon_a^T P\Phi_a(y)d(t)^T \\ &\leq -2 \| \epsilon_a \|^2 + \| P\Phi_a(y)d(t)^T \|^2 \end{aligned} \tag{7.36}$$

The problem of this chapter is to design an adaptive controller to make system (7.1) BIBO stable.

7.3 Control Design

7.3.1 Design Procedure

In this section, we present the adaptive control design using the backstepping technique with tuning functions in ρ steps. In order to avoid using the sign of the high frequency gain, we take the change of coordinates

$$z_1 = y - y_r \tag{7.37}$$

$$z_i = v_{m,i} - \alpha_{i-1}, \quad i = 2, 3, \ldots, \rho, \tag{7.38}$$

where α_{i-1} is the virtual control at each step and will be determined in later discussions. Before presenting the detail, a useful function is introduced. Firstly we define $s(x)$ as

$$s(x) = \begin{cases} x^2 & |x| \geq \delta \\ (\delta^2 - x^2)^\rho + x^2 & |x| < \delta \end{cases} \tag{7.39}$$

where δ is a positive design parameter. It can be shown that $s(x)$ is $(\rho - 1)$th order differentiable and bounded below for $|x| < \delta$. Based on $s(x)$, a function $H(z_1)$ is defined as follows

$$H(z_1) = \frac{\Phi_a(y)}{s(z_1)} = \begin{cases} \dfrac{\Phi_a(y)}{z_1^2} & |z_1| \geq \delta \\ \dfrac{\Phi_a(y)}{(\delta^2 - z_1^2)^\rho + z_1^2} & |z_1| < \delta \end{cases} \tag{7.40}$$

Clearly H is well defined and for $|z_1| < \delta$, H is bounded as $s(z_1)$ is bounded below.

Remark 7.2 *In [35], a similar function to (7.40) was used to design controllers* *for disturbance decoupling. However, the function is undefined at the time instants when $y = y_r$. Thus, the controller presented is undefined at these time instants.*

From (7.36) and (7.40) it can be shown that

$$\dot{V}_\epsilon \leq -2 \parallel \epsilon_a \parallel^2 + \frac{1}{2}s^4 \parallel PH \parallel^4 + \frac{1}{2} \parallel d(t) \parallel^4 \tag{7.41}$$

We now illustrate the backstepping design procedures using Nussbaum gain with details given for the first two steps.

Step.1

It follows from (7.21) and (7.37) that

$$\dot{z}_1 = b_m v_{m,2} + \beta + \bar{\omega}^T \theta + \epsilon_2 + d(t)\Phi_{a1}(y) - \dot{y}_r \tag{7.42}$$

Without using the sign of b_m, the following virtual control law α_1 is designed

$$\alpha_1 = N(\chi)\bar{\alpha}_1 e^{-ft} \tag{7.43}$$

$$N(\chi) = exp(\chi^2) \cos \frac{\pi}{2}\chi \tag{7.44}$$

where f is a positive real design parameter, χ is generated by

$$\dot{\chi} = z_1 \bar{\alpha}_1 \tag{7.45}$$

and $\bar{\alpha}_1$ is chosen to be

$$\begin{aligned} \bar{\alpha}_1 = & (c_1 + l_1 + (e_1^T \hat{\theta})^2) z_1 + \beta + \bar{\omega}^T \hat{\theta} - \dot{y}_r \\ & + z_1 h^2 \hat{q} + \frac{1}{4} z_1 \parallel \Phi_{a1}(y) \parallel^2 + \sum_{i=1}^{\rho} \frac{1}{8 l_i} z_1 s^3(z_1) \parallel PH \parallel^4 \end{aligned} \tag{7.46}$$

where c_1 and l_1 are two positive real design parameters, $\hat{\theta}$ and $\hat{q}$ denotes the estimate of θ and q. Notice that

$$b_m v_{m,2} = b_m(z_2 + \alpha_1) = \hat{b}_m z_2 + b_m \alpha_1 + \tilde{b}_m z_2 \tag{7.47}$$

where $\tilde{b}_m = b_m - \hat{b}_m$, $\hat{b}_m$ is the first element of $\hat{\theta}$, i.e. $\hat{b}_m = e_1^T \hat{\theta}$. Then from (7.42) and (7.46) we have

$$\begin{aligned} \dot{z}_1 - \bar{\alpha}_1 &= -(c_1 + l_1 + \hat{b}_m^2) z_1 + (\bar{\omega}^T + z_2 e_1^T)\tilde{\theta} + \epsilon_{a,2} + \epsilon_{b,2} - z_1 h^2 \hat{q} + \hat{b}_m z_2 + b_m \alpha_1 \\ &\quad + d(t)\Phi_{a1}(y) - \frac{1}{4} z_1 \parallel \Phi_{a1}(y) \parallel^2 - \sum_{i=1}^{\rho} \frac{1}{8l_i} z_1 s^3 \parallel PH \parallel^4 \end{aligned} \tag{7.48}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\epsilon_{a,2}$ and $\epsilon_{b,2}$ represent the second entry of ϵ_a and ϵ_b. To proceed, we define the Lyapunov function

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2}\tilde{q}^2 + \frac{1}{4l_1} V_\epsilon \tag{7.49}$$

where Γ is a positive definite matrix of $R^{(n+2)\times(n+2)}$. Then the derivative of V_1 along with (7.41), (7.43) and (7.48) is given by

$$\begin{aligned} \dot{V}_1 &= z_1(\dot{z}_1 - \bar{\alpha}_1) + z_1 \bar{\alpha}_1 + \tilde{\theta}^T \Gamma^{-1}(\dot{\theta} - \dot{\hat{\theta}}) + \tilde{q}\dot{\tilde{q}} + \frac{1}{4l_1}\dot{V}_\epsilon \\ &\le -(c_1 + \hat{b}_m^2) z_1^2 + \hat{b}_m z_1 z_2 + \tilde{\theta}^T \Gamma^{-1}(\tau_1 - \dot{\hat{\theta}}) - l_1 z_1^2 + \epsilon_{a,2} z_1 - \frac{1}{2l_1} \parallel \epsilon_a \parallel^2 \\ &\quad + \epsilon_{b,2} z_1 - \tilde{q}\dot{\hat{q}} - h^2 \hat{q} z_1^2 + d(t)\Phi_{a1}(y) z_1 - \frac{1}{4} z_1^2 \parallel \Phi_{a1}(y) \parallel^2 + b_m \alpha_1 z_1 + \bar{\alpha}_1 z_1 \\ &\quad + \frac{1}{8l_1} s^4 \parallel PH \parallel^4 - \sum_{i=1}^{\rho} \frac{1}{8l_i} z_1^2 s^3 \parallel PH \parallel^4 + \frac{1}{8l_1} \parallel d(t) \parallel^4 + \tilde{\theta}^T \Gamma^{-1} \dot{\theta} \end{aligned} \tag{7.50}$$

where

$$\tau_1 = \Gamma z_1(\bar{\omega} + z_2 e_1) \tag{7.51}$$

Here we know that

$$\epsilon_{b,2}z_1 - h^2\hat{q}z_1^2 \leq hq|z_1| - h^2\hat{q}z_1^2 \leq q(h^2z_1^2 + 1/4) - h^2\hat{q}z_1^2 = h^2\tilde{q}z_1^2 + \frac{q}{4}$$

Then we can get

$$\begin{aligned} \dot{V}_1 \leq & (b_m N(\chi)e^{-ft} + 1)\dot{\chi} - c_1 z_1^2 + \tilde{\theta}^T \Gamma^{-1}(\tau_1 - \dot{\hat{\theta}}) \\ & + \tilde{q}(\iota_1 - \dot{\hat{q}}) - \frac{1}{4l_1} \parallel \epsilon_a \parallel^2 + \frac{1}{4}z_2^2 + M_1 \end{aligned} \quad (7.52)$$

where

$$\iota_1 = h^2 z_1^2 \quad (7.53)$$

$$\begin{aligned} M_1 = & \parallel d(t) \parallel^2 + \frac{1}{8l_1} \parallel d(t) \parallel^4 - \sum_{i=2}^{\rho} \frac{1}{8l_i} s^4 \parallel PH \parallel^4 \\ & + \tilde{\theta}^T \Gamma^{-1} \dot{\theta} + \frac{1}{4} q + \bar{N} \end{aligned} \quad (7.54)$$

$$\bar{N} = \begin{cases} 0 & |z_1| \geq \delta \\ \sum_{i=1}^{\rho} \frac{1}{8l_i} (\delta^2 - z_1^2)^\rho s^3 \parallel PH \parallel^4 & |z_1| < \delta \end{cases} \quad (7.55)$$

From (7.40) we know that $\bar{N}$ is bounded.

Step.2

Now, we evaluate the dynamics of the second state z_2. Differentiating (7.38) for $i = 2$ and using (7.22), we have

$$\dot{z}_2 = v_{m,3} - k_2 v_{m,1} - \dot{\alpha}_1 \quad (7.56)$$

Note that α_1 is a function of $y, \hat{\theta}, \hat{q}, \xi, \Xi, \lambda, \chi$ and y_r and following from similar analysis to [30] by substituting (7.38) with $i = 3$ into (7.56), we get

$$\begin{aligned} \dot{z}_2 &= \alpha_2 - \beta_2 - \frac{\partial \alpha_1}{\partial y}\Big(\epsilon_2 + \omega^T\tilde{\theta} + d(t)\Phi_{a1}(y)\Big) + z_3 \\ &\quad -\frac{\partial \alpha_1}{\partial y}\omega^T\hat{\theta} - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} - \frac{\partial \alpha_1}{\partial \hat{q}}\dot{\hat{q}} \end{aligned} \tag{7.57}$$

where

$$\begin{aligned} \beta_2 &\triangleq k_2 v_{m,1} + \frac{\partial \alpha_1}{\partial y}\beta + \frac{\partial \alpha_1}{\partial \Pi}\dot{\Pi} + \sum_{j=1}^{m+1}\frac{\partial \alpha_1}{\partial \lambda_j}(-k_j\lambda_1 + \lambda_{j+1}) \\ &\quad +\frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \chi}\dot{\chi} \end{aligned} \tag{7.58}$$

where $\Pi = [\xi^T, Vec(\Xi)^T]^T$. Define the Lyapunov function and choose the virtual control for this step as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{4l_2}V_\epsilon \tag{7.59}$$

$$\begin{aligned} \alpha_2 &= -(c_2 + \frac{1}{4})z_2 + \frac{\partial \alpha_1}{\partial y}\omega^T\hat{\theta} - z_2 \parallel \frac{\partial \alpha_1}{\partial \hat{\theta}} \parallel^2 \parallel \tau_2 \parallel^2 - z_2 h^2 \hat{q} \parallel \frac{\partial \alpha_1}{\partial y} \parallel^2 \\ &\quad - z_2 \parallel \frac{\partial \alpha_1}{\partial \hat{q}} \parallel^2 \iota_2^2 - l_2 \parallel \frac{\partial \alpha_1}{\partial y} \parallel^2 z_2 + \beta_2 - \frac{z_2}{4} \parallel \frac{\partial \alpha_1}{\partial y}\Phi_{a1}(y) \parallel^2 \end{aligned} \tag{7.60}$$

$$\tau_2 = \tau_1 - \Gamma\frac{\partial \alpha_1}{\partial y}\omega z_2 \tag{7.61}$$

$$\iota_2 = \iota_1 + h^2 \parallel \frac{\partial \alpha_1}{\partial y} \parallel^2 z_2^2 \tag{7.62}$$

Using (7.52), (7.59) and (7.60), we have that

$$\begin{aligned} \dot{V}_2 &\le \dot{V}_1 + z_2\dot{z}_2 + \frac{1}{4l_2}\dot{V}_\epsilon \\ &\le -\sum_{i=1}^{2} c_i z_i^2 + (b_m N(\chi)e^{-ft} + 1)\dot{\chi} + z_2 z_3 - \sum_{i=1}^{2}\frac{1}{4l_i} \parallel \epsilon_a \parallel^2 + M_2 \\ &\quad +\tilde{\theta}^T\Gamma^{-1}(\tau_1 - \dot{\hat{\theta}}) - z_2\frac{\partial \alpha_1}{\partial y}\omega^T\tilde{\theta} + z_2^2 \parallel \frac{\partial \alpha_1}{\partial \hat{\theta}} \parallel^2 \parallel \dot{\hat{\theta}} \parallel^2 - z_2^2 \parallel \frac{\partial \alpha_1}{\partial \hat{\theta}} \parallel^2 \parallel \tau_2 \parallel^2 \end{aligned}$$

$$
\begin{aligned}
& +\tilde{q}(\iota_1 - \dot{\hat{q}}) + h^2\tilde{q} \parallel \frac{\partial \alpha_1}{\partial y} \parallel^2 z_2^2 + z_2^2 \parallel \frac{\partial \alpha_1}{\partial \hat{q}} \parallel^2 \dot{\hat{q}}^2 - z_2^2 \parallel \frac{\partial \alpha_1}{\partial \hat{q}} \parallel^2 \iota^2 \\
\leq & -\sum_{i=1}^{2} c_i z_i^2 + (b_m N(\chi)e^{-ft} + 1)\dot{\chi} + z_2 z_3 + \tilde{\theta}^T\Gamma^{-1}(\tau_2 - \dot{\hat{\theta}}) + \tilde{q}(\iota_2 - \dot{\hat{q}}) + M_2 \\
+ & \; z_2^2 \parallel \frac{\partial \alpha_1}{\partial \hat{\theta}} \parallel^2 (\parallel \dot{\hat{\theta}} \parallel^2 - \parallel \tau_2 \parallel^2) + z_2^2 (\frac{\partial \alpha_1}{\partial \hat{q}})^2 (\dot{\hat{q}}^2 - \iota_2^2) - \sum_{i=1}^{2} \frac{1}{4l_i} \parallel \epsilon_a \parallel^2 \quad (7.63)
\end{aligned}
$$

where

$$
\begin{aligned}
M_2 &= \sum_{i=1}^{2} \frac{1}{8l_i} \parallel d(t) \parallel^4 + 2 \parallel d(t) \parallel^2 - \sum_{i=3}^{\rho} \frac{1}{8l_i} s^4 \parallel PH \parallel^4 \\
& \quad + \tilde{\theta}^T \Gamma^{-1} \dot{\theta} + \frac{1}{2} + \frac{1}{2} q + \bar{N} \qquad (7.64)
\end{aligned}
$$

Remark 7.3 *Note that M_2 contains $s^4 \parallel PH \parallel^4$ and this term may not be bounded. As seen from our analysis, $\frac{1}{8l_2} s^4 \parallel PH \parallel^4$ disappears in M_2 due to the use of V_ϵ at step 2. If we use V_ϵ at each step, this term will disappear in M_ρ of the last step.*

Step.i $(i = 3, \ldots, \rho)$

We define the positive Lyapunov function V_i as

$$
V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{4l_i} V_\epsilon \qquad (7.65)
$$

and choose the virtual control law α_i as

$$
\begin{aligned}
\alpha_i = & -c_i z_i - l_i \parallel \frac{\partial \alpha_{i-1}}{\partial y} \parallel^2 z_i - z_{i-1} + \beta_i + \frac{\partial \alpha_{i-1}}{\partial y} \omega^T \hat{\theta} - \frac{z_i}{4} \parallel \frac{\partial \alpha_{i-1}}{\partial y} \Phi_{a1}(y) \parallel^2 \\
& - z_i \parallel \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \parallel^2 \parallel \tau_i \parallel^2 + \Big(\sum_{k=2}^{i-1} z_k^2 \parallel \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \parallel^2 \Big) (\tau_i + \tau_{i-1})^T \Gamma \frac{\partial \alpha_{i-1}}{\partial y} \omega \\
& - z_i \parallel \frac{\partial \alpha_{i-1}}{\partial \hat{q}} \parallel^2 \iota_i^2 - \Big(\sum_{k=2}^{i-1} z_k^2 \parallel \frac{\partial \alpha_{k-1}}{\partial \hat{q}} \parallel^2 \Big) (\iota_i + \iota_{i-1})^T h^2 \parallel \frac{\partial \alpha_{i-1}}{\partial y} \parallel^2 z_i \\
& - z_i h^2 \hat{q} \parallel \frac{\partial \alpha_{i-1}}{\partial y} \parallel^2 \qquad (7.66)
\end{aligned}
$$

$$\tau_i = \tau_{i-1} - \Gamma \frac{\partial \alpha_{i-1}}{\partial y} \omega z_i \tag{7.67}$$

$$\iota_i = \iota_{i-1} + h^2 \parallel \frac{\partial \alpha_{i-1}}{\partial y} \parallel^2 z_i^2 \tag{7.68}$$

where

$$\begin{aligned} \beta_i \triangleq\ & k_i v_{m,1} + \frac{\partial \alpha_{i-1}}{\partial y}\beta + \frac{\partial \alpha_{i-1}}{\partial \Pi}\dot{\Pi} + \frac{\partial \alpha_{i-1}}{\partial y_r}\dot{y}_r \\ & + \sum_{j=1}^{m+1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j}(-k_j\lambda_1 + \lambda_{j+1}) + \frac{\partial \alpha_{i-1}}{\partial \chi}\dot{\chi} \end{aligned} \tag{7.69}$$

Also note that

$$\begin{aligned} \parallel \tau_i \parallel^2 = \tau_i^T \tau_i &= \tau_i^T \tau_i - \tau_{i-1}^T \tau_{i-1} + \tau_{i-1}^T \tau_{i-1} \\ &= (\tau_i + \tau_{i-1})^T (\tau_i - \tau_{i-1}) + \tau_{i-1}^T \tau_{i-1} \\ &= -(\tau_i + \tau_{i-1})^T \Gamma \frac{\partial \alpha_{i-1}}{\partial y} \omega z_i + \tau_{i-1}^T \tau_{i-1} \\ \iota_i^2 &= (\iota_i + \iota_{i-1})^T h^2 \parallel \frac{\partial \alpha_{i-1}}{\partial y} \parallel^2 z_i^2 + \iota_{i-1}^2 \end{aligned} \tag{7.70}$$

Then the actual adaptive controller is obtained and given by

$$u(t) = \alpha_\rho - v_{m,\rho+1} \tag{7.71}$$

$$\dot{\hat{\theta}} = Proj(\tau_\rho) \tag{7.72}$$

$$\dot{\hat{q}} = Proj(\iota_\rho) \tag{7.73}$$

where Proj(.) is a smooth projection operation to ensure the estimates belong to compact sets for all time. Such an operation can be found in Appendix C.

