

# Compressive Sampling Methods for Superresolution Imaging

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## ABSTRACT

We investigate superresolution imaging using negative index metamaterials. Measurement of subwavelength scale features in the image domain is tedious and compressive sampling techniques are considered to alleviate this problem. A single detector (c.f. a single pixel camera geometry) is considered from which a high resolution image can be computed, which makes use of structured illumination for coding.

**Keywords:** Compressive sensing, superresolution, coded aperture, inverse scattering algorithm

## 1. INTRODUCTION

The suggestion by Pendry [1] that a slab of negative index material could transfer arbitrarily high spatial frequencies generated a considerable amount of excitement and controversy. There is no expectation that medium can be made with a perfectly uniform refractive index -1 but it remains a challenge for many to see how much improvement in resolution might be possible. A metamaterial with a negative index is expected to be narrow band and generally lossy which will truncate the transfer function, but an image with a resolution of  $\sim \lambda/5$  would be useful in many applications. The slab design may not be optimal and alternatives, include hyperlens systems which can convert evanescent waves to propagating waves have been widely studied [e.g 2]. In all cases, a metamaterial couples high  $k$  information from the object domain to a measurement domain. The object being imaged must therefore be located in the near field of the negative index material. Exponentially decaying waves carrying high spatial frequencies are coupled through the negative index medium and resonantly enhanced. This is illustrated in figure 1. We note that this could take a relatively long time, the higher the resolution we hope to achieve [3].

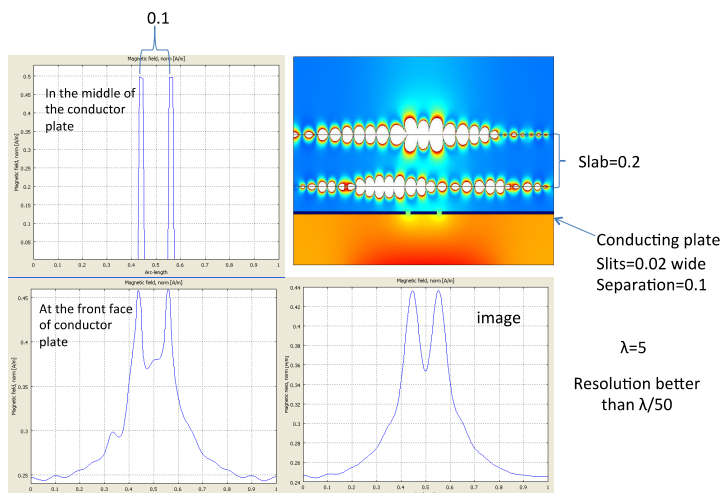


Figure 1: field intensities in the object plane, immediately behind the object plane and in the image plane. The image of the field exiting the object is reasonably well recovered, and the large amplitude surface waves can be seen on the front and back of the negative index slab.

Far field scattering is restricted to wave vectors  $k < k_0$  while near field scattering contains higher  $k$ , i.e.  $k > k_0$ . This obviously dictates that some mechanism on the opposite side of the metamaterial must probe the field in order to acquire the subwavelength scale information it possesses or it will be quickly lost. The alternative is to convert this evanescent information into propagating waves, which could comingle them with the propagating waves from the object, making image interpretation difficult. We consider the former in this paper.

