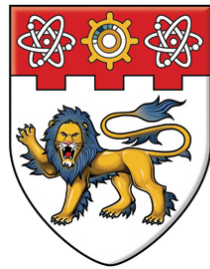


**An Integer Optimization Framework for Future
Air Traffic Flow Management**

by

Diao Xudong



**NANYANG
TECHNOLOGICAL
UNIVERSITY**

A thesis submitted to the School of Mechanical and Aerospace
Engineering in partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

at

NANYANG TECHNOLOGICAL UNIVERSITY

2016

Abstract

In this thesis, an integer optimization framework for future Air Traffic Flow Management (ATFM) is proposed. Here we address three major issues in ATFM systems: a) flexible rerouting operations, b) new airspace structure with large capacity, and c) efficient solution methodologies. All these issues focus on improving capacity and predictability of Air Traffic Management (ATM) systems while reducing various types of costs under safety considerations, which is the fundamental requirement of the future ATFM systems.

In the first part of this dissertation, we propose a binary integer optimization model to address the complex practical ATFM rerouting problem on a flight-by-flight basis. In some practical situations, the set of origin-destination (o-d) routes based on sectors must be represented by directed graphs with cycles and the advantage of our model is that it allows the existence of directed graphs with no limitations. Unrestricted directed graph is a type of general structure which allows not only complex o-d routes to be theoretically represented but also preserves global optimal solutions of the ATFM rerouting problem in some specific situations. Optimal traffic flow strategy which includes rerouting, ground-holding, airborne holding and speed control could be obtained directly for each individual flight by solving the model with commercial software. In order to improve the computational performance, we also propose two types of valid inequalities according to the model structure, and these inequalities could reduce solution time very significantly. The computational results indicate that the solution time can be controlled within 5 minutes for instances of a size which is comparable to that of the whole Southeast Asia ATM system.

In the second part, we introduce a type of new airspace structure to replace the traditional sector-based structure in order to improve the capacity and predictability of the ATFM system. The new airspace structure could ensure safety separation between flights and improve airspace capacity compared

with the traditional airspace structure. Then, we propose a new ATFM model based on the new airspace structure and in the model, operations like rerouting, ground-holding and cancellations are all considered on a flight-by-flight basis. In order to solve the model efficiently, we apply Danzig-Wolfe decomposition to decompose the original model formulation. After that, a distributed heuristic approach based on column generation is developed to generate conflict-free trajectories for each flight under airway entrance capacity constraints. By using commercial optimization software, integer solutions of good quality could be obtained in 20 minutes for the ATFM rerouting problems of the whole Southeast Asia region.

Acknowledgement

Four years ago, I came to Nanyang Technological University (NTU) to start an exciting academic research journey in Singapore. Now, I could complete my endeavor of earning a Ph.D. I would like to express my deepest gratitude to my supervisors Prof. Chen Chun-Hsien, Prof. Chen Songlin and Prof. Mao Jianfeng for their academic instructions.

My sincere gratitude also goes to my Thesis Advisory Committee members Prof. Jitamitra Desai, Prof. Nie Xiaofeng and Prof. Su Rong. They are always willing to give me their best suggestions.

I owe a lot to my parents who have provided the best opportunities for my life. They have fully stood by me during my tough childhood and always encourage me to behave like a man when there are difficulties. Last but not least, I would like to express my gratitude to NTU for offering me a chance to pursue my academic dream.

Contents

List of Figures	10
List of Tables	11
1 Introduction	13
1.1 Background	13
1.2 Air Traffic Management (ATM)	14
1.2.1 Next Generation Air Transportation System (NextGen)	15
1.2.2 Air Traffic Flow Management (ATFM)	16
1.2.3 Air Traffic Control (ATC)	20
1.2.4 Free-Flight	23
1.2.5 Collaborative Decision Making (CDM)	24
1.3 Research Objectives	24
1.4 Thesis Outline	25
2 Literature Review	27
2.1 Air Traffic Flow Management	27
2.1.1 Ground-Holding Problem (GHP)	28
2.1.2 ATFM problems	31
2.1.3 Fairness Issues in ATFM	35
2.2 Free-Flight	36
2.3 Collaborative Decision Making (CDM)	37
2.4 Conflict Detection and Resolution	37
2.5 Other ATM Topics	39

2.6	Starting Point: The Bertsimas-Lulli-Odoni Model	40
2.7	Research Gaps	46
3	An Integer Optimization Model for Practical Air Traffic Flow Management Rerouting Problem	49
3.1	Introduction	50
3.2	Model Formulation	53
3.3	Valid Inequalities	58
3.4	Size of Formulation	60
3.5	Numerical Result	60
3.5.1	Computational Test 1	60
3.5.2	Computational Test 2	63
3.6	Conclusion	66
4	A Sequence Model for Air Traffic Flow Management Rerouting Problem	69
4.1	Introduction	69
4.2	Motivation	70
4.3	Proposal	71
4.4	The Mathematical Model	73
4.4.1	Mathematical Model	73
4.4.2	The Advantage of SATFMRP	76
4.4.3	Complexity of the SATFMRP	79
4.5	Solution Approach	80
4.5.1	Danzig-Wolfe Decomposition	81
4.5.2	Subproblem	86
4.5.3	Heuristic Methodology	89
4.6	Numerical Results	92
4.7	Conclusion	96

5 Conclusion	99
5.1 Summary	99
5.2 Directions for Future Research	101
5.2.1 Data Uncertainty	101
5.2.2 Fairness	101
5.2.3 Dynamic Airway Configurations	102

List of Figures

1.1	On-Time Performance (Data Source: Bureau of Transportation Statistics (2014))	14
1.2	Snapshot of Flights (Data Source: Flightradar24.com)	14
1.3	Aviation Infrastructure	15
1.4	ATFM Tools	17
1.5	Ground Delay Program(Vossen and Ball (2002))	18
1.6	Rerouting (Bertsimas and Patterson (2000))	19
1.7	The general structure of ATC (Belobaba et al. (2009))	20
1.8	Organization of ATC centers in the FAA	21
1.9	Structure of airspace(Belobaba et al. (2009))	22
1.10	Horizontal separation standard	22
1.11	Vertical separation standard	23
2.1	Network diagram for minimum cost flow problem (Richetta and Odoni (1993))	29
2.2	Network structure for two-echelon inventory model (Ball et al. (2003))	30
2.3	Flight routes based on sectors (Bertsimas and Patterson (1998))	32
2.4	Airport structure (Bertsimas and Patterson (2000))	32
2.5	Airway network structure (Alonso-Ayuso et al. (2012a))	34
2.6	Framework of conflict risk models in different planning levels (Netjasov (2013))	39

2.7	Acyclic Graph Structure (Bertsimas et al. (2011b))	41
3.1	Snapshot of Flights in the Southeast Asia(Data Source: Fligh- tradar24.com)	51
3.2	Routes for Rerouting	52
3.3	Graph	61
3.4	Solution Time 1	61
3.5	Solution Time 2	61
3.6	Solution Time 3	62
3.7	Solution Time 4	62
4.1	Abstract Airway Structure	72
4.2	Abstract Airway Graph	74
4.3	Airway Configurations	77
4.4	Conflict between aircraft	77
4.5	Time period 1	78
4.6	Time period 2	78
4.7	Time period 3	79
4.8	Time period 4	79
4.9	Graph Demonstration	85
4.10	Flow Chart of the Heuristic Method	93
4.11	Solution Gap	94
4.12	Solution Time	95

List of Tables

- 3.1 Solution Results 66
- 4.1 Matrix representation of the restricted master problem 85
- 4.3 Restricted master problem after iteration 1 91
- 4.4 Restricted master problem after iteration 2 91

Chapter 1

Introduction

1.1 Background

Almost everyone has experienced late arrival or cancellation of flights. No matter for what reasons, it would make us upset since our travel plan is totally disrupted and we may miss some important personal affairs. The efficient operation of air traffic management systems is of critical importance, and there is always an urgent need for advanced tools to smooth air traffic delay and avoid congestions.

With the recovering global economies, air travel demand has been increasing rapidly. The Federal Aviation Administration ([FAA \(2013\)](#)) predicted that the U.S. carrier passenger growth rate is approximated to be 2.2% annually in the next 20 years, and the passenger demand over Asia-Pacific area is estimated to increase 4.3% per year. The rapid growth of global air transportation industry has already placed an enormous strain on the current ATM systems. Airspace congestion frequently happens because adverse weather often makes airspace unflyable. The U.S. Dept. of Transportation reported that approximately 21.6% of the flights within the United States were delayed in 2014 and another 2.25% were cancelled as shown in [Figure 1.1](#). The cancellation ratio hit record highs in 2014.

[Figure 1.2](#) is a snapshot of the National Airspace System (NAS) at 12 am

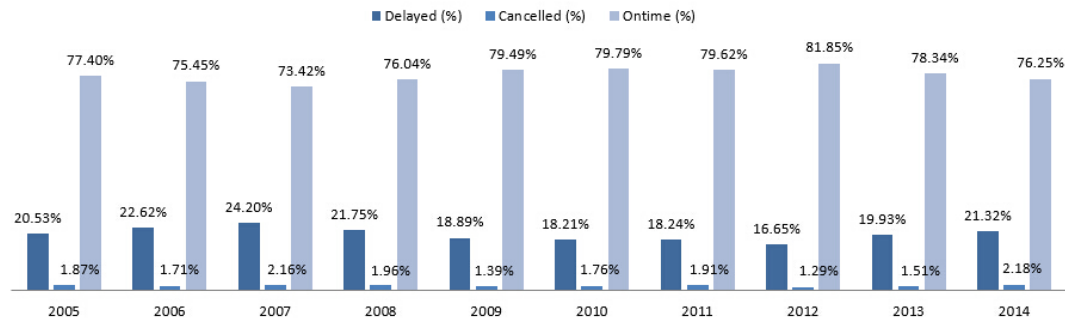


Figure 1.1: On-Time Performance (Data Source: [Bureau of Transportation Statistics \(2014\)](#))

and there were about 1500 flights which made the airspace congested. In order to improve the system capacity and predictability, The U.S. has proposed a program called the Next Generation Air Transportation System (NextGen).

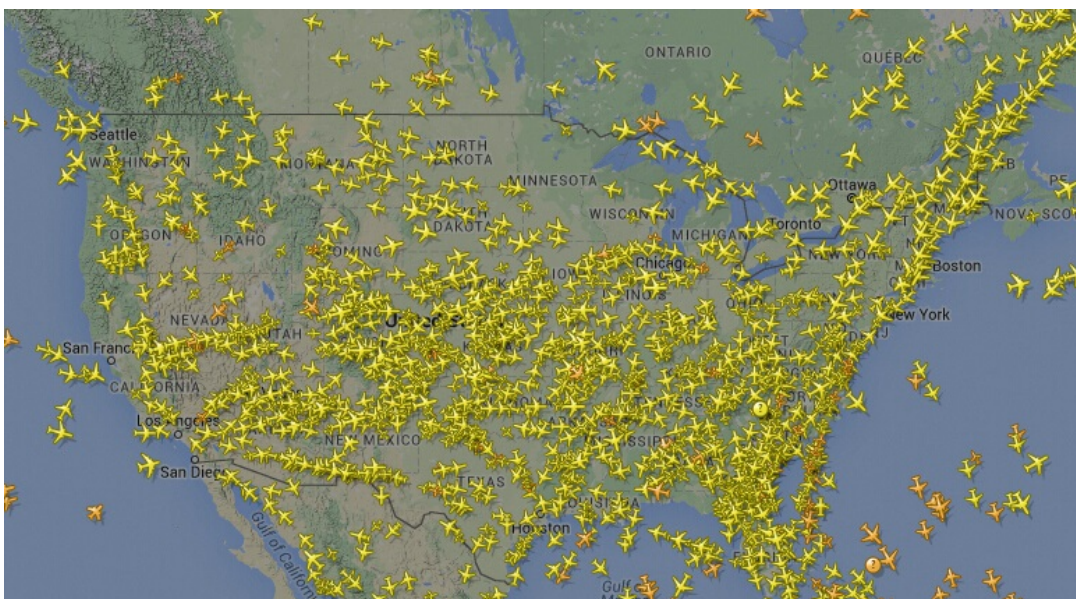


Figure 1.2: Snapshot of Flights (Data Source: [Flightradar24.com](#))

1.2 Air Traffic Management (ATM)

ATM is a type of air traffic service defined by EUROCONTROL as “The process, procedures and resources which come into play to make sure that aircraft are safely guided in the skies and on the ground.” [Barnhart et al. \(2003\)](#) presented

a framework in Figure 1.3 that ATM and airports are the principal components of the aviation infrastructure. ATM mainly includes two subsystems Air Traffic Control (ATC) and Air Traffic Flow Management (ATFM). ATFM focuses on minimizing delay cost on strategic level whereas ATC is in charge of tactical operations such as detection and resolution of conflicts.

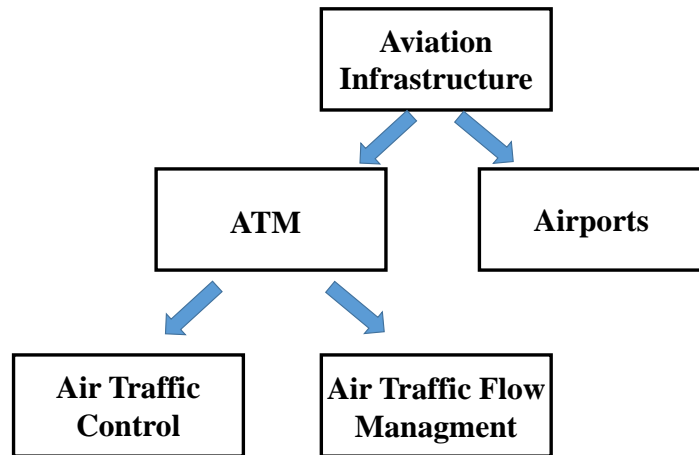


Figure 1.3: Aviation Infrastructure

1.2.1 Next Generation Air Transportation System (NextGen)

The continuous growth of air transportation industry has put huge pressure on the current ATM systems. Congestion arises almost daily and even a minor weather disturbance in busy airspace could cause a huge amount of delays over a large range of flights. Both the U.S. and the European nations have recognized the importance of upgrading ATM systems. In 2004, the European Commission launched the Single European Sky ATM Research Program (SESAR) to strengthen the European ATM system. In 2007, the Joint Planning and Development Office of the United States initiated a long-term program which is called

the Next Generation Air Transportation System (NextGen). The Motivation of NextGen is to improve ATM systems in various aspects such as terminal operations, coordination between all ATM subsystems, tracking on the airway and aircraft monitoring technology. Specifically, 4D trajectory operations are strongly proposed by NextGen since it could enhance the precision and predictability of the whole ATM system.

1.2.2 Air Traffic Flow Management (ATFM)

In order to protect ATM systems from overloading, a strategic planning activity called ATFM was introduced last century. ATFM is a control methodology which handles air traffic at an optimal flow rate while not exceeding the capacities of airports and other facilities. Recently, ATFM has become an important part of ATM with the fast growth of the air transportation industry. The current situations of ATFM in the U.S. and Europe are different: in the U.S., the limitation of ATFM system is the Airport Acceptance Rate, whereas, in Europe, sector capacity acts as the bottleneck.

There are two stakeholders in the framework of ATFM: **a)** air navigation service providers and **b)** airlines and military. Air navigation service providers take charge of the functioning of the whole ATFM system. In the U.S., the principal air navigation service provider is the FAA, and in Europe, EUROCONTROL plays the essential role to provide service. Airlines are the primary users of navigation service. The paramount objective that the air navigation service providers want to achieve is to ensure flight safety. Reducing workload for air traffic controller is the next important objective. The air navigation service providers must also consider the fairness issues in distributing resources because that affects the quality of their service.

The FAA has four effective ATFM tools as depicted in Figure 1.4 to smooth air traffic delay, and they are **Ground Delay Program, Airborne Holding**

Delay, Speed Control and Rerouting.

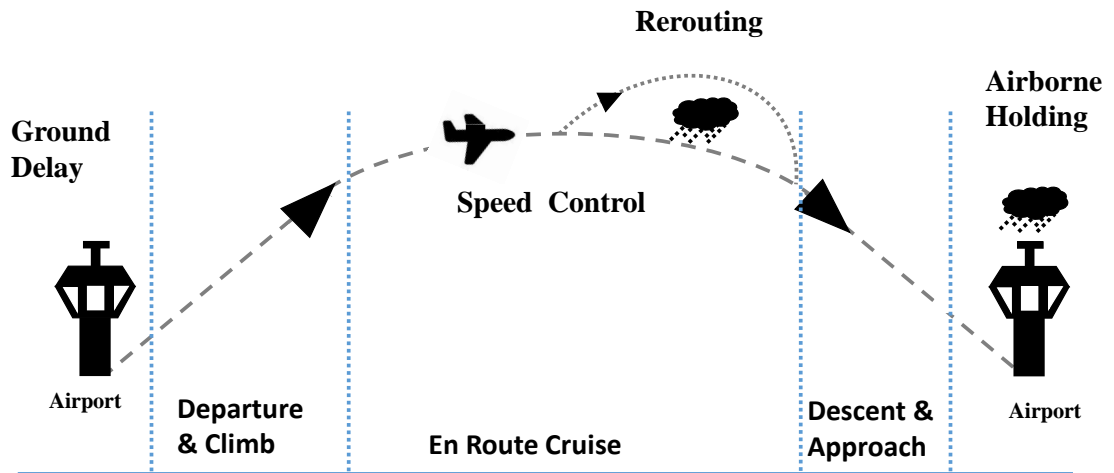


Figure 1.4: ATFM Tools

Ground Delay Program (GDP)

GDP is an ATFM methodology used to decrease the landing rate when the capacity of an airport cannot meet the projected arrival demand. GDP would be initiated on some airports when the Air Traffic Control System Command Center forecasts that there is an upcoming thunderstorm which would cause limited visibility. When GDP is initiated, certain flights would expect a delay of their scheduled departure times if their common destination airport is suffering from the adverse weather as shown in Figure 1.5. In practice, it is better for flights to experience ground-holding delays rather than to suffer airborne delays in the airspace. That's because the airborne delay is costly in safety and fuel consumption. The benefit of employing GDP is that it ensures less airborne holding cost while landing capacities are not violated at destination airports.

Airport Acceptance Rate is the principle factor in deciding whether to initiate a Ground Delay Program or not. It is obtained according to the weather

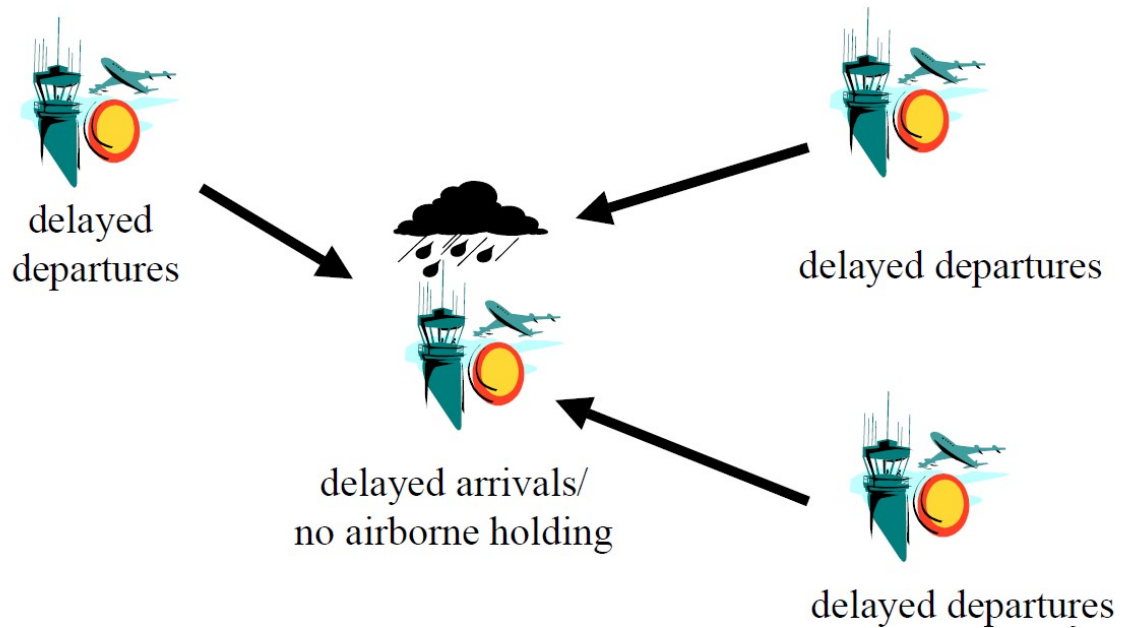


Figure 1.5: Ground Delay Program(Vossen and Ball (2002))

forecast and workload of air traffic control facilities. When landing capacity of an airport is expected to be reduced, the control facilities will decrease the Airport Acceptance Rate accordingly, and after that, GDP would be initiated by the Air Traffic Control System Command Center.

Airborne Holding Delay

In ATFM, airborne delays are allocated to flights which are in the phase of descent. It makes flights hover in the terminal airspace since the airport landing capacity is not enough to accommodate more flights. Compared with ground delay, airborne holding delay is costly.

Speed Control

Speed control is an efficient tool in controlling air traffic flow rate. By adjusting the speed of flights within a sector, mismatches in both capacity and controllers' workload could be smoothed in the connected sectors.

Rerouting

Rerouting is an ATFM tool which alters the original flight routes to avoid airspace congestion as shown in Figure 1.6. It is always employed when airspace is affected by bad weather or the air traffic control workload needs to be reduced for safety reasons. In the United States, updates of flight information which include speed, location and weather conditions could be collected by the corresponding control centers, and these centers issue rerouting commands to specific flights if necessary. Rerouting operations could improve the flexibility of ATM systems.

