Model Predictive Control of Discrete T-S Fuzzy Systems with Time-varying Delay

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***Abstract*—Robust model predictive control of discrete nonlinear systems with bounded time-varying delay and persistent disturbances is investigated in this paper. The T-S fuzzy systems are utilized to represent nonlinear systems. A Razumikhin-type Lyapunov function is adopted for time-delay systems due to its advantage in reducing the complexity especially for systems with large delays and disturbances. The robust positive invariance set theory for systems subjected to time-varying delay and disturbances is analyzed. In addition, the input-to-state stability is realized due to persistent disturbances. The controller synthesis conditions are derived by solving a sequence of matrix inequalities. Simulation on a continuous stirred-tank reactor (CSTR) is illustrated to verify the effectiveness of the proposed method.**

I. INTRODUCTION

Time delay occurs extensively in various industrial systems and has attracted many research interests. Generally, two kinds of Lyapunov functions have been investigated for stability analysis and controller design of time-delay systems: the Lyapunov-Krasovskii functional (LKF) and Lyapunov-Razumikhin function (LRF) [2]-[5]. For discrete-time systems, the Krasovskii approach uses an augmented state vector which involves all delayed states that, it yields the applications of classical Lyapunov approaches to an augmented system without delay. As a result, the controller synthesis and computation, especially for systems with large delays and disturbances, will become very complex [6]. Compared with the Krasovskii approach, the Razumikhin approach is conservative but it involves a Lyapunov function for the original non-augmented system. Therefore, it has a good potential to get rid of the complexity associated with the Krasovskii approach especially for systems with large delays and disturbances. It is noted that extensive works can be seen on stabilization of time-delay systems by adopting LKF, while LRF-based approaches have not been widely investigated until in recent years [6]-[9].

Moreover, compared with time-delay systems with constant delay, a more natural aspect is to consider systems with time-varying delay. However, LKF is utilized in most existing works for systems with time-varying delay [10]-[12]. Only a few results are based on LRF that only continuous systems are considered [13], [14].

On the other hand, as a kind of optimization control methods which can handle constraints, model predictive control (MPC) has been widely investigated and utilized in industrial processes. However, only a few results can be seen on MPC for time-delay systems. Early results can be mainly seen in [15]-[17] where LKF is adopted. Recently, an LRF-based stabilization method is proposed for linear delay difference inclusions (DDIs) with polytopic uncertainties in [8]. For nonlinear cases, a combination of LRF and LKF conditions is adopted for continuous time-delay systems [18]. MPC without terminal constraints can be seen in [19]. However, most of the aforementioned works requires the delay to be known and constant, except for [17].

In addition, the disturbance is not considered and only a few results are applied to nonlinear systems. Thus motivates this work for fuzzy model predictive control (FMPC) of discrete nonlinear systems with time-varying delay and persistent disturbances via LRF approach. The T-S fuzzy systems are adopted to approximate nonlinear systems with time-varying delay. Then LRF is adopted and conditions of robust positive invariance and input-to-state stability (ISS) under disturbances are derived.

The rest of this paper is organized as follows. The system dynamics, as well as the robust fuzzy model predictive control (RFMPC) problem, are introduced in Section II. Constructions of the robust positive invariant set and the terminal constraint set are provided in Section III, which is followed by the whole optimization control algorithm. In Section IV, simulation on a CSTR system is provided that the effectiveness of the proposed RFMPC method is verified. Finally, some conclusions are provided in Section V.

II. PRELIMINARIES

In this section, the system model, T-S fuzzy systems, and MPC control scheme for systems with time-varying delay are introduced in sequel.

A. System Model

Consider the following discrete-time nonlinear system with unknown time-varying state delay

$$x^{+} \in F(x(k), x_d(k), u(k), w(k)), \qquad k \in \mathbb{Z}_{+}, \tag{1}$$

where x^{+} represents the system state in the next time instant, $x(k), x_d(k) \in \mathbb{X}$, $u(k) \in \mathbb{U}$ and $w(k) \in \mathbb{W}$, with $\mathbb{X} \subseteq \mathbb{R}^n$, $\mathbb{U} \subseteq \mathbb{R}^m$ and $\mathbb{W} \subseteq \mathbb{R}^c$, represent system current and delayed states, inputs, and disturbances, respectively. $x_d(k) \in \mathbb{X}$ is defined as follows:

$$x_d(k) := x(k + d(k)), \qquad d(k) \in \mathbb{Z}_{[-h,\,-1]} \tag{2}$$

where h denotes the upper bound of delay, and the minimal delay is set as 1.