The difficulties in making a high resolution imaging system based on a negative index slab are many. The first, as mentioned above, is that making a metamaterial with a low loss that is capable of transferring evanescent waves into the image domain, has still to be convincingly demonstrated. Apart from early work in the uv, [4], which showed some promise using a thin silver film, realizing wave evanescent transfer at lower frequencies is plagued by metamaterials design and fabrication problems. An excellent review of the limitations of metamaterials superlenses was recently published by Liu and Alu [5]. They point out that image resolution is very sensitive to the granularity of the meta-atoms used and the finite dimensions and optical isotropy of the array. For any finite sized array, the arrangement of the meta-atoms and their mutual coupling and scattering between them, quite apart from losses, can make designing an effective homogenized refractive index close to the desired value of  $n = -1$  very difficult, [6,7]. Recognizing that the granularity and scattering from the metamaterial modifies the image field is not a fatal limitation of this approach. Recent work based on computational image reconstruction from knowledge of the transfer matrix of the metamaterial lens offers a means to address this, in principle [8]. Indeed there is much to be said for simply replacing the metamaterial with a random medium and characterizing the T-matrix of that medium, recognizing that increased scatter alone can improve image resolution, [9]. An important second problem that becomes clear from addressing this problem, is that one has to regard the object, metamaterial lens and the detector as a coupled system, because each is in the near field of the other [10]. A simple negative index slab treated in isolation is not going to provide high resolution fields in the image domain and care is required in estimating its actual transfer function. Evanescent wave transfer will also lead to evanescent wave coupling back and forth between scattering features in the lens, the object and the detector. For similar reasons, meta-atom periodicity and lens surface roughness will also contribute to the generation and scattering of evanescent waves [e.g. 11]. It is possible that some degree of roughness can enhance the coupling of evanescent waves from the object while suppressing the surface waves associated with their resonant enhancement to some extent, [11].

A more serious problem, only briefly mentioned here, is the fact that most objects of interest are not encoded in very thin perfectly conducting or perfectly absorbing masks, as shown in figure 1; real objects have some thickness and it is desirable to be able to acquire a superresolved image in 3D. As higher resolutions are sought, one can expect a penetrable 3D object to become increasingly strongly scattering, with evanescent waves playing a role in the multiple scattering that occurs. This confuses the interpretation of the image and an inverse scattering algorithm needs to be applied in order to recover from the superresolved measured field, an estimate of the fine structure of the physical object. This problem is the focus of a companion paper in this meeting ([12]). An example of this problem is shown in figure 2.

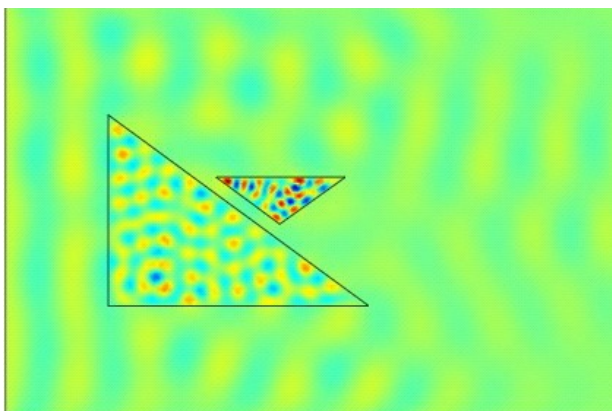


Figure 2: illuminating two transparent objects as shown, with a plane wave, leads to a complicated pattern for the total field both inside and outside these structures. Even assuming a perfect lens can replicate the object's field pattern, one still has to deduce from the fields alone in the image domain, what the refractive index distribution of the objects is. For simple objects such as these, the problem is not too difficult.

Moreover, object features closer to the front face of the lens will generate evanescent waves that have to be amplified more than those closer to the source. This follows since the amplitude of these high spatial frequencies, represent by the same exponentially decaying evanescent wave in the image domain, has to have the correct amplitude to restore those high resolution object features. Consequently, in the image domain, very large fields between the exit face of the lens and the image of those features, can be much larger than fields representing the rest of the object. This is illustrated in figure 3.

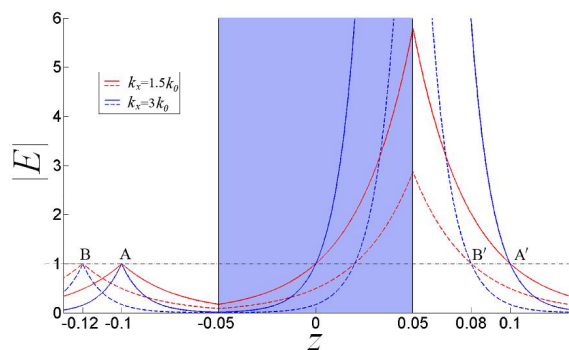


Figure 3: evanescent field masking shown here for two distinct spatial frequencies originating from two spatially separated points in the object, A and B. If the lens is capable of correctly transferring these spatial frequencies into their correct locations in the image domain, then necessarily fields contributing at A' will swamp those contributing at B'.