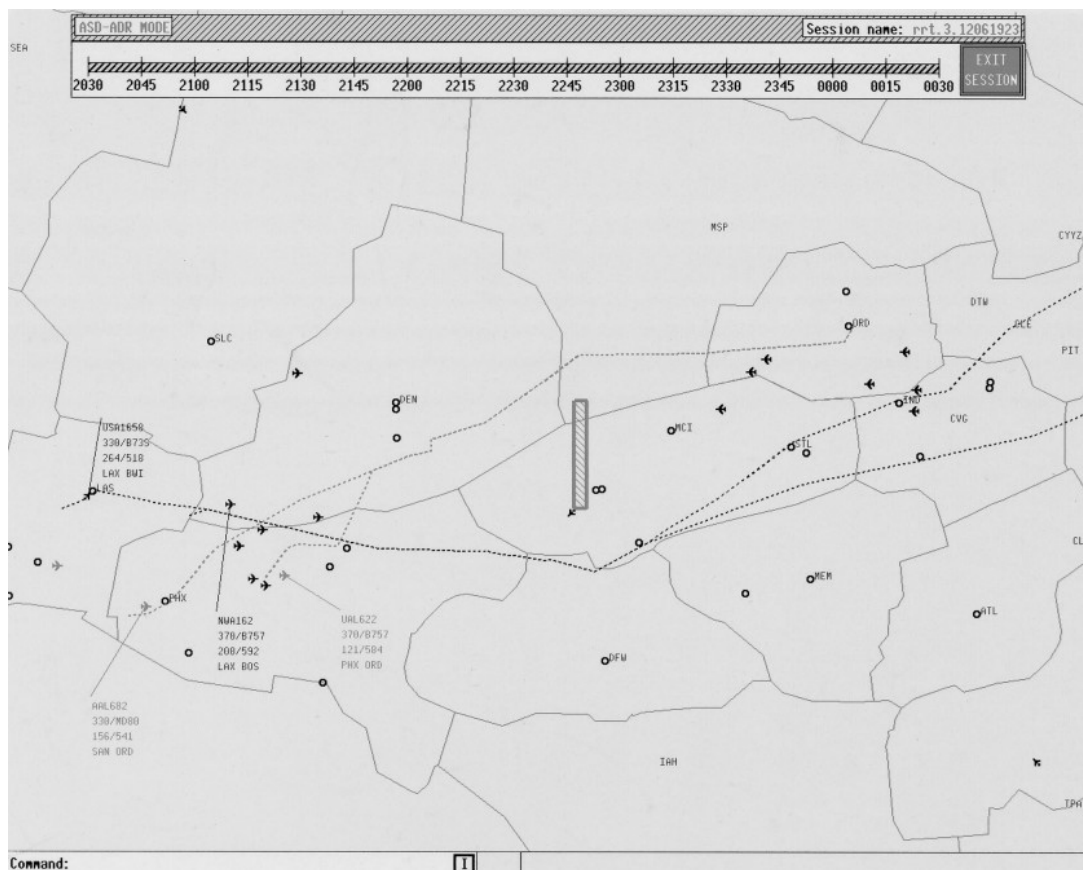


Figure 1.6: Rerouting (Bertsimas and Patterson (2000))

1.2.3 Air Traffic Control (ATC)

ATC is an important component of ATM system to guarantee the safety of flights both in airspace and on ground. The organization of a general ATC system is shown in Figure 1.7. It consists of surveillance systems, communications systems, navigation systems and other important systems. A controller obtains aircraft situations through the surveillance system and communicates with the aircraft via a communications system. The navigation system provides service for aircraft in cruise phase and updated weather data is sent to both pilots and controllers by weather information system.

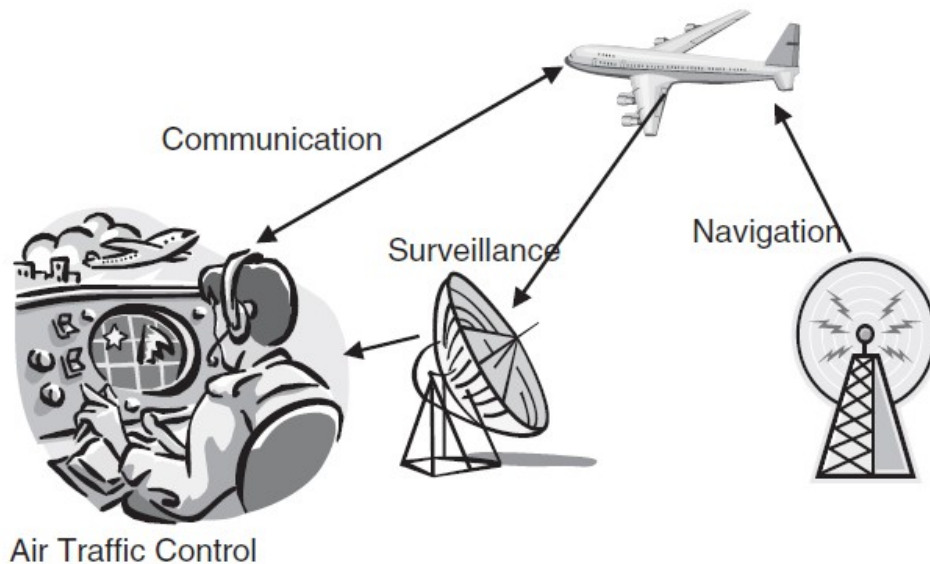


Figure 1.7: The general structure of ATC (Belobaba et al. (2009))

The FAA employs a multi-layered hierarchy of ATC centers to implement services as demonstrated in Figure 1.8. At the top of the pyramid is the central decision-making entity called Air Traffic Control System Command Center (ATCSCC). The ATCSCC executes ATFM functions on an operational and strategic level with the latest technology and equipment. The primary responsibility of ATCSCC is to monitor the current and projected demand continuously and estimate the system capacity simultaneously based on the conditions

of weather and runways. When capacity cannot meet demand, the ATCSCC would initialize control programs to resolve congestion. In the second layer of the pyramid, there are local control centers which are the so-called Air Route Traffic Control Centers (ARTCCs). ARTCCs are responsible for providing regional level ATC service for the aircraft in specific segments of airspace. ATCSCC coordinates all 22 ARTCCs which could cover the whole National Airspace System of the United States. When aircraft enters the terminal airspace, controllers in Terminal Radar Approach Control facilities (TRACONs) and Air Traffic Control Towers (ATCTs) would assume the responsibility to ensure flight safety.

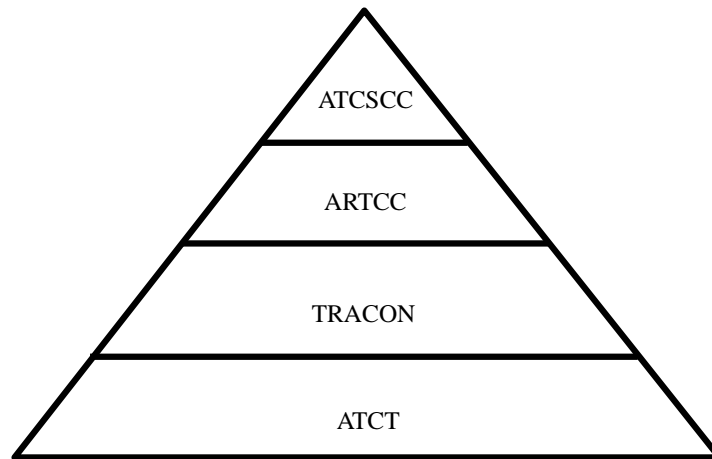


Figure 1.8: Organization of ATC centers in the FAA

In the framework of ATC, airspace consists of terminal airspace, en route airspace, tower airspace and oceanic airspace as shown in Figure 1.9. The criteria for minimum safety separation in different segments of airspace are different. For aircraft flying at the same altitude, the minimum radar separation in the terminal airspace is 3 nautical miles as depicted in Figure 1.10 and 5 nautical miles in en route airspace. This is due to the difference in the precision and frequency of terminal and en route radars. In practical situations, the minimum vertical separation is 1000 ft and 2000 ft for flights above Flight Level 290

(FL290) as shown in Figure 1.11.

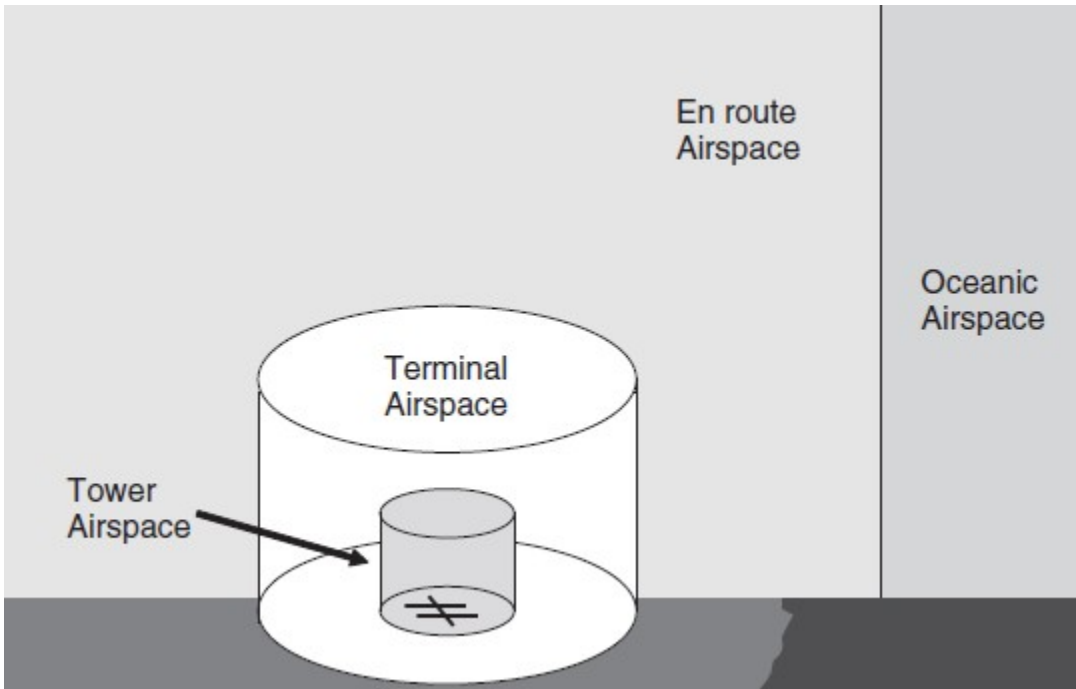


Figure 1.9: Structure of airspace(Belobaba et al. (2009))

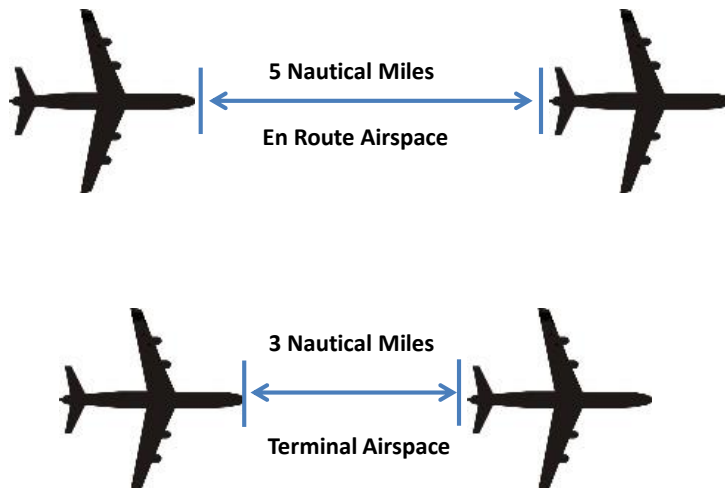


Figure 1.10: Horizontal separation standard

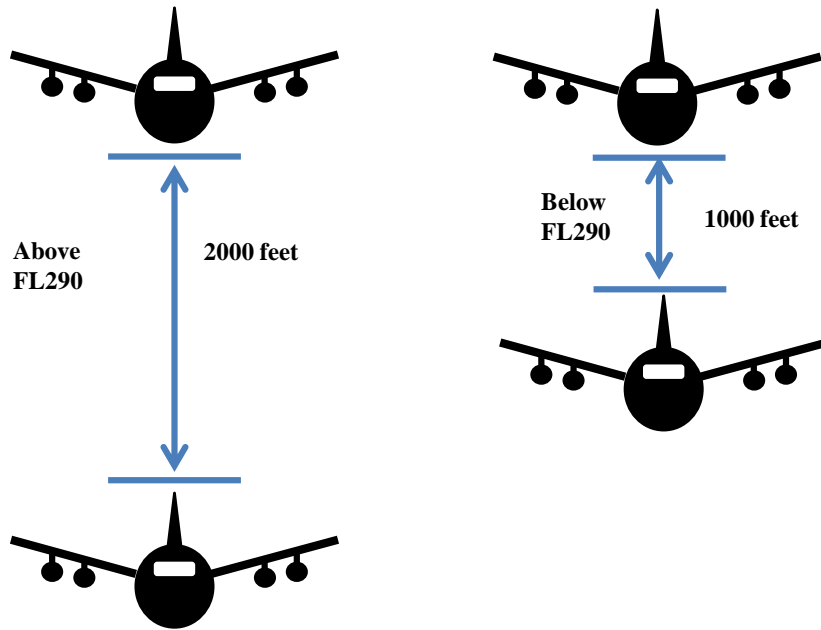


Figure 1.11: Vertical separation standard

In the future, ATC systems will integrate satellite navigation systems, surveillance systems based on ADS-B, and treat time as a control parameter for trajectory-based operations. They could also send information through system-wide information management (SWIM) while moving controllers to a more supervisory role.

1.2.4 Free-Flight

In airspace, routes for aircraft are fixed as highways on the ground and even there are no obstacles, aircraft cannot fly freely. With the development of technology, it is possible to upgrade the current routing system to a "Free-Flight" regime which enables aircraft to fly from their origins to their destinations directly. Free-flight is an innovative control methodology proposed to implement the above idea and enhance the efficiency of the National Airspace System without

compromising safety. The methodology changes the National Airspace System from a centralized control system between pilots and air traffic controller to a distributed control system, in which pilots have the flexibility to choose their favorite route and speed. Free-flight makes flight plans more flexible and economical.

1.2.5 Collaborative Decision Making (CDM)

CDM is a mechanism that the ATCSCC coordinates with airlines, airports and ATC facilities in the decision-making process. The motivation of CDM is to share information among all stakeholders and improve the efficiency of the ATM system. Recently, the FAA together with airlines has employed this mechanism to improve the Ground Delay Program. Prior to CDM, ATCSCC always got obsolete information from airlines and made bad decisions. For example, when the landing capacity of an airport was affected by bad weather, some airlines cancelled flights by themselves, but the cancellations would not be shared with ATCSCC. In such situations, ATCSCC overestimated the extent of the projected demand, and unnecessary commands which put restrictions on aviation system would be issued.

Nowadays, a variety of information-sharing techniques have been employed by CDM such as web-based flight schedule monitor and periodic teleconferences with the airlines. At the same time, lots of advanced mathematical models have been proposed in the framework of CDM in order to address the fairness issues in allocating the limited resources among airlines and other airspace users.

1.3 Research Objectives

Even there are lots of efficient decision tools, the ATFM systems are still under tremendous pressure for the continuous growth of air transportation industry. It is impossible for the current ATFM systems to meet the huge demand in

the future. In this thesis, we address three key issues for the ATFM systems: a) flexible rerouting operations, b) new airspace structure with large capacity, and c) effective solution methodologies. All these issues focus on improving precision and predictability of the whole system while reducing various types of costs under safety considerations, which is the fundamental requirement of the future ATFM systems.

1.4 Thesis Outline

The rest of dissertation includes 4 chapters as follows. Chapter 2 provides a literature review of the existed optimization models for ATM systems. Chapter 3 - 4 provides a framework of optimization models and algorithms for ATFM to improve system capacity and predictability under safety considerations. The model in chapter 3 is for traditional ATFM system and it extends the rerouting strategy based on practical situations. Chapter 4 provides an approach to improving the ATFM systems from a 4D trajectory-based perspective. Finally, several future research directions are presented in Chapter 5.

Chapter 2

Literature Review

2.1 Air Traffic Flow Management

[Agustin et al. \(2010\)](#) classified the existed ATFM problems into five methodologies: Single-Airport Ground-Holding Problem (SAGHP), Multi-Airport Ground-Holding Problem (MAGHP), Air Traffic Flow Management Problem (ATFMP), Air Traffic Flow Management Rerouting Problem (ATFMRP) and Air Traffic Flow Management Rerouting Problem (ATFMRP) under uncertainty. All these methodologies are developed to minimize delay cost on the strategic level. SAGHP is the simplest problem to find the optimal planning for a landing airport. The methodology of MAGHP takes into account the effect of delay propagation between multiple landing airports. ATFMP addresses practical situations in which the capacities of sectors are limited. ATFMRP considers more realistic situations that flights can change their original route to alternative ones in order to avoid congestions. ATFMRP is very sensitive to the fluctuations of system capacity, and ATFMRP under uncertainty could overcome the limitation.

2.1.1 Ground-Holding Problem (GHP)

The first model for GHP was proposed by [Odoni \(1987\)](#) and it was developed to schedule flights in order to minimize delay cost in real time. The model laid a solid foundation for the future ATFM research and soon after this work, several deterministic models were proposed to deal with the variants of GHP. Multi-Airport Ground Holding Problem (MAGHP) was first studied by [Vranas et al. \(1994b\)](#). In that work, an integer optimization model was developed to address GHP in a network of airports and the network effect was recognized as the main difference between MAGHP and SAGHP. Generally speaking, network effect is the propagation of delay in the aviation network. For example, if a flight is delayed, the connected flight performed by the same aircraft would also be delayed. Another common network effect at major hub airports is that a delayed flight may lead to additional ground-holding delays of several flights of the same airline because these flights will have to wait for those passengers on that late-arriving flight for boarding. Numerical results in their work demonstrated that if each flight has identical ground-holding cost coefficients, network effect would be of minor impact. [Andreatta and Brunetta \(1998\)](#) compared three types of deterministic MAGHP models with respect to computing time and analyzed each model's advantages and disadvantages through solution quality. [Andreatta et al. \(2000\)](#) improved the algorithm of static MAGHP and the optimal solution can be found quickly.

The stochastic model of GHP was first studied by [Terrab and Odoni \(1993\)](#) and they developed a multistage optimization model to address stochastic SAGHP. In that work, a dynamic programming approach was employed to obtain the optimal solution, but due to the huge scale of solving procedure, heuristic methods must be applied in the process of searching solutions. [Richetta and Odoni \(1993\)](#) proposed a new stochastic integer optimization model to address the static stochastic SAGHP. They adopted a type of network structure as demon-

strated in Figure 2.1 and applied the minimum cost flow algorithm which works very efficiently in computing the integer optimal solutions.

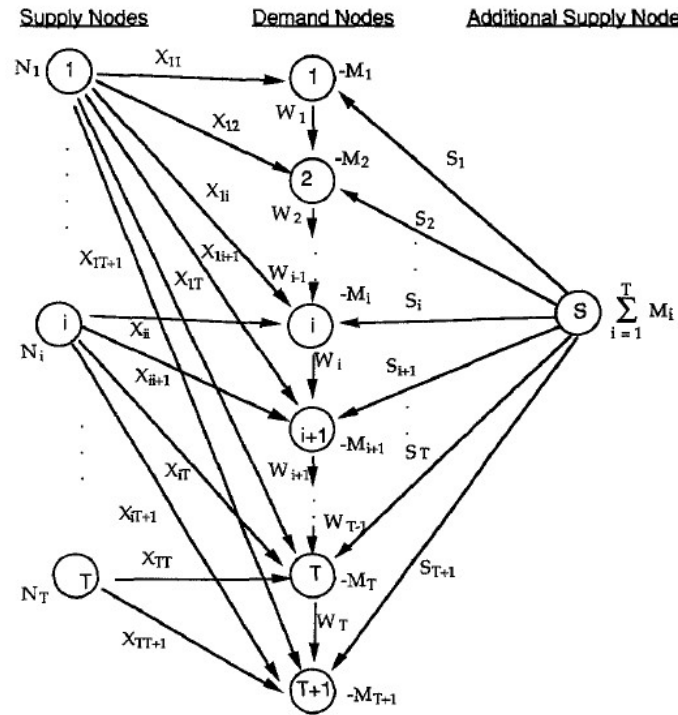


Figure 2.1: Network diagram for minimum cost flow problem (Richetta and Odoni (1993))

Since ground holding decisions are made only once at the planning horizon in static models, decisions cannot be changed according to the updated weather information after planning. Ball et al. (2003) developed a new stochastic integer optimization model with a higher level of aggregation of flights to address the static stochastic SAGHP. A two-echelon inventory model as shown in Figure 2.2 was applied in their work that the first echelon inventory is related to ground-holding flights whereas the airborne-holding flights are represented by the second echelon inventory. One amazing fact of their model is that the transpose of the constraint matrix is a network flow matrix and as a result totally unimodular, so the optimal integer solution could be reached quickly by solving the linear programming relaxation of the model.

Kotnyek and Richetta (2006) compared the above two static stochastic in-

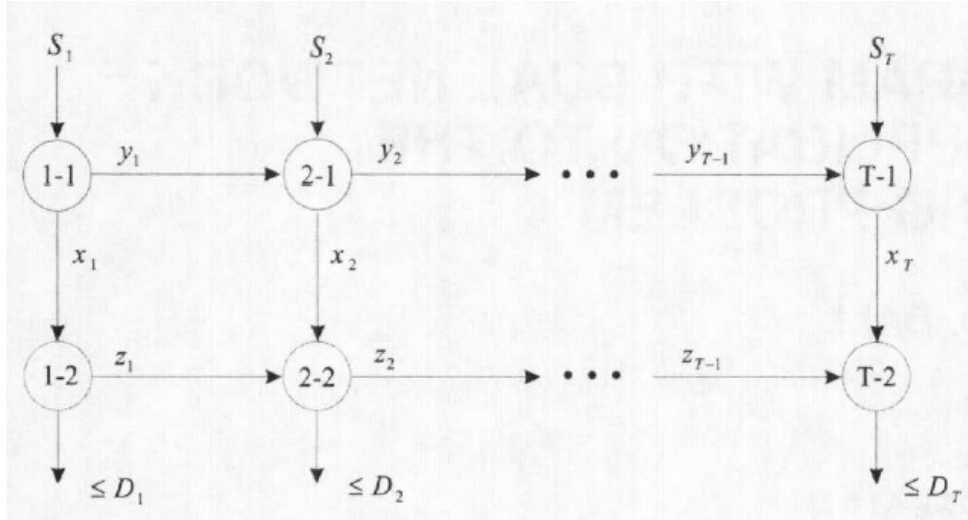


Figure 2.2: Network structure for two-echelon inventory model (Ball et al. (2003))

teger programming models and found there exists an interesting fact that if the ground holding cost function is linear, the two models are the same. Even if ground holding cost function is nonlinear and marginally increasing, the linear programming relaxation of the Richetta and Odoni (1993) 's model can also obtain integer optimal solutions.

Since static stochastic models have a limitation that revising decisions with updated information is not allowed, the scenarios of wasting capacities would always exist when the weather becomes clear after the planning horizon. In order to overcome this limitation, Richetta and Odoni (1994) proposed a multistage stochastic integer optimization model to handle dynamic SAGHP. They adopted a probabilistic binary decision tree to address the weather changing process and solved the model dynamically. One advantage of this approach is that the most up-to-date weather information could be included in the decision process once stochastic weather scenarios realized. Dynamic stochastic MAGHP was first studied by Vranas et al. (1994a). Mukherjee (2004) proposed a stochastic dynamic model for SAGHP. In their work, ground delays of non-departed flights could be revised based on updated information from branching the scenario tree, and fairness concern was also taken into account. Mukherjee and Hansen (2007)

found a limitation of [Richetta and Odoni \(1994\)](#)'s model that the decisions cannot be revised in some situations and they formulated a new dynamic stochastic model with the reformed structure to avoid these situations. [Mukherjee et al. \(2009\)](#) revised the dynamical stochastic SAGHP model and the information required by the new model is considerably less than that required by the existed models. [Glover and Ball \(2013\)](#) developed a stochastic programming model to address the efficiency and fairness issues in the stochastic SAGHP. [Mukherjee and Hansen \(2009\)](#) developed a stochastic integer optimization model to reroute flights in the vicinity of the terminal airspace dynamically.

