

# A Noisy Chaotic Neural Network for Solving Combinatorial Optimization Problems: Stochastic Chaotic Simulated Annealing

Lipo Wang, Sa Li, Fuyu Tian, and Xiuju Fu

**Abstract**—Recently Chen and Aihara have demonstrated both experimentally and mathematically that their chaotic simulated annealing (CSA) has better search ability for solving combinatorial optimization problems compared to both the Hopfield-Tank approach and stochastic simulated annealing (SSA). However, CSA may not find a globally optimal solution no matter how slowly annealing is carried out, because the chaotic dynamics are completely deterministic. In contrast, SSA tends to settle down to a global optimum if the temperature is reduced sufficiently slowly. Here we combine the best features of both SSA and CSA, thereby proposing a new approach for solving optimization problems, i.e., stochastic chaotic simulated annealing, by using a noisy chaotic neural network. We show the effectiveness of this new approach with two difficult combinatorial optimization problems, i.e., a traveling salesman problem and a channel assignment problem for cellular mobile communications.

**Index Terms**—Channel assignment, chaos, combinatorial optimization, neural network.

## I. INTRODUCTION

Chaotic neural networks have a richer spectrum of dynamic behaviors, such as stable fixed points, periodic oscillations, and chaos, in comparison with *static* neural network models. Recently, there have been extensive research interests and efforts in theory and applications of chaotic neural networks (for example, see [1]–[22]).

A chaotic neural network based on a modified Nagumo–Sato neuron model was proposed by Aihara *et al.* [4] in order to explain complex dynamics observed in a biological neural system. Nozawa [5] showed that the Euler approximation of the continuous-time Hopfield neural network [23] (EA-HNN) with a negative neuronal self-coupling exhibits chaotic dynamics and that this model is equivalent to a special case of a Aihara–Takabe–Toyoda chaotic neural network [4] after a variable transformation. Nozawa further showed [5], [7] that the EA-HNN has a much better searching ability in solving the traveling salesman problem (TSP), in comparison with the original Hopfield neural network [23]–[25], the Boltzmann machine, and the Gaussian machine.

Chen and Aihara [8], [9] proposed chaotic simulated annealing (CSA) by starting with a sufficiently large negative self-coupling in the Aihara–Takabe–Toyoda network when the dynamics is chaotic, and gradually decreasing the self-coupling so that the network eventually stabilizes, thereby obtaining a transiently chaotic neural network. Their computer simulations showed that CSA leads to good solutions for the TSP much more easily compared to the Hopfield-Tank approach [23], [24] and stochastic simulated annealing (SSA) [26]. Chen and Aihara [18] offered the following theoretical explanation for the global searching ability of the chaotic neural network: its attracting set contains all global and local optima of the optimization problem under certain conditions, and since the chaotic attracting set has a fractal structure and covers only a very small fraction of the entire state space,

L. Wang is with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 and also with Institute of Information Engineering, Xiangtan University, Xiangtan, Hunan, China (e-mail: elpwang@ntu.edu.sg).

S. Li, F. Tian and X. Fu are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.

CSA is more efficient in searching for good solutions for optimization problems compared to other global search algorithms such as SSA.

It is well-known that SSA tends to find a global optimum if the annealing process is carried out sufficiently slowly [27]. Practically speaking, this implies that SSA is able to find high-quality solutions (global optima or near-global-optima), if the annealing parameter (temperature) is reduced exponentially but with a sufficiently small exponent. However, for many applications, this may mean prohibitively long relaxation time in order to find solutions of acceptable quality, and conversely, reasonably long periods of time may still result in poor solutions. In this sense, SSA searches through the solution space in a much less efficient way compared to CSA, i.e., the stochastic search in SSA covers the entire solution space, rather than a fraction of the solution space covered by the search in CSA.

Although CSA searches in an efficient manner, CSA has completely *deterministic* dynamics and is not guaranteed to settle down at a global optimum no matter how slowly the annealing parameter (the neuronal self-coupling) is reduced [14]. In practical terms, this means that CSA may sometimes be unable to provide a good solution at the end of annealing for some initial conditions of the network, no matter how slowly annealing takes place.