We argue that there is little point in struggling to exploit this mode of superresolving imaging unless it clear offers some advantages. The potential for very high subwavelength-scale resolution is there, but we have cited a large number of difficulties that have to be overcome. In addition, there is the argument that if both the object and the detector need to be in the near field, and if the metamaterial lens is likely to be lossy, then what tangible advantage does this system have over a more conventional scanning probe imaging technique. The possibility of getting high resolution 3D images makes this approach compelling, but to date, procedures to do this have required extremely high quality (i.e. low noise) field measurements, for the reasons explained in figure 3, (see e.g. [13]). We therefore explore the possibility of simplifying the system, especially the data acquisition, in order to avoid any detector scanning, in order to compute a high resolution image of the field in the image domain. We propose to fix a single detector at the back focal plane of a focusing lens and illuminate the object with a set of different incident field patterns, i.e. employ structured wave illumination. We explain this in the next section. It is important to note that this does not avoid one important problem. The detector is still in the near field of the lens and is hence coupled to the lens and hence in turn to the object itself. By fixing its location, the consequences of this coupling should be quantifiable and correctable when calculating the high resolution image of the object. Just as an object's proximity to a dipole source can affect far field scattering [14], the presence of a detector in a complex field of propagating and evanescent waves can be modeled in order to determine its influence [15].

## 2. COMPRESSIVE SAMPLING CONCEPT

We assume that in the future the transfer of evanescent waves (i.e. spatial frequencies for which  $k > k_0$ ) will be possible through the use of practical metamaterials for high resolution imaging. The specific design of the metamaterial and how it couples to the object and the detector will be exploited to manage the transferred bandwidth, i.e. the transfer function. We have noted that due to the inevitable finite size of the meta-atoms comprising any metamaterial, one can regard the metamaterial's physical structure imprinting its own complex scattered field characteristics on the superresolved image. This corresponds to a spatial signature determined by the specific configuration of the meta-atoms intercepting the scattered field from the object. Using an example of an object consisting of two or three subwavelength sized discrete objects, we evaluate the projection of this object distribution function onto a series of incident wave patterns. This is analogous to the approach presented in the so-called single pixel camera first announced in 2006 by the Rice group, [16], see figure 4. In the same way as one can integrate the product of a signal function with a specific complex exponential function and obtain a number we call a Fourier coefficient, we can integrate an object function with a basis function or code to obtain a coefficient. Ideally the patterns or codes employed are in some sense optimized to reduce the number of

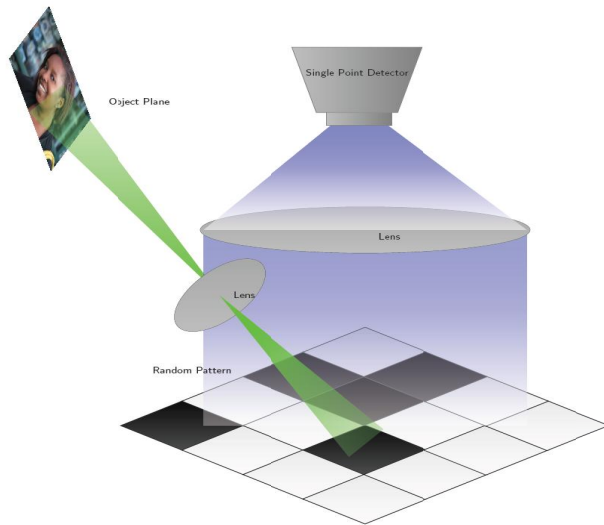


Figure 4: schematic of concept behind single pixel camera. An input scene is images onto a digital mirror device (DMD) which cycles through several different codes. The reflected light from the entire array is imaged onto a single point detector, giving a coefficient.