2.1.2 ATFM problems

According to the taxonomy of [Agustin et al. \(2010\)](#), ATFM problems include **Air Traffic Flow Management Problem (ATFMP)**, **Air Traffic Flow Management Rerouting Problem (ATFMRP)** and **ATFMRP with uncertainty**. The only difference between ATFMP and ATFMRP is that in ATFMRP, flights could select the alternative routes to avoid congestion on the original route.

[Bertsimas and Patterson \(1998\)](#) proved ATFMP is NP-hard and they first proposed an integer optimization model to address multi-airport ATFMRP under airspace capacity fluctuations. In their model, they assumed that a flight could be rerouted to the alternative sectors if the original flight plan is unusable for adverse weather. However, additional new decision variables must be added to represent rerouting operations. The routes in their work were based on sectors only as shown in [Figure 2.3](#). [Bertsimas and Patterson \(2000\)](#) extended the multi-airport ATFMRP with a dynamic network flow model. In this work, they reflected the airport by 4 nodes as demonstrated in [Figure 2.4](#) and represented flights by commodities in order to implement rerouting operations. The flow management decisions were made on an aggregated number of flights to

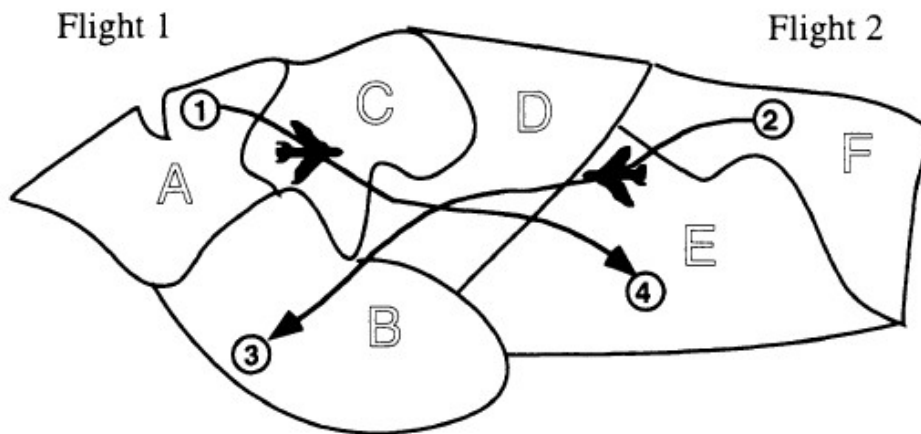


Figure 2.3: Flight routes based on sectors (Bertsimas and Patterson (1998))

minimize total delay cost. However, the computational performance is not sufficient to handle large-scale instances, and heuristic algorithm must be applied to decompose the solution into individual flight plans.

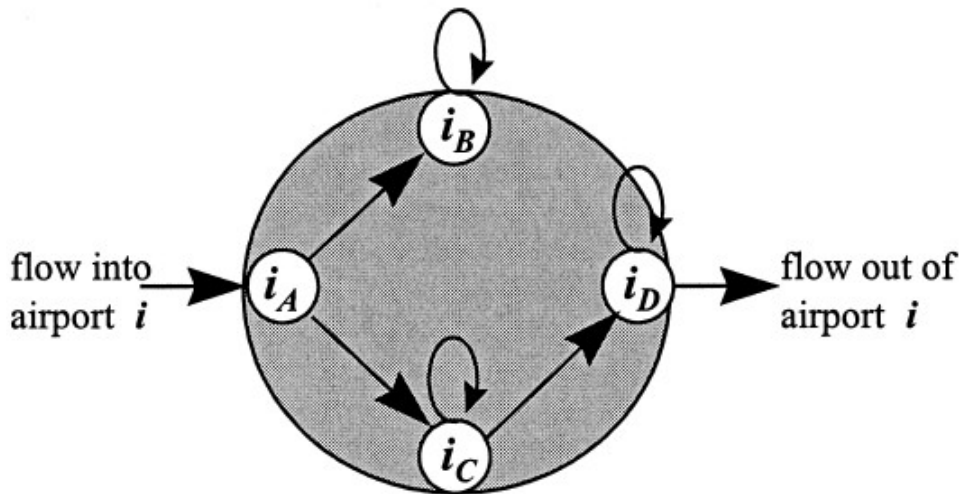


Figure 2.4: Airport structure (Bertsimas and Patterson (2000))

Bertsimas et al. (2011b) revised the existed ATFMRP model in order to handle large-scale problems efficiently. Compared with the former works, no additional variables were needed to address rerouting operations in their new model but only new constraints. Three classes of valid inequalities were also introduced to strengthen the model, and fairness issues on delay allocation were

also taken into account by using superlinear coefficients in the objective function. The model considered takeoff, cruising, and landing which covers all phases of flights. Optimal solutions including ground and airborne holding, rerouting and speed control on a flight-by-flight basis could be obtained by employing commercial software. ATFMP is computationally challenging and the reason is that there are too many sectors must be included in the problems, but most sectors are used far below their capacities. [Churchill et al. \(2009\)](#) introduced a new ATFM model based on a type of revised airspace structure which is called airspace volume. The benefit of adopting the airspace volume structure is that it could significantly reduce the computational time of sector-based ATFMP.

In Europe, sector capacity limitation is the bottleneck of the European ATM system. The ATFMP model addressing the peculiarities of European ATM systems was firstly studied by [Lulli and Odoni \(2007\)](#). The aviation network of Europe is quite complex, and there's little flexibility for rerouting operations in order to avoid congestion. A mixed integer optimization model was proposed in that work, and solutions demonstrated a counter-intuitive result that in Europe, airborne holding delay sometimes can reduce total delay cost even it is much more expensive than ground delay.

[Liang et al. \(2013\)](#) developed a sequence assignment model to pick the optimal flight sequence based on the tracks in the oceanic airspace. One distinguishing feature of the model is that it could guarantee conflict-free between flights and a branch-and-price algorithm was presented to compute the integer optimal solution under the framework of column generation.

Recently, ATFMP under uncertainty has drawn lots of academic attentions for practical reasons. Based on the work of [Bertsimas and Patterson \(1998\)](#), [Alonso et al. \(2000\)](#) proposed a multistage stochastic integer optimization model to address the weather uncertainty in ATFMP. By applying a fit-and-relax approach, the model preserved the high tightness of the deterministic model. [Andreatta et al. \(2011\)](#) developed an aggregate stochastic integer optimization

model to handle weather uncertainty in ATFMP. Several relevant issues were taken into account in their work: trade-off between airport arrivals and departures, airport capacities uncertainty and interactions between different hubs. Since the optimal solution only included the number of flights that can depart and arrive at each time period, it left airlines with flexibility to decide which flight should be delayed or cancelled. [Alonso-Ayuso et al. \(2012a,b\)](#) developed two ATFMRP models for both deterministic and stochastic capacity scenarios. In their models, the airspace structure became more detailed that rerouting decisions were based on a type of airway network as shown in Figure 2.5. Waypoints (nodes) must be added on the airways (arcs) if they cross the boundary of sectors. Another feature of their models is that there are several types of objective functions to be minimized, which are ground holding and airborne holding costs, number of flights exceeding a specific time delay, penalization of alternative routes to the original one, delay cost to arrive at each waypoints and penalization for advancing arrival over the schedule. However, conflicts between flights may happen on airways in their models.

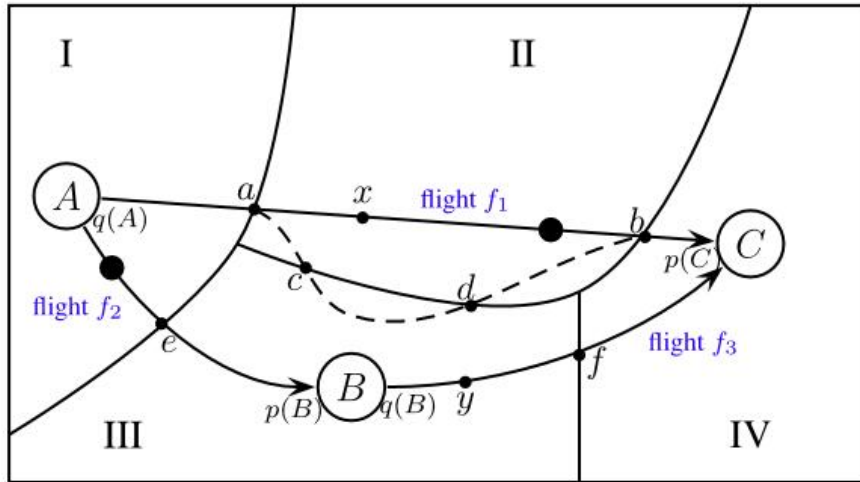


Figure 2.5: Airway network structure ([Alonso-Ayuso et al. \(2012a\)](#))

[Churchill and Lovell \(2012\)](#) presented a two-stage stochastic optimization model with the objective to allocate time slots in multiple congested resources

under capacity uncertainty. [Clare and Richards \(2012\)](#) first introduced chance constraints to address the weather uncertainty in ATFMRP. They developed two sets of chance constraints and provided a general approach. [Clarke et al. \(2012\)](#) pointed out that it is hard to implement stochastic ATFM models in practice because there are no valid algorithms mapping weather forecasts to real airspace capacity. They proposed a simulation-based method which could determine the stochastic airspace capacity quickly based on integrated weather-traffic models.

2.1.3 Fairness Issues in ATFM

[Bertsimas and Gupta \(2009\)](#) pointed out that ATFM models do not impose two essential fairness considerations: 1) the flight arrival order in the optimal solution is inconsistent with the flight ordering in the published schedules, i.e., there are a large number of pairwise reversals in the optimal solution; 2) Each airline should be taken into account as an individual stakeholder in the decision process, i.e., fairness should be considered on an airline basis. In that work, they developed an integer optimization model for ATFM to address the fairness issues on a national scale. ATFM problems on sector and airport reversals were studied by [Bertsimas and Gupta \(2015\)](#), the objective function of their model is to assign reversals fairly on an airline basis. That work could be adopted as a variant of Ration by Scheduling (RBS) policy in the Collaborative Decision Making. Nowadays, RBS is recognized as the criterion for time slot allocation in the industry because fairness can be guaranteed in the allocating process under RBS. [Ball et al. \(2010\)](#) designed a new slot allocation principle Ration-by-Distance (RBD) and compared it with RBS under stochastic weather clearance time. They proved RBD outperforms RBS in reducing total delay under some specific situations. [Manley and Sherry \(2010\)](#) compared six different rationing rules by analyzing the trade-off between passenger delays and excess surface fuel burn as well as airline equity and passenger equity. The numerical

result demonstrated that RBS is preferred when the objective is to minimize inequity among airlines and passengers. However, Ration-by-Passengers should be considered if we want to decrease total passenger delay. In the current airline industry, the accepted rules on how to allocate time slot all maintain a first-scheduled, first-served principle. [Barnhart et al. \(2012\)](#) designed a fairness metric which could measure deviation from first-scheduled, first-served criterion in situations of congestion, and more importantly, this metric can balance the tradeoff between equity and efficiency. In that work, they formulated an integer optimization model to minimize the metric directly, and the solutions numerically demonstrated this approach could save millions of dollars per year. [Castelli et al. \(2012\)](#) proposed a new approach to solving the time slots allocation problem in Europe and they also introduced a compensation mechanism for fairness. The approach maximizes the efficiency of the system by allocating slots to the preferred airlines. Numerical results demonstrated that even airlines with grandfather rights could be positively affected by using the approach and fairness mechanism.

2.2 Free-Flight

[Barnett \(2000\)](#) employed geometrical probability to assess how free-flight affects the safety issues in the context of current ATM systems, and simulation results demonstrated that revised structure of routes might reduce mid-air collision risk. [Sherali et al. \(2000\)](#) developed an airspace sector occupancy model and aircraft encounter model to estimate sector workload and potential conflict risk in the framework of free-flight. [Dell’Olmo and Lulli \(2003\)](#) formulated a two-level hierarchical mixed integer optimization model to maximize airway capacity based on free-flight.

2.3 Collaborative Decision Making (CDM)

[Hoffman et al. \(1999\)](#) reviewed the existed GHP models under CDM, and described the roles of FAA and airlines in the framework of CDM. [Vossen and Ball \(2006\)](#) designed a new slot trading mechanism that airlines could submit the so-called at-least, at-most offers for trading. [Castelli et al. \(2011\)](#) presented another slot trading mechanism between airlines based on market principles and demonstrated several good properties.

CDM could also be employed in selecting the best flight plans. [Sherali et al. \(2003, 2006\)](#) presented a large-scale mixed integer optimization model APCDM to enhance the U.S. National Airspace System in airspace planning and collaborative decision-making. The model could generate a set of good flight plans among all alternatives under fairness consideration without violating safe-separation and air traffic control workload constraints. [Sherali et al. \(2011\)](#) extended APCDM by integrating slot exchange mechanisms which allow airlines to trade their assigned slots to improve efficiency and flexibility of the whole system.

[Andersson et al. \(2003\)](#) first developed an integer optimization model named Arrival Sequencing Model to obtain the benefit of increasing collaboration between airlines and air traffic controllers in the landing process, and it significantly reduced the passengers' delay.

2.4 Conflict Detection and Resolution

In ATC, the majority of studies focus on conflict detection and resolution (CD&R). [Kuchar and Yang \(2000\)](#) presented a survey of the existed models for CD&R. The survey provided a taxonomy which includes six methodologies: dimensions of airspace (vertical, horizontal, or three-dimensional), aircraft state propagation (nominal, worst-case, or probabilistic), conflict detection threshold,

conflict resolution methodology (prescribed, optimized, force field, or manual), maneuvering methods (speed change, lateral, vertical, or combined maneuvers) and dimensions of multiple aircraft conflict resolution (pairwise or global).

[Pallottino et al. \(2002\)](#) proposed two integer optimization models to address the multi-aircraft conflict resolution problem. In the first model, velocity changes are the only admissible maneuvers, whereas the second model only allows heading angle adjustments. Optimal solutions of both models could be found within one minute in the situation that 15 aircraft cross the same waypoint at the same time. [Richards and How \(2002\)](#) developed a new paradigm for the multi-aircraft conflict resolution problem to obtain the trajectory of minimum travel time. In their work, linear constraints were generated by approximating aircraft dynamics. [Vela et al. \(2009b\)](#) formulated a two-stage stochastic optimization model to address conflict resolution problem under wind uncertainty. The optimal control plan could balance fuel consumption costs and conflict probability. [Vela et al. \(2009a\)](#) proposed a mixed integer optimization model which could change the flight-level and speed of aircraft in conflict resolution. [Alonso-Ayuso et al. \(2013\)](#) presented an integer optimization model to detect and avoid conflicts by changing speed and altitude. After that, [Alonso-Ayuso et al. \(2011\)](#) proposed a new model which could avoid false conflict detection whereas the former model cannot. [Alonso-Ayuso et al. \(2012c\)](#) developed a nonlinear integer optimization model to address the collision avoidance problem. The best strategy for an individual aircraft could be generated such that all conflicts in the airspace can be avoided. In order to solve the model in an efficient manner, a linear approximation approach was employed.

[Netjasov \(2012a,b, 2013\)](#) proposed a framework of conflict risk models in different planning levels as depicted in [Figure 2.6](#): strategic planning, tactical planning, and operational planning. The strategic planning model was intended to compare and analysis different airspace designs under air traffic flow levels. Tactical planning model was used for comparison of different flight plans for a

specific airspace sector, whereas operational planning model supported an air traffic manager in the decision-making process by the evaluation of conflict risk and air traffic controller task-load.

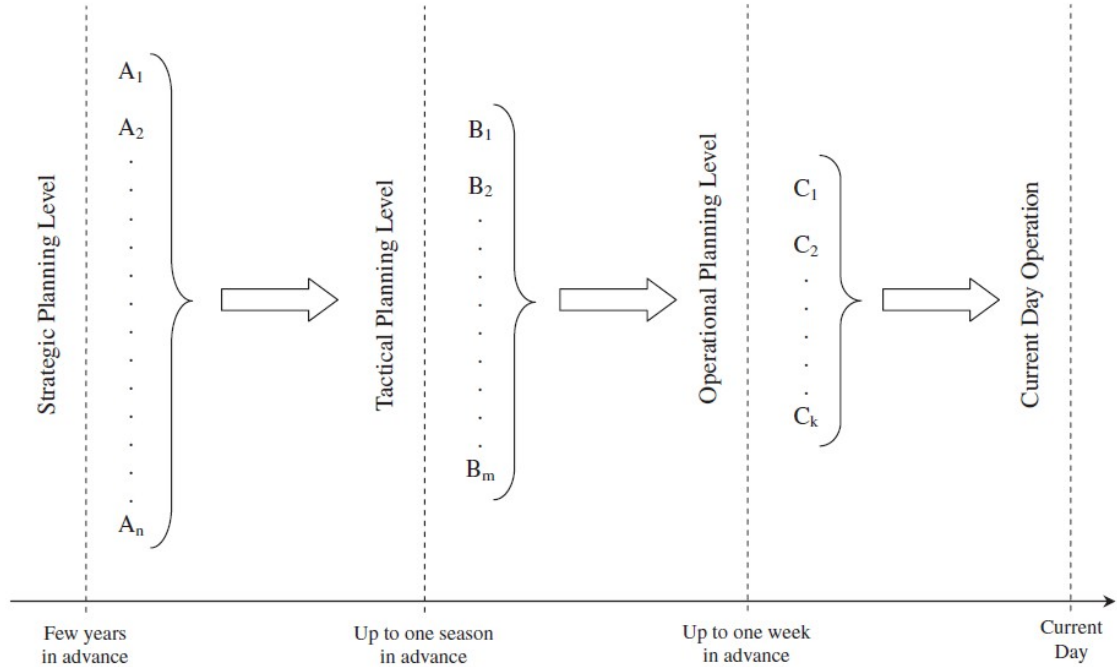


Figure 2.6: Framework of conflict risk models in different planning levels (Netjasov (2013))

In the proposals of NextGen, 4D trajectory-based operations would be implemented in ATM system to improve the capacity and predictability. Ruiz et al. (2013) found an efficient 4D trajectory CD&R algorithm to avoid conflicts in terminal areas. The numerical result demonstrated that the algorithm is quite fast, and conflict-free trajectories generated by the system were validated by a certified B738 Simulator.

2.5 Other ATM Topics

Clarke et al. (2008) proposed several methods to investigate and quantify the economic and environmental benefits of optimization tools that en-route air traffic controllers could apply. Mesgarpour et al. (2010) presented a brief review

of all aircraft landing models. [Sohoni et al. \(2011\)](#) designed two service-level metrics to guarantee the service quality of an airline. A stochastic integer optimization model was formulated to maximize service levels under the desired profitability. [Bachmat et al. \(2009\)](#) analyzed the passengers boarding process, and a clear link between boarding policies and congestion was found. [Bertsimas et al. \(2011a\)](#) presented a mixed integer optimization model to select the optimal sequence of runway configurations for an airport, and the optimal balance of arrivals and departures at each period could be determined simultaneously. [Prot et al. \(2010\)](#) applied graph theory to assess the feasibility of air traffic flow networks.

2.6 Starting Point: The Bertsimas-Lulli-Odoni Model

In this section, [Bertsimas et al. \(2011b\)](#)'s model is introduced as the starting point of all works proposed in this dissertation. We use BLO model to represent their work in the following part. One distinguishing feature of the BLO model is the revised structure of route which allows rerouting operations. The origin-destination routes for any individual flight can be represented by directed acyclic graphs. The nodes of these acyclic graphs represent airports or sectors which are the so-called capacitated elements of the airspace. The arcs of those directed graphs are not airways, and they just reflect the relationship of traveling order between those capacitated elements. If there is an arc from a node p to another node k , it means that flight could enter sector k immediately after it has traversed sector p . As demonstrated in [Figure 2.7](#), the directed graph consists of three routes between the airport of departure and arrival.

One strong assumption in BLO model is that the directed graphs of origin-destination routes must be acyclic. The relationship between the nodes of di-

rected acyclic graphs could be easily described by binary operations, and the set of routes could be mathematically associated with partially ordered sets. In these partially ordered sets, airports of departure are the minimum elements whereas the destination airports are the maximum elements, respectively.

In a view to make sure that each flight cannot select more than one route, BLO model introduces a new methodology to describe the travelling orders based on local relations among sectors. These relations are demonstrated in Figure 2.7. If there are multiple sectors precede one sector, it is defined as a joint structure as sector j in the illustration. Similarly, a fork structure is defined when there exist many subsequent sectors after one sector, and the fork structure in Figure 2.7 consists sector i and the set of its subsequent sectors. In BLO model, one requirement is that before entering a new sector, the aircraft must stay in one of its preceding sectors for a time duration which is at least equal to the minimum sector traversing time. This also means that the aircraft need to select one subsequent sector to enter when there are multiple choices.

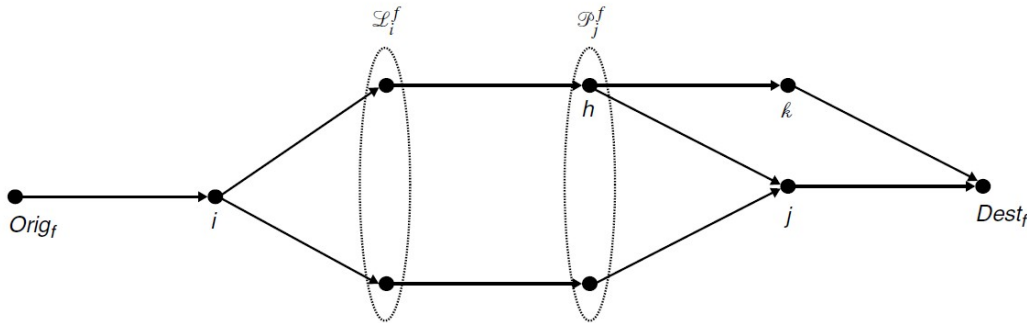


Figure 2.7: Acyclic Graph Structure (Bertsimas et al. (2011b))

Parameters, decision variables and mathematical formulation of BLO model are introduced in a detailed manner in the following.

Parameters:

- \mathcal{F} set of flights,
- \mathcal{K} set of airports,
- \mathcal{S} set of sectors,

$\mathcal{S}_f \subseteq \mathcal{S}$	set of sectors for flight f ,
\mathcal{C}	set of pairs of continued flights,
\mathcal{P}_i^f	set of sector i 's preceding sectors, $i \in \mathcal{S}_f$ for flight f ,
\mathcal{L}_i^f	set of sector i 's subsequent sectors, $i \in \mathcal{S}_f$ for flight f ,
\mathcal{T}	set of time periods,
d_f	scheduled departure time of flight f , $\forall f \in \mathcal{F}$,
a_f	scheduled arrival time of flight f , $\forall f \in \mathcal{F}$,
s_f	turnaround time of flight f ,
$orig_f$	departure airport for flight f , $orig_f \in \mathcal{K}$,
$dest_f$	landing airport for flight f , $dest_f \in \mathcal{K}$,
$D_k(t)$	departure capacity of airport k at time period t ,
$S_j(t)$	capacity of sector j at time t ,
$A_k(t)$	arrival capacity of airport k at time period t ,
l_{ij}^f	minimum time units that flight f must spend in sector i before entering in sector j ,
$T_j^f \equiv [\underline{T}_j^f, \bar{T}_j^f]$	set of feasible time periods for flight f to enter sector j ,
\underline{T}_j^f	the first time period in T_j^f ,
\bar{T}_j^f	the last time period in T_j^f ,
end_f	maximum duration of flight f .