Remark 7.4 *Note that the designed tuning functions are different from existing schemes in [30] as the projection operations are used in the parameter estimators.*

7.3.2 Stability Analysis

We construct the final Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{4l_i}V_\epsilon \tag{7.74}$$

By using the properties that $-\tilde{\theta}^T\Gamma^{-1}Proj(\tau) \leq -\tilde{\theta}^T\Gamma^{-1}\tau$ and $Proj(\tau)^T Proj(\tau) \leq \tau^T\tau$ the final Lynapunov function V_ρ satisfies

$$\begin{aligned}
\dot{V}_\rho \leq\ & -\sum_{k=1}^{\rho} c_k z_k^2 + (b_m N(\chi)e^{-ft} + 1)\dot{\chi} + M_\rho - \sum_{i=1}^{\rho}\frac{1}{4l_i} \parallel \epsilon_a \parallel^2 \\
& + \Big(\sum_{k=2}^{\rho} z_k^2 \parallel \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \parallel^2 \Big)(Proj(\tau_\rho)^T Proj(\tau_\rho) - \parallel \tau_\rho \parallel^2) \\
& + \tilde{\theta}^T\Gamma^{-1}(\tau_\rho - Proj(\tau_\rho)) + \tilde{q}(\iota_\rho - Proj(\iota_\rho)) \\
& + \Big(\sum_{k=2}^{\rho} z_k^2 (\frac{\partial \alpha_{k-1}}{\partial \hat{q}})^2\Big)(Proj(\iota_\rho)^2 - \iota_\rho^2) \\
\leq\ & -\sum_{k=1}^{\rho} c_k z_k^2 + b_m N(\chi)e^{-ft}\dot{\chi} + \dot{\chi} + M_\rho - \sum_{i=1}^{\rho}\frac{1}{4l_i} \parallel \epsilon_a \parallel^2
\end{aligned} \tag{7.75}$$

where

$$M_\rho = \sum_{i=1}^{\rho}\frac{1}{8l_i} \parallel d(t) \parallel^4 + \rho \parallel d(t) \parallel^2 + \tilde{\theta}^T\Gamma^{-1}\dot{\theta} + \frac{\rho-1}{2} + \frac{\rho}{4}q + \bar{N} \tag{7.76}$$

Integrating both sides of (7.75) over the interval $[0,t]$ gives

$$\begin{aligned}
\int_0^t \dot{V}_\rho e^{f\tau} d\tau \leq\ & -\int_0^t \sum_{k=1}^{\rho} c_k z_k^2 e^{f\tau} d\tau + \int_0^t b_m N(\chi)\dot{\chi} d\tau + \int_0^t \dot{\chi} e^{f\tau} d\tau \\
& + \int_0^t M_\rho e^{f\tau} d\tau - \int_0^t \sum_{i=1}^{\rho}\frac{1}{4l_i} \parallel \epsilon_a \parallel^2 e^{f\tau} d\tau
\end{aligned} \tag{7.77}$$

Note that $V_\epsilon \leq \parallel P \parallel \parallel \epsilon_a \parallel^2$. Then

$$\begin{aligned} V_\rho &= \sum_{k=1}^{\rho} \frac{1}{2} z_k^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2}\tilde{q}^2 + \sum_{i=1}^{\rho} \frac{1}{4l_i} V_\epsilon \\ &\leq \sum_{k=1}^{\rho} \frac{1}{2} z_k^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2}\tilde{q}^2 + \sum_{i=1}^{\rho} \frac{1}{4l_i} \parallel P \parallel \parallel \epsilon_a \parallel^2 \end{aligned} \quad (7.78)$$

This yields

$$\begin{aligned} 0 \leq V_\rho(t) \;\; &\leq \;\; V_\rho(0) + e^{-ft}\int_0^t b_m N(\chi)\dot{\chi} d\tau + \int_0^t \dot{\chi} e^{-f(t-\tau)} d\tau \\ &+ \int_0^t \frac{f}{2}(\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}) + \tilde{q}^2) e^{-f(t-\tau)} d\tau + \int_0^t M_\rho e^{-f(t-\tau)} d\tau \end{aligned} \quad (7.79)$$

where $f = min\{\frac{1}{\|P\|_2}, 2c_1, 2c_2, \ldots, 2c_\rho,\} > 0$. Due to the utilization of projection operations for $\hat{\theta}$ and $\hat{q}$, the boundedness of $\tilde{\theta}$ and $\tilde{q}$ can be guaranteed. Together with the boundedness $d(t)$, q and $\dot{\theta}$,the boundedness of M_ρ and $\int_0^t \frac{f}{2}(\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \tilde{q}^2) e^{-f(t-\tau)} d\tau + \int_0^t M_\rho e^{-f(t-\tau)} d\tau$ can be guaranteed. Thus by comparing with (7.4), f_0 is selected as the upper bound of $V_\rho(0) + \int_0^t \frac{f}{2}(\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \tilde{q}^2) e^{-f(t-\tau)} d\tau + \int_0^t M_\rho e^{-f(t-\tau)} d\tau, g_1 = b_m$ and $f_1 = f$. Using Lemma 7.1, we can conclude that $V_\rho(t)$ and $\chi(t)$, hence $z_i, (i = 1, \ldots, \rho)$ are bounded. Finally, from designed filter (7.13), (7.14), (7.18-7.20) the stability of the whole system can be established. To conclude this section, the results established are presented in the following theorem.

Theorem 7.1 *Consider the uncertain time-varying nonlinear system (7.1) satisfying Assumptions 1-4. With the application of the controller (7.71) and the parameter updating laws (7.72) and (7.73), the resulting closed loop system is BIBO stable.*

7.4 Simulation Examples

We now present simulation results to verify our proposed backstepping scheme. Two examples are considered. The results of simulation will verify that our adaptive controller will make the system stable and the tracking error will be bounded by a bound depending on the size of the disturbance. If the disturbance disappears, the output of the system can asymptotically track an arbitrary signal.

7.4.1 Example 1

To illustrate the proposed scheme, we consider the following second-order system

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta_{a1}(t)x_1^2 + d_1(t) \\ \dot{x}_2 &= b(t)u + \theta_{a2}(t)x_1 + d_2(t) \\ y &= x_1 \end{aligned} \tag{7.80}$$

where $\theta_{a1}(t) = x_1 e^{-0.5t}, \theta_{a2}(t) = 2 + \cos(t), b(t) = 3 + \sin(t)$, $d_1(t) = 0.6\sin(t)$ and $d_2(t) = 0.5\cos^2(t)$, actually these parameters are not needed to be known in controller design. with the application of the filters (7.13) and (7.14), the controller (7.71) and the parameter updating laws (7.72) and (7.73), the resulting closed-loop system is stable.

Since $\rho = 2 = n$, we have $v_\rho = v_n = \lambda$. The filters are designed as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} y \tag{7.81}$$

$$\begin{bmatrix} \dot{\Xi}_1 \\ \dot{\Xi}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \begin{bmatrix} \Xi_1 \\ \Xi_2 \end{bmatrix} + \begin{bmatrix} y^2 \\ y \end{bmatrix} \tag{7.82}$$

Since $b(t)$ is unknown, we define $\theta(t) = [b(t), \theta_{a1}(t), \theta_{a1}(t)]$ and the error ϵ satisfies

$$\dot{\epsilon} = A_0\epsilon + \Phi(y)D(t) \tag{7.83}$$

$$H(y, y_r) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{7.84}$$

$$P = \begin{bmatrix} \frac{1}{k_1} + \frac{k_2}{k_1} & -1 \\ -1 & \frac{1}{k_1} + \frac{k_2}{k_1} + \frac{1}{k_1 k_2} \end{bmatrix} \tag{7.85}$$

Following the steps presented in the controller design, we have

$$\begin{aligned} \alpha_1 &= -\omega_1^T\hat{\theta} - \dot{y}_r + \frac{1}{2}z_1 \parallel \Phi_{a1}(y) \parallel^2 N(\chi) \\ &\quad + \frac{1}{N(\chi)}\frac{1}{16l_1}z_1^3 \parallel PH(y, y_r) \parallel^4 \end{aligned} \tag{7.86}$$

$$\textit{with} \quad \omega_1 = [\bar{\omega}^T, c_1 z_1 + \chi^2 z_1 - \dot{y}_r + \beta]^T$$

$$\begin{aligned} u &= -c_2 z_2 + \beta_2 + 2N(\chi)z_1 e^{-ft} - l_2\Big(\frac{\partial\alpha_1}{\partial y}\Big)^2 z_2 + \frac{\partial\alpha_1}{\partial y}\omega^T\hat{\theta} \\ &\quad - \frac{z_2}{4} \parallel \frac{\partial\alpha_1}{\partial y}\Phi_{a1}(y) \parallel^2 - \sum_{i=2}^{\rho}\frac{1}{8l_i z_2}z_1^4 \parallel PH \parallel^4 \end{aligned} \tag{7.87}$$

and the adaptive law is obtained as

$$\dot{\hat{\theta}}_1 = Proj\big(-N(\chi)\Gamma_1\omega_1 z_1 e^{-ft}\big) \tag{7.88}$$

$$\dot{\hat{\theta}} = Proj\big(\tau_2\big) \tag{7.89}$$

$$\tau_2 = -\Gamma\frac{\partial\alpha_1}{\partial y}\omega z_2 \tag{7.90}$$

In the simulation, the design parameters were set as $c_1 = c_2 = 2$, $l_1 = l_2 = 1$, $k_1 = 5, k_2 = 4$, $\Gamma = I_3$. The desired trajectory was set as $y_r = \sin(2t) + \sin(t)$. Figure 7.1 and Figure 7.2 show the system output with reference signal and the control input. Clearly, simulation results verify the effectiveness of proposed scheme.

7.5 Summary

In this chapter, a scheme is proposed to design an adaptive output-feedback controller for uncertain time-varying nonlinear systems with unknown sign of high-frequency gains in the presence of disturbances. No growth conditions on system nonlinearities are imposed. In the design, certain well defined functions are used to cancel the effects of disturbances. To deal with the time variation problem, an estimator is used to estimate the bound of the variation rates. Furthermore, the overparamterization problem is also solved by using the concept of tuning functions. It is shown that the controller obtained by the proposed design scheme can make the whole adaptive control system stable.

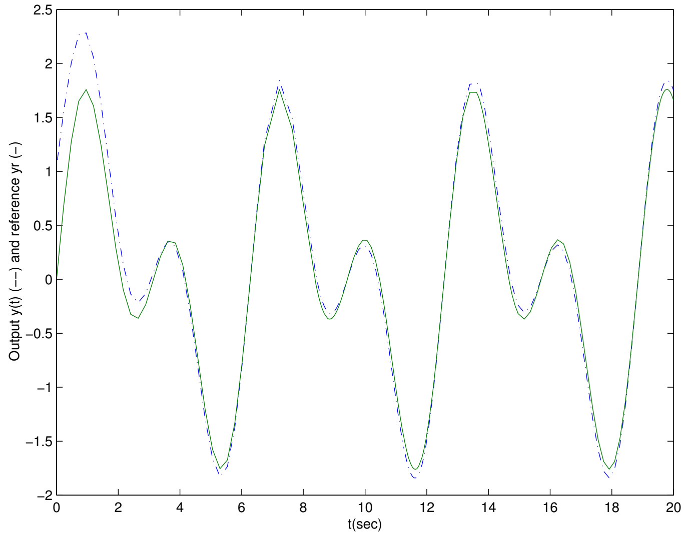


Figure 7.1: Output y and reference y_r.

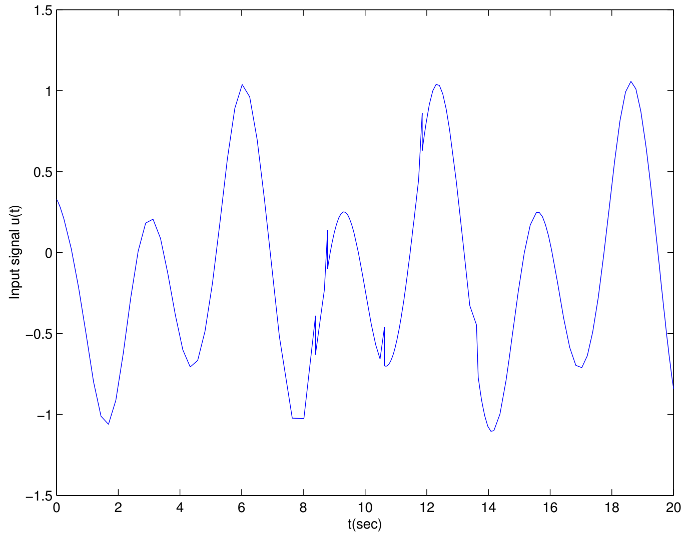


Figure 7.2: System input u.

Chapter 8

Multivariable Adaptive Control

In this chapter, adaptive output feedback control of a class of multiple-input multiple-output systems is considered in the presence of unknown disturbances. Except the signs of the term multiplying the control are assumed, no other knowledge on the unknown parameters is required. The control design is achieved by using backstepping, tuning functions, SDU factorization and estimating parameters. It is shown that the proposed controller can guarantee global uniform ultimate boundedness.

8.1 Introduction

In practice, most practical systems considered are multi-input multi-output (MIMO) systems. For such systems, the control problem is very complicated due to the coupling among various inputs and outputs. It becomes even more difficult to deal with when there exist uncertain parameters in the input or output coupling matrix. Due to these difficulties, it is noticed that, in comparison with the vast amount of results on controller design for SISO systems in control area, there are relatively fewer results available for a general class of MIMO systems. Adaptive backstepping control for a class of linear MIMO systems was studied in [147, 148, 149]. In [147] there exists a restrictive assumption about the high frequency gain B_m that a matrix S_m must be known such that $B_m S_m = (B_m S_m)^T > 0$. In [148] a similar restriction is relaxed using the factorization of high frequency gain. Recently a model-reference adaptive control was presented in [149] for MIMO linear systems without external disturbance using factorization of high frequency gain. In [150] a robust adaptive controller was designed for MIMO systems without disturbances by using switch functions. In [151] an output feedback control based on high-order sliding manifold approach was proposed for MIMO plants in presence of disturbances. The convergence of tracking error is not to zero, but to a small residual set.

In this chapter, a new scheme is developed for a class of MIMO system in the presence of unknown disturbances. With our scheme, a completely control solution to disturbance rejection is solved. In our design, the signs of the high frequency gains are known. To handle the disturbances and unknown parameters, we introduce new filters for state estimation and employ the internal model. As the parameters of the exosystem that generates external disturbances are unknown, an adaptive version of the internal model is proposed. Thus our estimator identifies the unknown parameters in both the system and the exosystem. It is shown that all closed-loop signals are bounded and the tracking error converges to zero.

8.2 Problem Formulation

The objective is to design an adaptive backstepping control scheme to generate the control $u(t)$ for the multivariable plant as

$$y(t) = G(p)\big(u(t) + \bar{d}(t)\big) \tag{8.1}$$

where $p = \frac{d}{dt}$, $u, y \in R^r, r > 1$, and $G(p)$ is an $r \times r$ strictly proper rational transfer matrix with unknown parameters, $\bar{d}$ is an unknown bounded disturbance.

A general MIMO plant $G(p)$ in (1) can be expressed as

$$
\begin{aligned}
G(p) &= D^{-1}N(p) = C_g(pI - A_g)^{-1}B_g \qquad (8.2)\\
D(p) &= p^v I_r + A_{v-1}p^{v-1} + \cdots + A_1 p + A_0\\
N(p) &= B_m p^m + \cdots + B_1 p + B_0\\
A_g &= \begin{bmatrix} -A_{v-1} & I_r & 0 & \dots & 0\\ -A_{v-2} & 0 & I_r & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -A_1 & 0 & 0 & \dots & I_r\\ -A_0 & 0 & 0 & \dots & 0 \end{bmatrix}, B_g = \begin{bmatrix} 0\\ \vdots\\ 0\\ B_p \end{bmatrix}, B_p = \begin{bmatrix} B_m\\ \vdots\\ B_1\\ B_0 \end{bmatrix} \qquad (8.3)\\
C_g &= [I_r\ 0\ \dots\ 0\ 0] \qquad (8.4)
\end{aligned}
$$

where v is the observability index of $G(p)$, $rv = n$, I_r is the $r \times r$ identity matrix, and $A_i, i = 0, \dots, v-1$, and $B_j, j = 0, \dots, m, m \le v-1$ are $r \times r$ matrices. Using (8.2), (8.1) can be expressed as the following feedback form

$$
\begin{aligned}
\dot{x} &= Ax + A_p y + \begin{bmatrix} 0\\ B_p \end{bmatrix} u + d(t) \qquad (8.5)\\
y &= C_g x
\end{aligned}
$$

where $x \in R^n$ is the system state, $A \in R^{n\times n}$ is the matrix A_g with the first r columns equal to zero, $A_p \in R^{n\times r}$ are the first r columns of A_g and $B_P \in R^{(m+1)r\times r}$, $d(t) = [0\ B_p]^T \bar{d}(t) = [d_1^T, \dots, d_v^T]^T \in R^n$ and $d_i \in R^r, (i = 1, \dots, v)$.
Suppose that the unknown disturbance is generated from the following unknown exosystem

$$
\dot{d}(t) = Sd(t) \qquad (8.6)
$$

where S is an unknown $n \times n$ matrix having distinct eigenvalues with zero real parts. The disturbance rejection problem in this chapter is based on the internal model principle.