their projections onto the object that are necessary to adequately represent it. Such codes would normally have to meet the requirements of a formal basis set and be orthogonal and complete. A judicious choice of codes allows a reconstruction of the object, i.e. an image of that object, from a weighted sum of those same basis functions, the weights for each basis being the coefficient derived from the detector's output. This, a point detector located in the image plane measures a coefficient representing the projection of the incident field onto the object, resulting in a scattered field that is integrated to give a single coefficient. Compressive sampling by exploiting projections of scattered and evanescent waves onto the metamaterial suggest a single pixel superresolving "camera" might be possible. Compressive methods applied to imaging scattered fields has been described in the literature, [17]. Our assumptions will be that the scattering is well approximated by the first Born approximation (i.e. weak scattering) which we have already argued is not likely as one strives for higher and higher resolution. Compressive methods were applied within the range of validity of the Rytov approximation, which requires object's permittivities to vary slowly, spatially, on the scale of the wavelength [16]. Also, random sparse measurements have been made in the near field of rough surfaces, with the intent to image surface corrugations [18].

An ideal perfect  $n=-1$  slab lens can in principle perfectly image a field distribution in one object plane to an associated image plane a distance  $2d$  away, for a lens of thickness  $d$ . The low loss negative index material is essential for successful transfer of evanescent waves to the image domain, which encode higher spatial frequencies. Using a negative index material to integrate a lightfield in a similar way to the convex lens in figure 4, requires the fabrication of a concave lens, as illustrated in figure 5.

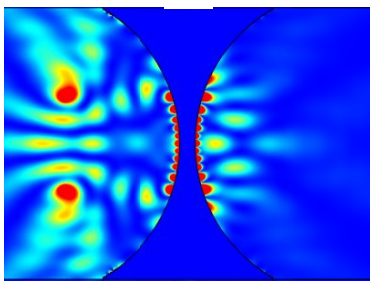


Figure 5: illustration of complex field propagating from left to right through a negative index concave lens. Note the surface waves present on the surface of the lens, indicating that some high spatial frequencies are present and being resonantly transferred across the lens and hence can contribute to the sum of fields in the image domain on the right of the lens.

### 3. COMPRESSIVE SAMPLING THEORY

Based on the early work of Papoulis, [19], his generalized sampling theorem indicated how a set of  $N$  independent low resolution signals could be combined to reliably generate a reconstruction of that signal with an  $N$ -times improvement in the resolution. The set of  $N$  independent images can be formed by projecting the image onto a set of patterns such as Hadamard codes having  $1/N^{\text{th}}$  the resolution. Since that time it has become evident that non-uniformly sampled signals, or more exactly, randomly and sub-sampled signals, can also provide accurate high resolution signal or image reconstructions from very few measured data. This was put on a firm theoretical footing by Candes et al [20] and is reviewed in Testorf and Fiddy [21]. Candes proposed that almost all signals and images are sparse in some (unknown) function space,  $Q$ , and that one can find a representation for that signal  $f(x) = \sum a_n q_n(x)$  with  $n = 1, 2, \dots, K \ll P$  for a signal that we require to be reliably represented at  $P$  points or pixels. It has been shown that an almost perfect estimate of  $f(x) = f(x)$  is possible from only  $\sim K \log(n)$  measurements where  $n$  is the dimension of the space in which  $f(x)$  resides, i.e. a finite dimensional function space. A complimentary view of this remarkable possibility is that optimal compression of a signal can result from the  $q_n(x)$  being appropriately chosen and that in many cases completely random functions, such as Gaussian white noise, are appropriate. If we consider that  $f(x) = \sum a_n q_n(x)$ ,  $n = 1, 2, \dots, P$ , where  $q_n(x)$  is a member of a "sparsity" basis and we gather data,  $d$ , using an orthogonal basis,  $\psi_n(u)$  then we can write  $d = \Psi f$  and  $f = Qa$ . A fast search or the use of a look-up table for the coefficients in the  $Q$  domain which have minimum  $L_1$  norm and that lead to the data  $d$  in the  $\Psi$  domain, provides the desired information from the signal in an extremely computationally efficient manner. In the superresolved imaging problem we describe here, our imaging system has a single detector, but it can cycle through a series of illuminating incident waves patterns, controlled by switching on and off different combinations of elements in an array of sources. At this time we do not control these sources to create a series of orthogonal field patterns to serve as a basis set, but only explore the possibility that a set of structured incident waves, under the assumption of relatively weak scattering, produces a set of coefficients from which an image can be calculated. More sophisticated computational imaging procedures can be considered in the future.