Decision Variables:

Fly-by Variable:

$$w_{j,t}^f = \begin{cases} 1 & \text{if flight } f \text{ has arrived at sector } j \text{ by time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Objective Function:

The objective function of BLO model to minimize the combination of airborne and ground-holding delay cost. One amazing feature of the objective function is that it is approximately equal to the difference between total delay cost and ground delay cost by applying superlinear coefficients. For example, if a flight is allocated a ground-holding delay for 2 time periods, the cost would be $2^{1+\epsilon_1}$ in calculation whereas ϵ_1 is a small positive real number. There is one distinguishing property of superlinear cost coefficients that it could address fairness issues in delay assignment on a flight-by-flight basis. For instance, if there are two units of delay to be assigned to two flights, it tends to allocate each of two flights for one time period of delay rather than allocate the total two time periods of delay to any one of the two flights. Similarly, superlinear coefficients are also applied for the airborne-holding delay, and a relatively larger $\epsilon_2 (> \epsilon_1)$ is adopted. The motivation is to make the objective function prefer ground-holding delay since the airborne delay is much more expensive with respect to fuel consumption and safety. The objective function is to minimize the summation of $AH_f^{1+\epsilon_2} + GH_f^{1+\epsilon_1}$ for all flights, where GH_f and AH_f are the ground-holding delay and the airborne delay allocated to flight f . Since both ϵ_1 and ϵ_2 are small, an approximation could be made as follows.

$AH_f^{1+\epsilon_2} + GH_f^{1+\epsilon_1} = AH_f^{1+\epsilon_2} + GH_f^{1+\epsilon_1} + GH_f^{1+\epsilon_2} - GH_f^{1+\epsilon_2} - (GH_f^{1+\epsilon_2} - GH_f^{1+\epsilon_1})$, where $TD_f = AH_f + GH_f$ is the total delay experienced by flight f .

The corresponding superlinear cost coefficients could be generalized as below. $c_{td}^f(t) = (t - a_f)^{1+\epsilon_2}$ is the total cost of delaying flight f for $(t - a_f)$ periods of time, $c_g^f(t) = (t - d_f)^{1+\epsilon_2} - (t - d_f)^{1+\epsilon_1}$ is the cost reduction to hold flight f on the ground for $(t - d_f)$ units of time ($\epsilon_2 > \epsilon_1 > 0$), where d_f and a_f are the original departure and arrival time slot for flight f , respectively. In view of the above, the objective function could be formulated as the following expression.

$$\text{Min} \sum_{f \in \mathcal{F}} \left(\sum_{t \in T_{dest_f}^f} c_{td}^f(t)(w_{dest_f,t}^f - w_{dest_f,t-1}^f) - \sum_{t \in T_{orig_f}^f} c_g^f(t)(w_{orig_f,t}^f - w_{orig_f,t-1}^f) \right)$$

The Constraints:

$$\sum_{f \in \mathcal{F}: orig_f=k} (w_{k,t}^f - w_{k,t-1}^f) \leq D_k(t) \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T}. \quad (2.1)$$

$$\sum_{f \in \mathcal{F}: dest_f=k} (w_{k,t}^f - w_{k,t-1}^f) \leq A_k(t) \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T}. \quad (2.2)$$

$$\sum_{f \in \mathcal{F}: j \in \mathcal{S}_f} (\max\{0, w_{j,t}^f - \sum_{j' \in \mathcal{L}_j^f} w_{j',t}^f\}) \leq S_j(t) \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T}. \quad (2.3)$$

$$w_{j,t}^f \leq \sum_{j' \in \mathcal{P}_j^f} w_{j',t-l_{j'}^f}^f \quad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}_f, j \neq orig_f, \forall t \in T_j^f. \quad (2.4)$$

$$w_{j,\bar{T}_j^f}^f \leq \sum_{j' \in \mathcal{L}_j^f} w_{j',\bar{T}_{j'}^f}^f \quad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}_f : j \neq dest_f. \quad (2.5)$$

$$\sum_{j' \in \mathcal{L}_j^f} w_{j',\bar{T}_{j'}^f}^f \leq 1 \quad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}_f : j \neq dest_f. \quad (2.6)$$

$$w_{orig_f,t}^f - w_{dest_{f'},t-s_f}^{f'} \leq 0, \quad \forall (f, f') \in \mathcal{C}, t \in T_{orig_f}^f : t - s_f \in T_{orig_{f'}}^{f'}. \quad (2.7)$$

$$w_{orig_f,t}^f - w_{dest_f,t+end_f}^f \leq 0, \quad \forall f \in \mathcal{F}, t \in T_{orig_f}^f : t + end_f \in T_{dest_f}^f. \quad (2.8)$$

$$w_{j,t-1}^f - w_{j,t}^f \leq 0, \quad \forall f \in \mathcal{F}, \forall t \in T. \quad (2.9)$$

$$w_{j,t}^f \in \{0, 1\}, \quad \forall f \in \mathcal{F}, \forall t \in T, \forall j \in \mathcal{S}_f. \quad (2.10)$$

The first three groups of constraints address the limitation of system capacity. Constraint set (2.1) states that the number of flights that take off from airport k at time t cannot exceed the departure capacity of airport k at that time. Similarly, Constraints (2.2) guarantee the number of flights that arrive at airport k at time t cannot exceed the arrival capacity. Constraints (2.3) state the total number of flights in Sector j at time t will not exceed the sector capacity at that

time. Constraints (2.4-2.6) ensure connectivity between sectors. Constraints (2.4) guarantee that if flight f arrives at sector j by time t , then flight f must have arrived one preceding sector j' at least $l_{j'j}^f$ earlier than time period t . This means that a flight cannot enter the subsequent sector on its path until it has stayed in one of the preceding sectors at least the minimum traversing time. If sector j have a joint structure and it is the only sector that could arrive from all preceding sectors, then inequality (2.4) are facet defining for the integer solution polytope of this model. Constraints (2.5) and (2.6) state that if flight f arrives at sector j , it must reach one of the subsequent sectors by the last time period of the time window. Constraints (2.7) represent connectivity between flights executed by the same aircraft. Constraints (2.8) ensure the total flight time cannot exceed the maximum acceptable duration. Constraints (2.9) demonstrate the non-decreasing property of fly-by variables with respect to time.

Three classes of strong valid inequalities are introduced in BLO model.

Proposition 2.1. *If sector j is a fork structure in the acyclic graph, then*

$$w_{j,t}^f \geq \sum_{j' \in \mathcal{L}_j^f: |\mathcal{P}_j^f|=1} w_{j',t+l_{jj'}^f}^f, \quad \forall f \in \mathcal{F}, \quad \forall t \in T_j^f \quad (2.11)$$

are valid inequalities for feasible solutions of this model. If sector j is a fork structure and it is the only entrance to all of its subsequent sectors, constraints (2.10) are facet defining for the integer solution polytope of the BLO model.

Proposition 2.2. *If sector j is a joint structure in the acyclic graph, then*

$$w_{j',\bar{T}_{j'}}^f \geq \sum_{j' \in \mathcal{P}_j^f: |\mathcal{L}_{j'}^f|=1} w_{j',\bar{T}_{j'}}^f, \quad \forall f \in \mathcal{F} \quad (2.12)$$

are valid inequalities for feasible solutions of the BLO model.

Proposition 2.3. *If \mathcal{A} is an anti-chain structure on \mathcal{S}_f ,*

$$\sum_{j \in \mathcal{A}} w_{j, \bar{T}_j}^f \leq 1. \quad (2.13)$$

are valid inequalities for feasible solutions of the BLO model.

These inequalities could also be applied in the model of chapter 3 of this thesis.

2.7 Research Gaps

This dissertation focuses on the current and future development in the ATFM system, aims to provide a framework of integer optimization models to get flexible rerouting operations, increase airspace capacity and design effective algorithm to generate flight plans under the capacitated airspace structures.

Chapter 3 proposes a new integer optimization model to address the traditional sector-based ATFM rerouting problem under complex practical situations. In practice, the set of o-d routes for flights must be represented by directed graphs with cycles, and the main contribution of the work is that it could handle ATFMRP on a flight-by-flight basis in which unrestricted directed graphs exist. Another benefit of applying unrestricted directed graphs is that the global optimal solutions could be preserved. In order to improve the computational efficiency, two types of strong valid inequalities are proposed according to the mathematical structure.

Chapter 4 introduces a type of new airspace structure to replace sector-based structure to support 4D trajectory-based operations. Then, we propose a new ATFM optimization model based on the new airspace structure which could ensure safety separation between flights and increase airspace capacity. The new ATFM model addresses rerouting, ground holds, fuel consumption and cancellations on a flight-by-flight basis. In order to solve the model efficiently,

Danzig-Wolfe decomposition is employed to decompose the original mathematical formulation. After that, we design a heuristic column generation algorithm to generate conflict-free trajectories for each flight under airway entrance capacity limitations. By using commercial optimization software, solution time could be controlled within 20 minutes for the ATFM rerouting problems of the Southeast Asia region.

Chapter 3

An Integer Optimization Model for Practical Air Traffic Flow Management Rerouting Problem

In this chapter, we propose a new integer optimization model to address the Air Traffic Flow Management Rerouting Problem (ATFMRP) under complex practical situations. Optimal traffic flow strategies which include rerouting, ground-holding, airborne holding and speed control can be obtained on a flight-by-flight basis by solving the model with commercial software. In some practical situations, the set of o-d routes for flights must be represented by directed graphs with cycles, and the main contribution of this work is that the model could handle ATFMRP on a flight-by-flight basis in which directed graphs with cycles exist. Directed graph without limitations is a type of general structure that allows more complex o-d routes to be theoretically represented. Two types of valid inequalities are proposed according to the model structure to improve the computational efficiency. The computational results indicate that solution time can be controlled in 5 minutes for instances of a size comparable to that of the whole Southeast Asia air traffic management system.

3.1 Introduction

With the recovering global economies, air travel demand has been increasing rapidly all over the world. The [FAA \(2013\)](#) predicts that the passenger demand over the whole Asia-Pacific region is estimated to increase 4.3% per year and this continuous growth has put tremendous pressure on the aviation system. Airspace congestion happens frequently there since adverse weather conditions often make airspace unflyable. During January 2015 alone, about 10,900 flights in this region were cancelled and another 192,000 flights were delayed. In the Southeast Asia region, sector capacity limitation has become the bottleneck since volatile weather applies huge pressure on air traffic controller and the outdated equipment. [Figure 3.1](#) shows a snapshot of flights in the Southeast Asia region at 3pm where there are about 600 flights during this period. Without advanced air traffic flow management tool, pilots sometimes take risky actions without permission to avoid congestion and bad weather situations in this region. The recent aviation accidents of the Southeast Asia critically affect the whole airline industry all over the world and there is an urgent need of developing advanced rerouting tools to improve the current situations. In order to improve these realities, this work aims to provide an efficient decision model in rerouting to smooth air traffic delay and avoid congestion.

Some previous studies propose unique approaches to solving ATFMRP based on the sector structure. However, when dealing with rerouting problems, the structure of practical o-d routes could be very complex based on sectors. [Bertsimas et al. \(2011b\)](#) proposed an integer optimization model for ATFMRP with excellent computational performance, but there is a restriction that their model can only handle route structures which must be represented by acyclic graphs. In fact, it is not uncommon to handle directed graphs with cycles practically. [Figure 3.2](#) shows a simple demonstration of a directed graph with cycles which could be encountered in real situations. In the figure, there are 4 sectors, 2



Figure 3.1: Snapshot of Flights in the Southeast Asia(Data Source: Flightradar24.com)

airports and 3 routes for rerouting operations.

$$\text{Route 1} = (a, A, B, C, D, b),$$

$$\text{Route 2} = (a, A, C, B, D, b),$$

$$\text{Route 3} = (a, A, C, D, b).$$

Route 1 requires flights to fly from sector B to C, whereas Route 2 is from sector C to B. If these two routes are considered as the rerouting routes simultaneously, a cycle between sector B and C would be formed as demonstrated in the figure. This case shows that the o-d routes based on sectors must be represented by directed graphs with cycles sometimes. Directed graph with cycles is a type of more general structure to represent complex practical ATFM

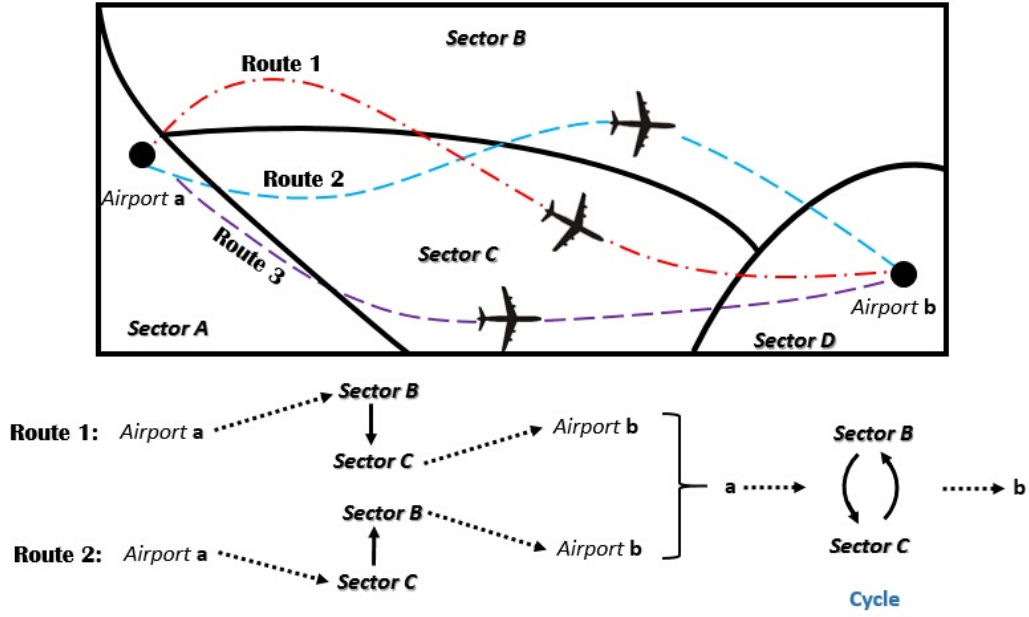


Figure 3.2: Routes for Rerouting

situations. There are also many other situations that the o-d routes must be represented by directed graphs with cycles. If we employ the model of [Bertsimas et al. \(2011b\)](#) to solve this problem, we can only pick at most one route from Route 1 & 2 to avoid cycles. In fact, it is difficult to judge which one should be included before solving the problem and it is possible to miss the global optimal solution once we choose the wrong route for rerouting. Till now, the only approach that allows unrestricted graphs based on sectors is developed by [Bertsimas and Patterson \(2000\)](#) and their approach employs a multi-commodity flow model to schedule flights. However, it is an aggregate model which can only obtain solutions of aggregated flows, and a heuristic algorithm must be applied to decompose the solutions into individual flight plans. The computational performance is also not good enough to handle practical problems. In this paper, we present a linear integer optimization model to address ATFMRP on a flight-by-flight basis which also allows the existence of directed graphs with cycles in the problem, and individual flight plan can be directly obtained by solving the model. We also propose advanced mathematical techniques to strengthen the

model structure and the computational performance is good enough to handle practical problems for the whole Southeast Asia region.

3.2 Model Formulation

In this section, we first provide a table to show the differences explicitly between this work and the BLO model.

Variables	Fly-by Variable (same as BLO model), Network Flow Variable
Constraints	Constraints 3.1-3.5 and 3.8 are new constraints and different from BLO model
Objective	Same as BLO model

Parameters, decision variables and model parts are introduced in a detailed manner in the following.

Parameters:

- \mathcal{F} set of flights,
- \mathcal{H} set of airports,
- \mathcal{S} set of sectors,
- $\mathcal{S}_f \subseteq \mathcal{S}$ set of sectors for flight f ,
- \mathcal{C} set of pairs of continued flights,
- d_f scheduled departure time of flight f ,
- a_f scheduled arrival time of flight f ,
- s_f turnaround time for flight f ,
- $orig_f$ departure airport of flight f , $orig_f \in \mathcal{H}$,
- $dest_f$ landing airport of flight f , $dest_f \in \mathcal{H}$,
- N set of directed arcs in the entire system,

$N_f \subseteq N$	set of directed arcs can be used by flight f ,
$D_k(t)$	departure capacity of airport k at time t ,
$A_k(t)$	arrival capacity of airport k at time t ,
$S_j(t)$	capacity of sector j at time t ,
l_{ij}^f	minimum time units that flight f must spend in sector i before entering in sector j ,
T	set of time periods,
$[\underline{T}_j^f, \overline{T}_j^f]$	set of feasible time periods for flight f to arrive in sector j ,
\underline{T}_j^f	the first feasible time period for flight f to arrive in sector j ,
\overline{T}_j^f	the last feasible time period for flight f to arrive in sector j ,
end_f	maximum duration of flight f .

Decision Variables:

Since directed graphs with cycles cannot be represented by partially ordered sets, we introduce network flow variable $x_{i,j}^f$ to generate the sector traversing order for each flight.

Network Flow Variable:

$$x_{i,j}^f = \begin{cases} 1 & \text{if flight } f \text{ flies from airport or sector } i \text{ to } j, \\ 0 & \text{otherwise.} \end{cases}$$

Fly-by Variable:

$$w_{j,t}^f = \begin{cases} 1 & \text{if flight } f \text{ has arrived at sector } j \text{ by time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Objective Function:

We employ the objective function of BLO model here for two practical reasons: fairness issues in delay allocation between flights and ground delay are preferred as compared to airborne delay. The objective is to minimize the combination of ground and airborne delays cost by approximating the difference of total delay cost and ground holding cost with superlinear coefficients. In the objective function, if a flight is allocated a ground-holding delay for 2 time periods, the cost would be $2^{1+\epsilon_1}$ in calculation whereas ϵ_1 is a small positive real number. There is one characteristic property of superlinear cost coefficients that it could address fairness issues in delay assignment on a flight-by-flight basis. For instance, if there are two units of delay to be assigned to two flights, it tends to allocate each of two flights for one time period of delay rather than allocate the total two time periods of delay to any one of the two flights. Similarly, superlinear coefficients are also applied for the airborne-holding delay, and a relatively larger $\epsilon_2 (> \epsilon_1)$ is adopted. The motivation is to make the objective function prefer ground-holding delay since the airborne delay is much more expensive with respect to fuel consumption and safety.

In our model, $c_{td}^f(t) = (t - a_f)^{1+\epsilon_2}$ is the total cost to delay flight f for $(t - a_f)$ units of time, and $c_g^f(t) = (t - d_f)^{1+\epsilon_2} - (t - d_f)^{1+\epsilon_1}$ is the cost reduction to hold flight f on the ground for $(t - d_f)$ units of time ($\epsilon_2 > \epsilon_1 > 0$), where a_f and d_f are the scheduled arrival and departure times of flight f , respectively.

The objective function is to minimize the summation of $AH_f^{1+\epsilon_2} + GH_f^{1+\epsilon_1}$ for all flights, where GH_f and AH_f are the ground-holding delay and the airborne delay allocated to flight f . Since both ϵ_1 and ϵ_2 are small, an approximation could be made as follows.

$$AH_f^{1+\epsilon_2} + GH_f^{1+\epsilon_1} = AH_f^{1+\epsilon_2} + GH_f^{1+\epsilon_1} + GH_f^{1+\epsilon_2} - GH_f^{1+\epsilon_2} \cong TD_f^{1+\epsilon_2} - (GH_f^{1+\epsilon_2} - GH_f^{1+\epsilon_1}),$$

where $TD_f = AH_f + GH_f$ is the total delay experienced by flight f .

In view of the above, the objective function is as follows:

$$\text{Min} \sum_{f \in F} \left(\sum_{t \in T_{dest_f}^f} c_{td}^f(t)(w_{dest_f,t}^f - w_{dest_f,t-1}^f) - \sum_{t \in T_{orig_f}^f} c_g^f(t)(w_{orig_f,t}^f - w_{orig_f,t-1}^f) \right)$$

The Constraints:

$$\sum_{j:(orig_f,j) \in N_f} x_{orig_f,j}^f = 1 \quad \forall f \in \mathcal{F}, \quad (3.1)$$

$$\sum_{j:(j,dest_f) \in N_f} x_{j,dest_f}^f = 1 \quad \forall f \in \mathcal{F}, \quad (3.2)$$

$$\sum_{j:(i,j) \in N_f} x_{i,j}^f - \sum_{j:(j,i) \in N_f} x_{j,i}^f = 0 \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{S}_f, i \neq orig_f, dest_f, \quad (3.3)$$

$$\sum_{t \in T} (w_{i,t}^f - w_{j,t}^f) \geq l_{ij}^f + M(x_{i,j}^f - 1) \quad \forall f \in \mathcal{F}, \forall (i,j) \in N_f, \quad (3.4)$$

$$\sum_{j:(i,j) \in N_f} x_{i,j}^f = w_{i,\bar{T}_i^f}^f \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{S}_f : i \neq dest_f, \quad (3.5)$$

$$\sum_{f \in F: orig_f=k} (w_{k,t}^f - w_{k,t-1}^f) \leq D_k(t) \quad \forall k \in \mathcal{K}, t \in T, \quad (3.6)$$

$$\sum_{f \in F: dest_f=k} (w_{k,t}^f - w_{k,t-1}^f) \leq A_k(t) \quad \forall k \in \mathcal{K}, t \in T, \quad (3.7)$$

$$\sum_{f \in F} (w_{i,t}^f - \sum_{j:(i,j) \in N_f} \min(x_{i,j}^f, w_{j,t}^f)) \leq S_i(t) \quad \forall i \in \mathcal{S}_f, \forall t \in T, \quad (3.8)$$

$$w_{orig_f,t}^f - w_{dest_f,t-s_f}^{f'} \leq 0, \quad \forall (f, f') \in \mathcal{C}, t \in T_{orig_f}^f : t - s_f \in T_{orig_{f'}}^{f'}, \quad (3.9)$$

$$w_{orig_f,t}^f - w_{dest_f,t+end_f}^f \leq 0, \quad \forall f \in \mathcal{F}, t \in T_{orig_f}^f : t + end_f \in T_{dest_f}^f, \quad (3.10)$$

$$w_{j,t-1}^f - w_{j,t}^f \leq 0, \quad \forall f \in \mathcal{F}, \forall t \in T, \quad (3.11)$$

$$w_{j,t}^f \in \{0, 1\}, \quad \forall f \in \mathcal{F}, \forall t \in T, \forall j \in \mathcal{S}_f, \quad (3.12)$$

$$x_{i,j}^f \in \{0, 1\}, \quad \forall f \in \mathcal{F}, \forall i, j \in \mathcal{S}_f. \quad (3.13)$$

The first three sets of constraints are network flow balance constraints. Constraint set (3.1) states that at the departure airports, every flight selects exactly one sector to enter and constraint set (3.2) ensures all flights arrive at their destinations airport from one sector. Constraints (3.3) keep network flow balance at sectors. Constraints (3.4) are if-then constraints which state that if flight f arrives at sector j from sector i , then flight f must arrive sector i at least l_{ij}^f earlier than sector j . This means that a flight cannot enter the subsequent sector on its selected route until it has stayed in the preceding sector for the minimum travelling time. In directed graphs without restriction, it is necessary to define sector order which is different from that of acyclic graphs. Constraint sets (3.1)-(3.4) guarantee each flight an o-d path without cycles. Constraints (3.5) ensure that if flight f arrives at sector i , it must arrive before \bar{T}_i^f . Constraints (3.6) guarantee the total flights that take off from airport k at time t cannot exceed the departure capacity of airport k at that time. Similarly, Constraints (3.7) state that the total flights that arrive at airport k at time t cannot exceed the arrival capacity. Constraint set (3.8) states the total number of flights in Sector j at time t will not exceed the sector capacity at that time. They are different from the sector capacity constraints in BLO model, and network variable $x_{i,j}^f$ must be employed to indicate which sector would be the subsequent one for flight f . The factor $\min(x, y)$ (x, y are all binary variables) could be represented by a logical constraint $2z \leq x + y \leq 1 + z$ which involves one additional binary variable z . Constraints (3.9) represent connectivity between flights executed by the same aircraft. Constraints (3.10) guarantee the total flight time cannot exceed the maximum flight duration and constraints (3.11) ensure the non-decreasing structure of fly-by variables with respect to time.