We attempt in this paper to combine the best of both SSA and CSA, i.e., stochastic wandering and efficient chaotic searching, by adding a decaying stochastic noise in the transiently chaotic neural network of Chen and Aihara [8], [9], [18]. We thus obtain a novel method for solving a general class of combinatorial optimization problems: stochastic chaotic simulated annealing (SCSA). Next, to demonstrate the effectiveness of the proposed SCSA, we apply the proposed SCSA to solving two difficult combinatorial optimization problems, i.e., a TSP and a channel assignment problem (CAP) for cellular mobile communications. Our simulation results show that SCSA leads to remarkable improvements over CSA.

This paper is organized as follows. Section II formulates SCSA. Sections III and IV present applications of SCSA to a TSP and a CAP, respectively, including problem statements, mappings of the problems onto chaotic neural networks, and results of computer simulations. Section V concludes the paper.

## II. SCSA

By adding decaying stochastic noise into Chen and Aihara’s transiently chaotic neural network [8], [9], [18], we propose SCSA as follows:

$$x_{ij}(t) = \frac{1}{1 + e^{-\frac{y_{ij}(t)}{\epsilon}}} \quad (1)$$

$$y_{ij}(t+1) = ky_{ij}(t) + \alpha \left( \sum_{k,l=1,k,l \neq i,j}^n w_{ijkl} x_{kl}(t) + I_{ij} \right) - z(t)(x_{ij}(t) - I_0) + n(t) \quad (2)$$

$$z(t+1) = (1 - \beta_1)z(t), \quad i, j, k, l = 1, \dots, n \quad (3)$$

$$A[n(t+1)] = (1 - \beta_2)A[n(t)] \quad (4)$$

where the variables are

$x_{ij}$	output of neuron $ij$ ;
$y_{ij}$	internal state of neuron $ij$ ;
$I_{ij}$	input bias of neuron $ij$ ;
$k$	damping factor of the nerve membrane ( $0 \leq k \leq 1$ );
$\alpha$	positive scaling parameter for the inputs;
$z(t)$	self-feedback neuronal connection weight or refractory strength ( $z(t) \geq 0$ );

- $\beta_1, \beta_2$  damping factors for the time-dependent neuronal self coupling and the added random noise, respectively ( $0 \leq \beta_1 \leq 1, 0 \leq \beta_2 \leq 1$ );
- $I_0$  positive parameter;
- $\varepsilon$  steepness parameter of the neuronal output function ( $\varepsilon > 0$ );
- $n(t)$  random noise injected into the neurons, with its actual value being in the range  $[-A, A]$  and with a uniform distribution, where  $A[n]$  is the noise amplitude;
- $w_{ijkl}$  connection weight from neuron  $kl$  to neuron  $ij$ .
- The connection weights can be obtained from

$$\sum_{k,l=1,k,l \neq i,j}^n w_{ijkl} x_{kl} + I_{ij} = -\frac{\partial E}{\partial x_{ij}} \quad (5)$$

where  $E$  is the energy function of the network or the cost function to be minimized in a give combinatorial optimization problem. In addition, the connection weights satisfy  $w_{ijkl} = w_{klij}$  and  $w_{ijij} = 0$ . In the absence of noise, i.e.,  $n(t) = 0$ , for all  $t$ , SCSA as proposed in (1)–(5) reduces to CSA of Chen and Aihara [8], [9], [18].

Furthermore, in the absence of noise and damping of the self-neuronal coupling, i.e.,  $n(t) = 0$ , for all  $t$ , and  $\beta_1 = 0$ , (1)–(5) become the Aihara–Takabe–Toyoda chaotic neural network [4], which is known to have a variety of different dynamical behaviors, including stable fixed points, periodic oscillations, and chaos, depending on the values of the network parameters.