Definition 1 (Robust Positively Invariant (RPI) Set, D-invariance): Consider system (1), a set Ω is called an RPI set for the closed-loop system corresponding to the control law π, for $-h \le d \le -1$, if $\forall x \in \Omega$, $\forall x_d \in \Omega$, and $\forall w \in \mathbb{W}$, it holds $x^{+} \in \Omega$.

Definition 2 (Input-to-state Stability (ISS)): Consider $x^{+} \in F(x(k), x_d(k), w(k))$, it is called ISS if there exist $\beta \in \mathcal{KL}$ and $\delta \in \mathcal{K}$, such that for all $k \in \mathbb{Z}_{+}$ it holds that

$$\|x(k)\| \le \beta(\left\|\mathbf{x}_{[-h,0]}\right\|, k) + \delta(\left\|\mathbf{w}_{[0,k-1]}\right\|), \tag{3}$$

where $\mathbf{x}_{[-h,0]} \in \mathbb{X}^{h+1}$ is the initial (delayed) states, $\mathbf{w}_{[0,k-1]} \in \mathbb{W}^k$ is the disturbance sequence.

Definition 3 (ISS Lyapunov Function): For LRF condition, suppose that there exists a positive definite function such that satisfies the following conditions:

(i) There exist $\alpha_1, \alpha_2 \in \mathcal{K}_{\infty}$ satisfying for all $x \in \mathbb{R}^n$,

$$\alpha_1(\|x\|) \le V(k, x) \le \alpha_2(\|x\|) \tag{4}$$

for all $(x, x_d, w, x_1) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^c \times F(x_0, x_d, w)$;

(ii) For all initial condition, there exists a function $\delta \in \mathcal{K}$ satisfying

$$V(k+1, x(k+1)) \le \max\left\{\bar{V}(k, x), \delta\left\|w\right\|\right\} \tag{5}$$

where $\bar{V}(k, x) = \max\{V(k, x(k)), V(k, x_d(k))\}$, then $V(k, x)$ is called an ISS Lyapunov-Razumikhin Function for $x^{+} \in F(x(k), x_d(k), w(k))$.

It is noted that detailed descriptions of ISS and ISS Lyapunov Function for discrete time-delay systems are introduced in [6].

B. T-S Fuzzy Time-delay Systems

Based on the aforementioned system model, nonlinear systems with time-varying delay can be represented as T-S fuzzy time-delay systems:

Plant Rule l**:**
IF: z_1 is F_1^l and …, z_v is F_v^l
THEN:

$$x(k+1) = A_l x(k) + A_{dl} x_d(k) + B_l u(k) + E_l w(k) \tag{6}$$

where, $z := [z_1, \ldots, z_v]$ are premise variables. F_1^l, …, F_v^l are fuzzy sets. $l \in \mathbb{Z}_{[1,\,L]}$, and L is the number of fuzzy rules. The T-S fuzzy system (6) can be written as,

$$x^{+} = A_{\mu} x(k) + A_{d\mu} x_d(k) + B_{\mu} u(k) + E_{\mu} w(k) \tag{7}$$

where $A_{\mu} := \sum_{l=1}^{L} \mu_l(z) A_l$, $A_{d\mu} := \sum_{l=1}^{L} \mu_l(z) A_{dl}$, $B_{\mu} := \sum_{l=1}^{L} \mu_l(z) B_l$, $E_{\mu} := \sum_{l=1}^{L} \mu_l(z) E_l$, $\mu_l(z)$ is normalized membership function, in what follows, denote $\mu_l(z)$ as μ_l for simplicity.

Control law for the aforementioned fuzzy system is given as follows:

Control Rule l**:**
IF: z_1 is F_1^l and …, z_v is F_v^l
THEN:

$$u(k) = K_l x(k)$$

Based on which, the final control output is formulated as

$$u(k) = K_{\mu} x(k) \tag{8}$$

where $K_{\mu} = \sum_{l=1}^{L} \mu_l K_l$.