We note that there is a growing trend in applying compressive sampling techniques as users increasingly accept that "full" signal recovery is not necessary. The community is moving instead toward the direct solution of so-called *inference* problems, such as detection, classification or estimation, which lends itself particularly well to this kind of computational approach. There is no reason why such ideas cannot be applied to classification of objects based on their subwavelength-scaled features.

Let us assume a set of  $N$  sources located to the left of the concave lens illustrated in figure 5. For a weakly scattering object, we can approximate the field in the object domain by  $V(r)\Psi_{Tj}(r)$  where  $V(r)$  is the relative refractive index distribution of the scattering object and  $\Psi_{Tj}(r)$  is the total field arising from the  $j$ th combination of sources being turned on. The integration of the product  $V(r)\Psi_{Tj}(r)$  over the aperture of the lens, gives a single output at the detector,  $c_j$ . Propagating and evanescent waves are well models by the first Born approximation,  $kVa \ll 1$ , for sufficiently thin or weakly scattering objects, but not otherwise, [12]. Here  $V$  is the maximum value for the index difference,  $a$  is a measure of the dimensions of the object and  $k = k_j > k_0$  where  $k_0$  is the free space wavenumber,  $2\pi/\lambda_0$ . At optical frequencies, we are unable to measure the phase of the scattered field without performing some kind of interferometric measurement, and so to be more precise,

$$c_j = \left| \int V(r)\Psi_{Tj}(r) dr \right|^2$$

### 4. NUMERICAL RESULTS

We conducted a series of numerical experiments in which a set of sources were sequentially turned on and off to provide the structured illumination to two subwavelength-scaled objects, as shown in figure 6.

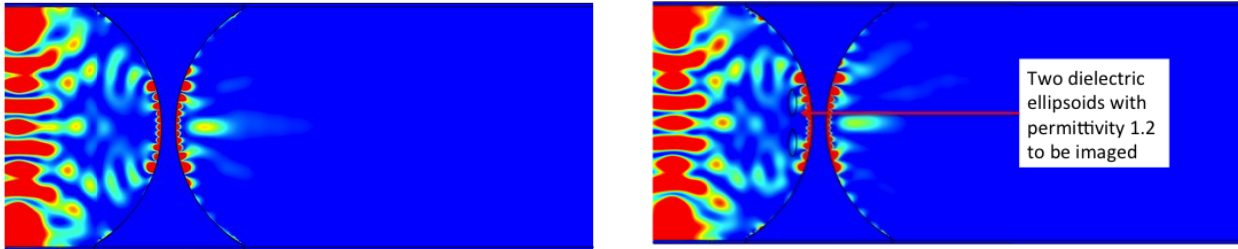


Figure 6: left: examples of  $N$  sources to left and the resulting field pattern on either side of the lens, both without (left) and with the scattering objects present (right).