For system (8.5) there exists an invariant manifold as stated below:

Lemma 8.1 *For the system (8.5) with an exosystem (8.6), there exists* $\pi(d) = \Pi d \in R^n$ *and* $\sigma(d) = \Lambda d \in R^r$ *such that*

$$\Pi S = A\Pi + I + \begin{bmatrix} 0 \\ B_p \end{bmatrix} \Lambda \tag{8.7}$$

where $\pi_1(d)$ *is the first* r *elements of* $\pi(d)$ *and satisfies* $\pi_1(d) = 0 \in R^r$*.*

Proof. The existence of Π and Λ in (8.7) follows from the fact that A is a shift matrix. Detailed is in Appendix D and also in [152, 153, 154].

With the invariant manifold $\pi(d)$, we define a state transformation as

$$\zeta = x - \pi(d) \tag{8.8}$$

It can be shown from (8.5), (8.6) and (8.7) that

$$\begin{aligned} \dot{\zeta} &= A\zeta + A_p y + \begin{bmatrix} 0 \\ B_p \end{bmatrix} (u - \sigma(d)) \\ y &= C_g \zeta \end{aligned} \tag{8.9}$$

Then disturbance rejection problem of (8.5) becomes the stabilization problem of (8.9).

The control objective is that the output $y(t)$ tracks a given bounded reference output $y_d(t)$ asymptotically and all closed-loop signals are bounded. Regarding the system and the reference signal, the following assumptions are made:

Assumption 1 The matrix B_m is nonsingular.

Assumption 2 The leading principle minors of B_m are nonzero and the signs are known.

Assumption 3 The reference output $y_d(t)$ is assumed to be known bounded function with bounded known derivatives.

8.3 Preliminary Results

We first perform a factorization of the high frequency gain B_m as stated in the following Lemma, which was proved in [148].

Lemma 8.2 *Every $m \times m$ real matrix B_m with nonzero leading principal minors, $\Delta_1, \Delta_2, \ldots, \Delta_m$, can be factored as*

$$B_m = S_1 D_1 U_1 \tag{8.10}$$

where S_1 is symmetric positive definite, U_1 is unity upper triangular, and $D_1 = \Gamma_D sign(D)$ is diagonal, with $D = diag\{\Delta_1, \Delta_2/\Delta_1, \ldots, \Delta_m/\Delta_{m-1}\}$, and Γ_D being an arbitrary positive diagonal matrix.

Remark 8.1 *The factorization $B_m = S_1 D_1 U_1$ is not unique because the diagonal matrix $\Gamma_D > 0$ is arbitrary and so D_1 is any diagonal matrix such that $sign(D_1) = sign(D)$.*

To construct a multivariable state observer using $u(t)$ and $y(t)$ for system (8.5), we choose the following matrix

$$K = [k_1 I_r, \ldots, k_v I_r]^T \in R^{n \times r} \tag{8.11}$$

where $k_i > 0, i = 1, \ldots, v$, such that the matrix

$$A_0 = A - KC_g \tag{8.12}$$

is stable. This is sufficient if $s^v + k_1 s^{v-1} + \cdots + k_{v-1}s + k_v$ is a Hurwitz polynomial. Define

$$E_i = e_i \otimes I_r \tag{8.13}$$

where $\otimes$ is the Kronecker product, and e_i is the ith coordinate vector in R^v. We also introduce the known vectors $\xi_v(t), \Xi(t)$, $v_j(t)$, as the outputs of the filters

$$\dot{\xi}_v = A_0\xi_v + Ky \tag{8.14}$$

$$\dot{\Xi} = A_0\Xi + E_{v-i}y \quad i = 0, 1, \ldots, v-1 \tag{8.15}$$

$$\dot{v}_j = A_0 v_j + E_{v-j}u \quad j = 0, 1, \ldots, m \tag{8.16}$$

where $\xi_v = [\xi_{v,1}^T, \ldots, \xi_{v,v}^T]^T$. Since A_0 is Hurwitz and the spectra of A_0 and S are disjoint, there exists $q(d) = Qd$ with $Q \in R^{n\times n}$ such that

$$QS = A_0 Q - \begin{bmatrix} 0 \\ B_p \end{bmatrix} \Lambda \tag{8.17}$$

With these filters, we construct a parameterized state observer for the system (8.9) as

$$\hat{\zeta}(t) = \xi_v(t) + q(d) - \sum_{i=0}^{v-1} \bar{A}_i \Xi(t) - \sum_{j=0}^{m} \bar{B}_j v_j(t) \tag{8.18}$$

where $\bar{A}_i = diag[A_i, \ldots, A_i] \in R^{n\times n}, \bar{B}_i = diag[B_i, \ldots, B_i] \in R^{n\times n}$.

Lemma 8.3 *The state observation error $\epsilon(t) = \zeta - \hat{\zeta}$ satisfies*

$$\dot{\epsilon}(t) = A_0\epsilon, \lim_{t\to\infty} \epsilon(t) = 0 \text{ exponentially} \tag{8.19}$$

Proof. Based on the special structures of $\bar{A}_I$ and A_0, we have

$$\bar{A}_i A_0 = A_0 \bar{A}_i, \quad (i = 0, 1, \ldots, v-1) \tag{8.20}$$

$$\bar{B}_j A_0 = A_0 \bar{B}_j, \quad (j = 0, 1, \ldots, m) \tag{8.21}$$

From (8.6),(8.9),(8.14-8.21) and from

$$A_p y(t) = -\sum_{i=0}^{v-1} \bar{A}_i E_{v-i} y(t) \tag{8.22}$$

$$\begin{bmatrix} 0 \\ B_p \end{bmatrix} u(t) = -\sum_{j=0}^{m} \bar{B}_j E_{v-j} u(t) \tag{8.23}$$

we obtain

$$\dot{\epsilon}(t) = \dot{\zeta} - \dot{\hat{\zeta}} = A_0 \epsilon \tag{8.24}$$

where A_0 is a stable matrix. So we get $\lim_{t\to\infty} \epsilon(t) = 0$ exponentially

Based on the parameterized canonical observer from (8.18), we now design a multivariable adaptive backstepping controller for the plant (8.9).

Let the state variable $\zeta(t)$ be partitioned as

$$\zeta = [\zeta_1^T, \ldots, \zeta_v^T]^T, \quad \zeta_i \in R^r \tag{8.25}$$

Then it follows from (8.4) and (8.9) that

$$y = \zeta_1 \tag{8.26}$$

$$\dot{y} = \zeta_2 - A_{v-1} y \tag{8.27}$$

Let $\epsilon(t), \Xi(t)$ and $v_j(t)$ be partitioned as

$$\begin{aligned} \epsilon &= [\epsilon_1^T, \ldots, \epsilon_v^T]^T, \quad \epsilon_i \in R^r \\ \Xi &= [\xi_{i,1}^T, \ldots, \xi_{i,v}^T]^T, \quad \xi_{i,k} \in R^r \\ v_j &= [v_{j,1}^T, \ldots, v_{j,v}^T]^T, \quad v_{j,k} \in R^r \end{aligned} \tag{8.28}$$

We also define

$$\begin{aligned} \Theta_a &= [-A_{v-1}, -A_{v-2}, \ldots, -A_1, -A_0], \\ \Theta_b &= [B_m, B_{m-1}, \ldots, B_1, B_0], \\ \xi_2 &= [\xi_{v-1,2}^T, \ldots, \xi_{0,2}^T]^T, \\ v_2 &= [v_{m,2}^T, \ldots, v_{0,2}^T]^T. \end{aligned} \tag{8.29}$$

From (8.18) and Lemma 3 we have

$$\zeta_2 = \xi_{v,2} + \Theta_a \xi_2 + \Theta_b v_2 + q_2 + \epsilon_2 \tag{8.30}$$

Substituting (8.30) into (8.31) yields

$$\begin{aligned} \dot{y} &= \xi_{v,2} + \Theta_a[\xi_2 + E_1 y] + \Theta_b v_2 + q_2 + \epsilon_2 \\ &= \xi_{v,2} + B_m v_{m,2} + \Theta\bar{\omega} + q_2 + \epsilon_2 \end{aligned} \tag{8.31}$$

where $E_1 = [T_r, 0_{r\times(v-1)r}]^T$, $\Theta = [\Theta_a, \Theta_b]$, $\bar{\omega} = [\xi_2^T + (E_1 y)^T, [0_{1,r}, v_{m-1,2}^T, \ldots, v_{0,2}^T]]^T$.

Remark 8.2 *The difficulty now is that the term $q(d)$ is not available, because the disturbance $d(t)$ and the matrix Q are unknown. For the adaptive backstepping approach proposed in [30], the state ζ_2 serves as the link between the output and the filter used for the output backstepping. The contribution of $\sigma(d)$ to ζ_2 is reflected by q_2 in ϵ. Following the treatment in [155] and [156], we reparameterize (8.6)* *for generating $q_2 = Q_2 d$, with Q_2 denoting the second r rows of Q. And we will introduce the new internal model and filters to handle it.*

Based on a lemma in [156], for any known controllable pair $\{F_i, G_i\}, i = 1, \ldots, r$ with $F_i \in R^{v\times v}$ being Hurwitz and $G_i \in R^v$, there exists a $\psi_i \in R^v$ such that

$$\begin{aligned} \dot{\eta}_i &= (F_i + G_i\psi_i^T)\eta_i \\ q_{2,i} &= \psi_i^T \eta_i, \quad i = 1, 2, \ldots, r \end{aligned} \tag{8.32}$$

where $q_{2,i}$ denotes the ith variable of $q_2 = [q_{2,1}, \ldots, q_{2,r}]^T$, the initial value $\eta_i(0)$ dependent on exogenerous variables. We define $\eta = [\eta_1^T, \ldots, \eta_r^T]^T$.
Based on the parametrization (8.32) of the initial model, we design an adaptive internal model as

$$\begin{aligned} \dot{\delta} &= \begin{bmatrix} \dot{\delta}_1 \\ \vdots \\ \dot{\delta}_r \end{bmatrix} = \begin{bmatrix} F_1\delta_1 - G_1\xi_{v,2,1} + F_1G_1y_1 \\ \vdots \\ F_r\delta_r - G_r\xi_{v,2,r} + F_rG_ry_r \end{bmatrix} \\ \dot{\hat{q}}_2 &= \begin{bmatrix} \dot{\hat{q}}_{2,1} \\ \vdots \\ \dot{\hat{q}}_{2,r} \end{bmatrix} = \begin{bmatrix} \hat{\psi}_1^T\delta_1 \\ \vdots \\ \hat{\psi}_r^T\delta_r \end{bmatrix} \end{aligned} \tag{8.33}$$

where $\hat{\psi}_i$ is the estimate of ψ_i, $\xi_{v,2} = [\xi_{v,2,1}, \ldots, \xi_{v,2,r}]^T$, and $y = [y_1, \ldots, y_r]^T$. To further exploit the stability of the internal model, we define the filters

$$\begin{aligned} \dot{\lambda} &= \begin{bmatrix} \dot{\lambda}_1 \\ \vdots \\ \dot{\lambda}_r \end{bmatrix} = \begin{bmatrix} F_1\lambda_1 + G_1\omega^T \\ \vdots \\ F_r\lambda_r + G_r\omega^T \end{bmatrix} \\ \dot{\lambda}_v &= \begin{bmatrix} \dot{\lambda}_{v,1} \\ \vdots \\ \dot{\lambda}_{v,r} \end{bmatrix} = \begin{bmatrix} F_1\lambda_{v,1} + F_1G_1v_{m,1}^T \\ \vdots \\ F_r\lambda_{v,r} + F_rG_rv_{m,1}^T \end{bmatrix} \end{aligned} \tag{8.34}$$

where $\omega = [\xi_2^T + (E_1 y)^T, [-(K_1 v_{m,1})^T, v_{m-1,2}^T, \ldots, v_{0,2}^T]]^T$.

We define the auxiliary error

$$e = \begin{bmatrix} e_1 \\ \vdots \\ e_r \end{bmatrix} = \eta - \delta + \begin{bmatrix} \lambda_1 \Theta_1^T \\ \vdots \\ \lambda_r \Theta_r^T \end{bmatrix} - \begin{bmatrix} G_1 y_1 \\ \vdots \\ G_r y_r \end{bmatrix} + \begin{bmatrix} G_1 B_{m,1} v_{m,1} \\ \vdots \\ G_r B_{m,r} v_{m,1} \end{bmatrix} + \begin{bmatrix} \lambda_{v,1} B_{m,1}^T \\ \vdots \\ \lambda_{v,r} B_{m,r}^T \end{bmatrix}$$

where $\Theta = [\Theta_a, \Theta_b] = \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_r \end{bmatrix}$, $B_m = \begin{bmatrix} B_{m,1} \\ \vdots \\ B_{m,r} \end{bmatrix}$.

Remark 8.3 *Note that the traditional filters in [30] cannot deal with the unknown disturbance generated from an unknown exosystem. Thus a new adaptive internal model (8.33) and new auxiliary filters (8.34) are introduced to achieve disturbance rejection.*

Lemma 8.4 *The auxiliary error e satisfies*

$$\dot{e} \;=\; = Fe + G\epsilon_2 \tag{8.35}$$

where $F = diag\{F_1, \ldots, F_r\}, G = diag\{G_1, \ldots, G_r\}$.

Proof. From (8.16), we have

$$\dot{v}_{m,1} = -K_1 v_{m,1} + v_{m,2} \tag{8.36}$$

From (8.31-8.34), it can be shown that

$$\dot{e} = \begin{bmatrix} F_1 e_1 \\ \vdots \\ F_r e_r \end{bmatrix} - \begin{bmatrix} G_1 \epsilon_{2,1} \\ \vdots \\ G_r \epsilon_{2,r} \end{bmatrix} \tag{8.37}$$

With the auxiliary error e, we can express q_2 as

$$\begin{aligned} q_2 &= \begin{bmatrix} \psi_1^T \eta_1 \\ \vdots \\ \psi_r^T \eta_r \end{bmatrix} \\ &= \begin{bmatrix} \psi_1^T e_1 + \psi_1^T \delta_1 + \psi_1^T G_1 y_1 - \Theta_{\psi,1} vec(\lambda_1) - B_{\psi,1}(G_1 \otimes v_{m,1} + vec(\lambda_{v,1})) \\ \vdots \\ \psi_r^T e_r + \psi_r^T \delta_r + \psi_r^T G_r y_r - \Theta_{\psi,r} vec(\lambda_r) - B_{\psi,r}(G_r \otimes v_{m,1} + vec(\lambda_{v,r})) \end{bmatrix} \\ &= \psi^T e + \psi^T G y + \psi^T \delta - \Theta_\psi \Lambda - B_\psi G \otimes v_{m,1} - B_\psi \Lambda_v \end{aligned} \tag{8.38}$$

where $\Theta_{\psi,i} = \psi_i^T \otimes \Theta_i$, $B_{\psi,i} = \psi_i^T \otimes B_{m,i}$, $i = 1, \ldots, r$, $\Lambda = [vec(\lambda_1)^T, \ldots, vec(\lambda_r)^T]^T$, $\Lambda_v = [vec(\lambda_{v,1})^T, \ldots, vec(\lambda_{v,r})^T]^T$, with $vec(.)$ denotes the vector obtained by rolling the column vectors of the matrix, and $\psi^T = diag\{\psi_1^T, \ldots, \psi_r^T\}$, $\Theta_\psi = diag\{\Theta_{\psi,1}, \ldots, \Theta_{\psi,r}\}$ and $B_\psi = diag\{B_{\psi,1}, \ldots, B_{\psi,r}\}$. Now all the terms in the right side of (8.38) are products of unknown parameters and unknown filtered signals or exponentially decaying signals.

8.4 Backstepping Design with SDU Factorization

In this section, we design an adaptive controller for the plant (8.9), under the given assumptions. With the filters designed in previous section, we will be able to deal with both unknown parameter Θ in the system and the unknown parameter ψ in the exosystem, to design the adaptive control input. From (8.31) and (8.38), we get

$$\begin{aligned} \dot{y} &= \xi_{v,2} + B_m v_{m,2} + \Theta \bar{\omega} + \epsilon_2 + \psi^T e + \psi^T G y + \psi^T \delta \\ &\quad - \Theta_\psi \Lambda - B_\psi G \otimes v_{m,1} - B_\psi \Lambda_v \\ &= \xi_{v,2} + \psi^T e + \epsilon_2 + \bar{\Theta} \Omega \end{aligned} \tag{8.39}$$

where $\bar{\Theta} = [\Theta, \psi^T, -\Theta_\psi, -B_\psi]$ and $\Omega = [\omega^T, (Gy+\delta)^T, \Lambda^T, (G \otimes v_{m,1} + \Lambda_v)^T]^T$. The design procedure is recursive and similar to that for the single-input single-output case introduced in [30]:

$$
\begin{aligned}
z_1 &= y - y_d && (8.40)\\
z_i &= v_{m,i} - \alpha_{i-1}, \quad i = 2, \ldots, \rho && (8.41)
\end{aligned}
$$

where y_d is the reference output, α_i is the virtual control and will be designed based on the following procedures.