C. MPC Scheme

In this subsection, control scheme about MPC for the aforementioned T-S fuzzy time-delay systems with disturbances is introduced. The prediction model is provided by

$$\begin{aligned} x(k+s+1|k) = A_{\mu} x(k+s|k) + A_{d\mu} x_d(k+s|k) + \\ B_{\mu} u(k+s|k) + E_{\mu} w(k+s|k) \end{aligned} \tag{9}$$

Usually, MPC optimizes a finite horizon cost function under some constraints,

$$J(k) = \sum_{s=0}^{N-1} \ell(k+s|k) + V_T(x(k+N|k)) \tag{10}$$

where $\ell(k+s|k)$ and $V_T(x(k+N|k))$ are called the stage cost and the terminal cost at the predicted instant, respectively. And $V_T(\cdot)$ is a positive definite function. In this case where persistent disturbances are involved, the stage cost is given by

$$\begin{aligned} \ell(k+s|k) = x^T(k+s|k) Q x(k+s|k) + \\ u^T(k+s|k) R u(k+s|k) - \tau w^T(k+s|k) w(k+s|k) \end{aligned} \tag{11}$$

where Q and R are positive matrices, τ is a positive scalar. One can see that the disturbance is included in the stage cost, and it is worth mentioning that this kind of cost function can be seen in [21] where the concept of "H_{∞} model predictive control" is mentioned. Since unknown persistent disturbance is involved, a robust terminal constraint set is employed so that the system state will enter the terminal set at the end of prediction. The MPC optimization is formulated as solving the following min-max problem

$$\begin{cases} \min\limits_{u(k+s|k)} \; \max\limits_{w(k+s|k)} \quad J(k), \\ u(k+s|k) \in \mathbb{U}, \\ w(k+s|k) \in \mathbb{W}, \\ x(k+N|k) \in \Omega_T, \end{cases}$$

where $s \in \mathbb{Z}_{[1,N-1]}$, $\Omega_T \subseteq \mathbb{X}$ is the robust terminal constraint set. Assume that the disturbance is bounded, and control input should satisfy some constraints, thus

$$w \in \mathbb{W} := \{w | w^T w \le \eta^2\}$$

$$u \in \mathbb{U} := \{u | \, |u_t| \le u_{t,\max}\}$$

where η and $u_{t,\max}$ are known constants. And u_t is the t-th element of the input vector, with $t \in \mathbb{Z}_{[1,m]}$.

III. MAIN RESULTS

In this section, the conditions of RPI property for MPC of T-S fuzzy systems with time-varying delay and disturbances are discussed at first, which is followed by the construction of the terminal constraint set.

A. RPI Set

Denote $\Omega \subseteq \mathbb{X}$ as an RPI set which is described in Definition 1, and π as the corresponding control law. Let quadratic LRF $V(k) := x^T(k) P_\mu x(k)$, define Ω as the following ellipsoidal set

$$\Omega := \{\{x(k), x_d(k)\} | V(k) \le \xi, V(k+d) \le \xi\} \tag{12}$$

and the corresponding control law

$$\pi(x_0, x_d) := \{K_\mu x_0\} \tag{13}$$

Lemma 1: Consider system (7), the set Ω is an RPI set if it holds that

$$\tfrac{1}{\xi} V(x^+) - \tfrac{1-\lambda}{\xi} \bar{V}(k,x) - \tfrac{\lambda}{\eta^2} w^T w \le 0 \tag{14}$$

where $\lambda \in \mathbb{R}_{(0,1)}$, and $\bar{V}(k,x)$ is defined in (5). It is noted that the RPI approach involving disturbances is extended from the case of non-delayed systems in [20]. And the proof is very similar to the one in [20], thus it is omitted here.

In what follows, the satisfaction of RPI property and input constraint will be given by solving matrix inequalities.