3.3 Valid Inequalities

Two classes of strong valid inequalities are presented in this section based on general graph structure without restrictions and the purpose is to strengthen the mathematical structure of the formulation. The valid inequalities (2.10-2.12) proposed in BLO model could also be employed in our model once such specific graph structures are encountered, but the valid inequalities proposed in the following are designed for general graphs.

Proposition 3.1.

$$w_{i,t}^f \geq \sum_{j:(i,j) \in N_f} \min(x_{i,j}^f, w_{j,t+l_{ij}^f}^f), \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{S}_f : i \neq \text{dest}_f, \forall t \in [\underline{T}_j^f, \bar{T}_j^f] \quad (3.14)$$

are valid inequalities for feasible solutions of the ATFMRP model.

Proof. According to constraint set (3.5), it follows that $\forall f \in \mathcal{F}, \forall i \in \mathcal{S}_f,$

$\sum_{j:(i,j) \in N_f} x_{i,j}^f$ could be either 0 or 1.

(i) Suppose $\sum_{j:(i,j) \in N_f} x_{i,j}^f = 0$, then

$$\sum_{j:(i,j) \in N_f} \min(x_{i,j}^f, w_{j,t+l_{ij}^f}^f) = \sum_{j:(i,j) \in N_f} x_{i,j}^f = 0.$$

The proposition is true since $w_{i,t}^f \in \{0, 1\}$.

(ii) Suppose $\sum_{j:(i,j) \in N_f} x_{i,j}^f = 1$, then there exists one $\tilde{j} : (i, \tilde{j}) \in N_f$ such that $x_{i,\tilde{j}}^f = 1$, and for any other $j \neq \tilde{j}$, $x_{i,j}^f$ must be 0. Thus

$$\sum_{j:(i,j) \in N_f} \min(x_{i,j}^f, w_{j,t+l_{ij}^f}^f) = \min(x_{i,\tilde{j}}^f, w_{\tilde{j},t+l_{i\tilde{j}}^f}^f) = w_{\tilde{j},t+l_{i\tilde{j}}^f}^f.$$

According to the constraint set (3.4), it follows that

$$\sum_{t \in T} (w_{i,t}^f - w_{\tilde{j},t}^f) \geq l_{i\tilde{j}}^f,$$

and by the non-decreasing property of the fly-by variable $w_{i,t}^f$ with respect to time, $w_{i,t}^f \geq w_{\tilde{j},t+l_{ij}^f}^f$.

The proposition is true since

$$\sum_{j:(i,j) \in N_f} \min(x_{i,j}^f, w_{j,t+l_{ij}^f}^f) = \min(x_{i,\tilde{j}}^f, w_{\tilde{j},t+l_{i\tilde{j}}^f}^f) = w_{\tilde{j},t+l_{i\tilde{j}}^f}^f \leq w_{i,t}^f.$$

□

Proposition 3.2.

$$w_{j,t}^f \leq \sum_{i:(i,j) \in N_f} \min(x_{i,j}^f, w_{i,t-l_{ij}^f}^f), \quad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}_f : j \neq \text{orig}_f, \forall t \in [\underline{T}_j^f, \bar{T}_j^f] \quad (3.15)$$

are valid inequalities for feasible solutions of the ATFMRP model.

Proof. According to the constraint set (3.3) and (3.5), it follows that

$$\forall f \in \mathcal{F}, \forall j \in \mathcal{S}_f : j \neq \text{orig}_f, \quad \sum_{i:(i,j) \in N_f} x_{i,j}^f = w_{j,\bar{T}_j^f}^f,$$

which means $\sum_{i:(i,j) \in N_f} x_{i,j}^f$ could be either 0 or 1.

(i) Suppose $\sum_{i:(i,j) \in N_f} x_{i,j}^f = w_{j,\bar{T}_j^f}^f = 0$, then

$$\sum_{i:(i,j) \in N_f} \min(x_{i,j}^f, w_{i,t-l_{ij}^f}^f) = 0.$$

The proposition is true since

$$w_{j,t}^f \leq w_{j,\bar{T}_j^f}^f = \sum_{i:(i,j) \in N_f} \min(x_{i,j}^f, w_{i,t-l_{ij}^f}^f) = 0.$$

(ii) Suppose $\sum_{i:(i,j) \in N_f} x_{i,j}^f = 1$. Then there exist only one $\tilde{i} : (\tilde{i}, j) \in N_f$ such that $x_{\tilde{i},j}^f = 1$, and any other $i \neq \tilde{i}$ $x_{i,j}^f$ must be 0. Thus

$$\sum_{i:(i,j) \in N_f} \min(x_{i,j}^f, w_{i,t-l_{ij}^f}^f) = \min(x_{\tilde{i},j}^f, w_{\tilde{i},t-l_{i\tilde{j}}^f}^f) = w_{\tilde{i},t-l_{i\tilde{j}}^f}^f.$$

According to constraint set (3.4), it follows that

$$\sum_{t \in T} (w_{\tilde{i},t}^f - w_{j,t}^f) \geq l_{i\tilde{j}}^f,$$

and by the non-decreasing property of the fly-by variable $w_{i,t}^f$ with respect to time, $w_{i,t-l_{ij}^f}^f \geq w_{j,t}^f$.

The proposition is true since

$$\sum_{j:(i,j) \in N_f} \min(x_{i,j}^f, w_{i,t-l_{ij}^f}^f) = \min(x_{i,j}^f, w_{i,t-l_{ij}^f}^f) = w_{i,t-l_{ij}^f}^f \geq w_{j,t}^f.$$

□

3.4 Size of Formulation

The introduction of network flow variables has increased the total number of decision variables compared with that of the BLO model. There are $\sum_{\forall f \in \mathcal{F}, \forall i \in \mathcal{S}_f} |T| + \sum_{\forall f \in \mathcal{F}} |N_f|$ decision variables. The number of additional binary variables is $2 \sum_{\forall f \in \mathcal{F}} |N_f| |T|$.

The number of total constraints is bounded above by $2|\mathcal{F}| + 4|\mathcal{F}||\mathcal{S}_f| + |\mathcal{F}||N_f| + 2|\mathcal{K}||T| + |\mathcal{S}_f||T| + 2|\mathcal{F}||T| + |\mathcal{C}||T|$. The number of valid inequalities are included above.

3.5 Numerical Result

In this section, we report the computational performance of the mathematical model with valid inequalities in Propositions 1-2. In order to compute optimal solutions, we use CPLEX dynamical search method 12.4 (CPLEX-MIP), implemented under CPLEX IDE with Optimization Programming Language. All test instances are solved on a laptop with an Intel I7-3630QM processor, 2.40 GHz, 8 GB RAM with Windows 7 platform.

3.5.1 Computational Test 1

The first test demonstrates the power of the valid inequalities in Propositions 3.1-3.2 by analyzing the solution time. The parameters for this computational

test are as follows: $\mathcal{K} = \{o, d\}$, $\mathcal{S} = \{A, B, C, D, E, F, G, H\}$, 10 time periods and the set of o-d routes are represented by a directed graph with cycles as shown in Figure 3.3. The graph structure does not detract from the generality of this model since all types of graphs can be employed in the model. In the figure, there is a directed cycle between sectors B and E.

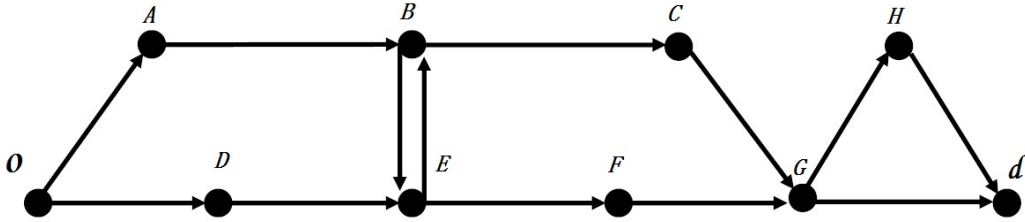


Figure 3.3: Graph

In order to demonstrate the computational performance under congestions, the capacities of all sectors are limited to half of the number of total flights. All flights are homogeneous with same schedules and solution times are collected from instances of different sizes. Comparison of solution time between our model with and without valid inequalities of proposition 3.1 is demonstrated in Figure 3.4.

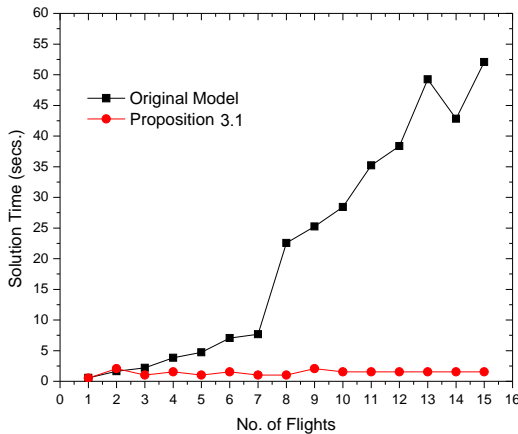


Figure 3.4: Solution Time 1

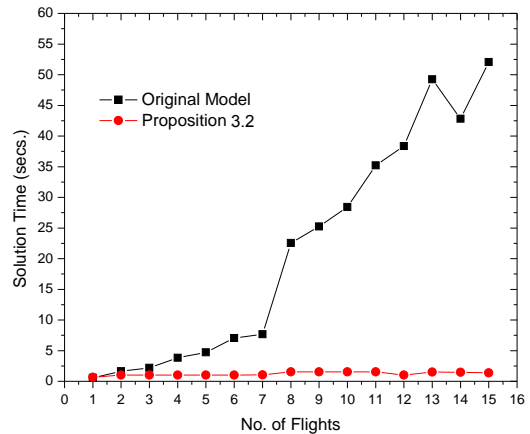


Figure 3.5: Solution Time 2

One obvious trend in the figure is that by adding valid inequalities proposed in Proposition 3.1, solution time could be significantly reduced with the growth

of total flights. The solution time of instances of 15 flights can be controlled within 2 seconds under Proposition 3.1. However, the solution time grows very fast that it exceeds 50 seconds without adding these valid constraints. The inequalities in proposition 3.2 have almost the same power as proposition 3.1 which can be seen in Figure 3.5.

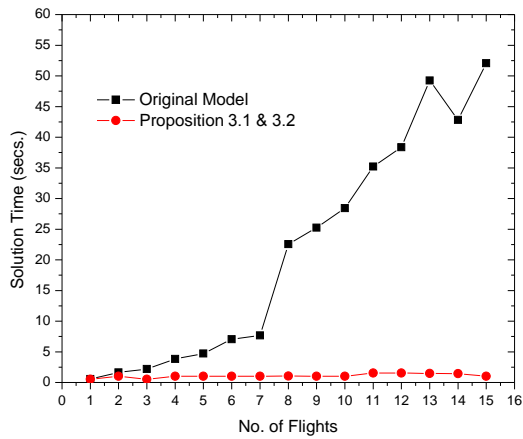


Figure 3.6: Solution Time 3

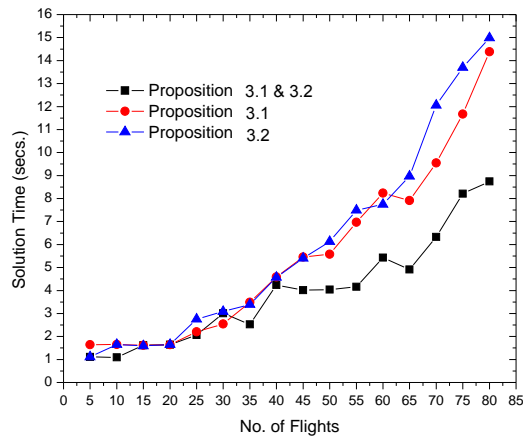


Figure 3.7: Solution Time 4

Figure 3.6 demonstrates the combination effect of inequalities in both proposition 3.1 & 3.2, and there is no difference with adding just one type of valid constraints. In order to check whether the combination of two kinds of inequalities in proposition 3.1 & 3.2 can work together well or not, a large test is done, and the result is shown in Figure 3.7. Apparently, the computational efficiency is improved by adding both classes of constraints. Below 40 flights, there is no significant differences in solution time, but the solution time grows much slower when the number of flights exceeds 40 if we employ both propositions simultaneously. We can see in Figure 3.7 that when the number of flights increases to 80, the solution time could be reduced approximate 40% with both proposition-s. Another feature is that by applying the two propositions, we can obtain the optimal solution much faster when the MIP gap is within a small range.

3.5.2 Computational Test 2

The test in this section is to demonstrate the computational results of instances which are comparable to the practical ATFM RP of the whole Southeast Asia region. In the experimental setup, we subdivide the airspace into rectangular sectors of same dimensions which form a grid and the minimum traversing times for each sector are all the same for each flight. The regular shape of sectors does not critically affect the model's generality since sectors of arbitrary shape can be accommodated by the model. We also assume that all flights are homogeneous.

The size of the instance depends on the number of flights, discrete time periods, sectors and airports and all these parameters of test instances in this part are set according to the real situations in that region. We consider a five-hour horizon and divide it equally into 20 time periods with 15 minutes for each period. We also assume all flights must take off and land within the 20 periods. The minimum traversing time for each sector is 1. This instance consists of 2050 flights, 110 sectors, 20 time periods and 13 major airports which include Hong Kong International Airport, Singapore Changi Airport, etc. All airports locate in their real positions, and the nominal routes for each flight only include the sectors on the shortest o-d routes. If there are multiple shortest o-d routes, we include all sectors on these routes for the flights of that o-d pair. We assume that the capacities of 15 sectors are affected by adverse weather, and if the nominal path crosses these sectors, alternative o-d routes (not the shortest path) are included to avoid possible congestions. Out of these 15 sectors, 10 of them are busy sectors which are crossed by the nominal routes of multiple o-d pairs. The departure and arrival capacities are generated by a normal distribution from 80%-100% of peak capacity level to simulate real airports. For the sector capacity, the default setting is 25 per period which is quite close to the actual level. With all the above configurations, the ATFM model has a total of 1,430,000 constraints and 595,000 binary decision variables. In this study, we

accept solutions within a relative MIP gap of 0.5%. The numerical results are listed in Table 1, and the first column is the proportion of connected flights. Sector capacities and solution times are in column 2 and 3.

The information in column 3 shows all the solutions could be obtained within 5 minutes, and the average solution times are quite similar no matter the connected proportion is high or low. When the capacity of those affected sectors drops to 5, there is no feasible solution, and that is because some flights cannot land within the time domain. It is evident that the solution time increases faster when the capacity drops to 6. Another trend is that with the gradually dropping of the sector capacity under different connected proportions, the delay costs are the same at the beginning. However, when the capacity reduction is severe enough, the instance with more connected flights would suffer larger delay costs. From the table, we can find that when the sector capacity is 6, the instance with 53.7% connected flights suffers the largest total delay cost than that of other instances. The reason is that if those connected flights cannot arrive in time, the subsequent flights couldn't depart and then delays propagate in the whole system. Based on the optimal solution of the model, it seems that the optimal strategy to avoid delay propagation under severe congestion is to allocate delay to those flights without connected subsequent flights. However, this policy is impractical because it associates with equity issues that airlines with less connected flights would suffer larger delay cost.

Connected Flights (%)	Sector Capacity	Solution Time (secs.)	GAP (%)	Objective
14.5	25	133.51	0.00	164.70
	20	140.07	0.00	184.97
	18	114.86	0.00	194.92
	16	174.83	0.00	205.09
	14	179.00	0.00	241.27

	12	144.40	0.00	302.79
	11	141.03	0.01	344.16
	10	103.32	0.26	396.21
	9	109.45	0.22	460.48
	8	101.57	0.00	540.90
	7	218.26	0.00	646.93
	6	267.91	0.00	791.81
	5	Infeasible		
<hr/>				
29.0	25	113.15	0.21	164.81
	20	119.62	0.00	185.04
	18	101.90	0.00	194.97
	16	103.16	0.00	205.15
	14	110.96	0.00	241.40
	12	149.59	0.00	303.11
	11	101.70	0.00	347.00
	10	151.29	0.00	399.19
	9	152.97	0.00	469.26
	8	175.72	0.00	556.93
	7	161.24	0.00	666.01
	6	189.46	0.00	816.43
	5	Infeasible		
<hr/>				
43.9	25	78.17	0.00	164.70
	20	73.69	0.19	184.73
	18	94.30	0.00	194.97
	16	94.35	0.17	208.07
	14	111.38	0.00	245.55
	12	114.58	0.00	310.02
	11	104.51	0.00	355.82

	10	72.57	0.00	411.29
	9	102.43	0.00	483.08
	8	158.14	0.00	571.83
	7	90.45	0.00	689.70
	6	150.54	0.00	862.10
	5	Infeasible		
53.7	25	105.25	0.00	165.42
	20	94.16	0.00	185.69
	18	94.27	0.00	196.65
	16	102.21	0.00	208.63
	14	98.00	0.00	246.08
	12	112.51	0.00	310.47
	11	81.74	0.00	356.27
	10	79.39	0.24	412.69
	9	94.99	0.45	487.50
	8	136.25	0.00	577.66
	7	110.79	0.00	696.52
	6	165.42	0.00	870.92
	5	Infeasible		

Table 3.1: Solution Results

3.6 Conclusion

The work in this chapter introduces a new binary integer optimization model to address complex practical ATFMRP. One distinguishing feature of this job is that it could address sector-based ATFMRP on a flight-by-flight basis which allows the existence of directed graphs with cycles. The model keeps the advantages of the existed ATFM models that it addresses all phases of

flights from takeoff to landing. Two classes of valid inequalities are proposed to strengthen the model which can improve the solution time significantly. In the computational experiments, good solutions could be obtained within a reasonable computation time for large-size instances which are comparable to that of the whole Southeast Asia region.

Even there are lots of excellent studies on traditional sector-based ATFMRP, it's hard for the traditional ATFM system to handle the continuously increasing demand of air transportation industry in the future. In order to upgrade the system, we must design new tools based on the proposals of NextGen to improve the predictability and capacity of current ATM systems. Recently, there is a trend that the gap between ATFM and ATC becomes closer and closer, and that's because integrating the control methodologies of ATC in flow management could enhance the current ATM systems. The issues of airspace capacity, conflict-resolution and fuel consumption could also be improved. In the next chapter, a new ATFM model is proposed to address all these matters.

Chapter 4

A Sequence Model for Air Traffic Flow Management Rerouting Problem

4.1 Introduction

With the rapid growth of the global air transportation industry, the current ATM systems can hardly meet the demand recently. In order to improve the capacity and predictability of the current ATM system, new decision tools must be developed based on the proposals of NextGen. In order to achieve the goal, an ATFM model based on new airspace structures is proposed in this chapter which integrates 4D trajectory-based operations in controlling flights. In the first step, we introduce one type of airspace structure to replace the traditional sector-based structure. One distinguishing feature of the new airspace structure is that it could ensure safety separation between flights under a sequencing mode, and airspace capacity can be improved compared with the traditional sector-based structure. Based on the new airspace structure, an ATFM model is proposed in which the issues of reroutes, ground holds, fuel consumption and cancellations are all addressed on a flight-by-flight basis. In order to solve

the model efficiently, Danzig-Wolfe decomposition is employed to decompose the original mathematical formulation into master problems and subproblems. After that, a distributed heuristic approach via column generation is developed to generate 4D conflict-free trajectories for each flight. By using commercial optimization software, integer solutions could be obtained in 20 minutes for the ATFM rerouting problems of the Southeast Asia region.

4.2 Motivation

Even there are lots of existed excellent studies on traditional sector-based ATFM-RP, the current systems still need to be improved in order to meet the huge demand in the future. One major limitation of the current ATM system is that the sector-based structure is the bottleneck in improving airspace capacity. Even under lower sector capacity, conflicts could still happen and the workload of air traffic controller is high. Another limitation is that the problem of fuel consumption cannot be addressed by the existed sector-based ATFM models. In order to improve these practical issues, there is an urgent need for advanced tools to overcome these limitations.

Under the framework of NextGen, the capacity and predictability of the ATM system would be increased by introducing 4D trajectory-based operations. According to the proposals of NextGen, each individual flight would be assigned a conflict-free 4D trajectory in the future, and in such situations, time windows and capacity would be efficiently utilized to minimize total delay. The workload of air traffic controllers would also be reduced because there are no conflicts between each pair of trajectories.

In order to upgrade the current ATM system, we consider proposing a type of new airspace structure to replace the sector structure and incorporating 4D trajectory-based operations into the ATFM models. The issues of energy-efficiency and conflict-free could also be addressed.

4.3 Proposal

The traditional ATFM system based on sectors is a type of control systems on a macroscopic level, whereas ATC is on the microscopic level. In this chapter, we try to combine them together to obtain the expected benefits. In the first step, we introduce a new airspace structure which could ensure safety separation between flights under a sequencing mode and improve airspace capacity compared with traditional sector-based structure. Based on the new airspace structure, we incorporate 4D trajectory-based operations into ATFM systems. Then, a revised ATFM model is proposed in which operations such as rerouting, ground-holding delay, fuel consumption and cancellations are taken into account on a flight-by-flight basis. The proposals and model assumptions are listed as follows.

- 1) The new airways structure called abstract airway structure replaces the sector-based structure.

One simple demonstration of the new abstract airway structure is shown in Figure 4.1. There are three types of representations of airways in the illustration. The strategic airway between waypoints C and D is only one arc and this structure is employed in the traditional sector-based ATFM models by [Alonso-Ayuso et al. \(2012a,b\)](#), but it is impossible to track the flight levels and locations of the aircraft. Conflict-free cannot be ensured in strategic airway structure also. In the tactical level, there are 3 airways between waypoints C and D which represent practical airways of different directions and flight levels, but we are not able to represent such structure by planar graphs. However, in abstract airway structure, we apply planar graphs to depict the tactical airway structure by applying 2D waypoints. The topological structure of tactical airways could be preserved by such planar graphs. In abstract airway structure, multiple directed arcs between two waypoints are allowed as demonstrated in the figure. The topological structure of 3 practical airways between C and D in the abstract

airway structure is identical to that of the tactical structure.