Step 1. From equations (8.39-8.41), and using Lemma 2, we get

$$
\begin{aligned}
\dot{z}_1 &= \xi_{v,2} + B_m v_{m,2} + \bar{\Theta}\bar{\Omega} + \psi^T e + \epsilon_2 - \dot{y}_d\\
&= \xi_{v,2} + S_1 D_1 U_1 v_{m,2} + \bar{\Theta}\bar{\Omega} + \psi^T e + \epsilon_2 - \dot{y}_d\\
&= S_1 D_1 z_2 + S_1 D_1[\alpha_1 + (U_1 - I)v_{m,2}] + \xi_{v,2} + \bar{\Theta}\bar{\Omega} + \psi^T e + \epsilon_2 - \dot{y}_d && (8.42)
\end{aligned}
$$

where $\bar{\Omega} = [\bar{\omega}^T, (Gy+\delta)^T, \Lambda^T, (G \otimes v_{m,1} + \Lambda_v)^T]^T$ and $(U_1 - I)$ is strictly upper triangular matrix. In order to remove the zero entries from the above parametrization, we introduce new parameter vectors χ_k and regressor matrix γ_k $(k = 1, \ldots, r)$ as in [148] via the identity

$$
(U_1 - I)v_{m,2} = v_{m,2_2}\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}[U_{1,2}] + \cdots + v_{m,2_r}\begin{bmatrix} I_{r-1} \\ 0_{1,r-1} \end{bmatrix}\begin{bmatrix} U_{1,r} \\ \vdots \\ U_{r-1,r} \end{bmatrix} = \sum_{k=2}^{r} \gamma_k \chi_k \qquad (8.43)
$$

where $\gamma_k = v_{m,2_k}\begin{bmatrix} I_{k-1} \\ 0_{r-k+1,k-1} \end{bmatrix} \in R^{r\times(k-1)}, \chi_1 = 0, \chi_k = [U_{1,k}\ U_{2,k}\ \ldots\ U_{k-1,k}]^t \in R^{k-1}(k = 2, \ldots, r)$ and $v_{m,2_k}$ indicates the kth component of vector $v_{m,2}$. Since $S_1 D_1$ is nonsingular, we can introduce $P = (S_1 D_1)^{-1}$ and the new matrix parameters $\theta = P\bar{\Theta}$ and $\Psi = P\psi^T$. Using (8.43) and choosing positive constants C_1 and $\bar{l}_1$, adding and subtracting $(C_1 + \bar{l}_1)z_1$ to equation (8.42) and multiplying both sides by S_1^{-1} we get

$$\begin{aligned} S_1^{-1} z_1 &= -S_1^{-1} C_1 z_1 - S_1^{-1} \bar{l}_1 z_1 + D_1 z_2 + D_1(\alpha_1 + P(C_1 z_1 + \bar{l}_1 z_1 + \xi_{v,2} - \dot{y}_d)) \\ &+ D_1 \sum_{k=2}^{r} \gamma_k \chi_k + D_1 \Theta \bar{\Omega} + D_1 \Psi e + S_1^{-1} \epsilon_2 \end{aligned}$$

Remark 8.4 *The reason for using two positive constants C_1 and $\bar{l}_1$ is to have uniformity with subsequent steps of the backstepping procedure where $\bar{l}_1$ is a coefficient of a damping term countering ϵ_2.*

We define the signal

$$\phi = C_1 z_1 + \bar{l}_1 z_1 + \xi_{v,2} - \dot{y}_d \tag{8.44}$$

and introduce $\hat{P}, \hat{\chi}_k, \hat{\theta}$ and $\hat{L}_i (i = 1, \ldots, \rho)$ as estimates of P, χ_k, θ and the upper bound of $\| \Psi \|^2$, respectively. We can choose the first virtual control law α_1 as

$$\alpha_1 = -\hat{P}\phi - \sum_{k=2}^{r} (\gamma_k \hat{\chi}_k) - \hat{\theta}\bar{\Omega} - D_1^T \hat{L}_1 z_1 \tag{8.45}$$

Choose the Lyapunov function as

$$\begin{aligned} V_1 &= \frac{1}{2} Tr(S_1^{-1} z_1 z_1^T) + \frac{1}{2} Tr(\tilde{\theta}\tilde{\theta}^T) + \frac{1}{2} Tr(\tilde{P}\tilde{P}^T) \\ &+ \frac{1}{\bar{l}_1} \epsilon^T P_\epsilon \epsilon + \frac{1}{2} \tilde{L}_1^2 + \frac{1}{2} e^T P_e e + \frac{1}{2} [\sum_{k=2}^{r} Tr(\tilde{\chi}_k^T \tilde{\chi}_k)] \end{aligned} \tag{8.46}$$

where $\tilde{\theta} = \theta - \hat{\theta}, \tilde{P} = P - \hat{P}, \tilde{\chi}_k = \chi_k - \hat{\chi}_k$, $\tilde{L}_i = L_i - \hat{L}_i, i = 1, \ldots, \rho$, $P_\epsilon = P_\epsilon^T \in R^{n \times n}$ is the solution of $P_\epsilon A_0 + A_0^T P_\epsilon = -diag(S_1^{-1}, \ldots, S_1^{-1}) - I$ for stable matrix A_0 in (8.19), $P_e = P_e^T \in R^{n \times n}$ is the solution of $P_e F + F^T P_e = -2(\rho + 2)I$.

Using the following update laws for $\hat{\theta}, \hat{P}, \hat{\chi}_k$ and $\hat{L}_1$

$$\dot{\hat{\theta}} = D_1 z_1 \bar{\Omega}^T \tag{8.47}$$

$$\dot{\hat{P}} = D_1 z_1 \phi^T \tag{8.48}$$

$$\dot{\hat{\chi}}_k = \gamma_k^T D_1 z_1, \quad (k = 2, 3, \ldots, r) \tag{8.49}$$

$$\dot{\hat{L}}_1 = Tr(D_1 D_1^T z_1 z_1^T) \tag{8.50}$$

It can be verified that

$$\dot{V}_1 \leq -C_1 Tr(S_1^{-1} z_1 z_1^T) + Tr(D_1 z_2 z_1^T) - \frac{1}{\bar{l}_1} M_1(\epsilon) - \rho e^T e \tag{8.51}$$

where $M_1(\epsilon) = Tr(S_1^{-1}\epsilon_1\epsilon_1^T + \frac{3}{4}S_1^{-1}\epsilon_2\epsilon_2^T + \cdots + S_1^{-1}\epsilon_v\epsilon_v^T)$.
Furthermore, we set constant positive reals $\bar{l}_1$ satisfying the following conditions:

$$\frac{1}{\bar{l}_1} \geq \frac{\| P_e G \|^2}{4} \tag{8.52}$$

Notice that D_1 is a known matrix. By Lemma 2, D_1 is any diagonal matrix such that $\text{sign}(D_1) = \text{sign}(D)$.

Step i. $(i = 2, \ldots, \rho), \rho = v - m$, we introduce the signal $z_i = v_{m,i} - \alpha_{i-1}$. The corresponding time derivatives can be expressed

$$\dot{z}_i = v_{m,i+1} + \beta_i - \frac{\partial \alpha_{i-1}}{\partial y}(\bar{\Theta}\Omega - \epsilon_2 - \psi^T e) - \iota_i$$

where

$$\frac{\partial \alpha_i}{\partial y} = \begin{bmatrix} \frac{\partial \alpha_{i,1}}{\partial y} \\ \vdots \\ \frac{\partial \alpha_{i,r}}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial \alpha_{i,1}}{\partial y_1} & \cdots & \frac{\partial \alpha_{i,1}}{\partial y_r} \\ \vdots & \ddots & \vdots \\ \frac{\partial \alpha_{i,r}}{\partial y_1} & \cdots & \frac{\partial \alpha_{i,r}}{\partial y_r} \end{bmatrix}$$

$$\begin{aligned}
\iota_i &= \big(Tr(\dot{\hat{\bar{\Theta}}}\frac{\partial \alpha_{i-1,1}}{\partial \hat{\bar{\Theta}}}), \ldots, Tr(\dot{\hat{\bar{\Theta}}}\frac{\partial \alpha_{i-1,r}}{\partial \hat{\bar{\Theta}}})\big)^T \\
(\frac{\partial \alpha_{l,k}}{\partial \hat{\bar{\Theta}}})_{i,j} &= \frac{\partial \alpha_{l,k}}{\partial \hat{\bar{\Theta}}_{j,i}}, \qquad (k = 1, 2, \ldots, r)
\end{aligned}$$

and β_i represents all the known terms except $v_{m,i+1}$. We choose the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}Tr(z_2 z_2^T) + \frac{1}{l_2}\epsilon^T P_0 \epsilon + \frac{1}{2}Tr\big(\tilde{\bar{\Theta}}\Gamma^{-1}\tilde{\bar{\Theta}}^T\big) + \frac{1}{2}\tilde{L}_2^2 \tag{8.53}$$

$$V_i = V_{i-1} + \frac{1}{2}Tr(z_i z_i^T) + \frac{1}{l_i}\epsilon^T P_0 \epsilon + \frac{1}{2}\tilde{L}_i^2 \tag{8.54}$$

where $\tilde{\bar{\theta}} = \bar{\Theta} - \hat{\bar{\Theta}}$, Γ is a positive definite matrix, $P_0 = P_0^T > 0$ is the solution of $P_0 A_0 + A_0^T P_0 = -I$ for the stable matric A_0.

And choose the adaptive control laws as

$$u = \alpha_\rho - v_{m,\rho-1} - \sum_{k=2}^{r} u_k \begin{bmatrix} I_{k-1} \\ 0_{r-k+1,k-1} \end{bmatrix} \hat{\chi}_k \tag{8.55}$$

$$\dot{\hat{\bar{\Theta}}}^T = \tau_\rho \tag{8.56}$$

$$\begin{aligned}
\alpha_i &= -c_i z_i - l_i(\frac{\partial \alpha_{i-1}}{\partial y})^T \frac{\partial \alpha_{i-1}}{\partial y} z_i - \beta_i - z_{i-1} + \frac{\partial \alpha_{i-1}}{\partial y}\hat{\bar{\Theta}}\Omega \\
&\quad - \sum_{k=2}^{j-1}\{o_{k,j-i}\} + \hat{\iota}_i - \hat{L}_i(\frac{\partial \alpha_{i-1}}{\partial y})^T \frac{\partial \alpha_{i-1}}{\partial y} z_i
\end{aligned} \tag{8.57}$$

$$\dot{\hat{L}}_i = Tr[(\frac{\partial \alpha_{i-1}}{\partial y})^T \frac{\partial \alpha_{i-1}}{\partial y} z_i z_i^T] \tag{8.58}$$

with

$$\tau_i^T = \tau_{i-1}^T - \Gamma\Omega z_i^T \frac{\partial \alpha_{i-1}}{\partial y} \tag{8.59}$$

$$\hat{\iota}_i = \big(Tr(\tau_i \frac{\partial \alpha_{i-1,1}}{\partial \hat{\bar{\Theta}}}), \ldots, Tr(\tau_{i-1}\frac{\partial \alpha_{i-1,r}}{\partial \hat{\bar{\Theta}}})\big)^T \tag{8.60}$$

where $Tr(\frac{\partial \alpha_{j-1,1}}{\partial y})^T z_k \Omega^T \Gamma^T \frac{\partial \alpha_{k-1,i}}{\partial \hat{\bar{\Theta}}})$ as the ith element of $o_{k,j-1} \in R^r$.

Note that the term $\sum_{k=2}^{r} \left(u_k \begin{bmatrix} I_{k-1} \\ 0_{r-k+1,k-1} \end{bmatrix} \hat{\chi}_k\right)$ appears because the signal $v_{m,2_k}$ is used in the regressor matrix γ_k to define the first stabilizing functions α_1 and is passed to the following steps into signal $\beta_j, (j = 2, \ldots, \rho)$ which is formed by the known terms of the derivatives of $\alpha_l, (l = 1, \ldots, j-1)$.

Upon some algebraic manipulation, we can write $\dot{V}_\rho$ as

$$
\begin{aligned}
\dot{V}_\rho &\leq \sum_{j=1}^{\rho} \{ -Tr(c_j z_j z_j^T) - \frac{1}{l_i} M(\epsilon) - l_i Tr[(\frac{\partial \alpha_{j-1}}{\partial y} z_j + \frac{\epsilon_2}{2l_j})(\frac{\partial \alpha_{j-1}}{\partial y} z_j + \frac{\epsilon_2}{2l_j})^T] \} \\
&- e^T e \leq 0 \qquad (8.61) \\
M(\epsilon) &= Tr(\epsilon_1 \epsilon_1^T + \frac{3}{4} \epsilon_2 \epsilon_2^T + \cdots + \epsilon_v \epsilon_v^T)
\end{aligned}
$$

where $c_1 = C_1 S_1^{-1}, \frac{1}{l_1} = \frac{1}{l_1} S_1^{-1}$. Since $\dot{V}_\rho \leq 0$, we conclude that the complete system $z_1, \ldots, z_\rho, \hat{\theta}, \hat{P}, \hat{\chi}_k, \hat{L}_k, \hat{\Theta}$ are bounded and so the plant output $y(t)$ is bounded. Then it can be shown that all closed-loop signals are bounded and the tracking error $z_1 = y - y_d$ converges to zero.

Theorem 8.1 *Consider the MIMO system (8.5) satisfying Assumptions 1-3. With the application of controller (8.55) and the parameter update laws (8.47-8.50), (8.56) and (8.58), all closed-loop signals are bounded and the tracking error converges to zero.*

8.5 Simulation Studies

In this section, we illustrate the above method on a simple MIMO system. Consider the 2×2 plant described by

$$
G(p) = \begin{bmatrix} \frac{2(p+1)}{(p-2)^3} & \frac{-(p+1)}{(p-2)^2} \\ \frac{(p+2)}{(p-2)^2(p+1)} & \frac{(p-2)^2}{(p+1)} \end{bmatrix}, \quad \dot{\bar{d}}(t) = \begin{bmatrix} 0 & 1 \\ -9\sigma^2 & 0 \end{bmatrix} \bar{d}(t) \qquad (8.62)
$$

where $\bar{d}(t)$ is external disturbance, σ is unknown parameter which ranges between 1 and 4. The only information necessary for the design are the observability index $v = 3$, the order of $N(s)$, $m = 1$, and the signs of the leading principal minors of s. We choose the following design parameters $k_1 = 6, k_2 = 12, k_3 = 8, D_1 = I_2, C_2 = I_2, c_1 = l_1 = l_2 = 1, \Gamma = I_{10}$. The reference signal is given by $y_d = \frac{1}{(p+1)^2}[2\sin(t/2),\ 4\cos(t)]^T$. The plant initial condition is such that $y(0) = [0.4, 0.4]^T$. All other initial conditions are zero. The update laws and the control law are given by (8.47-8.50), (8.56), (8.58) and (8.55). The simulation results presented in the Figure 8.1 and Figure 8.2 show the systems output y and the desired trajectory signal y_d. Clearly, system output y can completely track the trajectory y_d. The result verifies our theoretical findings and show the effectiveness of the control schemes.

8.6 Summary

In this chapter, a new scheme is proposed to design an adaptive output-feedback controller for uncertain MIMO systems in the presence of disturbances. In order to reject disturbances generated from an unknown exosystems, new filters for state estimation are constructed and an adaptive internal model is employed. It is shown that the proposed controller can ensure all the signals in the closed-loop system bounded and the tracking error to converge to zero.

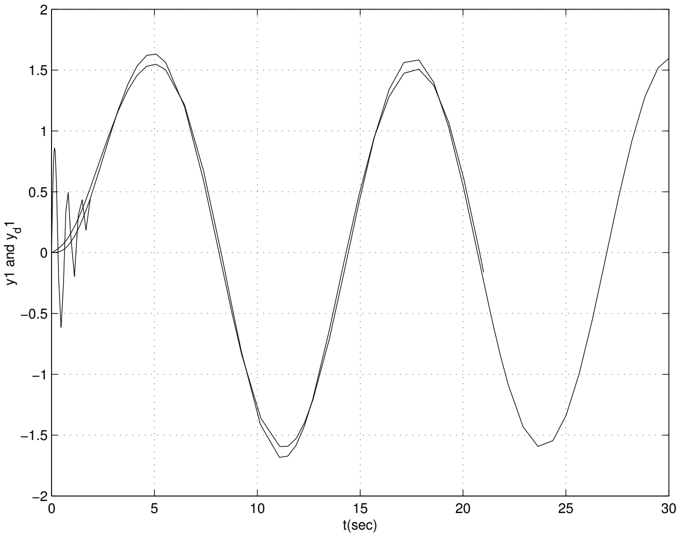


Figure 8.1: Output y_1 and trajectory y_{d1}.

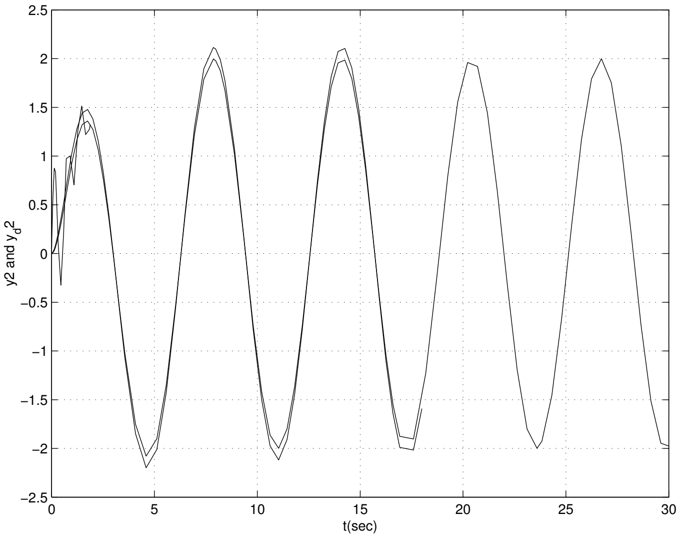


Figure 8.2: Output y_2 and trajectory y_{d2}.