Theorem 1: Consider T-S fuzzy system (7), if there exist positive definite matrix X_a (or X_b, X_l), matrices Y_b, G_b, Z, positive scalar $\lambda \in \mathbb{R}_{(0,1)}$, such that the following matrix inequalities are feasible

$$\begin{bmatrix} \gamma\Phi & * & * & * \\ 0 & \gamma_d\Phi & * & * \\ 0 & 0 & -\frac{\lambda}{\eta^2} I & * \\ A_a G_b + B_a Y_b & A_{da} G_b & E_a & -X_l \end{bmatrix} \le 0 \tag{15}$$

$$\begin{bmatrix} Z & * \\ {Y_b}^T & G_b + {G_b}^T - X_b \end{bmatrix} \ge 0, \; Z_{tt} \le u_{t,\max}^2, \; t \in \mathbb{Z}_{[0,m]} \tag{16}$$

where $\Phi = (1-\lambda)(X_a - G_b - {G_b}^T)$, $a, \; b, \; l \in \mathbb{Z}_{[1,L]}$, Z_{tt} is the t-th diagonal element of matrix Z, γ and γ_d are pre-set valued positive numbers that satisfy $\gamma + \gamma_d = 1$, then the set $\Omega = \{x | x^T P_\mu x \le \xi\}$, with $P_\mu = \sum_{l=1}^{L} \mu_l(x) P_l$ and $P_l = \xi X_l^{-1}$, is an RPI set that corresponds to the control law $\pi = K_\mu x(k)$, with $K_\mu = \sum_{l=1}^{L} \mu_l K_l$ and $K_l = Y_l {G_l}^{-1}$.

Proof: According to the dilation lemma in [22],

$$-{G_b}^T {X_a}^{-1} G_b \le X_a - {G_b}^T - G_b \tag{17}$$

The following inequality is obtained from (15),

$$\begin{bmatrix} \gamma\Delta & * & * & * \\ 0 & \gamma_d\Delta & * & * \\ 0 & 0 & -\frac{\lambda}{\eta^2} I & * \\ A_a G_b + B_a Y_b & A_{da} G_b & E_a & -X_l \end{bmatrix} \le 0 \tag{18}$$

where $\Delta = (\lambda - 1){G_b}^T {X_a}^{-1} G_b$. By multiplying $diag\{{G_b}^{-T}, {G_b}^{-T}, I, I\}$ and its transpose from both sides of (18), respectively, yields that

$$\begin{bmatrix} \gamma(\lambda-1){X_a}^{-1} & * & * & * \\ 0 & \gamma_d(\lambda-1){X_a}^{-1} & * & * \\ 0 & 0 & -\frac{\lambda}{\eta^2} I & * \\ A_a + B_a K_b & A_{da} & E_a & -X_l \end{bmatrix} \le 0 \tag{19}$$

From (19), one can get that

$$\begin{bmatrix} \gamma(\lambda-1){X_\mu}^{-1} & * & * & * \\ 0 & \gamma_d(\lambda-1){X_\mu}^{-1} & * & * \\ 0 & 0 & -\frac{\lambda}{\eta^2} I & * \\ A_\mu + B_\mu K_\mu & A_{d\mu} & E_\mu & -X_\mu \end{bmatrix} \le 0 \tag{20}$$

Applying schur complement to (20), and then multiplying $[x^T, {x_d}^T, w^T]$ and its transpose from both sides, respectively, yields that

$$\begin{aligned} &\varpi \begin{bmatrix} \gamma(\lambda-1){X_\mu}^{-1} & & \\ & \gamma_d(\lambda-1){X_\mu}^{-1} & \\ & & -\frac{\lambda}{\eta^2} I \end{bmatrix} \varpi^T \\ &+ \varpi \Lambda {X_\mu}^{-1} \Lambda^T \varpi^T \le 0 \end{aligned} \tag{21}$$

where $\varpi = [x^T, {x_d}^T, w^T]$, $\Lambda = [(A_\mu + B_\mu K_\mu), A_{d\mu}, E_\mu]^T$.

Substituting X_μ^{-1} with P_μ/ξ, (21) is equal to

$$\tfrac{1}{\xi} V(x^+) - \tfrac{\lambda}{\eta^2} w^T w \le \tfrac{1-\lambda}{\xi} (\gamma V(x(k)) + \gamma_d V(x_d(k))) \tag{22}$$

With $\gamma + \gamma_d = 1$, it is easy to get $\gamma V(x(k)) + \gamma_d V(x_d(k)) \le \bar{V}(k,x)$, put it to (22) and then (14) is obtained.

The input constraint is guaranteed by (16) which has been widely investigated. Thus the proof is completed.