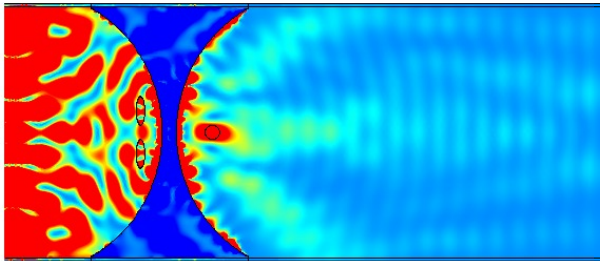


Figure 7: we see the scattered field in and around the two objects and the location (black circle) of the area in which the integrated incident and scattered field on the left of the lens is detected.

In the figures below (figure 8) we see the change in the observed coefficients as a function of changes in the object's permittivity.

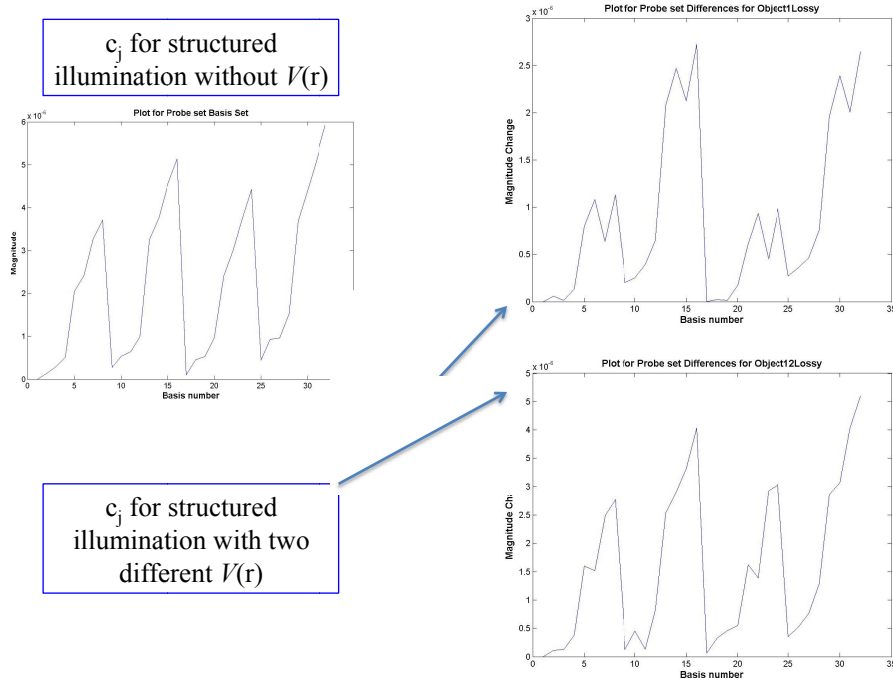


Figure 8: left: a plot of the coefficient values with no objects present for 32 combinations of source points being on; right shows the *differences* between the coefficient values on the left and those measured after the different objects were introduced (right top and right lower).

Finally, we compare the changes in coefficients for a single object placed in front of the lens, as a function of its permittivity, using values of 2,3,4 and 5. As can be seen in figure 9, a systematic change in these coefficients is observed as a function of increasing permittivity, which is to be expected.

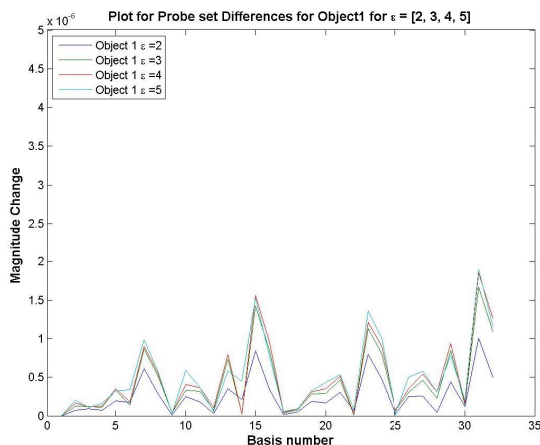


Figure 9: coefficient values as a function of increasing object permittivity.