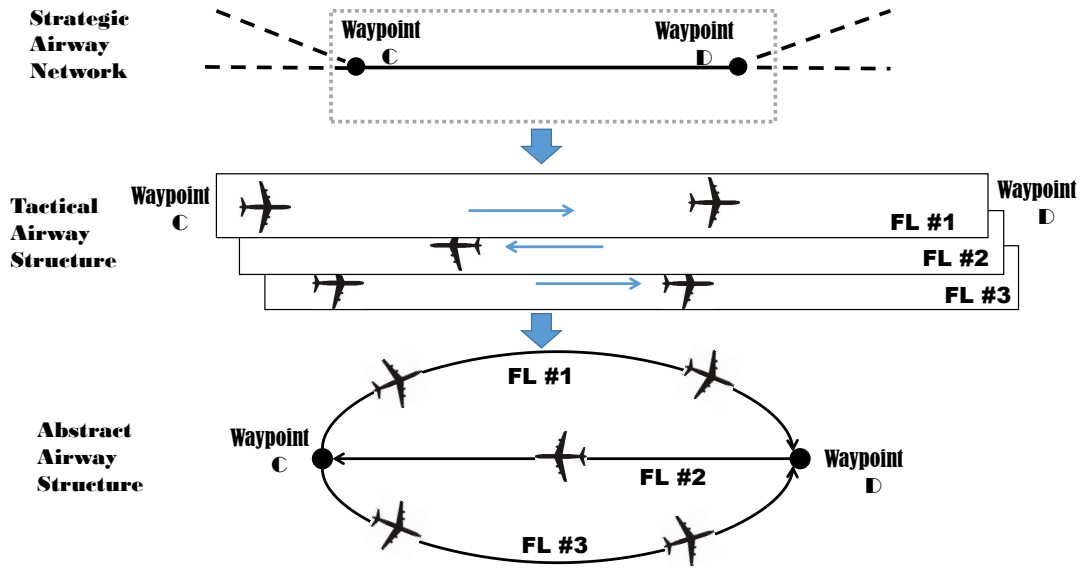


Figure 4.1: Abstract Airway Structure

2) Flights must traverse the airway if they enter that airway, and they are allowed to change airway only at waypoints. This assumption is based on practical situations that flights cannot arbitrarily change flight levels or enter other airways without permissions.

3) Each airway prescribes a unique constant speed for aircraft to traverse, so overtakes cannot happen on airways. Since airways are at different flight levels, the fuel consumption costs for aircraft to traverse different airways are different. Each airway could prescribe the optimal constant speed according to its flight level to control the fuel consumption cost. Another benefit is that safety separation between aircraft on the same airway could be guaranteed just by allocating enter time slots.

Based on 1)-3), the predictability of the current ATM would be improved since information about the positions and speed of all aircraft at any time period

could be obtained by our model.

4.4 The Mathematical Model

In this part, a **Sequence Model for Air Traffic Flow Management Rerouting Problem (SATFMRP)** is developed based on abstract airway structure, and we assume each airway has an optimal constant speed to guarantee safety separation. In SATFMRP, the abstract airway structure includes all information about flight levels, directions and fuel consumption costs. The nodes in the graph represent airports or waypoints. Specifically, there are modifications in the abstract airway graph to depict landing airports as shown in Figure 4.2. We add a dummy node sa_f for the node of landing airport, and between the two nodes, a dummy arc called arrival airway is employed to track the arrival time. The arrival airway has no entrance capacity limitation since it is a virtual airway. In the illustration, (sd_f, a, b, c, d, sa_f) could define an o-d route.

4.4.1 Mathematical Model

Parameters:

\mathcal{F}	set of flights.
\mathcal{A}	set of airways in the system.
$\mathcal{A}^f \subseteq \mathcal{A}$	set of airways for flight f .
\mathcal{T}	set of time intervals.
$[\underline{T}_j^f, \overline{T}_j^f] \equiv T_j^f$	set of feasible time periods for flight f to enter airway j .
\underline{T}_j^f	first time period of the set T_j^f .
\overline{T}_j^f	last time period of the set T_j^f .
\mathcal{S}	set of waypoints and airports.
$\mathcal{S}_f \subseteq \mathcal{S}$	set of waypoints and airports for flight f .

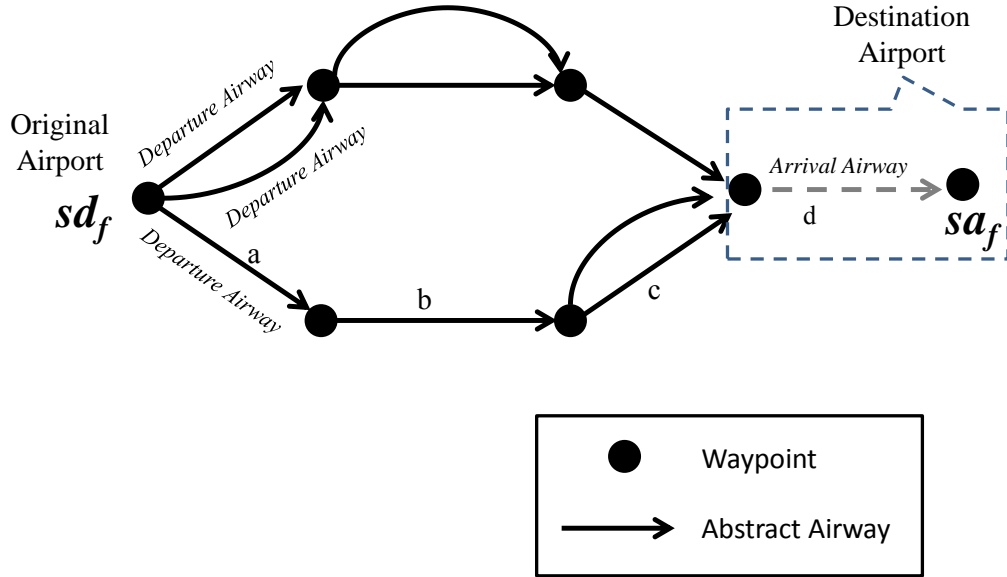


Figure 4.2: Abstract Airway Graph

- $\mathcal{A}_{out(s)}^f$ set of airways (arcs) outbound from waypoint s .
- $\mathcal{A}_{in(s)}^f$ set of airways (arcs) inbound to waypoint s .
- $I_a(t)$ entrance indicator of airway a at time t .
- d_f scheduled departure time for flight f .
- a_f scheduled arrival time for flight f .
- dep_f set of departure airways for flight f .
- arr_f arrival airway for flight f .
- sd_f departure waypoint for flight f .
- sa_f arrival waypoint for flight f .
- l_a the traversing time of airway a .
- C_a^f fuel consumption cost to traverse airway a for flight f .

$C_{td}^f(t)$ total delay cost for flight f to land at time t .
 D^f cancellation cost for flight f .

Airway entrance indicator:

$$I_a(t) = \begin{cases} 1 & \text{If airway } a \text{ is not closed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Decision Variables:

Airway enter variable:

$$e_{a,t}^f = \begin{cases} 1 & \text{If flight } f \text{ enters airway } a \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

For the objective function, we minimize the summation of fuel consumption cost $\sum_{f \in \mathcal{F}} \sum_{a \in \mathcal{A}^f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f$, the total delay cost $\sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_{td}^f(t) e_{arr_f,t}^f$ and the flight cancellation cost $\sum_{f \in \mathcal{F}} (1 - \sum_{t \in \mathcal{T}} e_{arr_f,t}^f) D^f$.

Model:

$$Min \sum_{f \in \mathcal{F}} \sum_{a \in \mathcal{A}^f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f + \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_{td}^f(t) e_{arr_f,t}^f + \sum_{f \in \mathcal{F}} (1 - \sum_{t \in \mathcal{T}} e_{arr_f,t}^f) D^f$$

The Constraints:

$$\sum_{t \in [T_{arr_f}^f, \bar{T}_{arr_f}^f]} e_{arr_f,t}^f = \sum_{t \in [T_{dep_f}^f, \bar{T}_{dep_f}^f]} e_{dep_f,t}^f \leq 1, \quad \forall f \in \mathcal{F}, \quad (4.1)$$

$$\sum_{f \in \mathcal{F}} e_{a,t}^f \leq I_a(t) \quad \forall t \in \mathcal{T}, \quad \forall a \in \mathcal{A}_f, \quad (4.2)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f = \sum_{b \in \mathcal{A}_{in}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f \quad \forall f \in \mathcal{F}, \quad \forall s \in \mathcal{S}_f : s \neq sa_f, sd_f, \quad (4.3)$$

$$\sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_{out}^f(s)} e_{a,t}^f \leq 1 \quad \forall f \in \mathcal{F}, \quad \forall s \in \mathcal{S}_f, s \neq sa_f, \quad (4.4)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} e_{b,t}^f = \sum_{a \in \mathcal{A}_{in}^f(s)} e_{a,t-l_a}^f \quad \forall f \in \mathcal{F}, \quad \forall t \in T, \quad \forall s \in \mathcal{S}_f : s \neq sa_f, sd_f, \quad (4.5)$$

$$e_{a,t}^f \in \{0, 1\}, \quad \forall f \in \mathcal{F}, \quad \forall t \in \mathcal{T}, \quad \forall a \in \mathcal{A}_f. \quad (4.6)$$

Constraint set (4.1) states that any flight could take off or be cancelled. The flight is cancelled when the RHS is 0. Constraint set (4.2) ensures that at most one flight could enter one airway in a time period when the entrance indicator of that airway is 1. Constraint set (4.3) guarantees the network flow balance at each waypoint which means that if a flight enters one airway inbounds to a waypoint, it will enter one subsequent airway outbound from that waypoint. Constraints (4.4) state that at each waypoint, each flight could enter at most one subsequent airway. Constraints (4.5) are the connectivity constraints which ensure after traversing an airway inbound to a waypoint, aircraft enters the subsequent airway outbound from that waypoint immediately.

4.4.2 The Advantage of SATFMRP

In this section, we demonstrate the advantage of SATFMRP in improving system capacity. One simple case (consisting of two homogeneous aircraft, 4 time periods) is employed here.

Parameters are listed as follows:

$$\mathcal{T} = \{1; 2; 3; 4\},$$

$$\mathcal{F} = \{f_1; f_2\},$$

$$\mathcal{A} = (\textit{origin}; \textit{destination}),$$

$$l_a = 2.$$

The scheduled departure time is 1 and arrival time is 3. The airway configuration is shown in Figure 4.3. The length of the airway is 2 and its prescribed constant speed is 1, so the traversing time is 2. The first 1.5 length of the airway is in sector 1, and the rest is in sector 2.

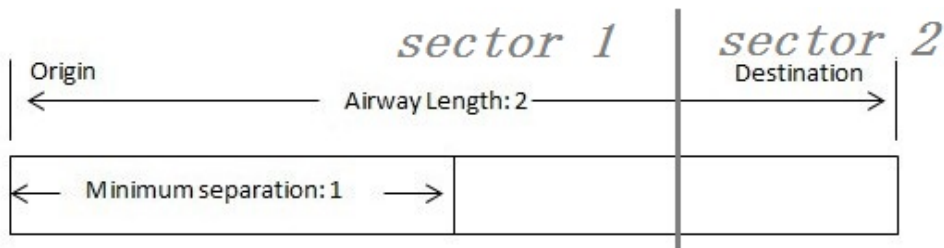


Figure 4.3: Airway Configurations

For traditional sector-based ATFM models, the capacities of sectors 1& 2 cannot exceed 2 under the above airway configurations. The traditional ATFM models only consider the limitations on sector capacity, and if we set the sector capacities greater than 2, two flights would take off and enter the airway at the same time period. A conflict between them occurred as shown in Figure 4.4, so the sector capacity can only be 1.

We set the sector capacity to 1 for the sector-based ATFM model in the following comparison.

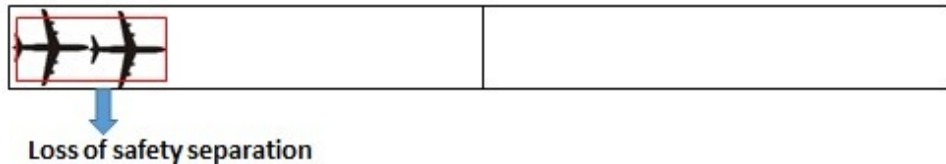


Figure 4.4: Conflict between aircraft

We demonstrate and compare the optimal solutions of both model period by period, A and B are employed to represent traditional sector-based ATFM model and SATFMRP, respectively. At the beginning of time period 1, we demonstrate the optimal solution in Figure 4.5, and there's no difference between optimal solutions of both model. Only one aircraft could depart because of safety separation and the other one must be delayed on the ground.

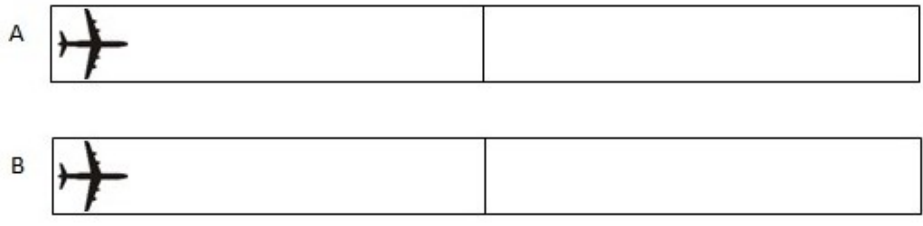


Figure 4.5: Time period 1

In Figure 4.6, one obvious difference is that the delayed aircraft in SATFMRP is taking off at the beginning of time period 2 without loss of safety separation, but the traditional ATFM model still holds the delayed aircraft on the ground.

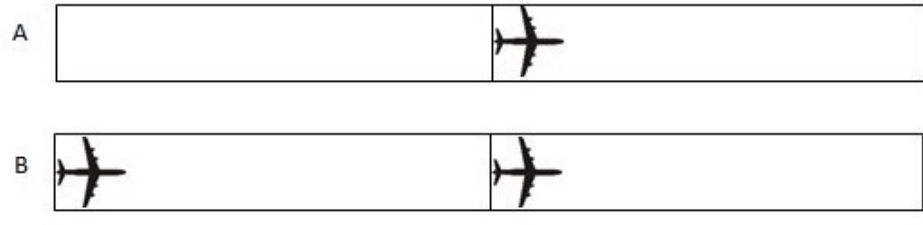


Figure 4.6: Time period 2

In Figure 4.7, it is apparent that the delayed aircraft in traditional ATFM model is taking off at the beginning of time period 3.

From optimal strategy of the traditional ATFM model, we can find that even the aircraft could take off safely, it must be delayed because of the limitations of sector capacity. SATFMRP could overcome the limitation and increase system capacity.

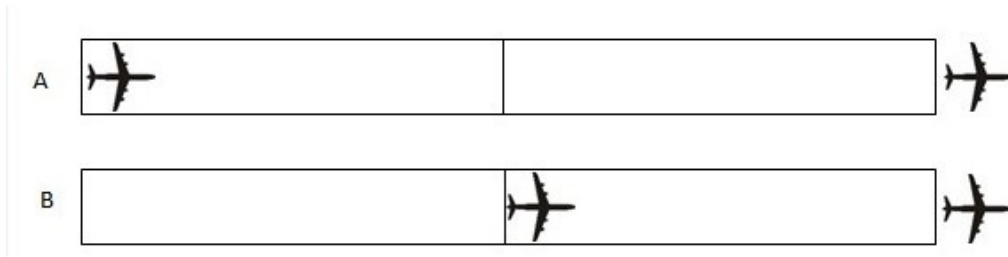


Figure 4.7: Time period 3

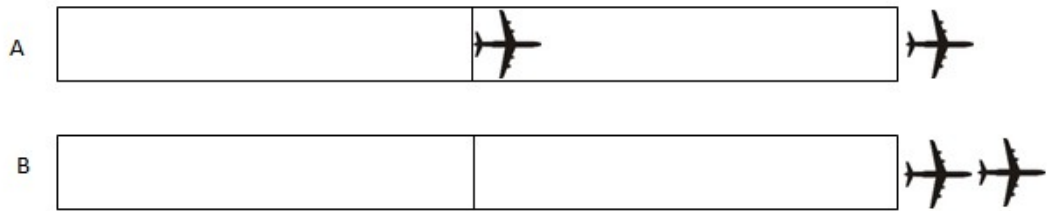


Figure 4.8: Time period 4

4.4.3 Complexity of the SATFMRP

In this section, we prove that the SATFMRP is NP-hard.

Theorem 4.1. *The Sequence Model for Air Traffic Flow Management Problem (SATFMP) with all airway traversing times equal to 1 is NP-hard.*

Proof. Job-Shop Scheduling Problem (JSP) can be reduced to SATFMP.

JSP consists of m processors and a job set J . In each job $j \in J$, there is an ordered collection of tasks $t_k[j]$, $1 \leq k \leq n_j$ (n_j is the number of tasks in job j). For each task t , there is a processing time $l(t) \in \mathbb{Z}_0^+$ ($l(t) = 1$ in this problem) and a processor $p(t) \in \{1, 2, \dots, m\}$. In JSP, $p(t_k[j]) \neq p(t_{k+1}[j]) \forall j \in J, 1 \leq k < n_j$, which means that two consecutive tasks must be processed by different processors. The objective of JSP is to minimize the deadline D which is a positive integer. JSP subjects to the following 3 restrictions: a) once a machine starts processing one task, it must finish this task before processing other ones. b) At any given time period, each machine can only process at most one task. c) The tasks of a job must be processed in the required order.

The decision problem for JSP is that whether there is a possible schedule

meet the overall deadline.

For each job in the set, we create a flight and each airway of our proposed structure corresponds to a processor. For each task $t_k[j]$ of job j , we associate a time segment for flight j to traverse an airway. Furthermore, the starting time of task $t_k[j]$ corresponds to the airway enter time for flight j . Then, we obtain an ordered collection of airways, $(A_j^1, A_j^2, \dots, A_j^k, \dots, A_j^{n_j})$, and an ordered sequence of time segments accordingly, $(t_{A_j}^1, t_{A_j}^2, \dots, t_{A_j}^{n_j})$. For each flight j , the relationship could be described as follows:

$$A_j^1 = p(t_j[1]), t_{A_j}^1 = l(t_j[1]), \dots, A_j^{n_j} = p(t_j[n_j]), t_{A_j}^{n_j} = l(t_j[n_j])$$

If we can find a feasible solution for the SATFMP in which each flight is assigned a conflict-free trajectory and arrives before the deadline D , a corresponding job-shop schedule that meets the required conditions would be obtained.

The starting time of the subsequent task is

$$\sigma_{i'}(t_{k+1}[j]) = \sigma_i(t_k[j]) + l(t_k[j]) = \sigma_i(t_k[j]) + 1 \quad (i' = p(t_{k+1}[j]), i = p(t_k[j])).$$

According to the above relationship, no two tasks would be processed simultaneously on one processor, which is equivalent to limit the entrance capacity of airways to one and no two flights could enter the same airway at the same time period. The relationship also requires that a task is not qualified to be processed unless the previous one has been processed and this ensures the connectivity between airways. Thus, if there is a feasible solution to the SATFMP, a feasible job-shop schedule could be found. \square

Since the traversing time of each airway is not necessarily equal to one in practice, we could convert each long airway into a sequence of short airways with traversing time of one. Thus, we could show that SATFMRP is NP-hard.

4.5 Solution Approach

We have proved SATFMRP is an NP-hard problem, and it is impossible to handle large-scale instances by solving the original model directly with commercial

software. The reason is that the scale of original formulation grows quickly with the number of flights, airways and time periods. Designing heuristic algorithms for large-scale applications is necessary. In this section, we propose an efficient heuristic methodology based on column generation. We first apply Danzig-Wolfe decomposition algorithm to decompose the original formulation into pricing subproblems and a restricted master problem. Each pricing subproblem is responsible for generating trajectories for a specific flight. The master problem takes charges of flight cancellation and conflict resolution. Based on the decomposed formulation, a heuristic column generation algorithm is proposed to obtain the integer solutions for large-scale applications.

4.5.1 Danzig-Wolfe Decomposition

In the original problem, there are an exponentially large number of possible 4D trajectories, and we resort to Danzig-Wolfe decomposition algorithm to reformulate the original problem. The integer restricted master problem which addresses flight cancellation and conflict resolution could be formulated as follows:

Restricted Master Problem

Parameters:

\mathcal{T}	Set of time intervals.
\mathcal{F}	Set of flights.
\mathcal{A}	Set of airways in the system.
$\mathcal{A}^f \subseteq \mathcal{A}$	Set of airways can be chosen by flight f .
\mathcal{R}	Set of 4D trajectories.
$\mathcal{R}_f \subseteq \mathcal{R}$	Set of 4D trajectories can be chosen by flight f .
$\mathcal{R}_a^f(t) \subseteq \mathcal{R}_f$	Set of 4D trajectories that flight f enters airway a at time t .
c_r^f	Delay and fuel consumption cost for flight f to choose trajectory r .

D^f Cancellation cost for flight f .

Airway entrance indicator:

$$I_a(t) = \begin{cases} 1 & \text{If airway } a \text{ is not closed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Decision Variable:

Trajectory selection variable:

$$x_r^f = \begin{cases} 1 & \text{if trajectory } r \text{ is selected by flight } f, \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Min} \sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_f} c_r^f x_r^f + \sum_{f \in \mathcal{F}} (1 - \sum_{r \in \mathcal{R}_f} x_r^f) D^f$$

The Constraints:

$$\sum_{r \in \mathcal{R}_f} x_r^f \leq 1 \quad \forall f \in \mathcal{F}, \quad (4.7)$$

$$\sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_a^f(t)} x_r^f \leq I_a(t) \quad \forall t \in \mathcal{T}, \forall a \in \mathcal{A}_f, \quad (4.8)$$

$$x_r^f \in \{0, 1\}. \quad (4.9)$$

The objective function could be reformulated and the mathematical formu-

lation of the whole model can be expressed as follows,

$$Max \sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_f} (D^f - c_r^f) x_r^f - \sum_{f \in \mathcal{F}} D^f$$

The Constraints:

$$\sum_{r \in \mathcal{R}_f} x_r^f \leq 1 \quad \forall f \in \mathcal{F}, \quad (4.10)$$

$$\sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_a^f(t)} x_r^f \leq I_a(t) \quad \forall t \in \mathcal{T}, \forall a \in \mathcal{A}_f, \quad (4.11)$$

$$x_r^f \in \{0, 1\}. \quad (4.12)$$

Constraints (4.10) state that each flight selects at most one 4D trajectory and if it selects nothing, the flight is cancelled. Constraint set (4.11) ensures all trajectories that are chosen by flights must be conflict-free, which means that any two different chosen trajectories cannot include the same enter time for the same airway. In column generation, we solve a linear programming relaxation of restricted master problem with a subset of the decision variables and apply the dual values from the solution in identifying new columns in subproblems. When there are good columns, we send them to the column set of the restricted master problem. In order to solve the linear programming relaxation of the master problem, we could replace constraint (4.9) by $x_r^f \geq 0, \forall r \in \mathcal{R}_f$. The upper bound is not needed because these 0-1 variables are bounded by constraint (4.10).