B. Terminal Constraint Set

Definition 4: The terminal constraint set Ω_T should be an RPI set. In addition, there must exist $\alpha_3, \alpha_4 \in \mathcal{K}_\infty$, and a positive definite function $V(k)$ such that $\forall \, x \in \Omega_T$,

$$\alpha_3(\|x(k)\|) \le V(k) \le \alpha_4(\|x(k)\|) \tag{23}$$

$$\begin{aligned} V(x^+) - \bar{V}(k,x) \le& -x(k)^T Q x(k) - u^T(k) R u(k) \\ &+ \tau w^T w \end{aligned} \tag{24}$$

Definition 4 is concluded based on [6]. In the following, the condition which guarantees Definition 4 is derived.

Theorem 2: Consider system (7), Ω_T is a terminal constraint set if it holds that

$$\begin{bmatrix} \gamma\Xi & * & * & * & * & * \\ 0 & \gamma_d\Xi & * & * & * & * \\ 0 & 0 & -\tau\xi I & * & * & * \\ \Psi & A_{da}G_b & E_a & -X_l & * & * \\ QG_b & 0 & 0 & 0 & -\xi Q & * \\ RY_b & 0 & 0 & 0 & 0 & -\xi R \end{bmatrix} \leq 0 \tag{25}$$

where $\Xi = X_a - G_b - {G_b}^T$, $\Psi = A_aG_b + B_aY_b$. X_a (or X_b, X_l), matrices Y_b, G_b, are the same as shown in Theorem 1.

Proof: it can be easily seen that (23) is satisfied by resorting the eigenvalues of the positive definite matrices. Then the main concentration is focused on derivation of (24). Considering (17), then multiplying $diag\left\{{G_b}^{-T}, {G_b}^{-T}, I, I, I, I\right\}$ and its transpose from both sides of (25), respectively, which is equivalent to

$$\begin{bmatrix} -\gamma {X_a}^{-1} & * & * & * & * & * \\ 0 & -\gamma_d {X_a}^{-1} & * & * & * & * \\ 0 & 0 & -\tau\xi I & * & * & * \\ \Upsilon & A_{da} & \xi E_a & -X_l & * & * \\ Q & 0 & 0 & 0 & -\xi Q & * \\ RK_b & 0 & 0 & 0 & 0 & -\xi R \end{bmatrix} \leq 0 \tag{26}$$

where $\Upsilon = A_a + B_aK_b$. (26) is equal to

$$\begin{bmatrix} -\gamma\xi {X_a}^{-1} & * & * & * & * & * \\ 0 & -\gamma_d\xi {X_a}^{-1} & * & * & * & * \\ 0 & 0 & -\tau I & * & * & * \\ \Upsilon & A_{da} & E_a & -\frac{X_l}{\xi} & * & * \\ Q & 0 & 0 & 0 & -Q & * \\ RK_b & 0 & 0 & 0 & 0 & -R \end{bmatrix} \leq 0 \tag{27}$$

Substituting $X^{-1} = \xi^{-1}P$ to (27), and rewriting (27) as

$$\begin{bmatrix} Q + {K_\mu}^T RK_\mu - \gamma P_\mu & * & * & * \\ 0 & -\gamma_d P_\mu & * & * \\ 0 & 0 & -\tau I & * \\ A_\mu + B_\mu K_\mu & A_{d\mu} & E_\mu & -{P_\mu}^{-1} \end{bmatrix} \leq 0 \tag{28}$$

Applying schur complement to (28), then multiplying $\left[x^T, {x_d}^T, w^T\right]$ and its transpose respectively yields that

$$\begin{aligned} &{x^+}^T Px^+ + x^TQx + x^T{K_\mu}^T RK_\mu x - \tau w^Tw \\ &-(\gamma x^TP_\mu x + \gamma_d {x_d}^TP_\mu x_d) \leq 0 \end{aligned} \tag{29}$$

With $\gamma + \gamma_d = 1$ and $\bar{V}(k,x)$ defined in (5), it is easy to get $\gamma x^TP_\mu x + \gamma_d {x_d}^TP_\mu x_d \leq \bar{V}(k,x)$, thus

$$\begin{aligned} {x^+}^T Px^+ - \bar{V}(k,x) \leq -x(k)^TQx(k) - u(k)^TRu(k) \\ + \tau w^Tw \end{aligned} \tag{30}$$

Proof is completed.