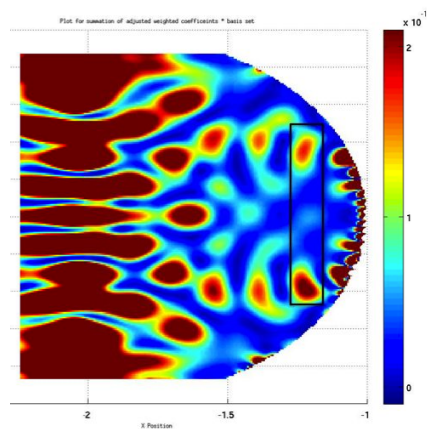


Figure 10: example reconstruction of  $\sum_j V \Psi_j$

Figure 10 shows an example reconstruction. While in principle this is encouraging, the limited range of differences between the various structured illumination wave patterns is not providing the diversity and independence between basis functions to span the space that would provide a good representation of the image. In the examples shown here, the source locations are uniformly spaced and in phase with each other. By varying the location and relative phase of the point sources, one can address this.

## CONCLUSIONS

We have described some of the difficulties in developing a practical imaging system capable of subwavelength resolution using a negative index slab lens. Scanning the image domain in the near field to measure subwavelength field patterns is both tedious and, in this case, problematic because of the coupling that can occur between the detector, the lens and the object. We proposed using a single detector to measure the projection of a sequence of structured fields onto the object, the scattered field being collected by a (concave) negative index focusing lens. The negative index material is still important since it provides a mechanism for the transfer of high spatial frequencies (evanescent waves) from the object to the detector. For a weak scatterer, each illuminating incident field is regarded as a coded wave pattern that multiplies the object. Thus in the same manner as for other examples of imaging by compressive sampling, we measure a series of coefficients of the projection of the incident field on the scattering object and then reconstruct the object using the original field patterns and the measured coefficients. This examples presented here are very crude but we have investigated the basic idea. For higher resolution and consequently more strongly interacting objects, we would expect that further computational processing would be required to extract the object's index profile from the measured field pattern using an inverse scattering algorithm.

Results were presented indicating that there is a systematic change in the measured coefficients as a function of object size, shape and intrinsic permittivity. With this particular arrangement of sources, some combinations did not provide useful illumination of the chosen object and no change in coefficient values was observed before and after the objects

were introduced. There has also been no attempt so far to select a set of incident field patterns that one could argue satisfied some fundamental requirements that they represent a formal set of basis functions. Clearly the final resolution achieved will be a function of the spatial resolution present in the object domain and this originates both from the structured illumination and scattering from the object. Nevertheless, that differences were observed in measured coefficients as a function of the illumination indicates that information about the objects has transferred through the lens. It follows that this approach could encode and hence transfer very high spatial frequency information, up to some maximum value for  $k$  determined by the transfer function of the negative index lens. In practice that limit will be a function of the meta-atom size in the metamaterial, the metamaterial lens's losses and the proximity of the scattering features of interest to the surface of the lens.

In summary, we investigated a different approach for exploiting the potential for high resolution (subwavelength scale) imaging using a negative index metamaterial. To avoid the coupling that occurs between a detecting probe and the object, via the lens, we proposed a single fixed detector and studied how structured illumination might provide the diversity necessary to reconstruct a superresolved image. For a fixed array of dipole sources, we demonstrated that changing combinations of which sources were turned on does provide information consistent with changes introduced to the scattering object. We conclude that this approach has a lot of promise for a more practical superresolving imaging system, but the structured illumination needs to provide more spatial diversity (randomness) in order to represent a better set of basis functions for this kind of compressive imaging.