Chapter 9

Decentralized Adaptive Stabilization in the Presence of Unknown Backlash-Like Hysteresis

In this chapter, a new scheme is proposed to address an output feedback control problem: decentralized adaptive stabilization of a class of interconnected subsystems with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity, where the hysteresis is modelled by a differential equation. Each local controller is designed by using backstepping technique and consists of a new robust control law and a new estimator to estimate the unknown parameters. For the implementation of the controller, no knowledge is assumed on the bounds of unknown system parameters and the effect contributed by the hysteresis. There is no structure requirement on the model of each subsystem such as an upper triangular form, since a general transfer function is considered. Also the interactions between subsystems are allowed to satisfy a nonlinear bound. It is shown that all the signals are bounded. A root mean square type of bound is obtained for the system states as a function of design parameters. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters. This implies that the transient system performance can be adjusted by choosing suitable design parameters.

9.1 Introduction

In the control of a large scale system, one usually faces poor knowledge on the plant parameters and interactions between subsystems. Thus the adaptive control technique in this case is an appropriate strategy to be employed. If some subsystems distribute distantly, it is difficult for a centralized controller to gather feedback signals from these subsystems. Also the design and implementation of the centralized controller are complicated. Therefore decentralized controllers, designed independently for local subsystems and using local available signals for feedback, are proposed to overcome such problems. The resulting decentralized controllers are also reliable in the sense that when some local controllers are out of order, the rest can still be in operation. Such decentralized controllers, however, should be robust against the ignored interactions. In the context of decentralized adaptive control, only a limited number of results have been obtained, see for examples [16, 19, 21, 23, 24, 31, 32, 33, 157, 158, 159]. The scheme presented in [31] is the first result using backstepping technique to relax the requirement on the relative degree of subsystems. But the result is only applicable to interactions satisfying a first-order type of bound and transient performance is not established. In the case that the input of each loop is preceded by unknown backlash-like hysteresis, there is still no result available.

Hysteresis exists in a wide range of physical systems and devices, such as biology optics, electro-magnetism, mechanical actuators, electronic relay circuits and other systems. Control of such systems is typically challenging. For backlash hysteresis, several adaptive control schemes have recently been proposed, see for examples [43, 44, 45, 56, 55]. In [44], [56] and [55] an inverse hysteresis nonlinearity was constructed. An adaptive hysteresis inverse cascaded with the plant was employed to cancel the effects of hysteresis. In [43], a dynamic hysteresis model is used to pattern a backlash-like hysteresis rather than constructing an inverse model to mitigate the bounded effects of the hysteresis. In the chapter, an adaptive state feedback control scheme is developed for a class of nonlinear systems. In the design, the term multiplying the control and the uncertain parameters of the system must be within a known compact set and a bound for the effects from hysteresis must also be available, in order to implement the projection operation in the estimator. Strictly speaking, only local stability is ensured in the sense that the initial values of the parameter estimates, which can be considered as part of system state variables, must be chosen from the compact set. If the true parameters are outside the set or the hysteresis effect is not bounded by the given bound, system stability cannot be ensured. In [121], a state feedback control for a special structure of nonlinear systems with backlash-like hysteresis is developed using backstepping methodology.

Due to difficulties to consider the effects of interconnections, extension of single loop results to multi-loop interconnecting systems is challenging, which is why the number of available results is still limited, especially for the case when the relative degree of each subsystem is greater than 2. In this chapter, we develop new output feedback decentralized stabilizers for a class of interconnected systems with subsystem having arbitrary relative degrees and with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity, where the backlash hysteresis is modelled by a differential equation as in [43]. Also the interactions between subsystems are allowed to satisfy a nonlinear bound. By using backstepping technique, new decentralized control laws and new parameter estimation schemes are derived, so that each local adaptive controller only uses the local information available. In our design, the term multiplying the control and the system parameters are not assumed to be within known intervals. There is no structure requirement on the model of each subsystem, such as an upper triangular form in [43], [121] and [159], since we consider a general transfer function. It is shown that the proposed adaptive controller can guarantee all the signals in the closed-loop adaptive system globally uniformly ultimately bounded. A root mean square type of bound is obtained for the system states as a function of design parameters. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters. This implies that the transient system performance can be adjusted by choosing suitable design parameters.

9.2 Problem Formulation

A system consisting of N interconnected subsystems modelled below is considered.

$$\dot{x}_{oi} = A_{oi}x_{oi} + b_{oi}u_i + \sum_{j=1}^{N} \bar{f}_{ij}(t, x_{oj}) \tag{9.1}$$

$$y_j = c_{oi}^T x_{oi}, \ for \ i = 1, \ldots, N \tag{9.2}$$

where $x_{oi} \in R^{n_i}$, $u_i \in R^1$ and $y_i \in R^1$ are the states, input and output of the ith subsystem, respectively, $\bar{f}_{ij}(t, x_{oj}) \in R^{n_i}$ denotes the nonlinear interactions from the jth subsystem to the ith subsystem for $j \neq i$, or a nonlinear un-modelled part of the ith subsystem for $j = i$. The matrices and vectors in (9.1) and (9.2) have appropriate dimensions, and their elements are constant but unknown.

Usually each loop has a backlash-like hysteresis nonlinearity and u_i is the output of such hysteresis described by

$$u_i(t) = BH_i(w_i(t)) \tag{9.3}$$

where $w_i(t)$ is the input of the hysteresis, $BH_i(w_i(t))$ is the backlash hysteresis operator. In this chapter we consider the hysteresis given in [43] which is described by a continuous-time dynamic model

$$\frac{du_i}{dt} = \alpha_i^{'} \left|\frac{dw_i}{dt}\right| (c_i^{'} w_i - u_i) + h_i \frac{dw_i}{dt} \tag{9.4}$$

where $\alpha_i^{'}$, $c_i^{'}$ and h_i are constants, $c_i^{'} > 0$ is the slope of the lines satisfying $c_i^{'} > h_i$. Based on the analysis in [43], this equation can be solved explicitly

$$\begin{aligned} u_i(t) &= c_i^{'} w_i(t) + \bar{\bar{d}}_i(t) && (9.5)\\ \bar{\bar{d}}_i(t) &= [u_i(0) - c_i^{'} w_i(0)] e^{-\alpha_i^{'}(w_i - w_i(0))\text{sign}\dot{w}_i} \\ &\quad + e^{-\alpha_i^{'} w_i \text{sign}\dot{w}_i} \int_{w_i(0)}^{w_i} [h_i - c_i^{'}] e^{\alpha_i^{'}\xi(\text{sign}\dot{w}_i)} d\xi && (9.6) \end{aligned}$$

Now substituting (9.5) to (9.1) gives

$$\begin{aligned} \dot{x}_{oi} &= A_{oi} x_{oi} + \bar{b}_{oi} w_i + \sum_{j=1}^{N} \bar{f}_{ij}(t, x_{oj}) + \bar{d}_i(t) && (9.7)\\ y_j &= c_{oi}^T x_{oi} && (9.8) \end{aligned}$$

where $\bar{b}_{oi} = b_{oi} c_i^{'}$ and $\bar{d}_i(t) = b_{oi} \bar{\bar{d}}_i(t)$. For each decoupled local system, we make the following assumptions.

Assumption 1: n_i is known;

Assumption 2: The triple $(A_{oi},\ \bar{b}_{oi},\ c_{oi})$ are completely controllable and observable;

Assumption 3: In the transfer function

$$\begin{aligned} G_i(s) &= c_{oi}^T (sI - A_{oi})^{-1} \bar{b}_{oi} = \frac{N_i(s)}{D_i(s)} \\ &= \frac{b_i^{m_i} s^{m_i} + \cdots + b_i^1 s + b_i^0}{s^{n_i} + a_i^{n_i - 1} + \cdots + a_i^1 s + a_i^0} && (9.9) \end{aligned}$$

$N_i(s)$ is a Hurwitz polynomial. The sign of $b_i^{m_i}$ and the relative degree $\rho_i (= n_i - m_i)$ of $G_i(s)$ are known;

Assumption 4: The nonlinear interaction terms satisfy

$$\| \bar{f}_{ij}(t, x_{oj}) \| \leq \bar{\gamma}_{ij} \left| y_j \psi_j(y_j) \right| \qquad (9.10)$$

where $\|\cdot\|$denotes the Euclidean norm, $\bar{\gamma}_{ij}$ are constants denoting the strength of the interaction, and $\psi_j(y_j), j = 1, 2, \ldots, N$ are known nonlinear functions.

Remark 9.1 *It is allowed that the interaction $\bar{f}_{ij}$ contains states x_{oj}, as long as it satisfies (9.10).*

Remark 9.2 *The class of systems considered in [31] and [159] is a special case as their interactions satisfy the Lipschitz condition which implies $\psi_j(y_j) = 1$.*

The control objective is to design totally decentralized adaptive controllers for system (9.1) and (9.4) satisfying Assumptions 1-4 such that the closed-loop system is stable.

9.3 Local State Estimation Filters

Clearly, there exists a nonsingular matrix T_i, such that under transformation $x_{oi} = T_i x_i$, (9.7) and (9.8) can be transformed to

$$\dot{x}_i = A_i x_i + a_i y_i + \begin{bmatrix} 0 \\ b_i \end{bmatrix} w_i + f_i + d_i \tag{9.11}$$

$$y_i = (e_i^1)^T x_i, \quad for\ i = 1, \ldots, N \tag{9.12}$$

where

$$A_i = \begin{bmatrix} 0 & & \\ \vdots & I & \\ 0 & \ldots & 0 \end{bmatrix}, \; a_i = \begin{bmatrix} -a_i^{n_i-1} \\ \vdots \\ -a_i^0 \end{bmatrix}, \; b = \begin{bmatrix} b_i^{m_i} \\ \vdots \\ b_i^0 \end{bmatrix} \tag{9.13}$$

$$f_i = \sum_{j=1}^{N} T_i^{-1} \bar{f}_{ij}, \; d_i = T_i^{-1} \bar{d}_i(t) \tag{9.14}$$

and e_i^k denotes the kth coordinate vector in R^{n_i}.

For state estimation, by following the standard procedures as in [30], we can obtain

$$\dot{\lambda}_i = A_i^0\lambda_i + e_i^{n_i}w_i \quad (9.15)$$

$$\dot{\eta}_i = A_i^0\eta_i + e_i^{n_i}y_i \quad (9.16)$$

$$\Omega_i^T = [v_i^{m_i}, \ldots, v_i^1, v_i^0, \Xi_i] \quad (9.17)$$

$$v_i^j = (A_i^0)^j\lambda_i, \qquad j = 0, \ldots, m_i \quad (9.18)$$

$$\Xi_i = -[(A_i^0)^{n_i-1}\eta_i, \ldots, A_i^0\eta_i, \eta_i] \quad (9.19)$$

$$\xi_i^{n_i} = -(A_i^0)^{n_i}\eta_i \quad (9.20)$$

where the vector $k_i = [k_i^1, \ldots, k_i^{n_i}]^T$ is chosen so that the matrix $A_i^0 = A_i - k_i(e_i^1)^T$ is Hurwitz. Hence there exists a P_i such that $P_iA_i^0 + A_i^0P_i^T = -4I,\ P_i = P_i^T > 0$. With these designed filters our state estimate is

$$\hat{x}_i = \xi_i^{n_i} + \Omega_i^T\theta_i \quad (9.21)$$

$$\theta_i^T = [a_i^T, b_i^T] \quad (9.22)$$

and the state estimation error $\epsilon_i = x - \hat{x}$ satisfies

$$\dot{\epsilon}_i = A_i^0\epsilon_i + f_i + d_i \quad (9.23)$$

Let $V_{\epsilon_i} = \epsilon_i^TP_i\epsilon_i$. It can be shown that

$$\begin{aligned} \dot{V}_{\epsilon_i} &= \epsilon_i^T\left[P_iA_i^0 + (A_i^0)^TP_i\right]\epsilon_i + 2\epsilon_i^TP_i(f_i + d_i) \\ &\leq -2\epsilon_i^T\epsilon_i + \| P_id_i \|^2 + \| P_if_i \|^2 \end{aligned} \quad (9.24)$$

Then system (9.11) can be expressed as

$$\dot{y}_i = b_i^{m_i}v_i^{m_i,2} + \xi_i^{n_i,2} + \bar{\delta}_i^T\theta_i + \epsilon_i^2 + f_i^1 + d_i^1 \quad (9.25)$$

$$\dot{v}_i^{m_i,\ q} = v_i^{m_i,q+1} - k_i^q v_i^{m_i,1}, \qquad q = 2,\dots,\rho_i - 1 \tag{9.26}$$

$$\dot{v}_i^{m_i,\ \rho_i} = v_i^{m_i,\ \rho_i+1} - k_i^{\rho_i} v_i^{m_i,1} + w_i \tag{9.27}$$

where

$$\delta_i = [v_i^{m_i,2}, v_i^{m_i-1,2}, \dots, v_i^{0,2}, \Xi_i^{(2)} - y_i(e_i^1)^T]^T \tag{9.28}$$

$$\bar{\delta}_i = [0, v_i^{m_i-1,2}, \dots, v_i^{0,2}, \Xi_i^{(2)} - y_i(e_i^1)^T]^T \tag{9.29}$$

and $v_i^{m_i,2}, \epsilon_i^2, \xi_i^{n_i,2}$ denote the second entries of $v_i^{m_i}, \epsilon_i, \xi_i^{n_i}$ respectively, f_i^1 and d_i^1 are the first element of vector f_i and d_i. All states of the local filters in (9.15) and (9.16) are available for feedback.

9.4 Design of Adaptive Controllers

As usual in backstepping approach, the following change of coordinates is made.

$$z_i^1 = y_i \tag{9.30}$$

$$z_i^q = v_i^{m_i,q} - \alpha_i^{q-1}, \ q = 2, 3, \dots, \rho_i \tag{9.31}$$

where α_i^{q-1} is the virtual control at the *qth step* of the ith loop and will be determined in later discussion.

To illustrate the controller design procedures, we now give a brief description on the first step.

• *Step* 1: We start with the equations for the stabilization error z_i^1 obtained from (9.25), (9.30) and (9.31) to get

$$\dot{z}_i^1 = b_i^{m_i}\alpha_i^1 + \xi_i^{n_i,2} + \bar{\delta}_i^T\theta_i + \epsilon_i^2 + f_i^1 + d_i^1 + b_i^{m_i} z_i^2 \tag{9.32}$$

We design the virtual control law α_i^1 as

$$\alpha_i^1 = \hat{p}_i \bar{\alpha}_i^1 \tag{9.33}$$

$$\bar{\alpha}_i^1 = -c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 \psi_i^2(z_i^1) - \xi_i^{n_i,2} - \bar{\delta}_i^T \hat{\theta}_i \tag{9.34}$$

where c_i^1 and l_i^1 are positive design parameters, $\hat{\theta}_i$ is the estimate of θ_i, $\hat{p}_i$ is an estimate of $p_i = 1/b_{m_i}$.

Remark 9.3 *The term $l_i^* z_i^1 \psi_i^2(z_i^1)$ in (9.34) is designed to compensate the effects of interactions from other subsystems or the un-modelled part of its own sussystem. Note that the scheme in [31] dose not have such a term and thus the result of [31] is not applicable to the systems considered here.*

In the following, we split l_i^1 into $3\bar{l}_i^1$ to make the presentation easier in the stability analysis. Each of $\bar{l}_i^1$ will deal with the terms having ϵ_i^2, d_i^1, or f_i^1 in the evaluation of $\dot{V}_i^1$, respectively.

From (9.32) and (9.33) we have

$$\begin{aligned} \dot{z}_i^1 &= -c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 \psi_i^2(z_i^1) + \epsilon_i^2 + \bar{\delta}_i^T \tilde{\theta}_i - b_i^{m_i} \bar{\alpha}_i^1 \tilde{p}_i + b_i^{m_i} z_i^2 + f_i^1 + d_i^1 \\ &= -c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 \psi_i^2(z_i^1) + \epsilon_i^2 + (\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_i^1)^T \tilde{\theta}_i - b_i^{m_i} \bar{\alpha}_i^1 \tilde{p}_i + \hat{b}_i^{m_i} z_i^2 \\ &\quad + f_i^1 + d_i^1 \end{aligned} \tag{9.35}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and using $\tilde{p}_i = \frac{1}{b_i^{m_i}} - \frac{1}{\hat{b}_i^{m_i}}$, we have

$$b_i^{m_i} \alpha_i^1 = b_i^{m_i} \hat{p}_i \bar{\alpha}_i^1 = \bar{\alpha}_i^1 - b_i^{m_i} \tilde{p} \bar{\alpha}_i^1 \tag{9.36}$$

$$\begin{aligned} \bar{\delta}_i^T \tilde{\theta}_i + b_i^{m_i} z_i^2 &= \bar{\delta}_i^T \tilde{\theta} + \tilde{b}_i^{m_i} z_i^2 + \hat{b}_i^{m_i} z_i^2 \\ &= \bar{\delta}_i^T \tilde{\theta}_i + (v_i^{m_i,2} - \alpha_i^1)(e_i^1)^T \tilde{\theta}_i + \hat{b}_i^{m_i} z_i^2 \\ &= (\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_i^1)^T \tilde{\theta}_i + \hat{b}_i^{m_i} z_i^2 \end{aligned} \tag{9.37}$$

We consider the Lyapunov function

$$V_i^1 = \frac{1}{2}(z_i^1)^2 + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{|b_i^{m_i}|}{2\gamma_i}\tilde{p}_i^2 + \frac{1}{2\bar{l}_i^1} V_{\epsilon_i} \tag{9.38}$$

where Γ_i is a positive definite design matrix and γ_i is a positive design parameter. We now examine the derivative of V_i^1

$$\begin{aligned} \dot{V}_i^1 \leq & -c_1(z_i^1)^2 + \hat{b}_i^{m_i} z_i^1 z_2 - |b_i^{m_i}|\tilde{e}\frac{1}{\gamma}[\gamma \text{sign}(b_i^{m_i})\bar{\alpha}_1 z_i^1 + \dot{\hat{p}}_i] - l_i^*(z_i^1)^2 \psi_i^2(z_i^1) \\ & + \tilde{\theta}_i^T \Gamma_i^{-1}[\Gamma_i(\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_i^1) z_i^1 - \dot{\hat{\theta}}_i] - \frac{1}{\bar{l}_i^1}\epsilon_i^T \epsilon_i + \frac{1}{\bar{l}_i^1}(\parallel P_i d_i \parallel^2 + \parallel P_i f_i \parallel^2) \\ & + \frac{1}{4\bar{l}_i^1}(\parallel d_i^1 \parallel^2 + \parallel f_i^1 \parallel^2) \end{aligned} \tag{9.39}$$

Now we choose

$$\dot{\hat{p}}_i = -\gamma_i \text{sign}(b_i^{m_i})\bar{\alpha}_i^1 z_i^1 - \gamma_i l_i^p(\hat{p}_i - p_i^0) \tag{9.40}$$

$$\tau_i^1 = (\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_i^1) z_i^1 \tag{9.41}$$

where l_i^p and p_i^0 are two positive design constants.