C. MPC Algorithm

The online MPC algorithm is provided in this subsection based on the aforementioned results. From the aforementioned RPI property, the terminal constraint set $\bar{V}(k,x)$ should satisfy that

$$\bar{V}(k,x) \leq \xi \tag{31}$$

thus $x(k)$, $x_d(k) \in \Omega_m$. In addition, minimization is given to $\bar{V}(k,x)$ to get an optimized control performance

$$\min \quad \xi, \ subjected \ to \ \bar{V}(k,x) \leq \xi$$

$\bar{V}(k,x) \leq \xi$ is satisfied by

$$\begin{bmatrix} 1 & x^T(k+j) \\ x(k+j) & X_l \end{bmatrix} \geq 0, \ j \in \mathbb{Z}_{[-h,0]}, \ l \in \mathbb{Z}_{[1,L]} \tag{32}$$

Remark 1: In (5), $\bar{V}(k,x)$ is only involved with $x(k)$ and $x_d(k)$, i.e., $x(k) \in \Omega_k$ and $x_d(k) \in \Omega_k$. However, all possibly delayed states are employed in (32), which indicates $x(k+j) \in \tilde{\Omega}_k$, $j \in {}_{[-h,0]}$. It is easy to get $\Omega_k \subseteq \tilde{\Omega}_k$. Although this choice may be less optimal because it may lead to a larger terminal constraint set, it is essential to guarantee the positive invariance of terminal constraint set for systems with time-varying delay and it will be utilized for the feasibility analysis later in this section.

As τ is used to represent the disturbance attenuation level in (25), it is noted that τ also can be optimized. Thus ξ and τ can be optimized together. Inspired by [20], thus a new variable $\varepsilon = \tau\xi$ is minimized instead. Then rewrite inequality (25) as

$$\begin{bmatrix} \gamma\Xi & * & * & * & * & * \\ 0 & \gamma_d\Xi & * & * & * & * \\ 0 & 0 & -\varepsilon I & * & * & * \\ \Psi & A_{da}G_b & E_a & -X_l & * & * \\ QG_b & 0 & 0 & 0 & -\xi Q & * \\ RY_b & 0 & 0 & 0 & 0 & -\xi R \end{bmatrix} \leq 0 \tag{33}$$

Therefore, the MPC algorithm comes out as solving (15), (16), (32), and (33), under the minimization of ε.

The feasibility and ISS are very similar with the results in [20] where system without delay is investigated. Thus the proofs are omitted here.

IV. ILLUSTRATIVE EXAMPLE

Consider the well-known CSTR system in [23]. The sampling time is set as $T_s = 0.2$ minutes. There are three nominal states when $u = 0$: $x_{s1} = [0.1440; 0.8862]$, $x_{s2} = [0.4472; 2.7520]$, x_{s3}=[0.7646;4.7052]. Suppose the time delay is time-varying and unknown, and the upper bound of delay is $h = 10$. Then the discrete T-S fuzzy time-delay systems can be described as follows,

Rule 1: If $x_2(k)$ is 0.8862, then

$$x(k+1) = A_1x(k) + A_{d1}x_d(k) + B_1u(k) + E_1w(k)$$

Rule 2: If $x_2(k)$ is 2.7520, then

$$x(k+1) = A_2x(k) + A_{d2}x_d(k) + B_2u(k) + E_2w(k)$$

Rule 3: If $x_2(k)$ is 4.7052, then

$$x(k+1) = A_3x(k) + A_{d3}x_d(k) + B_3u(k) + E_3w(k)$$

where

$$A_1 = \begin{bmatrix} 0.7500 & 0.0119 \\ -0.2238 & 0.8262 \end{bmatrix}, A_2 = \begin{bmatrix} 0.6203 & 0.0762 \\ -1.2337 & 1.3265 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.3068 & 0.0442 \\ -3.6621 & 1.0765 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.0435 & 0.0003 \\ -0.0061 & 0.0455 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 0.0403 & 0.0019 \\ -0.0312 & 0.0581 \end{bmatrix}, A_{d3} = \begin{bmatrix} 0.0310 & 0.0013 \\ -0.1037 & 0.0528 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.0004 \\ 0.0546 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0023 \\ 0.0698 \end{bmatrix}, B_3 = \begin{bmatrix} 0.0015 \\ 0.0634 \end{bmatrix},$$
$$E_1 = E_2 = E_3 = \begin{bmatrix} 0 \\ 0.00001 \end{bmatrix}.$$