### III. SOLVING THE TSP USING SCSA

The TSP is a classical combinatorial optimization problem. The goal of the TSP is to find the shortest route through  $n$  cities, visiting each city once and only once, and returning to the starting point. Since Hopfield and Tank [18] first applied their neural networks to solving the TSP, the TSP is often used as a benchmarking problem for testing neural network approaches to solving combinatorial optimization problems.

Hopfield and Tank [24] mapped the solution of an  $n$ -city TSP to a network with  $n \times n$  neurons.  $x_{ij} = 1$  represents the fact that city  $i$  is visited in visiting order  $j$ , whereas  $x_{ij} = 0$  represents that city  $i$  is not visited in visiting order  $j$ . The energy function to be minimized consists of two parts, as follows:

$$E = \frac{W_1}{2} \left\{ \sum_{i=1}^n \left( \sum_{j=1}^n x_{ij} - 1 \right)^2 + \sum_{j=1}^n \left( \sum_{i=1}^n x_{ij} - 1 \right)^2 \right\} + \frac{W_2}{2} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n (x_{kj+1} + x_{kj-1}) x_{ij} d_{ik} \quad (6)$$

where  $x_{i0} = x_{in}$  and  $x_{in+1} = x_{i1}$ .  $d_{ij}$  is the distance between city  $i$  and city  $j$ . The first two terms in (6) (inside  $\{\}$ ) represent the constraints, i.e., one and only one  $x_{ij}$  is 1 for each  $j$ , and one and only one  $x_{ij}$  is 1 for each  $i$  (each city is visited once and only once). The last term in (6) (without the coefficient  $W_2$ ) represents the total length of the tour. Coefficients  $W_1$  and  $W_2$  reflect the relative strength of the constraint and the tour length terms. Thus, a global minimum of  $E$  represents a *shortest valid* tour.

We note that Hopfield and Tank’s prescription of mapping the TSP onto a neural network as described above (6) is *not* the most effective way for solving the TSP using either neural networks or chaotic dynamics. Because of the need for  $n^2$  neurons for an  $n$ -city TSP, the size of the TSP that can be handled by this prescription is limited. Other prescriptions specially tailored for the TSP can significantly increase the size of the TSP that can be handled (e.g., [22]). In this paper, we shall not attempt to adopt other mapping prescriptions in order to solve

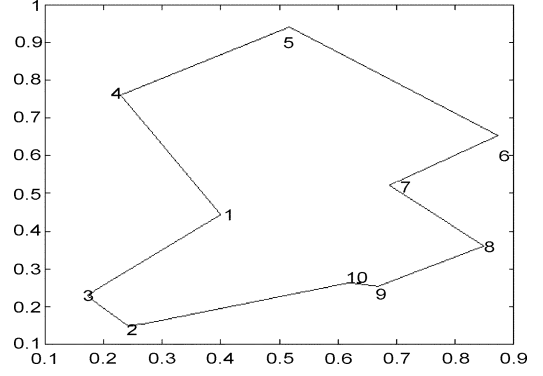


Fig. 1. The optimal tour of the Hopfield-Tank 10 city TSP.

larger TSP’s. Rather, the purpose of the present work is to demonstrate the improved solving ability of SCSA over CSA for a given objective function. In other words, neither the 10-city and the 21-city nor 52-city and 70-city TSP studied below may be considered difficult; however, *finding the global optima for the objective functions given by (6) with parameters specified below* (i.e., parameters drawn for the 10-city TSP, 21-city TSP, 52-city TSP and 70-city TSP) is indeed nontrivial and can therefore be used as benchmarking optimization problems to compare various optimization algorithms, such as CSA and SCSA. Hence, a more precise title for this Section would be “Searching for Global Minima of the Function Given by (6) Using SCSA”.