Without loss of generality, we assume that the flight cancellation cost D_f is larger than the fuel consumption and delay cost c_r^f of any 4D trajectory. That's because we are willing to give a heavy penalty for flight cancellation. Thus, $D^f - c_r^f$ is a positive number for all 4D trajectories and it can be treated as the profit of choosing a trajectory. The objective function is to maximize the

total profit of choosing 4D trajectories for all flights. We could ignore $\sum_{f \in \mathcal{F}} D^f$ since it is a constant number. In the solving process, the formulation becomes a standard linear optimization model as follows,

$$Max \sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_f} (D^f - c_r^f) x_r^f$$

The Constraints:

$$\sum_{r \in \mathcal{R}_f} x_r^f \leq 1 \quad \forall f \in \mathcal{F}, \quad (4.13)$$

$$\sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_a^f(t)} x_r^f \leq I_a(t) \quad \forall t \in \mathcal{T}, \forall a \in \mathcal{A}_f, \quad (4.14)$$

$$x_r^f \geq 0. \quad (4.15)$$

ϕ_f The dual variables for constraints (4.13).

$\psi_a(t)$ The dual variables for constraints (4.14).

In order to demonstrate the matrix form of restricted master problem, we employ a simple problem. The problem consists of two homogeneous flights with the same flight plans. Their original departure time is 1, and the traversing time of each airway is 2. The airway configurations in Figure 4.9 are listed as follows:

$$\mathcal{T} = \{1, 2, 3, 4\},$$

$$\mathcal{F} = \{f_1, f_2\},$$

$$\mathcal{A} = \{a_1, a_2\},$$

a_3 is the dummy airway for tracking the landing time slots,

\mathcal{R}_{f_1} is in the second column of Table 4.1,

\mathcal{R}_{f_2} is in the third column of Table 4.1.

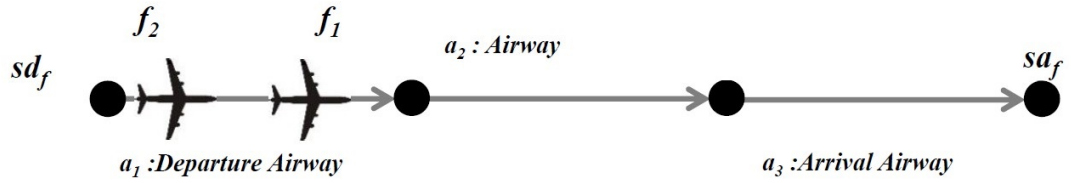


Figure 4.9: Graph Demonstration

The matrix representation of restricted master problem is in Table 4.1

	$x_{r_1}^{f_1}$	$x_{r_2}^{f_2}$	RHS	Dual	Objective Coefficients
	$D^{f_1} - c_{r_1}^{f_1}$	$D^{f_2} - c_{r_2}^{f_2}$			
Flight f_1	1	0	≤ 1	ϕ_{f_1}	Each flight selects at most one trajectory
Flight f_2	0	1	≤ 1	ϕ_{f_2}	
a_1 Time Period 1	1	0	≤ 1	$\psi_{a_1}(t_1)$	Airway entrance capacity
a_1 Time Period 2	0	1	≤ 1	$\psi_{a_1}(t_2)$	
a_1 Time Period 3	0	0	≤ 1	$\psi_{a_1}(t_3)$	
a_1 Time Period 4	0	0	≤ 1	$\psi_{a_1}(t_4)$	
a_2 Time Period 1	0	0	≤ 1	$\psi_{a_2}(t_1)$	
a_2 Time Period 2	0	0	≤ 1	$\psi_{a_2}(t_2)$	
a_2 Time Period 3	1	0	≤ 1	$\psi_{a_2}(t_3)$	
a_2 Time Period 4	0	1	≤ 1	$\psi_{a_2}(t_4)$	
a_3					Dummy Airway

Table 4.1: Matrix representation of the restricted master problem

Corollary 4.1. ϕ_f and $\psi_a(t)$ are nonnegative.

Proof. If we apply the linear programming relaxation to the integer restricted master problem, the formulation could be illustrated as a primal problem with all positive objective coefficients c_i .

Primal Problem:

$$\begin{aligned} & \text{Max} \quad \sum_i c_i x_i \\ \text{s.t.} \quad & \sum_i a_{i,j} x_i \leq 1, \quad \forall j \\ & x_i \geq 0, \quad \forall i \end{aligned}$$

from the duality theory in linear programming, we can obtain the dual problem from the primal problem.

Dual Problem:

$$\begin{aligned} & \text{Min} \quad \sum_j y_j \\ \text{s.t.} \quad & \sum_i a_{i,j} y_j \geq c_i, \quad \forall i \\ & y_j \geq 0, \quad \forall j \end{aligned}$$

The values of y_j in the dual problem correspond to that of ϕ_f and $\psi_a(t)$. \square

4.5.2 Subproblem

In this section, we present the formulation of the subproblem. In the framework of column generation, each flight associates a subproblem to generate 4D trajectories for itself. After the linear programming relaxation of the restricted master problem is solved, the dual values are employed in the subproblems as the coefficients in the objective function. Then, we could solve the subproblems to generate good columns and add them to the column set of the restricted master problem. The subproblem can be formulated as follows:

Parameters:

\mathcal{T}	set of time intervals,
$\mathcal{A}^f \subseteq \mathcal{A}$	set of airways can be chosen by flight f ,
$\mathcal{S}_f \subseteq \mathcal{S}$	set of waypoints by flight f ,
$\mathcal{A}_{out(s)}^f$	set of airways (arcs) outbound from waypoint s ,
$\mathcal{A}_{in(s)}^f$	set of airways (arcs) inbound to waypoint s ,
$[\underline{T}_j^f, \overline{T}_j^f] \equiv T_j^f$	set of feasible time periods for flight f to enter airway j ,
\underline{T}_j^f	first time period in the set T_j^f ,
\overline{T}_j^f	last time period in the set T_j^f ,
$I_a(t)$	entrance indicator of airway a at time t ,
d_f	scheduled departure time of flight f ,
a_f	scheduled arrival time of flight f ,
dep_f	set of departure airways for flight f ,
arr_f	arrival airway for flight f ,
sd_f	departure waypoint for flight f ,
l_a	traversing time of airway a ,
C_a^f	fuel consumption cost of traversing airway a for flight f ,
$C_{id}^f(t)$	total delay cost for flight f to land at time t ,
D^f	cancellation cost for flight f ,
ϕ_f	dual variables for constraints (4.13),
$\psi_a(t)$	dual variables for constraints (4.14).

Decision Variable:

Airway enter variable:

$$e_{a,t}^f = \begin{cases} 1 & \text{flight } f \text{ enters airway } a \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

After the restricted master problem is solved, we get the values of dual variables ϕ_f and $\psi_a(t)$. The increased profit of choosing a 4D trajectory by flight f can be calculated by **Flight cancellation cost - Fuel consumption cost-Arrival delay cost - Dual of constraints (4.13) -Dual of constraint (4.14)**.

The mathematical expression is

$$D_f - \sum_{a \in \mathcal{A}^f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f - \sum_{t \in \mathcal{T}} C_{td}^f(t) e_{arr_f,t}^f - \phi_f - \sum_{a \in \mathcal{A}^f} \sum_{t \in \mathcal{T}} \psi_a(t) e_{a,t}^f$$

The objective of subproblems is to search a 4D trajectory that could maximize the increased profit for each flight. The formulation of subproblem for each flight can be formulated as follows:

$$Max \quad D_f - \sum_{a \in \mathcal{A}^f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f - \sum_{t \in \mathcal{T}} C_{td}^f(t) e_{arr_f,t}^f - \phi_f - \sum_{a \in \mathcal{A}^f} \sum_{t \in \mathcal{T}} \psi_a(t) e_{a,t}^f$$

The Constraints:

$$\sum_{t \in \mathcal{T}} e_{dep_f,t}^f = 1 \quad \forall t \in [\underline{T}_{dep_f}^f, \bar{T}_{dep_f}^f], \quad (4.16)$$

$$\sum_{t \in \mathcal{T}} e_{arr_f,t}^f = 1 \quad \forall t \in [\underline{T}_{arr_f}^f, \bar{T}_{arr_f}^f], \quad (4.17)$$

$$\sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_{out(s)}^f} e_{a,t}^f \leq 1 \quad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}_f : s \neq sa_f, \quad (4.18)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} e_{b,t}^f = \sum_{a \in \mathcal{A}_{in}^f(s)} e_{a,t-l_{fa}}^f \quad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}_f : s \neq sa_f, sd_f, \quad (4.19)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f = \sum_{b \in \mathcal{A}_{in}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f \quad \forall s \in \mathcal{S}_f : s \neq sa_f, sd_f, \quad (4.20)$$

$$e_{a,t}^f \in \{0, 1\}, \forall f \in \mathcal{F}, \forall t \in \mathcal{T}, \forall a \in \mathcal{A}_f. \quad (4.21)$$

The first and second constraint sets require the flight must select feasible time slots for departure and landing. Constraint set (4.18) ensures that at each waypoint, the flight could enter at most one subsequent airway. Constraints (4.19) are the connectivity constraints which ensure that after traversing an airway inbounds to a waypoint, the flight would enter one subsequent airway outbound from that waypoint immediately. Constraint set (4.20) guarantees the network flow balance at each waypoint which means that if the flight enters one airway inbound to a waypoint, it will enter one subsequent airway outbound from that waypoint.

4.5.3 Heuristic Methodology

In this section, we present a heuristic method to handle large-scale instances. In the beginning, the default set of columns (4D trajectories) for the restricted master problem is empty, and we first develop a preprocessing algorithm in Algorithm 1 to generate one trajectory for each flight. In the algorithm, we apply dynamical search method 12.4 (CPLEX-MIP) to solve the integer subproblem since it is already very fast.

Initialization: Sorting the scheduled departure times of all flights in ascending order, and labeling each flight with a number n from 1 to $|F|$ according to the departure order;

Set $n = 1$, $\phi_f = 0$, $\forall f \in F$ and $\psi_a(t) = 0$, $\forall a \in A$, $\forall t \in T$. θ is a large positive integer;

while $n \leq |F|$ **do**

Solve the integer subproblem for flight n by CPLEX and obtain the solution $e_{a,t}^n$;
Pick the solution to the column set of the restricted master problem;
Set $\psi_a(t) = \theta * e_{a,t}^n + \psi_a(t)$;
$n = n + 1$;

end

Algorithm 1: Preprocessing Algorithm

We employ a simple demonstration to show how the preprocessing algorithm works on the problem in Figure 4.9. The demonstration is in Table 4.3 & 4.4. Since two flights have identical departure times, we break the tie arbitrarily and label f_1 and f_2 with numbers 1 and 2, respectively. Then we set $\phi_f = 0$, $\forall f \in F$; $\psi_a(t) = 0$, $\forall a \in A$, $\forall t \in T$; $\theta = 1$. After that, we apply $\psi_a(t)$ as the coefficients of the subproblem to get a trajectory for flight 1 which is in the second column of Table 4.3, and we update the values of $\psi_a(t)$ by $\psi_a(t) = \theta * e_{a,t}^1 + \psi_a(t)$. The new dual values are in the fifth column of Table 4.3. In the next step, the updated values of $\psi_a(t)$ are applied in the subproblem of flight 2. A trajectory for flight 2 is obtained which is listed in the third column of Table 4.4 after solving the subproblem. The final values of $\psi_a(t)$ are updated again which are listed in the fifth column of Table 4.4.

The preprocessing algorithm is based on the insight of First-Departure-First-Schedule (FDFS), and according to Corollary 4.1, all dual variables must be non-negative. With the growth of $\psi_a(t)$, the flights that take off later would avoid entering airway a at time t if $\psi_a(t) > 0$, and that's because it is a positive

	$x_{r_1}^{f_1}$	$x_{r_2}^{f_2}$	RHS	Pseudo Dual
	$D^{f_1} - c_{r_1}^{f_1}$	$D^{f_2} - c_{r_2}^{f_2}$		
Flight f_1	1	0	≤ 1	$\phi_{f_1} = 0$
Flight f_2	0	0	≤ 1	$\phi_{f_2} = 0$
a_1 Time Period 1	1	0	≤ 1	$\psi_{a_1}(t_1) = 1$
a_1 Time Period 2	0	0	≤ 1	$\psi_{a_1}(t_2) = 0$
a_1 Time Period 3	0	0	≤ 1	$\psi_{a_1}(t_3) = 0$
a_1 Time Period 4	0	0	≤ 1	$\psi_{a_1}(t_4) = 0$
a_2 Time Period 1	0	0	≤ 1	$\psi_{a_2}(t_1) = 0$
a_2 Time Period 2	0	0	≤ 1	$\psi_{a_2}(t_2) = 0$
a_2 Time Period 3	1	0	≤ 1	$\psi_{a_2}(t_3) = 1$
a_2 Time Period 4	0	0	≤ 1	$\psi_{a_2}(t_4) = 0$

Table 4.3: Restricted master problem after iteration 1

	$x_{r_1}^{f_1}$	$x_{r_2}^{f_2}$	RHS	Pseudo Dual
	$D^{f_1} - c_{r_1}^{f_1}$	$D^{f_2} - c_{r_2}^{f_2}$		
Flight f_1	1	0	≤ 1	$\phi_{f_1} = 0$
Flight f_2	0	1	≤ 1	$\phi_{f_2} = 0$
a_1 Time Period 1	1	0	≤ 1	$\psi_{a_1}(t_1) = 1$
a_1 Time Period 2	0	1	≤ 1	$\psi_{a_1}(t_2) = 1$
a_1 Time Period 3	0	0	≤ 1	$\psi_{a_1}(t_3) = 0$
a_1 Time Period 4	0	0	≤ 1	$\psi_{a_1}(t_4) = 0$
a_2 Time Period 1	0	0	≤ 1	$\psi_{a_2}(t_1) = 0$
a_2 Time Period 2	0	0	≤ 1	$\psi_{a_2}(t_2) = 0$
a_2 Time Period 3	1	0	≤ 1	$\psi_{a_2}(t_3) = 1$
a_2 Time Period 4	0	1	≤ 1	$\psi_{a_2}(t_4) = 1$

Table 4.4: Restricted master problem after iteration 2

penalty. If θ is large enough, the preprocessing algorithm is FDFS, which means that the initial feasible solution is provided by FDFS. We could implement the preprocessing algorithm again under a random flight sequence to get more columns if necessary.

The flow chart of the whole algorithm is shown in Figure 4.10. After the preprocessing algorithm, the column set of restricted master problem is nonempty. Linear programming relaxation algorithm can be applied to the restricted master problem in calculating the values of dual variables. Then, all subproblems

update the coefficient according to the dual values, and we solve them one by one to search good columns. In the searching step, we set a threshold, and if the objective value of the generated column is greater than the threshold, we send the column to the column set of the restricted master problem for the next iteration. When the optimal value of the linear programming relaxation cannot be improved, we stop the iterations and apply CPLEX to solve the integer version of the restricted master problem by limiting the decision variables to be 0-1.

4.6 Numerical Results

In this section, we report the computational results of the heuristic algorithm proposed in the above section. In the solving process, we employ CPLEX 12.4 as the principle solver, implemented under CPLEX IDE with Optimization Programming Language. All instances are solved on a laptop with an Intel I7-3630 QM processor, 2.40 GHz, 8 GB RAM with a platform of Windows 7. In the process, when we encounter integer subproblems and integer master problem, we apply CPLEX dynamical search method 12.4 (CPLEX-MIP) to handle the problem with the MIP gap 0.0001%.

The first computation test is to show the gaps between the heuristic algorithm's solution and the exact optional solution. The test instances include 15 airways that connect 7 airports, 15 time periods and 30-50 flights. The results are plotted in Figure 4.11, and the solution gap is less than 10%.

The second test in this section is to demonstrate the computational performance with respect to large-scale instances which are capable of reflecting the whole air traffic flow management system of the Southeast Asia. In the experiments, the sector-based airspace structure is replaced by abstract airway structure, and we assume that all flights are homogeneous.

The size of the tested instance depends on the number of flights, discrete time periods, airways and airports. We use simulated data, and all parameters

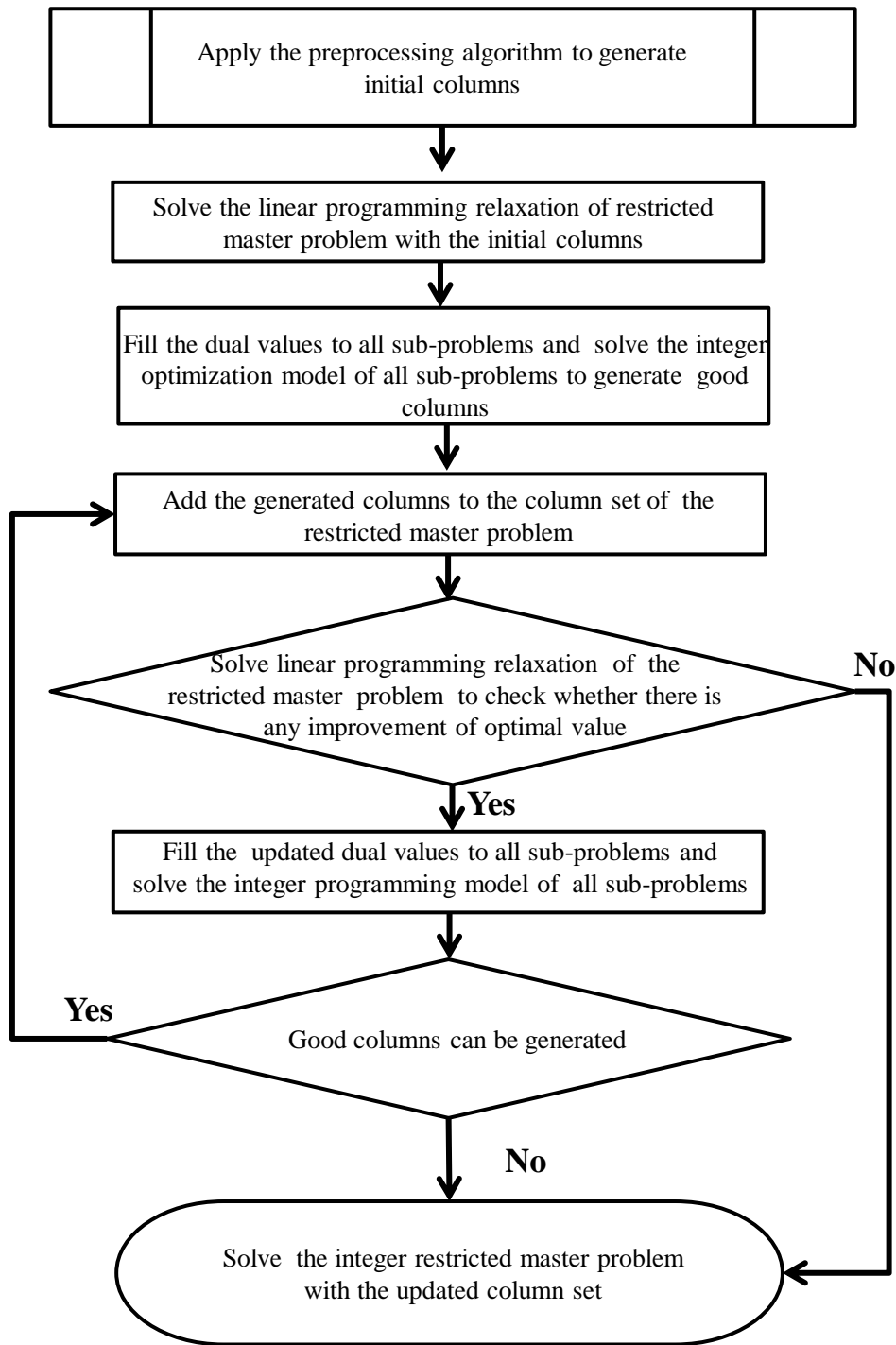


Figure 4.10: Flow Chart of the Heuristic Method

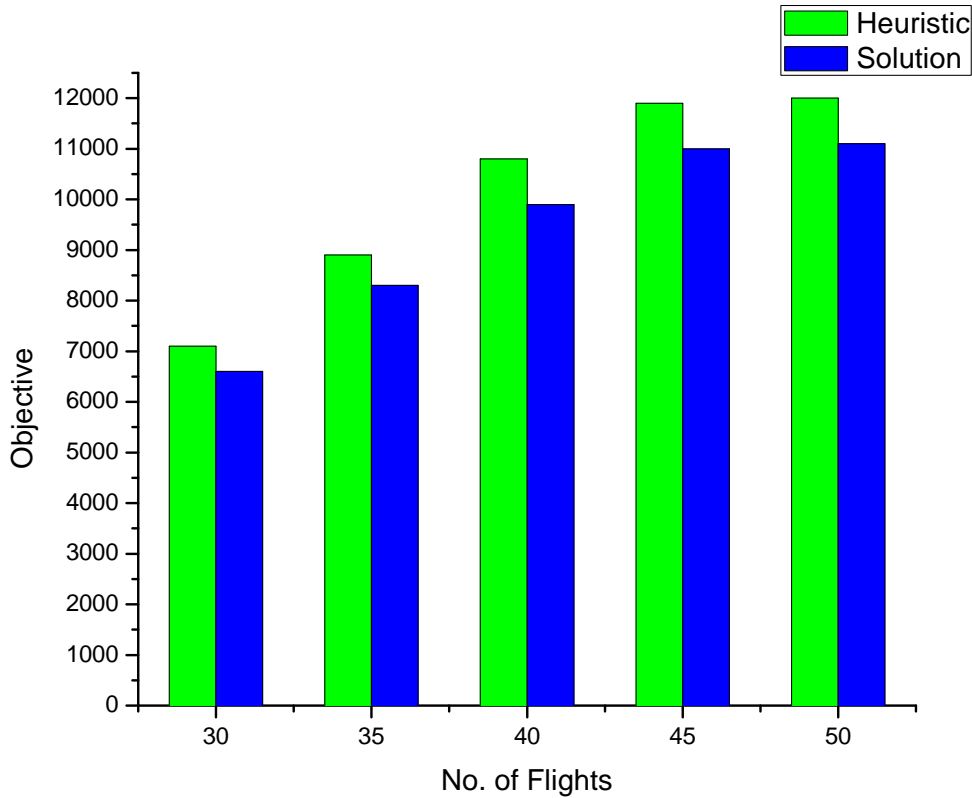


Figure 4.11: Solution Gap

of the instances in this part are based on practical situations of the Southeast Asia region. We consider a five-hour horizon and divide it equally into 60 time periods with 5 minutes each. We also assume all flights must take off and land within 60 periods if they are not cancelled. The instances consist of 300-500 flights, 105 abstract airways, 60 time periods and 7 main international airports of the Southeast Asia region. The 7 airports include Singapore Changi Airport and Hong Kong International Airport. All these airports are located according to their real positions in the test. Flights of the same o-d pairs adopt same airway networks, and the computational results for instances with different numbers of flights is reported in Figure 4.12.