From the choice, the following useful property can be obtained:

$$\begin{aligned} l_i^p \tilde{p}_i(\hat{p}_i - p_i^0) = & -l_i^p(\hat{p}_i - p_i)(\frac{1}{2}(\hat{p}_i - p_i) + \frac{1}{2}(\hat{p}_i + p_i) - p_i^0) \\ & -\frac{1}{2} l_i^p \tilde{p}_i^2 + \frac{1}{2} l_i^p (p_i - p_i^0)^2 \end{aligned} \tag{9.42}$$

Then the following derivation for the derivative of V_i^1 can be carried out by using (9.40)-(9.42)

$$\begin{aligned} \dot{V}_i^1 \leq & -c_i^1(z_i^1)^2 + \hat{b}_i^{m_i} z_i^1 z_i^2 - \frac{|b_i^{m_i}|}{2} l_i^p \tilde{p}_i^2 - \frac{1}{l_i^1}\epsilon_i^T \epsilon_i + \frac{|b_i^{m_i}|}{2} l_i^p (p_i - p_i^0)^2 - l_i^*(z_i^1)^2 \psi_i^2(z_i^1) \\ + \ \tilde{\theta}_i^T & (\tau_i^1 - \Gamma_i^{-1}\dot{\hat{\theta}}_i) + \frac{1}{\bar{l}_i^1} \parallel P_i d_i \parallel^2 + \frac{1}{4\bar{l}_i^1} \parallel d_i^1 \parallel^2 + \frac{1}{\bar{l}_i^1} \parallel P_i f_i \parallel^2 + \frac{1}{4\bar{l}_i^1} \parallel f_i^1 \parallel^2 \end{aligned} \tag{9.43}$$

Remark 9.4 *Note that a new term $\gamma_i l_i^p(\hat{p}_i - p_i^0)$ is introduced in the parameter update law (9.40) compared with the traditional estimator using backstepping. This term is used to mitigate the hysteresis effect for system stability as shown in later discussion.*

- *Step q ($q = 2, \ldots, \rho_i$, $i = 1, \ldots, N$)*: Choose virtual control laws

$$\alpha_i^2 = -\hat{b}_i^{m_i} z_1 - \Big[c_i^2 + l_i^2\big(\frac{\partial \alpha_i^1}{\partial y_i}\big)^2\Big] z_2 + \bar{B}_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i}\Gamma_i \tau_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i}\Gamma_i l_i^\theta(\hat{\theta}_i - \theta_i^0) \quad (9.44)$$

$$\alpha_i^q = -z_i^{q-1} - \Big[c_i^q + l_i^q\big(\frac{\partial \alpha_i^{q-1}}{\partial y_i}\big)^2\Big] z_i^q + \bar{B}_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i}\Gamma_i \tau_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i}\Gamma_i l_i^\theta(\hat{\theta}_i - \theta_i^0)$$
$$-\Big(\sum_{k=2}^{q-1} z_i^k \frac{\partial \alpha_i^{k-1}}{\partial \hat{\theta}_i}\Big)\Gamma_i \frac{\partial \alpha_i^{q-1}}{\partial y_i}\delta_i \quad (9.45)$$

$$\tau_i^q = \tau_i^{q-1} - \frac{\partial \alpha_i^{q-1}}{\partial y_i}\delta_i z_i^q \quad (9.46)$$

where $c_i^q, l_i^q, q = 3, \ldots, \rho_i$ are positive design parameters, and $\bar{B}_i^q, q = 2, \ldots, \rho_i$ denotes some known terms and its detailed structure can be found in [30]. Then the adaptive controller and parameter update laws are finally given by

$$w_i = \alpha_i^{\rho_i} - v_i^{m_i, \rho_i + 1} \quad (9.47)$$

$$\dot{\hat{\theta}}_i = \Gamma_i \tau_i^{\rho_i} + \Gamma_i l_i^\theta(\hat{\theta}_i - \theta_i^0) \quad (9.48)$$

where l_i^θ and θ_i^0 are positive design constants.

The designed adaptive controllers are summarized in Table 9.1.

Remark 9.5 *When going through the details of the design procedures, we note that in the equations concerning $\dot{z}_i^q, q = 1, 2, \ldots, \rho_i$, just functions f_i^1 from the interactions and d_i^1 due to the hysteresis effect appear, and they are always together with ϵ_i^2. This is because only $\dot{y}_i$ from the plant model (9.11) was used in the calculation of $\dot{\alpha}_i^q$ for steps $q = 2, \ldots, \rho_i$.*

Table 9.1: Decentralized Adaptive Backstepping Controller

Adaptive Control Laws:

$$
\begin{aligned}
\alpha_i^1 &= \hat{p}_i \bar{\alpha}_i^1 && \text{(T.1)} \\
\bar{\alpha}_i^1 &= -c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 \psi_i^2(z_i^1) - \xi_i^{n_i,2} - \bar{\delta}_i^T \hat{\theta}_i && \text{(T.2)} \\
\alpha_i^2 &= -\hat{b}_i^{m_i} z_1 - \Big[c_i^2 + l_i^2 \Big(\frac{\partial \alpha_i^1}{\partial y_i}\Big)^2\Big] z_2 + \bar{B}_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i} \Gamma_i \tau_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i} \Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0) && \text{(T.3)} \\
\alpha_i^q &= -z_i^{q-1} - \Big[c_i^q + l_i^q \Big(\frac{\partial \alpha_i^{q-1}}{\partial y_i}\Big)^2\Big] z_i^q + \bar{B}_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i} \Gamma_i \tau_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i} \Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0) \\
&\quad - \Big(\sum_{k=2}^{q-1} z_i^k \frac{\partial \alpha_i^{k-1}}{\partial \hat{\theta}_i}\Big) \Gamma_i \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta_i, \quad q = 2, \ldots, \rho_i, \; i = 1, \ldots, N && \text{(T.4)} \\
w_i &= \alpha_i^{\rho_i} - v_i^{m_i, \rho_i + 1} && \text{(T.5)}
\end{aligned}
$$

Parameter Update Laws:

$$
\begin{aligned}
\dot{\hat{p}}_i &= -\gamma_i \text{sign}(b_i^{m_i}) \bar{\alpha}_i^1 z_i^1 - \gamma_i l_i^p (\hat{p}_i - p_i^0) && \text{(T.6)} \\
\dot{\hat{\theta}}_i &= \Gamma_i \tau_i^{\rho_i} + \Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0) && \text{(T.7)} \\
\textit{with} \quad \tau_i^q &= \tau_i^{q-1} - \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta_i z_i^q && \text{(T.8)} \\
\tau_i^1 &= (\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_i^1) z_i^1 && \text{(T.9)}
\end{aligned}
$$

9.5 Stability Analysis

In this section, the stability of the overall closed-loop system consisting of the interconnected plant and decentralized controllers will be established. Firstly, a mathematical model for each local closed-loop control system is derived from (9.35) and the rest of the design steps $2, \ldots, \rho_i$.

$$\begin{bmatrix} \dot{z}_i^1 \\ \dot{z}_i^2 \\ \vdots \\ \dot{z}_i^{\rho_i} \end{bmatrix} = A_i^c \begin{bmatrix} z_i^1 \\ z_i^2 \\ \vdots \\ z_i^{\rho_i} \end{bmatrix} + [\delta_i^T \tilde{\theta}_i + \epsilon_i^2 + f_i^1 + d_i^1] \begin{bmatrix} 1 \\ -\frac{\partial \alpha_i^1}{\partial y_i} \\ \vdots \\ -\frac{\partial \alpha_i^{\rho_i - 1}}{\partial y_i} \end{bmatrix} + \tilde{p}_i \begin{bmatrix} -b_i^{m_i} \bar{\alpha}_i^1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \tilde{b}_i^{m_i} \begin{bmatrix} -v_i^{m_i,2} \\ z_i^1 \\ \vdots \\ 0 \end{bmatrix} \tag{9.49}$$

where A_i^c is a matrix having the same structure as in the scalar systems given in [30].

Now we define a Lyapunov function of the overall decentralized adaptive control system as

$$V = \sum_{i=1}^{N} V_i \tag{9.50}$$

where

$$V_i = \sum_{q=1}^{\rho_i} \left(\frac{1}{2}(z_i^q)^2 + \frac{1}{2\bar{l}_i^q} V_{\epsilon_i}\right) + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{|b_i^{m_i}|}{2\gamma_i} \tilde{p}_i^2 \tag{9.51}$$

Note that

$$\Gamma_i \tau_i^{q-1} - \dot{\hat{\theta}}_i = \Gamma_i \tau_i^{q-1} - \Gamma_i \tau_i^q + \Gamma_i \tau_i^q - \dot{\hat{\theta}}_i = \Gamma_i \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta z_i^q + (\Gamma_i \tau_i^q - \dot{\hat{\theta}}_i) \tag{9.52}$$

$$l_i^\theta \tilde{\theta}_i^T(\hat{\theta}_i - \theta_i^0) \leq -\frac{1}{2} l_i^\theta \parallel \tilde{\theta}_i \parallel^2 + \frac{1}{2} l_i^\theta \parallel \theta_i - \theta_i^0 \parallel^2 \quad (9.53)$$

From (9.44) - (9.48), the derivative of the V_i in (9.51) satisfies

$$\begin{aligned}
\dot{V}_i &= \sum_{q=1}^{\rho_i} z_i^q \dot{z}_i^q - \tilde{\theta}_i^T \Gamma^{-1} \dot{\hat{\theta}}_i - \frac{|b_i^{m_i}|}{\gamma_i} \tilde{p}_i \dot{\hat{p}}_i + \sum_{q=1}^{\rho_i} \frac{1}{2\bar{l}_i^q} \dot{V}_{\epsilon_i} \\
&\leq -\sum_{q=1}^{\rho_i} c_i^q (z_i^q)^2 - \frac{1}{2} l_i^\theta \parallel \tilde{\theta}_i \parallel^2 + \frac{1}{2} l_i^\theta \parallel \theta_i - \theta_i^0 \parallel^2 + \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i^q} (\parallel P_i d_i \parallel^2 + \frac{1}{4} \parallel d_i^1 \parallel^2) \\
&\quad - \frac{|b_i^{m_i}|}{2} l_i^p \tilde{p}_i^2 + \frac{|b_i^{m_i}|}{2} l_i^p (p_i - p_i^0)^2 - \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i} \epsilon_i^T \epsilon_i - l_i^*(z_i^1)^2 \psi_i^2(z_i^1) \\
&\quad + \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i^q} (\parallel P_i f_i \parallel^2 + \frac{1}{4} \parallel f_i \parallel^2) - \sum_{q=1}^{\rho_i} \bar{l}_i^q (z_i^q \frac{\partial \alpha_i^{q-1}}{\partial y_q} + \frac{1}{2\bar{l}_i^q} f_i^1) \\
&\leq -l_i^*(z_i^1)^2 \psi_i^2(z_i^1) - \sum_{q=1}^{\rho_i} c_i^q (z_i^q)^2 - \frac{1}{2} l_i^\theta \parallel \tilde{\theta}_i \parallel^2 - \frac{|b_i^{m_i}|}{2} l_i^p (\tilde{p}_i)^2 - \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i} \epsilon_i^T \epsilon_i \\
&\quad + \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i^q} (\parallel P_i f_i \parallel^2 + \frac{1}{4} \parallel f_i \parallel^2) + M_i^* \quad (9.54)
\end{aligned}$$

where the inequality $ab \leq (a^2 + b^2)/2$ was used, $D_{i,max}$ denotes the bound of $d_i(t)$, and

$$M_i^* = M_i + \sum_{q=1}^{\rho_i} \frac{1}{4\bar{l}_i^q} (4 \parallel P_i \parallel^2 + 1) D_{i,max}^2 \quad (9.55)$$

$$M_i = \frac{|b_i^{m_i}|}{2} l_i^p (p_i - p_i^0)^2 + \frac{1}{2} l_i^\theta \parallel \theta_i - \theta_i^0 \parallel^2 \quad (9.56)$$

Remark 9.6 *Due to the presence of hysteresis, an extra term M_i^* appears in (9.54) comparing to the analysis in [31]. The handling of M_i^* is elaborated after (9.60).*

From Assumption 4, we can show that

$$\sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i^q} (\parallel P_i f_i \parallel^2 + \frac{1}{4} \parallel f_i \parallel^2) \leq \sum_{j=1}^{N} \gamma_{ij} \left| z_j^1 \psi_j(z_j^1) \right|^2 \quad (9.57)$$

where $\gamma_{ij} = O(\bar{\gamma}_{ij})$ and indicating the coupling strength from the jth subsystem to the ith subsystem, depending on $\bar{l}_i^q, \| P_i \|$. Clearly there exist γ_{ij}^* such that for all $\gamma_{ij} \leq \gamma_{ij}^*$,

$$l_i^* \geq \sum_{j=1}^{N} \gamma_{ji}^* \tag{9.58}$$

Now taking the summation of the first term in (9.54) into account and using (9.57) and (9.58), we get

$$\begin{aligned}
& \sum_{i=1}^{N} -[l_i^*(z_i^1)^2\psi_i^2(z_i^1) - \sum_{k=1}^{\rho_i} \frac{1}{\bar{l}_i^k}(\| P_i f_i \|^2 + \frac{1}{4} \| f_i \|^2)] \\
\leq \; & \sum_{i=1}^{N} -l_i^*(z_i^1)^2\psi_i^2(z_i^1) + \sum_{i=1}^{N}(\sum_{j=1}^{N} \gamma_{ij}^* \left|z_j^1\psi_j(z_j^1)\right|^2) \\
= \; & \sum_{i=1}^{N} -l_i^*\left|z_i^1\psi_i(z_i^1)\right|^2 + \sum_{j=1}^{N}(\sum_{i=1}^{N} \gamma_{ij}^*)\left|z_j^1\psi_j(z_j^1)\right|^2 \\
= \; & \sum_{i=1}^{N} -l_i^*\left|z_i^1\psi_i(z_i^1)\right|^2 + \sum_{i=1}^{N}(\sum_{j=1}^{N} \gamma_{ji}^*)\left|z_i^1\psi_i(z_i^1)\right|^2 \\
& (\textbf{In this step, we exchange i for j and exchange j for i.}) \\
= \; & \sum_{i=1}^{N} -[l_i^* - \sum_{j=1}^{N} \gamma_{ji}^*]\left|z_i^1\psi_i(z_i^1)\right|^2 \leq 0
\end{aligned} \tag{9.59}$$

Then

$$\dot{V} \leq \sum_{i=1}^{N} [-\sum_{q=1}^{\rho_i} c_i^q(z_i^q)^2 - \frac{1}{2} l_i^\theta \| \tilde{\theta}_i \|^2 - \frac{|b_i^{m_i}|}{2} l_i^p(\tilde{p}_i)^2 - \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i} \epsilon_i^T \epsilon_i + M_i^*] \tag{9.60}$$

Remark 9.7 *The summation in (9.59) is one of the key steps in the stability analysis. Note that this results in the cancellation of the interaction effects from other subsystems. The approach in [31] cannot be applied here due to non-Lipschitz type nonlinear interactions.*

Notice that

$$-\sum_{q=1}^{\rho_i} c_i^q (z_i^q)^2 - \frac{1}{2} l_i^\theta \parallel \tilde{\theta}_i \parallel^2 - \frac{|b_i^{m_i}|}{2} l_i^p (\tilde{p}_i)^2 - \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i} \epsilon_i^T \epsilon_i \leq -f_i^- \bar{V}_i \tag{9.61}$$

and

$$V_i = \sum_{q=1}^{\rho_i} \frac{1}{2}(z_i^q)^2 + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{|b_i^{m_i}|}{2\gamma_i} \tilde{p}_i^2 + \sum_{q=1}^{\rho_i} \frac{1}{2\bar{l}_i^q} {\epsilon_i}^T P_i \epsilon_i \leq f_i^+ \bar{V}_i \tag{9.62}$$

where

$$\bar{V}_i = \sum_{q=1}^{\rho_i} (z_i^q)^2 + \tilde{\theta}_i^T \tilde{\theta}_i + \tilde{p}_i^2 + \sum_{q=1}^{\rho_i} {\epsilon_i}^T \epsilon_i \tag{9.63}$$

$$f_i^- = min\{c_i^q,\ \frac{1}{2} l_i^\theta,\ \frac{|b_i^{m_i}|}{2} l_i^p,\ \frac{1}{\bar{l}_i^q}\} \tag{9.64}$$

$$f_i^+ = max\{\frac{1}{2},\ \frac{1}{2}\lambda_i^{q,max}(\Gamma_i),\ \frac{|b_i^{m_i}|}{2\gamma_i},\ \frac{1}{2\bar{l}_i^q}\lambda_i^{q,max}(P_i)\},\quad q = 1, \ldots, \rho_i \tag{9.65}$$

where $\lambda_i^{q,max}(P_i)$ and $\lambda_i^{q,max}(\Gamma_i)$ are the maximum eigenvalues of P_i and Γ_i, respectively. Therefore, from (9.60) we obtain

$$\dot{V} \leq -f^* V + M^* \tag{9.66}$$

where $f^* = \sum_{i=1}^N f_i^- / \sum_{i=1}^N f_i^+$, $M^* = \sum_{i=1}^N M_i^*$ is a bounded term. By direct integrations of the differential inequality (9.66), we have

$$V \leq V(0)e^{-f^* t} + \frac{M^*}{f^*}(1 - e^{-f^* t}) \leq V(0) + \frac{M^*}{f^*} \tag{9.67}$$

This shows that V is uniformly bounded. Thus $z_i^1, z_i^2, \ldots, \hat{p}_i, \hat{\theta}_i$ and ϵ_i are bounded. Since z_i^1 are bounded, y_i is also bounded. Then from (9.15) and (9.16) we can show that λ_i, η_i and x_i are bounded as in [30]. Therefore boundedness of all signals in the system is ensured as formally stated in the following theorem.