From (2), (5), and (6), we derive the dynamics of the SCSA for the TSP as follows:

$$y_{ij}(t+1) = k y_{ij}(t) - z(t) (x_{ij}(t) - I_0) + \alpha \left\{ -W_1 \left[ \sum_{l \neq j}^n x_{il}(t) + \sum_{k \neq i}^n x_{kj}(t) \right] + W_1 - W_2 \sum_{k \neq i}^n (x_{kj+1}(t) + x_{kj-1}(t)) d_{ik} \right\} + n(t). \quad (7)$$

We first minimize the function given by (6) derived from the Hopfield-Tank 10-city TSP (Fig. 1) [24] using our SCSA. To compare the performance with that of CSA, we use a set of  $k, \alpha, \beta, \varepsilon, W_1, W_2$  that are the same as Chen and Aihara’s [9]

$$k = 0.90; \varepsilon = 0.004; I_0 = 0.65; z(0) = 0.10 \\ \alpha = 0.015; \beta_1 = \beta_2 = 0.01; W_1 = W_2 = 1. \quad (8)$$

In SCSA, we choose  $A[n(0)] = 0.002$ . We repeat the simulations with 5000 different initial conditions of  $y_{ij}$  generated randomly in the region  $[-1, 1]$ . The results are summarized in Table I.

As shown in Table I, the performances of CSA and SCSA are about the same for such a relatively small problem.

Next, we minimize the energy function given by (6) with parameters drawn from a 21-city TSP [28]. The optimal tour length is known to be 2707 [28], which is the global minimum of the energy function. The distance matrix  $d_{ij}$  is given as follows (only  $d_{ij}$  with  $i \geq j$  are shown—we assume  $d_{ij} = d_{ji}$ , i.e., a symmetric TSP):

The system parameters for the noisy chaotic neural network, also shown at the bottom of the next page, are set as follows:

$$k = 0.90; \varepsilon = 0.004; I_0 = 0.5; z(0) = 0.10 \\ \alpha = 0.015; \beta_1 = 5 \times 10^{-5}; W_1 = W_2 = 1. \quad (9)$$





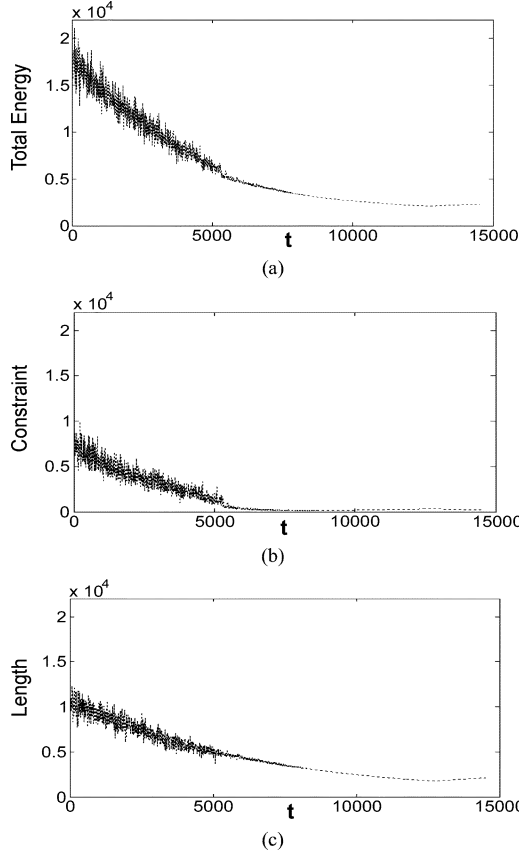


Fig. 5. Same as Fig. 3, for CSA.