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## REFERENCES

- [1] J. B. Pendry, "Negative index makes a perfect lens", *Phys. Rev. Lett.*, 85, 18, pp 3966-3969, 2000.
- [2] Z. Jacob et al, "Optical hyperlens: far field imaging beyond the diffraction limit", *Optics Express*, 14, 18, pp 8247-8256, 2006.
- [3] R. Hegde et al, "The dynamics of nanoscale superresolution imaging with the superlens", *IEEE MTT* 59, 10 pp 2612- 2623, 2011.
- [4] N. Fang et al, "Sub-diffraction-limited optical imaging with a silver superlens", *Science* 308, issue 5721, pp 534-537, 2005.
- [5] X-X Liu and A. Alu, "Limitations and potentials of metamaterial lenses", *J. of Nanophotonics*, Vol 5, 053509-1-13, 2011.
- [6] Yuan Zhang, Yi-Chen Chuang and M. A. Fiddy, "Study of scattering patterns and sub-wavelength scale imaging based on finite-sized metamaterials", *Applied Physics A* DOI 10.1007/s00339-01106738-9, (2011)
- [7] Yi-Chen Chuang, R. Dudley and M. A. Fiddy, "Optimal arrangement of meta-atoms composing metamaterials", *SPIE Paper 8269 82691D1-7*, *Photonic and Phononic Properties of Engineered Nanostructures II*, Ed. A. Adibi, S-Y Lin and A. Scherer, *Photonics West* January 2012.
- [8] C. P. Moore et al, "An improved transfer-matrix model for optical superlenses", *Optics Express* 17, no 16, pp14260-14269, 2009.
- [9] A. P. Mosk et al, "Controlling waves in space and time for imaging and focusing in complex media", *Nature Photonics*, Vol. 6, pp 283-292, 2012.
- [10] Z. Liu, et al, "Coupling effect in a near field object-superlens system", *Appl. Phys. A.*, 107, pp83-88, 2012.
- [11] S. Huang et al, "Subwavelength imaging enhancement through a three-dimensional plasmon superlens with rough surface", *Optics Letters*, vol 37, no 8, pp 1295-1297, 2012.
- [12] Yi-Chen Chuang, R. Dudley and M. A. Fiddy, "Superresolved image reconstruction from incomplete data", *SPIE Paper 8500-8*, *Image Recovery from Incomplete Data VII*, Ed. P.J. Bones, M. A. Fiddy and R. Millane, August 2012.
- [13] R. S. Hegde, G. Hotan, E.P. Li, W. J. R. Hoefler and M. A. Fiddy, "Towards the reconstruction of three dimensional sub-wavelength objects imaged by a Veselago-Pendry superlens", in *COMPUMAG 2011 Digest*, Sydney, Australia.

- [14] A. Kolim and G. S. Agarwal, "Resolution of objects within subwavelength range by using the near field of a dipole", *Optics Letters*, vol 37, issue 12, pp2313-2315, 2012.
- [15] H. F. Arnoldus and J. T. Foley, "Spatial separation of the traveling and evanescent parts of dipole radiation", *Optics Letters*, vol 28, no. 15, pp 1299- 1301, 2003.
- [16] M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk. Single-pixel imaging via compressive sampling. *IEEE Sig. Proc. Magazine*, vol 25(2), pp 83–91, 2008.
- [17] G. Oliveri et al, "Bayesian compressive optical imaging within the Rytov approximation", *Optics Letters*, vol 37, no 10, pp 1760-1762, 2012.
- [18] A. Fannjiang and H-C. Tseng, "Compressive imaging of subwavelength structures: periodic rough surfaces", *J. Opt. Soc. A.*, vol 29, no 4., pp 617-626, 2012.
- [19] A. Papoulis, "Generalized sampling expansion", *IEEE Transactions on Circuits and Systems, CAS-24*, pp 652–654, 1977.
- [20] E. Candes, J. Romberg, J and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information", *IEEE Transactions on Information Theory*, 52(2), pp 489–509, 2006.
- [21] M. E. Testorf and M. A. Fiddy, "Superresolution Imaging: Revisited", *Advances in Imaging and Electron Physics*, Ed. P. W. Hawkes, Vol 163, ch 5, pp166-218, Ed. P. W. Hawkes, Academic Press, 2010