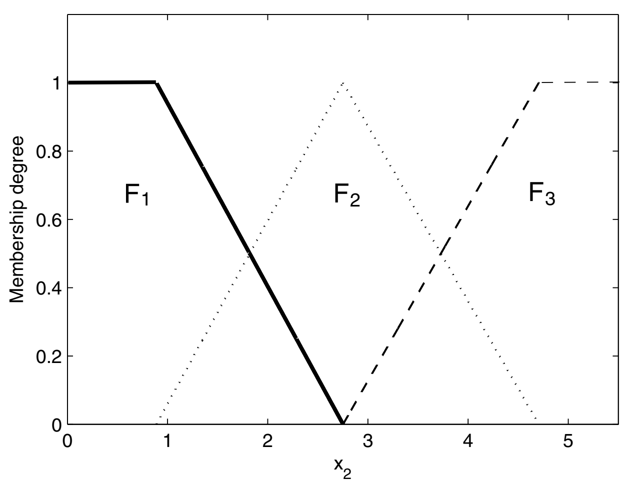

Fig. 1. Membership functions.

The membership functions (see, Fig. 1) are given as follows:

$$\begin{cases} F_1 = \begin{cases} 1 & if \quad x_2 \leq 0.8862 \\ \frac{2.7520-x_2}{2.7520-0.8862} & if \quad 0.8862 < x_2 \leq 2.7520 \\ 0 & if \quad x_2 > 2.7520 \end{cases} \\ F_2 = \begin{cases} 1 - F_1 & if \quad 0.8862 < x_2 \leq 2.7520, \\ 1 - F_3 & if \quad 2.7520 < x_2 \leq 4.7052. \end{cases} \\ F_3 = \begin{cases} 0 & if \quad x_2 \leq 2.7520 \\ \frac{2.7520-x_2}{4.7052-2.7520} & if \quad 2.7520 < x_2 \leq 4.7052 \\ 1 & if \quad x_2 > 4.7052 \end{cases} \end{cases}$$

The weighting matrices Q and R in the cost function are selected as $diag\{1e-6, 1e-9\}$ and 0.001, respectively. The bounded disturbance is set as $\|w\| \leq 1$. And the input constraint is given as $\|u\| \leq 10$. Since γ_0 and γ_{-1} are positive scalars and should satisfy the requirement $\gamma_0 + \gamma_{-1} = 1$, a feasible solution of these two parameters for the simulation is given as $\gamma_0 = 0.8$, $\gamma_{-1} = 0.2$.

The simulation results for the online approach are shown in Fig. 2. The following piecewise reference signal is given,

$$x_{ref} = \begin{cases} x_{s2} & 0 \leq t < 4min, \\ x_{s1} & 4 \leq t < 8min, \\ x_{s2} & 8 \leq t < 12min, \\ x_{s3} & 12 \leq t \leq 16min. \end{cases}$$