Theorem 9.1 *Consider the closed-loop adaptive system consisting of the plant (9.1) under Assumptions 1-4, the controller (9.47), the estimator (9.40), (9.48), and the filters (9.15) and (9.16). There exist* γ_{ij}^* *such that for all* $\gamma_{ij} \leq \gamma_{ij}^*$, $i, j = 1, \ldots, N$, *all the signals in the system are globally uniformly bounded.*

Remark 9.8 *The condition that* $\gamma_{ij} \leq \gamma_{ij}^*$ *has the following two implications:*
(1) If we know $\bar{\gamma}_{ij}$*, then we can design* l_i^* *according to (9.57) and (9.58). This means that the coupling strength of the interconnection between subsystems is not necessary to be weak.*
(2) If we do not know $\bar{\gamma}_{ij}$*, then the designed local controllers are able to stabilize any interconnected system with coupling strength satisfying (9.58). This implication is similar to the interpretations of the results in [31]-[32], [21], [23] where sufficiently weak interactions are allowed.*

We now derive a bound for the vector $z_i(t)$ where $z_i(t) = [z_i^1, z_i^2, \ldots, z_i^{\rho_i}]^T$. Firstly, the following definitions are made.

$$c_i^0 = min_{1 \leq q \leq \rho_i} c_i^q, \quad d_0 = \sum_{i=1}^{N} \sum_{q=1}^{\rho_i} \frac{1}{4\bar{l}_i^q} \tag{9.68}$$

$$\| z_i \|_{[0,T]} = \sqrt{\frac{1}{T} \int_0^T \left(z_i(t)\right)^2 dt} \tag{9.69}$$

This definition is similar to the root mean square value used in electric circuit. Then from (9.54), we have

$$\dot{V} \leq -c_i^0 \| z_i \|^2 + M^* \tag{9.70}$$

Integrating both sides, we obtain

$$\begin{aligned} \| z_i \|_{[0,T]} \leq \; & \frac{1}{c_i^0} [\frac{|V(0) - V(T)|}{T} + \sum_{i=1}^{N} M_i \\ & + d_0 \frac{1}{T} \sum_{i=1}^{N} \rho_i (4 \| P_i \|^2 + 1) \int_0^T d_i^2(t) dt] \end{aligned} \tag{9.71}$$

On the other hand, from (9.66), we have

$$\begin{aligned}
&\frac{|V(0)-V(T)|}{T} \\
\leq\ & \frac{1-e^{-f^*T}}{T}\left(\frac{M}{f^*}+V(0)\right)+\frac{1}{T}d_0\sum_{i=1}^{N}\rho_i(4\parallel P_i\parallel^2+1)\int_0^T e^{-f^*(T-t)}d_i^2(t)dt \\
\leq\ & M+f^*V(0)+\frac{1}{T}d_0\sum_{i=1}^{N}\rho_i(4\parallel P_i\parallel^2+1)\int_0^T e^{-f^*(T-t)}d_i^2(t)dt \\
& \qquad\qquad for\ all\ T\geq 0, \qquad (9.72)
\end{aligned}$$

where we have used the fact that $e^{-f^*(T-t)} \leq 1$ and $\frac{1-e^{-f^*T}}{T} \leq f^*$, and $M = \sum_{i=1}^{N} M_i$.

By setting $z_i(0) = 0$, the initial value of the Lyapunov function is

$$V(0) = \sum_{i=1}^{N}\frac{1}{2}\left[\parallel\tilde{\theta}_i(0)\parallel^2_{\Gamma_i^{-1}}+\frac{|b_i^{m_i}|}{2\gamma_i}|\tilde{p}_i(0)|^2+d_i^0|\epsilon_i(0)|^2_{P_i}\right] \qquad (9.73)$$

where $d_i^0 = \sum_{q=1}^{\rho_i}\frac{1}{l_i^q}$.

Then a bound for $\parallel z_i \parallel_{[0,T]}$ is established and stated in the following theorem.

Theorem 9.2 *The bound* $\parallel z_i \parallel_{[0,T]}$ *satisfies*

$$\begin{aligned}
\parallel z_i\parallel_{[0,T]} \leq\ & \sum_{i=1}^{N}\parallel\tilde{\theta}_i(0)\parallel^2_{\Gamma_i^{-1}}+\frac{|b_i^{m_i}|}{\gamma_i}|\tilde{p}_i(0)|^2+d_i^0|\epsilon_i(0)|^2_{P_i} \\
&+\frac{1}{c_i^0}\sum_{i=1}^{N}(|b_i^{m_i}|l_i^p(p_i-p_i^0)^2+l_i^\theta\parallel\theta_i-\theta_i^0\parallel^2) \\
&+\frac{1}{c_i^0}d_0\sum_{i=1}^{N}2\rho_i(4\parallel P_i\parallel^2+1)D^2_{i,max} \qquad (9.74)
\end{aligned}$$

Proof: Using (9.64), (9.65) and (9.71) - (9.73), the fact that $f^*/c_i^0 \leq 2$, (9.74) can be obtained.

Remark 9.9 *Regarding the above bound, the following conclusions can be drawn:*

- *The transient performance in the sense of truncated norm given in (9.74) de-*

pends on the initial estimate errors $\tilde{\theta}_i(0), \tilde{p}_i(0)$ and $\epsilon_i(0)$. The closer the initial estimates to the true values, the better the transient performance.

- *This bound can also be systematically reduced by increasing $\Gamma_i, \gamma_i, c_i^0, \bar{l}_i^q$ and decreasing l_i^p, l_i^θ.*
- *The bound is depending on the effect of hysteresis. If the system has no hysteresis, then $d_i(t) = 0$ and we have the following corollary.*

Corollary: Consider the closed-loop decentralized adaptive control system consisting of the plant (9.1) without input hysteresis under Assumptions 1-4. All the states of the system asymptotically approach to zero and the bound $\| z_i \|_2$ is given by

$$\| z_i \|_2 \leq \frac{1}{2\sqrt{c_i^0}} \Big(\sum_{i=1}^{N} \| \tilde{\theta}_i(0) \|^2_{\Gamma_i^{-1}} + \frac{|b_i^{m_i}|}{\gamma_i} |\tilde{p}_i(0)|^2 + d_i^0 |\epsilon_i(0)|^2_{P_i} \Big)^{1/2} \tag{9.75}$$

Proof: In the absence of hysteresis the term $d_i(t) = 0$, so $M_i^* = 0$ in (9.60). We have

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^{N} \Big[-\sum_{q=1}^{\rho_i} c_i^q (z_i^q)^2 - \frac{1}{2} l_i^\theta \| \tilde{\theta}_i \|^2 - \frac{|b_i^{m_i}|}{2} l_i^p (\tilde{p}_i)^2 - \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_i} \epsilon_i^T \epsilon_i \Big] \\ &\leq -c_i^0 \| z_i \|_2^2 \end{aligned} \tag{9.76}$$

where $\| z_i \|_2^2 = \int_0^\infty \| z_i \|^2 d\tau$. Clearly (9.75) can be obtained.

Remark 9.10 *This result further extends that presented in [31], where only first order interactions considered and no transient performance like (9.75) is available.*

9.6 An Illustrative Example

We consider the following simple system with two subsystems

$$\begin{aligned} \dot{x}_1 &= a_1x_1 + b_1u_1 + f_1 \\ \dot{x}_2 &= a_2x_2 + b_2u_2 + f_2 \qquad (9.77) \\ y_1 &= x_1, \quad y_2 = x_2 \\ u_1 &= BH_1(w_1) \quad , u_2 = BH_2(w_2) \qquad (9.78) \end{aligned}$$

where $a_1 = 1, b_1 = 1, a_2 = 0.5, b_2 = 1$, the nonlinear interaction terms $f_1 = y_2 + \sin(y_2), f_2 = 0.2y_1^2$, $BH_1(w_1)$ and $BH_2(w_2)$ are the backlash hysteresis described by (9.4) with parameters $\alpha_1^{'} = 1, c_1^{'} = 2, h_1 = 0.2$, $\alpha_2^{'} = 1, c_2^{'} = 1, h_2 = 0.2$. These parameters are not needed to be known in the controller design. The objective is to stabilize system (9.77) and (9.78). The controller (9.47) and the estimator (9.40), (9.48) are implemented, where $\hat{p}_i$ and $\hat{\theta}_i$ are estimates of $p_i = 1/b_ic_i^{'}$ and $\theta_i = a_i, i = 1, 2$, respectively. The design parameters are chosen as $c_1^1 = c_2^1 = 10, l_1^1 = l_1^2 = 5, l_i^* = 5, \gamma_1 = 2, \gamma_2 = 2, \Gamma_1 = \Gamma_2 = 1$. The initials are set as $y_1(0) = 0.3, y_2(0) = 0.5$. The simulation results presented in the Figure 9.1 and Figure 9.2 show the system outputs y_1 and y_2. Figure 9.3 and Figure 9.4 show the control signals $w_1(t)$ and $w_2(t)$. These simulation results clearly verify that our proposed scheme is effective to cope with hysteresis nonlinearity and high order nonlinear interactions.

9.7 Sumarry

This chapter addresses an output feedback control problem: decentralized adaptive stabilization of a class of interconnected subsystems with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity, where the hysteresis is modelled by a differential equation. The controllers are designed by using back-stepping technique. A new robust control law and a new estimator to estimate the

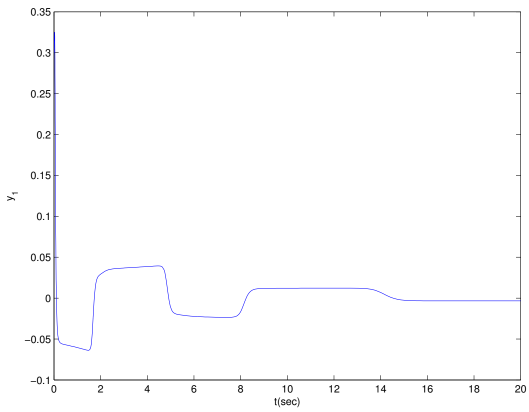


Figure 9.1: Output y_1.

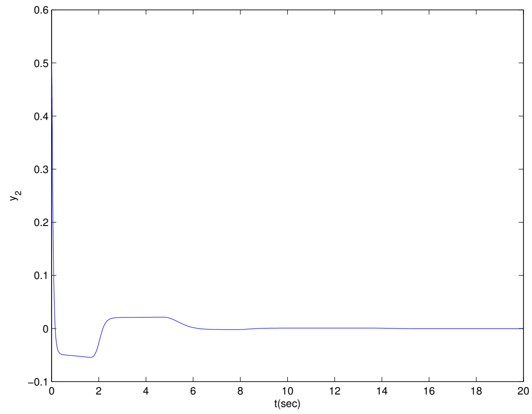


Figure 9.2: Output y_2.

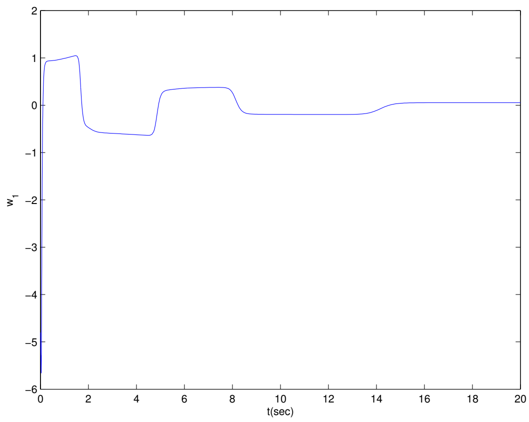


Figure 9.3: Control input w_1.

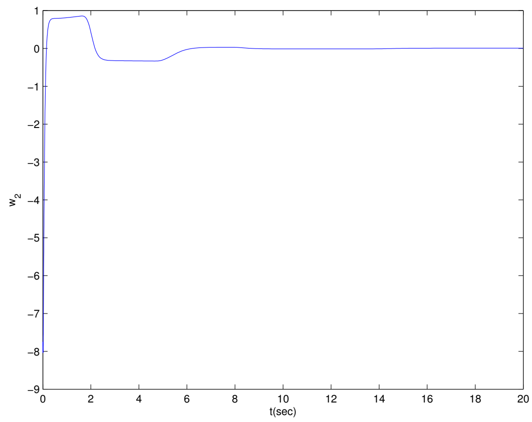


Figure 9.4: Control input w_2.

unknown parameters are derived. Also the interactions between subsystems are allowed to satisfying a nonlinear bound. There is no structure requirement on the model of each subsystem such as an upper triangular form, since a general transfer function is considered. For the implementation of the controller, no knowledge is assumed on the bounds of unknown system parameters and the effect contributed by the hysteresis. It is shown that adaptive control system is global stable in the sense that all the signals are bounded. Also a root mean square type of bound is obtained for the system states as a function of the design parameters. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters. As these bounds can give quantification of the system transient performance, thus the transient performance is adjustable by choosing suitable parameters.

Chapter 10

Conclusion and Recommendations

10.1 Conclusion

Adaptive control is popular in engineering and science. However, it still faces many important challenges, such as the handling of nonsmooth nonlinearity. This thesis has addressed numerous issues pertaining to design of uncertain systems. Our main concern is how to develop more intelligent approaches to compensate nonsmooth nonlinear industrial characteristics, including backlash, dead-zone, hysteresis and saturation. These nonsmooth nonlinearities are common in mechanical connection, hydraulic servo valves, piezoelectric translators, electric servomotors and other industrial control systems. They often limit system performance. Control of systems with nonsmooth nonlinearities is an important area of control system research.

In this thesis, we have presented adaptive backstepping control schemes for uncertain systems with four types of nonsmooth nonlinear industrial characteristics: backlash, dead-zone, hysteresis and saturation. We have shown how these nonsmooth nonlinearities can adaptively compensated and how desired system performance is achieved.

Usually parameters of practical systems are changing with time. To meet the fast growth of adaptive control applications, we develop new backstepping methodology to time-varying nonlinear systems. Most practical systems are multi-input multi-output (MIMO) systems. For such systems, the control problem is very complicated due to the coupling among various inputs and outputs. It becomes even more difficult to deal with when there exist unknown parameters in the input or output coupling matrix. We have developed adaptive control laws for MIMO systems. In the control of a large scale system, one usually faces poor knowledge on the interactions between subsystems. If some subsystems distribute distantly, it is difficult for a centralized controller to gather feedback signals from these subsystems. Also the design and implementation of the centralized controller are complicated. Therefore decentralized controllers, designed independently for local subsystems and using local available signals for feedback, are proposed to overcome such problems. The resulting decentralized controllers are also reliable in the sense that when some local controllers are out of order, the rest can still be in operation. In this thesis, a new scheme is proposed to address an output feedback control problem: decentralized adaptive stabilization of a class of interconnected subsystems.

• We begin by considering adaptive control of systems with unknown backlash. Two types of robust adaptive backstepping control algorithms: state feedback control of a class of nonlinear system with unknown backlash and output feedback control of a class of linear system with unknown backlash. For state feedback control two backstepping adaptive controller design schemes are developed. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. The developed backstepping controls do not require the model parameters within known intervals and the knowledge on the bound of 'disturbance-like' term is not required. Besides showing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters. For output feedback control an adaptive control scheme with certain modifications to the existing backstepping control design is proposed to achieve tracking. For the implementation of the controller, no a priori knowledge on the bounds of all the unknown parameters and the hysteresis effect is required.

It is shown that adaptive control system is global stable in the sense that all the signals are bounded.

• We presents an output feedback backstepping adaptive controller design scheme for a class of uncertain nonlinear SISO system preceded by uncertain dead-zone actuator nonlinearity. We propose a new smooth adaptive inverse to compensate the effect of the unknown dead-zone. Such an inverse can avoid possible chattering phenomenon which may be caused by nonsmooth inverse. The inverse function is employed in the backstepping controller design. For the design and implementation of the controller, no knowledge is assumed on the unknown system parameters and nonlinearity. Besides showing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters. Simulation results illustrates the effectiveness of our schemes.