It is evident in Figure 4.12 that CPLEX could solve large-scale instances which include 500 flights in 20 minutes with the heuristic algorithm. So the methodology could be applied to practical applications. The model provides conflict-free flight plans that minimize the combination of fuel consumption,

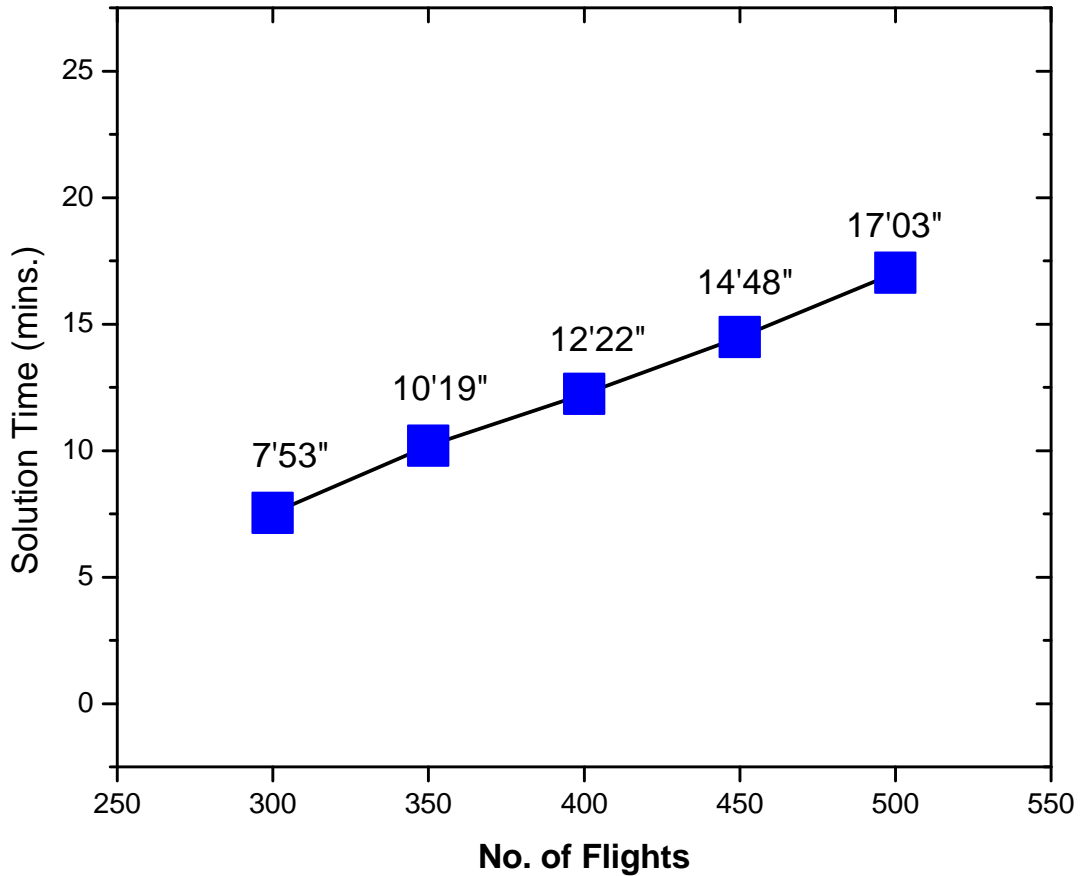


Figure 4.12: Solution Time

delay and cancellation cost for 5 hours. One obvious trend in the figure is that the solution time grows linearly with the number of flights, and it becomes tractable under the framework of the heuristic algorithm. One reason is that the preprocessing algorithm could provide initial solutions of high quality, and the linear programming relaxation of restricted master problem could be solved to optimality very fast. Another reason is that the solution time mainly depends on the number and solution time of the subproblems. We need to solve all integer subproblems and pick good columns into the master problem, so the solution time grows linearly with the number of flights since each flight associates an integer subproblem. When the optimal value of the linear relaxed restricted master problem can no longer be improved, the number of columns for each flight is not very large. CPLEX could solve the integer master problem very quickly

and the solution time could be ignored compared with that of all subproblems. Thus, we can get a conclusion that the solution time mainly depends on the initial solution, solution time of each individual subproblem and the number of subproblems.

4.7 Conclusion

In this chapter, we propose a new sequence model for ATFM RP which has three distinguishing features. The first one is that the model introduced a type of new airspace structure based on real airways to replace the sector-based airspace structure. Under the new airspace structure, a simple case demonstrates that the potential capacity of the ATFM system could be improved. At the same time, the case also shows the sector-based airspace structure plays an insignificant role in supporting the proposals of NextGen. The ATFM models based on new airspace structure could address all phases of a flight from departure to landing. Each flight could be assigned a conflict-free 4D trajectory in the new airspace structure.

The second feature is the excellent computational performance. One critical contribution of this work is that we also proposed a heuristic algorithm that could be employed to handle large-scale instances. The algorithm could guarantee the quality of solutions since the initial solution is provided by FDFS and the solution time grows linearly with the number of flights. The computational time could be controlled within 20 minutes for instances which are capable of reflecting the Southeast Asia ATFM system. The model could track all the details of flights during 5 hours, so it could be solved only 4-5 times during the course of a day with the updated weather information.

The long-term goal of ATFM is to develop advanced decision tools according to the proposal of NextGen. Another feature is that our model could address the issue of fuel consumption, and it minimizes the total fuel consumption cost of

the whole system to achieve energy-efficient which is also the goal of NextGen.

The NextGen motivates our future research in ATFM. One challenging work is to collect more realistic data, and the data includes time periods, abstract paths and airports which could reflect the whole ATM system. Then, we can test the model and refine it according to the real situations. Secondly, we still need to improve the computational performance further and develop more efficient algorithms. In the next step, we can resort to powerful computational equipment and advanced optimization software. The third future work is to propose the stochastic version of ATFM model and design efficient methodology to solve it. Finally, a future direction is to address the issues of fairness in ATFM model in both delay allocation and fuel consumption on a flight-by-flight basis.

Chapter 5

Conclusion

In this chapter, contributions of this thesis will be pointed out, and we also provide some directions for the future research in ATFM.

5.1 Summary

Our aspiration in this thesis is to provide a framework of integer optimization approaches to address three critical issues of the current ATFM system in order to cover the research gaps, namely:

- 1. Flexible Rerouting.** For sector-based ATFM model, we apply flexible rerouting operations based on unrestricted graphs to handle practical problems on a flight-by-flight basis. In some situations, the set of origin-destination routes for flights must be represented by directed graphs with cycles, but the existing disaggregated ATFM models could only provide rerouting strategy via acyclic graphs. It is possible to miss the global optimal solution if we apply these approaches. Unrestricted graph is a type of general structure that allows more complex o-d routes to be theoretically represented. The main contribution of our sector-based ATFM model is that it permits the existence of unrestricted directed graphs to make the rerouting operations more flexible.

- 2. Airspace Structure with Large Capacity.** Subsequently, we make

further improvements on the ATFM system that a type of new airspace structure based on the real airways is introduced to replace the sector-based structure. Under the framework of the new airspace structure, we propose a new ATFM model based on a sequencing mode to improve the system capacity and predictability. A simple problem demonstrates that the capacity of the ATFM system could be increased by employing the new airspace structure. At the same time, the problem also shows that the sector capacity constraints play an insignificant role in exploring the system capacity to support the NextGen. Since the new airspace structure keeps the topological property of the real airways, the issues of fuel consumption could also be addressed, and the model could minimize the total fuel consumption cost of the whole system to achieve energy-efficient.

3. Efficient Solution Methodologies. ATFM problems are proved to be NP-hard, so it is necessary to develop efficient solution methodologies to reduce the solution time. In our first model, we proposed two types of strong valid inequalities based on the structure of unrestricted directed graphs. The computational test demonstrates that the solution time could be reduced significantly by adding these inequalities. The computation time could be controlled within 5 minutes for instances which are comparable to the whole Southeast Asia ATFM system. For the second ATFM model, we design a heuristic algorithm under the framework of column generation since it is hard to solve the original model directly. We first apply Danzig-Wolfe decomposition to decompose the original formulation into a restricted master problem and many pricing subproblems. The master problem is responsible for resolution of conflicts, and each subproblem takes charge of generating 4D trajectories for a specific flight. Then we apply a heuristic column generation algorithm to get integer solution, and the solution quality is excellent. One amazing fact is that the solution time grows linearly with the number of flights and it could be controlled within 20 minutes for instances which could reflect that of the region of the Southeast Asia. The

model could track all flights for 5 hours so that it could be employed at most five times with the updated weather information in a whole day.

5.2 Directions for Future Research

In the future, more and more challenges and opportunities would arise with the continuous growth of air transportation industry. In this section, we conclude some directions for future research which build on the ATFM models in this dissertation.

5.2.1 Data Uncertainty

First and foremost, one simple assumption in our ATFM models is that all data is known in advance without uncertainty, so our models only address deterministic situations. This assumption is reasonable if the weather information is precise enough and no irregular operations happen on flights. In practical situations, the precision of weather forecast is not enough and many possible scenarios may happen. As far as we know, even minor weather disturbances may change the safety separation standard of the airways or shut down airways for a long time period. It is necessary to consider rerouting problems in the presence of airway entrance uncertainty in the future. Thus, a stochastic version of ATFM model which provides rerouting plans under airway entrance uncertainty would be appealing from a research standpoint. Both robust and stochastic optimization approaches could be employed in handling all these uncertainties.

5.2.2 Fairness

Fairness issues on how to allocate all types of costs on a flight-by-flight basis are very critical in the future. Recently, there is an urgent need for advanced metrics in minimizing the difference of ground-holding delay allocation on a

flight-by-flight basis. In order to avoid delay propagation under severe congestions, the existed ATFM models prefer to allocate more ground-holding delays to the flights without connected subsequent flights. However, this strategy is impractical because it associates with equity issues that airlines with less connected flights would suffer larger delay cost. How to allocate fuel consumption cost is another important fairness issue since rerouting may incur additional cost for aircraft. In the next step, designing a suitable metric for ATFM models which could distribute fuel consumption cost equally among flights that compete for the same airway would be appealing.

5.2.3 Dynamic Airway Configurations

In the future ATM systems, the system capacity should become more and more robust under weather uncertainty. In order to achieve this goal, one approach is that the airway configurations could be changed dynamically with the updated weather information. The idea is that the air traffic control center could generate new airways and close some old airways according to the weather forecast. Dynamic airway configurations are appealing from a practical standpoint, and since new airways could be opened to replace the closed airways, the system capacity would not be so sensitive with respect to weather conditions.

Bibliography

- Agustin, A., Alonso-Ayuso, A., Escudero, L. F., and Pizarro, C. (2010). Mathematical optimization models for air traffic flow management: a review. *Studia Informatica Universalis*, 8(2):141–184.
- Alonso, A., Escudero, L. F., and Teresa Ortuno, M. (2000). A stochastic 0-1 program based approach for the air traffic flow management problem. *European Journal of Operational Research*, 120(1):47–62.
- Alonso-Ayuso, A., Escudero, L., Olasso, P., and Pizarro, C. (2013). Conflict avoidance: 0-1 linear models for conflict detection and resolution. *Top*, 21(3):485–504.
- Alonso-Ayuso, A., Escudero, L., and Pizarro, C. (2012a). On air traffic flow management with rerouting. part i: Deterministic case. *European Journal of Operational Research*, 219(1):156–166.
- Alonso-Ayuso, A., Escudero, L., and Pizarro, C. (2012b). On air traffic flow management with rerouting. part ii: Stochastic case. *European Journal of Operational Research*, 219(1):167–177.
- Alonso-Ayuso, A., Escudero, L. F., and Martin-Campo, F. J. (2011). Collision avoidance in air traffic management: A mixed-integer linear optimization approach. *Intelligent Transportation Systems, IEEE Transactions on*, 12(1):47–57.

- Alonso-Ayuso, A., Escudero, L. F., and Martin-Campo, F. J. (2012c). A mixed 0-1 nonlinear optimization model and algorithmic approach for the collision avoidance in atm: Velocity changes through a time horizon. *Computers and Operations Research*, 39(12):3136–3146.
- Andersson, K., Hall, W., Atkins, S., and Feron, E. (2003). Optimization-based analysis of collaborative airport arrival planning. *Transportation Science*, 37(4):422–433.
- Andreatta, G. and Brunetta, L. (1998). Multiairport ground holding problem: A computational evaluation of exact algorithms. *Operations Research*, 46(1):57–64.
- Andreatta, G., Brunetta, L., and Guastalla, G. (2000). From ground holding to free flight: An exact approach. *Transportation Science*, 34(4):394–401.
- Andreatta, G., Dell’Olmo, P., and Lulli, G. (2011). An aggregate stochastic programming model for air traffic flow management. *European journal of operational research*, 215(3):697–704.
- Bachmat, E., Berend, D., Sapir, L., Skiena, S., and Stolyarov, N. (2009). Analysis of airplane boarding times. *Operations research*, 57(2):499–513.
- Ball, M. O., Hoffman, R., and Mukherjee, A. (2010). Ground delay program planning under uncertainty based on the ration-by-distance principle. *Transportation Science*, 44(1):1–14.
- Ball, M. O., Hoffman, R., Odoni, A. R., and Rifkin, R. (2003). A stochastic integer program with dual network structure and its application to the ground-holding problem. *Operations Research*, 51(1):167–171.
- Barnett, A. (2000). Free-flight and en route air safety: a first-order analysis. *Operations research*, 48(6):833–845.

- Barnhart, C., Belobaba, P., and Odoni, A. R. (2003). Applications of operations research in the air transport industry. *Transportation science*, 37(4):368–391.
- Barnhart, C., Bertsimas, D., Caramanis, C., and Fearing, D. (2012). Equitable and efficient coordination in traffic flow management. *Transportation Science*, 46(2):262–280.
- Belobaba, P., Odoni, A., and Barnhart, C. (2009). *The global airline industry*, volume 23. John Wiley and Sons.
- Bertsimas, D., Frankovich, M., and Odoni, A. (2011a). Optimal selection of airport runway configurations. *Operations research*, 59(6):1407–1419.
- Bertsimas, D. and Gupta, S. (2009). Fairness in air traffic flow management. In *INFORMS Meeting, San Diego, CA, Usa*, volume 32, page 7.
- Bertsimas, D. and Gupta, S. (2015). Fairness and collaboration in network air traffic flow management: An optimization approach. *Transportation Science*, 50(1):57–76.
- Bertsimas, D., Lulli, G., and Odoni, A. (2011b). An integer optimization approach to large-scale air traffic flow management. *Operations Research*, 59(1):211–227.
- Bertsimas, D. and Patterson, S. S. (1998). The air traffic flow management problem with enroute capacities. *Operations research*, 46(3):406–422.
- Bertsimas, D. and Patterson, S. S. (2000). The traffic flow management rerouting problem in air traffic control: A dynamic network flow approach. *Transportation Science*, 34(3):239–255.
- Bureau of Transportation Statistics (2014). On Time Performance.

- Castelli, L., Pellegrini, P., and Pesenti, R. (2012). Airport slot allocation in europe: economic efficiency and fairness. *International journal of revenue management*, 6(1):28–44.
- Castelli, L., Pesenti, R., and Ranieri, A. (2011). The design of a market mechanism to allocate air traffic flow management slots. *Transportation Research Part C: Emerging Technologies*, 19(5):931–943.
- Churchill, A. M. and Lovell, D. J. (2012). Coordinated aviation network resource allocation under uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):19–33.
- Churchill, A. M., Lovell, D. J., and Ball, M. O. (2009). Evaluating a new formulation for large-scale traffic flow management. In *Proceedings of the 8th USA/Europe Air Traffic Seminar (ATM09)*.
- Clare, G. and Richards, A. (2012). Air traffic flow management under uncertainty: application of chance constraints. In *Proceedings of the 2nd International Conference on Application and Theory of Automation in Command and Control Systems*, pages 20–26. IRIT Press.
- Clarke, J.-P., Lowther, M., Ren, L., Singhose, W., Solak, S., Vela, A., and Wong, L. (2008). En route traffic optimization to reduce environmental impact. *PARTNER Project*, 5.
- Clarke, J.-P. B., Solak, S., Ren, L., and Vela, A. E. (2012). Determining stochastic airspace capacity for air traffic flow management. *Transportation Science*, 47(4):542–559.
- Dell’Olmo, P. and Lulli, G. (2003). A new hierarchical architecture for air traffic management: Optimisation of airway capacity in a free flight scenario. *European Journal of Operational Research*, 144(1):179–193.

- FAA (2013). Aerospace forecast: Fiscal years 2013-2033. *US Department of Transportation Federal Aviation Administration Aviation Policy and Plans*.
- Glover, C. N. and Ball, M. O. (2013). Stochastic optimization models for ground delay program planning with equity-efficiency tradeoffs. *Transportation Research Part C: Emerging Technologies*.
- Hoffman, R., Hall, W., Ball, M., Odoni, A., and Wambsganss, M. (1999). Collaborative decision making in air traffic flow management. *manuscript in preparation*.
- Kotnyek, B. and Richetta, O. (2006). Equitable models for the stochastic ground-holding problem under collaborative decision making. *Transportation Science*, 40(2):133–146.
- Kuchar, J. K. and Yang, L. C. (2000). A review of conflict detection and resolution modeling methods. *Intelligent Transportation Systems, IEEE Transactions on*, 1(4):179–189.
- Liang, Z., Chaovalitwongse, W. A., and Elsayed, E. A. (2013). Sequence assignment model for the flight conflict resolution problem. *Transportation Science*, 48(3):334–350.
- Lulli, G. and Odoni, A. (2007). The european air traffic flow management problem. *Transportation Science*, 41(4):431–443.
- Manley, B. and Sherry, L. (2010). Analysis of performance and equity in ground delay programs. *Transportation Research Part C: Emerging Technologies*, 18(6):910–920.
- Mesgarpour, M., Potts, C. N., and Bennell, J. A. (2010). Models for aircraft landing optimization. In *Proceedings of the 4th international conference on research in air transportation (ICRAT2010)*.

- Mukherjee, A. (2004). *Dynamic stochastic optimization models for air traffic flow management*. PhD thesis, University of California Berkeley.
- Mukherjee, A. and Hansen, M. (2007). A dynamic stochastic model for the single airport ground holding problem. *Transportation Science*, 41(4):444–456.
- Mukherjee, A. and Hansen, M. (2009). A dynamic rerouting model for air traffic flow management. *Transportation Research Part B: Methodological*, 43(1):159–171.
- Mukherjee, A., Hansen, M., and Grabbe, S. (2009). Ground delay program planning under uncertainty in airport capacity. *Transportation Planning and Technology*, 35(6):611–628.
- Netjasov, F. (2012a). Framework for airspace planning and design based on conflict risk assessment: Part 1: Conflict risk assessment model for airspace strategic planning. *Transportation research part C: emerging technologies*, 24:190–212.
- Netjasov, F. (2012b). Framework for airspace planning and design based on conflict risk assessment: Part 2: Conflict risk assessment model for airspace tactical planning. *Transportation research part C: emerging technologies*, 24:213–226.
- Netjasov, F. (2013). Framework for airspace planning and design based on conflict risk assessment: Part 3: Conflict risk assessment model for airspace operational and current day planning. *Transportation Research Part C: Emerging Technologies*, 32:31–47.
- Odoni, A. R. (1987). The flow management problem in air traffic control. *Flow control of congested networks*, Springer, pages 269–288.
- Pallottino, L., Feron, E. M., and Bicchi, A. (2002). Conflict resolution problems

- for air traffic management systems solved with mixed integer programming. *Intelligent Transportation Systems, IEEE Transactions on*, 3(1):3–11.
- Prot, D., Rapine, C., Constans, S., and Fondacci, R. (2010). Using graph concepts to assess the feasibility of a sequenced air traffic flow with low conflict rate. *European Journal of Operational Research*, 207(1):184–196.
- Richards, A. and How, J. P. (2002). Aircraft trajectory planning with collision avoidance using mixed integer linear programming. In *American Control Conference, 2002. Proceedings of the 2002*, volume 3, pages 1936–1941. IEEE.
- Richetta, O. and Odoni, A. R. (1993). Solving optimally the static ground-holding policy problem in air traffic control. *Transportation Science*, 27(3):228–238.
- Richetta, O. and Odoni, A. R. (1994). Dynamic solution to the ground-holding problem in air traffic control. *Transportation Research Part A: Policy and Practice*, 28(3):167–185.
- Ruiz, S., Piera, M. A., and Del Pozo, I. (2013). A medium term conflict detection and resolution system for terminal maneuvering area based on spatial data structures and 4d trajectories. *Transportation Research Part C: Emerging Technologies*, 26:396–417.
- Sherali, H. D., Hill, J. M., McCrea, M. V., and Trani, A. A. (2011). Integrating slot exchange, safety, capacity, and equity mechanisms within an airspace flow program. *Transportation Science*, 45(2):271–284.
- Sherali, H. D., Smith, J. C., Trani, A. A., and Sale, S. (2000). National airspace sector occupancy and conflict analysis models for evaluating scenarios under the free-flight paradigm. *Transportation science*, 34(4):321–336.
- Sherali, H. D., Staats, R. W., and Trani, A. A. (2003). An airspace planning and

- collaborative decision-making model: Part 1-probabilistic conflicts, workload, and equity considerations. *Transportation Science*, 37(4):434–456.
- Sherali, H. D., Staats, R. W., and Trani, A. A. (2006). An airspace-planning and collaborative decision-making model: Part 2-cost model, data considerations, and computations. *Transportation Science*, 40(2):147–164.
- Sohoni, M., Lee, Y.-C., and Klabjan, D. (2011). Robust airline scheduling under block-time uncertainty. *Transportation Science*, 45(4):451–464.
- Terrab, M. and Odoni, A. R. (1993). Strategic flow management for air traffic control. *Operations Research*, 41(1):138–152.
- Vela, A., Solak, S., Singhose, W., and Clarke, J.-P. (2009a). A mixed integer program for flight-level assignment and speed control for conflict resolution. In *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pages 5219–5226.
- Vela, A. E., Salaun, E., Solak, S., Feron, E., Singhose, W., and Clarke, J.-P. (2009b). A two-stage stochastic optimization model for air traffic conflict resolution under wind uncertainty. In *Digital Avionics Systems Conference, 2009. DASC'09. IEEE/AIAA 28th*, pages 2. E. 5–1–2. E. 5–13.
- Vossen, T. and Ball, M. (2002). *Fair allocation concepts in air traffic management*. PhD thesis, University of Maryland, College Park, Md.
- Vossen, T. W. and Ball, M. O. (2006). Slot trading opportunities in collaborative ground delay programs. *Transportation Science*, 40(1):29–43.
- Vranas, P. B., Bertsimas, D., and Odoni, A. R. (1994a). Dynamic ground-holding policies for a network of airports. *Transportation Science*, 28(4):275–291.

Vranas, P. B., Bertsimas, D. J., and Odoni, A. R. (1994b). The multi-airport ground-holding problem in air traffic control. *Operations Research*, 42(2):249–261.