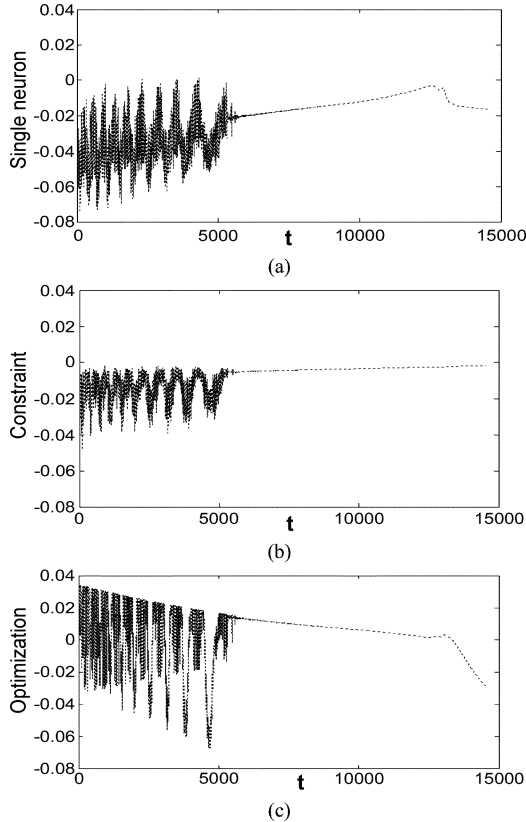


Fig. 6. Same as Fig. 4, for CSA.

To make our work more convincing, a 70-city TSP (ST70) [28] is further used. The best-known tour length listed in TSPLIB is 675. We

TABLE III  
RESULTS FOR BERLIN52 TSP USING SCSA WITH VARIOUS  $W_2$  AND 200 DIFFERENT INITIAL CONDITIONS

$W_2$	Minimum Tour Length	Average Tour Length (not include the invalid)	The number of valid (%)
1.0	7599	8570.4	175 (87.5%)
1.2	7544	7965.1	70 (35%)
1.4	7559	8002.3	105 (52.5%)
1.6	7525	7885.6	68 (34%)
1.7	7530	7914.5	39 (19.5%)
2.0	7549	7855.5	77 (38.5%)
2.5	7636	7997.1	136 (68%)

TABLE IV  
RESULTS OF CSA AND SCSA IN BERLIN52 USING VARIOUS  $\beta_2$  WITH 200 DIFFERENT INITIAL CONDITIONS IN  $W_1 = 1, W_2 = 1.6$

		Minimum Tour Length	Average Tour Length (not include the invalid)	The number of valid (%)
CSA ( $\beta_2 = 1$ )		7555	7789.3	5 (2.5%)
SCSA	$\beta_2 = 10^{-5}$	7991	8131.5	15 (7.5%)
	$\beta_2 = 7 \times 10^{-6}$	7549	7909.7	69 (34.5%)
	$\beta_2 = 3 \times 10^{-6}$	7525	7885.6	68 (34%)
	$\beta_2 = 10^{-6}$	7601	7942.4	64 (32%)

did the simulations by SCSA with 20 different randomly selected initial  $y_{ij}$  in the region  $[-1, 1]$ . The system parameters are set as same as in (15).

While fixing  $\beta_1 = 3 \times 10^{-6}$ , different performances of CSA and SCSA are compared by using various  $\beta_2$  in Table V. In Table V, SCSA not only finds the valid tours more frequently than CSA, but also obtains the minimum tour length 666 that is far better than that of CSA, 722. Here, we also provide the optimal tour of our simulation

[25 - 45 - 39 - 61 - 40 - 9 - 43 - 17 - 21 - 34 -  
12 - 33 - 62 - 54 - 48 - 67 - 11 - 56 - 65 - 64 -  
51 - 60 - 52 - 53 - 5 - 10 - 50 - 58 - 37 - 47 -  
16 - 23 - 1 - 36 - 29 - 13 - 31 - 70 - 35 - 69 -  
38 - 59 - 22 - 66 - 63 - 57 - 15 - 24 - 19 - 7 -  
2 - 4 - 18 - 6 - 41 - 42 - 32 - 3 - 8 - 26 - 55 -  
49 - 28 - 14 - 20 - 30 - 44 - 68 - 27 - 46].

The simulation results show that CSA performs as well as SCSA in the small-size TSPs, such as 10-city, 21-city. But when used on the larger size TSP's, such as 52-city and 70-city, SCSA indeed achieves a much better performance than CSA.