• We addres a new scheme to design adaptive backstepping controller for a class of uncertain nonlinear systems in the presence of input saturation. A new control law has been proposed to compensate the effect of the saturation nonlinearity using backstepping technique. The developed backstepping controls do not require the model parameters within known intervals. Besides showing global stability, we also give an explicit bound on the performance of the tracking error in terms of design parameters. Simulation results illustrate the effectiveness of our proposed scheme. Also improvement of system performance over a backstepping adaptive controller designed without considering saturation is observed.

• We develop two backstepping adaptive controllers for a second-order uncertain building structural system found in base isolation scheme for seismic active protection of building structures. The hysteretic nonlinear behavior is described by the so-called Bouc-Wen model. The developed backstepping controls do not require the model parameters within known intervals.In the first scheme, the partial effect of the hysteresis is treated as a bounded disturbance. In the second scheme, we further take the structure of the hysteresis into account in our controller design. It is shown that the proposed controllers can guarantee global uniform ultimate bounded and achieve tracking to a desired precision. Numerical results show that the adaptive control law is working satisfactorily in that the response induced by seismic action is significant reduced.

• A new scheme is designed for single-input single-output uncertain time-varying nonlinear systems with unknown sign of high-frequency gains in the presence of unknown disturbances. To deal with the time variation problem, an estimator is used to estimate the bound of the variation rates. Furthermore, the overparameterization problem is also solved by using the concept of tuning functions. It is shown that the controller obtained by the proposed design scheme can make the whole adaptive control system stable.

• A new scheme is developed for a class of multi-input multi-output system in the presence of unknown disturbances. With our scheme, a completely control solution to disturbance rejection is solved. In our design, the signs of the high frequency gains are known. To handle the disturbances and unknown parameters, we introduce new filters for state estimation and employ the internal model. As the parameters of the exosystem that generates external disturbances are unknown, an adaptive version of the internal model is proposed. Thus our estimator identifies the unknown parameters in both the system and the exosystem. It is shown that all closed-loop signals are bounded and the tracking error converges to zero.

• We address an output feedback control problem: decentralized adaptive stabilization of a class of interconnected subsystems with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity. The controllers are designed by using backstepping technique. A new robust control law and a new estimator to estimate the unknown parameters are derived. Also the interactions between subsystems are allowed to satisfying a nonlinear bound. For the implementation of the controller, no knowledge is assumed on the bounds of unknown system parameters and the effect contributed by the hysteresis. It is shown that adaptive control system is global stable in the sense that all the signals are bounded. Also a root mean square type of bound is obtained for the system states as a function of the design parameters. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters. As these bounds can give quantification of the system transient performance, thus the transient performance is adjustable by choosing suitable parameters.

10.2 Recommendations for Further Research

In this section, we present some research directions which can be pursued in the future.

(1) The adaptive backstepping methodology studied in our work has obtained some basic and theoretical results. Our proposed schemes can be applied to more general systems or plants and obtain a complete and systematic theory or scheme. From the standpoint of the theory, proofs of general system signal boundedness should be explored, such as a car-like mobile robot.

(2) It is an interest work of systematic treatment for actuator and sensor nonlinearities, illustrations of system performance improvement and application to practical systems, such as mechanical, hydraulic, magnetic, and biological of system components. Focusing on dead-zone, backlash, hysteresis and saturation, it will be shown how nonlinear industrial characteristics can be adaptively compensated and how desired system performance can be achieved in the presence of such nonlinearities. In many applications their approach avoids the need for costly and specialized hardware. In our approaches we have chosen linear and nonlinear model with a sufficient number of adjustable parameters which provide significant flexibility in

matching real situations. This flexibility will be exploited for control schemes with practical systems.

(3) In Chapter 3, to further improve system performance such as the tracking error, especially in the case without using sign functions, it is worthy to take the system hysteresis into account in the controller design, instead of only considering its effect like bounded disturbances. The first step of achieving this is perhaps to obtain an efficient adaptive hysteresis inverse which is still unclear and currently under investigation

(4) In Chapter 4, the disturbance bound D is estimated as each step of backstepping, therefor there are several estimates for the same bound, which is so-called over-parametrization. It would be interesting to eliminate the estimation of the disturbance bound D at each step of backstepping, just leaving it in the last step.

(5) In chapter 5, even though the boundedness of control v is guaranteed, whether its bound is confined inside the saturation limit is unclear and still under investigation. It would be interesting to study the effect of the introduced input compensation system on the system performance, to specify the conditions under which the desired system performance is ensured. In this case, the specification of the desired performance is also an interesting issue, in the presence of system input saturation. Input constraints, such as input amplitude saturation constraints and input rate saturation constraints, represent the most encountered nonlinearities in real-world control problems. In this chapter, we only address the case on the adaptive control of input amplitude saturation. The case of the rate constrained adaptive control or both of them deserves further investigation.

(6) As we know, time delay exists in most practical systems and the presence of time delay has a significant influence on system performance. In the face of uncertainties in the delay and other parameters, the controller design is rather

challenging and the techniques developed in this thesis need to be extended to study systems with time delay. Friction is another type of nonlinearities in practical systems. It would be interesting to systematically study adaptive designs for friction nonlinearity.

(7) In Chapter 9, decentralized adaptive stabilization of a set of interconnected dynamic systems is studied. How to use other techniques developed in the previous chapters to further study this decentralized adaptive control problem would be an interesting further research topic.

Appendices

Appendix A

Lyapunov Stability[30]

For all control systems and adaptive control systems in particular, stability is the primary requirement. Consider the time-varying system

$$\dot{x} = f(x, t) \tag{A.1}$$

where $x \in R^n$, and $f: \; R^n \times R_+ \to R^n$ is piecewise continuous in t and locally Lipschiz in x. The solution of (A.1) which starts from the point x_0 at time $t_0 \geq 0$ is denoted as $x(t; x_0, t_0)$ with $x(t_0; x_0, t_0) = x_0$. If the initial condition x_0 is perturbed to $\tilde{x}_0$, then, for stability, the resulting perturbed solution $x(t; \tilde{x}_0, t_0)$ is required to stay close to $x(t; x_0, t_0)$ for all $t \geq t_0$. In addition, for asymptotic stability, the error $x(t; \tilde{x}_0, t_0) \to x(t; x_0, t_0)$ is required to vanish as $t \to \infty$. So the solution $x(t; x_0, t_0)$ of (A.1) is

• *bounded*, if there exists a constant $B(x_0, t_0) > 0$ such that
$|x(t; x_0, t_0)| < B(x_0, t_0), \quad \forall \; t \geq t_0$;

• *stable*, if for each $\epsilon > 0$ there exists a $\delta(\epsilon, t_0) > 0$ such that
$|\tilde{x}_0 - x_0| < \delta, \quad |x(t; \tilde{x}_0, t_0) - x(t; x_0, t_0)| < \epsilon, \quad \forall \; t \geq t_0$;

• *attractive*, if there exists a $r(t_0) > 0$ and, for each $\epsilon > 0$, a $T(\epsilon, t_0) > 0$ such that
$|\tilde{x}_0 - x_0| < r, \quad |x(t; \tilde{x}_0, t_0) - x(t; x_0, t_0)| < \epsilon, \quad \forall \; t \geq t_0 + T$;

• *asymptotically stable*, if it is stable and attractive; and

• *unstable*, if it is not stable.

Theorem A.1 (Uniform Stability) Let $x = 0$ be an equilibrium point of (A.1) and $D = \{x \in R^n \mid |x| < r$. Let $V:\ D \times R^n \rightarrow R_+$ be a continuously differentiable function such that $\forall t \geq 0,\ \forall x \in D$, such that

$\gamma_1(|x|) \leq V(x,t) \leq \gamma_2(|x|)$

$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) \leq -\gamma_3(|x|)$,

The the equilibrium $x = 0$ is

- uniformly stable, if γ_1 and γ_2 are class κ functions on $[0, r)$ and $\gamma_3(.) \geq 0$ on $[0, r)$;
- uniformly asymptotically stable, if γ_1, γ_2 and γ_3 are class κ functions on $[0, r)$;
- exponentially stable, if $\gamma_i(\rho) = k_i \rho^\alpha$ on $[0, r), k_i > 0, \alpha > 0, i = 1, 2, 3$;
- globally uniformly stable, if $D = R^n$, γ_1 and γ_2 are class κ_∞ functions, and $\gamma_3(.) \geq 0$ on R_+;
- globally unifromly asymptotically stable, if $D = R^n$, γ_1 and γ_2 are class κ_∞ functions, and γ_3 is a class of κ function on R_+; and
- globally exponentially stable, if $D = R^n$ and $\gamma_i(\rho) = k_i \rho^\alpha$ on $R_+, k_i > 0, \alpha > 0, i = 1, 2, 3$.

Appendix B

LaSalle-Yoshizawa Theorem [30]

Theorem B.1 (LaSalle-Yoshizawa) Let $x = 0$ be an equilibrium point of (A.1) and suppose f is locally Lipschitz in x uniformly in t. Let $V:\ R^n \times R_+ \rightarrow R_+$ be a continuously differentiable function such that

$$\gamma_1(|x|) \leq V(x,t) \leq \gamma_2(|x|) \tag{B.1}$$

$$\dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) \leq -W(x) \leq 0 \tag{B.2}$$

$\forall\ t \geq 0,\ \forall\ x \in R^n$, where γ_1 and γ_2 are class k_∞ functions and W is a continuous function. Then, all solutions of (A.1) are globally uniformly bounded and satisfy

$$lim_{t\to\infty} W(x(t)) = 0 \tag{B.3}$$

In addition, if $w(x)$ is positive definite, then the equilibrium $x = 0$ is globally uniformly asymptotically stable.

Appendix C

Parameter Projection[30]

Defining the following convex set

$$II_\epsilon = \{\hat{\theta} \in IR^p | P(\hat{\theta}) \leq \epsilon\}, \quad II = \{\hat{\theta} \in IR^p | P(\hat{\theta}) \leq 0\} \tag{C.1}$$

which is a union of the set II and an $O(\epsilon)$-boundary layer around it. Let us denote the interior of II_ϵ by II^o and observe that $\nabla_{\hat{\theta}} P$ represents an outward normal vector at $\hat{\theta} \in \partial II_\epsilon$. The standard projection operator is

$$Proj\{\tau\} = \begin{cases} \tau & \hat{\theta} \in II^o \ or \ \nabla_{\hat{\theta}} P^t \tau \leq 0 \\ (I - c(\hat{\theta})\Gamma \frac{\nabla_{\hat{\theta}} P \nabla_{\hat{\theta}} P^T}{\nabla_{\hat{\theta}} P^T \Gamma \nabla_{\hat{\theta}} P})\tau & \hat{\theta} \in II_\epsilon / II^o \ and \ \nabla_{\hat{\theta}} P^T \tau > 0 \end{cases} \tag{C.2}$$

$$c(\hat{\theta}) = min\{1, \frac{P(\hat{\theta})}{\epsilon}\} \tag{C.3}$$

where Γ belongs to the set G of all positive definite symmetric $p \times p$ matrices. It is helpful to note that $c(\partial II_\epsilon) = 1$.

Theorem C.1 (**Projection Operator**) The following are the properties of the projection operator (C.2):

(i). The mapping $Proj$: $IR^p \times II_\epsilon \times G \to IR^p$ is locally Lipschiz in its arguments $\tau, \hat{\theta}, \Gamma$.

(ii). $Proj\{\tau\}^T \Gamma^{-1} Proj\{\tau\} \leq \tau^T \Gamma^{-1} \tau, \quad \forall \, \hat{\theta} \in II_\epsilon$.

(iii). Let $\Gamma(t), \tau(t)$ be continuously differentiable and

$\dot{\hat{\theta}} = Proj\{\tau\}, \quad \hat{\theta}(0) \in II_\epsilon.$

Then, on its domain of definition, the solution $\hat{\theta}(t)$ remains in II_ϵ.

(iv). $-\tilde{\theta}^T \Gamma^{-1} Proj\{\tau\} \leq -\tilde{\theta}^T \Gamma^{-1} \tau, \quad \forall \hat{\theta} \in II_\epsilon, \ \theta \in II.$

Appendix D

Consider w generated by an exosystem

$$\dot{w} = Sw \tag{D.1}$$

where S is an unknown matrix having distinct eigenvalues with zero real parts. Such as

$$S = \begin{bmatrix} S_1 & \dots & 0 \\ . & \dots & . \\ 0 & \dots & S_m \end{bmatrix}, \quad S_1 = \begin{bmatrix} 0 & \beta_1 \\ -\beta_1 & 0 \end{bmatrix} \ \dots \ S_m = \begin{bmatrix} 0 & \beta_m \\ -\beta_m & 0 \end{bmatrix} \tag{D.2}$$

where $w = \mathrm{col}(w_{11}, w_{12}, \dots, w_{m1}, w_{m2})$, $\beta_1, \dots, \beta_m$ are constants.

Lemma D.1 Let A be a $n \times n$ matrix having all eigenvalues with nonzero real part and S be a matrix which the eigenvalues are zero real parts and distinct as in (D.2). Let $\mathcal{P}$ denote the set of all homogeneous polynomials of degree p in $w_{11}, w_{12}, \dots, w_{m1}, w_{m2}$ with coefficients in $\mathcal{R}$. For any $q(w) \in \mathcal{P}^n$, the equation

$$\frac{\partial \pi(w)}{\partial w} Sw = A\pi(w) + q(w) \tag{D.3}$$

has a unique solution $\pi(w)$, which is an element of $\mathcal{P}^n$.

Proof. Follows the proof as in [160]. $\mathcal{P}$ is indeed a vector space over $\mathcal{R}$, of finite dimension $d(p, m)$. Set

$$X_i = w_{i1} - jw_{i2}, \quad \bar{X}_i = w_{i1} + jw_{i2} \tag{D.4}$$

and note that any $b(w) \in \mathcal{P}$ can be written as

$$b(w) = \sum_{i_1+j_1+\cdots+i_m+j_m=p} b_{i_1j_1\ldots i_mj_m} X_i^{i_1}\bar{X}_1^{j_1}\ldots X_m^{i_m}\bar{X}_m^{j_m} \tag{D.5}$$

where the $b_{i_1j_1\ldots i_mj_m}$'s are unique determined and

$$b_{i_1j_1\ldots i_mj_m} = \bar{b}_{j_1i_1\ldots j_mi_m} \tag{D.6}$$

because the coefficients of $b(w)$ are real numbers. Choose any order for the set of indices $i_1j_1\ldots i_mj_m$ and write $b(w)$ in the form

$$b(w) = BW \tag{D.7}$$

where W is $d(p,m)\times 1$ vector consisting of all products of the form the $X_i^{i_1}\bar{X}_1^{j_1}\ldots X_m^{i_m}\bar{X}_m^{j_m}$, while B is a $1\times d(p,m)$ vector consisting of the corresponding $b_{j_1i_1\ldots j_mi_m}$'s. In the notation thus established, elements $q(w)$ and $\pi(w)$ of $\mathcal{P}^n$ can be expressed in the form

$$q(w) = QW, \quad \pi(w) = \Pi W, \tag{D.8}$$

where Q and Π are $n\times d(p,m)$ matrices.

Note that

$$\frac{\partial X_i^{i_1}\bar{X}_1^{j_1}\ldots X_m^{i_m}\bar{X}_m^{j_m}}{\partial w} Sw = \lambda_{i_1j_1\ldots i_mj_m} X_i^{i_1}\bar{X}_1^{j_1}\ldots X_m^{i_m}\bar{X}_m^{j_m}, \tag{D.9}$$

where

$$\lambda_{i_1j_1\ldots i_mj_m} = j((i_1-j_1)\beta_1+\cdots+(i_m-j_m)\beta_m). \tag{D.10}$$

Thus,

$$\frac{\partial W}{\partial w} Sw = \tilde{S} W \tag{D.11}$$

where $\tilde{S}$ is a $d(p,m) \times d(p,m)$ diagonal matrix having all the eigenvalues on the imaginary axis.

In the notation introduced above, the equation (D.3) becomes

$$\Pi \tilde{S} = A \Pi W + QW \tag{D.12}$$

and this in turn reduces to the Sylvester equation

$$\Pi \tilde{S} = A \Pi + Q \tag{D.13}$$

Since the spectra of $\tilde{S}$ and A are disjoint, this equation has a unique solution Π.

$\triangleleft \triangleleft \triangleleft$

Using this property it is possible to prove the following result.

Proposition D.2 Let $F(x, u, w) = Ax + Bu + Dw$ and S as in (D.2). Assume that all matrices A_i have eigenvalues with negative real part. The the equation

$$\frac{\partial \pi(w)}{\partial w} Sw = F(\pi(w), \alpha(w), w), \quad \pi(0) = 0 \tag{D.14}$$

having a globally defined solution $\pi(w)$, whose entries are polynomials, in the components of w.

Proof. Set $\pi(w) = \Pi w$, $\alpha(w) = \Lambda w$, where Π and Λ are matrices of appropriate dimensions. Then observe that the equation

$$\frac{\partial \pi(w)}{\partial w} Sw = A\pi(w) + B\Lambda w + Dw \tag{D.15}$$

reduces to a Sylvester equation of the form

$$\Pi S = A\Pi + B\Lambda + D \tag{D.16}$$

which indeed has a unique solution Π because the spectra of S and A are disjoint. Thus according to Lemma D.1, It is easy to show the existence and uniqueness of the solution $\pi(w)$ of (D.14), whose entries are homogeneous polynomials.

$\triangleleft \triangleleft \triangleleft$

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(3) J. Zhou, C. Wen and Y. Zhang, "Adaptive Output Control of Nonlinear Systems with Uncertain Dead-zone Nonlinearity", *IEEE Transactions on Automatic Control*, accepted for publication.

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