#### IV. SOLVING THE CAP USING SCSA

In this section, we test SCSA in another combinatorial optimization problem, i.e., the CAP in cellular mobile communications. Due to rising demand and limited frequency channels available, an effective solution to the CAP is very important to the telecommunications

TABLE V  
RESULTS OF CSA AND SCSA IN ST70 USING VARIOUS  $\beta_2$  WITH 20  
DIFFERENT INITIAL CONDITIONS

		Minimum Tour Length	Average Tour Length (not include the invalid)	The number of valid (%)
CSA ( $\beta_2 = 1$ )		722	722	1 (0.5%)
SCSA	$\beta_2 = 10^{-5}$	--	--	0 (0%)
	$\beta_2 = 7 \times 10^{-6}$	--	--	0 (0%)
	$\beta_2 = 3 \times 10^{-6}$	668	677.5	4 (20%)
	$\beta_2 = 10^{-6}$	666	686.1	7 (35%)

industry and many excellent results have been obtained using different algorithms (e.g. [29]–[35]).

CAP's are often divided into two categories, i.e., CAP1 and CAP2 [34]. CAP1 is to minimize the span of channels subject to demand and interference-free constraints. CAP2 is to minimize interference subject to demand constraints. In this paper, we are concerned with only CAP2 because it is more useful in practical cases compared to CAP1 due to limited frequency channels available and high demands in mobile communications.

Suppose a mobile radio network has  $N$  cells and  $M$  frequency channels available. The number of calls in cell  $i$  is  $D_i$ . The constraints specify the minimum distance in the frequency domain by which two calls must be separated in order to guarantee an acceptably low signal/interference ratio in each cell. These minimum distances are stored in an  $N \times N$  symmetric *compatibility matrix*  $C$ , i.e.,  $C_{ij}$  is the required separation in frequency channels between a call in cell  $i$  and another call in cell  $j$  for the two calls to have no interference with each other.

Following Smith and Palaniswami [34], we map the CAP2 onto a neural network with  $N \times M$  neurons. Assume  $x_{jk}$  the output of neuron  $jk$  and

$$x_{jk} = \begin{cases} 1, & \text{if cell } j \text{ is assigned to channel } k \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

for  $j = 1, \dots, N$  and  $k = 1, \dots, M$ .

If  $x_{jk} = x_{il} = 1$ , i.e., cell  $j$  is assigned to channel  $k$  and at the same time, cell  $i$  is assigned to channel  $l$ , the interference should be at its maximum when  $k = l$  and decreases until the two channels are far enough that no interference exists. For simplicity, we assume linear reduction in interference with respect to channel distance [34]. The interference caused by such assignments is therefore given by the following cost tensor  $P_{ji(m+1)}$  (where  $m = |l - k|$  is the distance in the channel domain between channels  $k$  and  $l$ ):

$$P_{ji(m+1)} = \max(0, P_{jim} - 1), \quad \forall m = 1, \dots, M - 1 \quad (17)$$

$$P_{ji1} = C_{ji}, \quad \forall j, i \neq j \quad (18)$$

$$P_{jj1} = 0, \quad \forall j. \quad (19)$$

Then the CAP2 can be formulated to minimize the following cost:  
minimize

$$f(x) = \sum_{j=1}^N \sum_{k=1}^M x_{jk} \sum_{i=1}^N \sum_{l=1}^M P_{ji(|k-l|+1)} x_{il} \quad (20)$$

subject to

$$\sum_{k=1}^M x_{jk} = D_j, \quad \forall j = 1, \dots, N \quad (21)$$

$$x_{jk} \in \{0, 1\}, \quad \forall j = 1, \dots, N, \quad \forall k = 1, \dots, M \quad (22)$$

where  $f(x)$  is the total interference and  $x \equiv \{x_{jk}\}$ .

TABLE VI  
TOTAL INTERFERENCE OBTAINED FROM SSA, CSA, AND SCSA  
FOR VARIOUS CAP2S

	SSA		CSA		SCSA	
	Ave	Min	Ave	Min	Ave	Min
HEX1	0.0	0	0.0	0	<b>0.0</b>	<b>0</b>
HEX2	0.1	0	0.0	0	<b>0.0</b>	<b>0</b>
HEX1	50.7	49	48.1	47	<b>47.1</b>	<b>46</b>
HEX2	20.4	19	18.9	18	<b>17.5</b>	<b>17</b>
HEX3	82.9	79	77.1	76	<b>76.8</b>	<b>74</b>
HEX4	21.0	17	17.7	17	<b>16.4</b>	<b>16</b>

With (20) and (21), the following computational energy may be defined as a sum of the total interferences and constraints:

$$E = \frac{W_1}{2} \sum_{j=1}^N \left( \sum_{k=1}^M x_{jk} - D_j \right)^2 + \frac{W_2}{2} \sum_{j=1}^N \sum_{k=1}^M x_{jk} \sum_{i=1}^N \sum_{l=1}^M P_{ji(|k-l|+1)} x_{il} \quad (23)$$

which is similar to (6), in the previous section.  $W_1$  and  $W_2$  are the weighting coefficients corresponding to the constraints and severity of interferences, respectively.

Connection weight  $W_{jkil}$  between neuron  $jk$  and neuron  $il$  can be obtained similarly using (5) and (23). Thus, the network dynamics of the SCSA for the CAP is as follows:

$$y_{jk}(t+1) = k y_{jk}(t) - z(t) (x_{jk}(t) - I_0) + \alpha \left\{ -W_1 \sum_{q \neq k}^M x_{jq} + W_1 D_j - W_2 \times \sum_{p=1, p \neq j}^N \sum_{q=1, q \neq k}^M P_{j,p,|k-q|+1} x_{pq} \right\} + n(t). \quad (24)$$

Here, we use the data set of a 21-cell cellular system, i.e., HEX1 in [29], [34], to test our algorithm. In HEX1, the number of cells  $N$  is 21 and the number of available channels  $M$  is 37. The demand is  $D_1^T = (2, 6, 2, 2, 2, 2, 4, 4, 13, 19, 7, 4, 4, 7, 4, 9, 14, 7, 2, 2, 4, 2)$ , where  $A^T$  stands for the transposed matrix of matrix  $A$ . The minimum distance in frequency domain between two channels in the same cell is 2. A smaller test problem with 4 cells and 11 channels, i.e., the EX problems in Table V.

Table V shows the simulation results of the HEX and EX CAP's using SCSA and we also show the results obtained using SSA and CSA for a comparison. Each of those heuristics is run from ten different random initial conditions and an average is calculated. In Table V, "Min" means the minimum total interference (20) found during these ten times, and "Ave" is the average total interference for the ten runs. These results show that the overall interference obtained from SCSA is lower than the interference obtained from CSA, i.e., SCSA results in better channel assignments compared to CSA. The network parameters are set in Table VI.

## V. CONCLUSIONS

In this paper, we proposed a noisy chaotic neural network (NCNN) or SCSA by adding noise to Chen and Aihara's transiently chaotic neural network. Application of this noisy chaotic neural network to a TSP and a CAP showed marked improvements over CSA. In contrast to the conventional SSA, SCSA restricts the random search to a subspace

TABLE VII  
PARAMETERS FOR VARIOUS CAP2S

Problem	$k$	$\varepsilon$	$I_0$	$\beta_1, \beta_2$	$\alpha$	$Z(0)$	$A[n(0)]$	$W_1$	$W_2$
HEX1	0.9	1/250	0.65	0.0005	0.005	0.1	0.5	1.0	0.02
HEX2	0.9	1/250	0.65	0.0005	0.005	0.1	0.5	1.0	0.02
HEX1	0.9	1/250	0.05	0.0005	0.05	0.1	0.05	1.0	0.25
HEX2	0.9	1/250	0.05	0.0005	0.05	0.1	0.05	1.0	0.25
HEX3	0.9	1/250	0.05	0.0005	0.05	0.1	0.05	1.0	0.25
HEX4	0.9	1/250	0.05	0.0005	0.05	0.1	0.05	1.0	0.25

of chaotic attracting sets which is much smaller than the entire state space searched by SSA. In contrast to CSA, SCSA is not completely deterministic and continues to search after the disappearance of chaos. SCSA can be a powerful approach to solving a general class of combinatorial optimization problems and our future work will include applications of SCSA to other practical optimization problems.

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