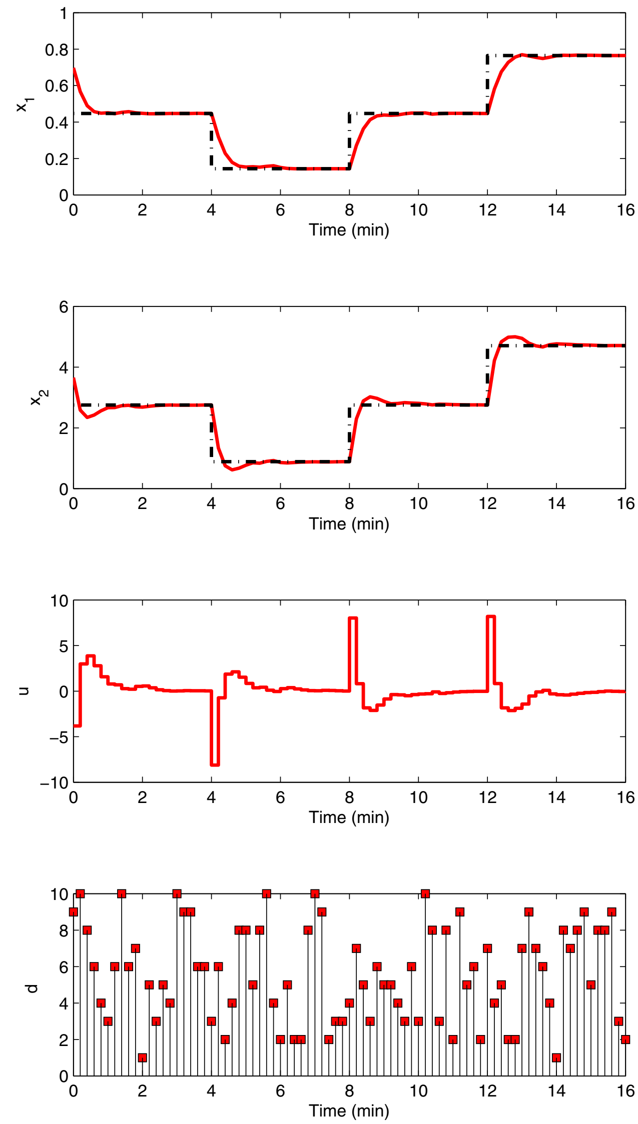

Fig. 2. Simulation results.

From Fig. 2, one can see that, despite the persistent disturbances, the state trajectories can track well with the variance of different reference signals. In addition, the input signal satisfies the constraint. Moreover, the time-varying delay at each sampling instant is also shown in Fig. 2.

V. CONCLUSIONS

FMPC of discrete-time nonlinear systems with time-varying delay, persistent disturbances and input constraints is investigated in this paper. Based on the Lyapunov-Razumikhin approach, new conditions of input-to-state stability and robust positive invariance under disturbances for nonlinear systems subjected to time-varying delay are provided. Simulation result shows that the proposed FMPC method is effective.

REFERENCES

[1] E. Fridman, and U. Shaked, “A descriptor system approach to H_∞ control of linear time-delay systems,” IEEE Trans. Autom. Control, vol. 47, no. 2, pp. 253-270, 2002.

[2] P. Pepe, “Input-to-state stabilization of stabilizable, time-delay, control-affine, nonlinear systems,” IEEE Trans. Autom. Control, vol. 54, no. 7, pp. 1688-1693, 2009.

[3] E. Fridman, M. Dambrine, and N. Yeganefar, “On input-to-state stability of systems with time-delay: a matrix inequalities approach,” Automatica, vol. 44, no. 9, pp. 2364-2369, 2008.

[4] M. Jankovic, “Control Lyapunov-Razumikhin functions and robust stabilization of time delay systems,” IEEE Trans. Autom. Control, vol. 46, no. 7, 1048-1060, 2001.

[5] A. R. Teel, “Connections between Razumikhin-type theorems and the ISS nonlinear small gain theorem,” IEEE Trans. Autom. Control, vol. 43, no. 7, pp. 960-964, 1998.

[6] B. Liu, and D. J. Hill, “Input-to-state stability for discrete time-delay systems via the Razumikhin technique,” Syst. Control Lett., vol. 58, no. 8, pp. 567-575, 2009.

[7] B. Liu, and H. J. Marquez, “Razumikhin-type stability theorems for discrete delay systems,” Automatica, vol. 43, no. 7, pp. 1219-1225, 2007.

[8] R. H. Gielen, and M. Lazar, “Stabilization of polytopic delay difference inclusions via the Razumikhin approach,” Automatica, vol. 47, no. 12, pp. 2562-2570, 2011.

[9] R. H. Gielen, M. Lazar, and S. V. Rakovic, “Necessary and sufficient Razumikhin-type conditions for stability of delay difference equations,” IEEE Trans. Autom. Control, vol. 58, no. 10, pp. 2637-2642, 2013.

[10] S. H. Song, J. K. Kim, C. H. Yim, and H. C. Kim, “H_∞ control of discrete-time linear systems with time-varying delays in state,” Automatica, vol. 35, no. 9, pp. 1587-1591, 1999.

[11] H. Gao, J. Lam, C. Wang, and Y. Wang, “Delay-dependent output-feedback stabilisation of discrete-time systems with time-varying state delay,” IEE Proc.-Control Theory Appl., vol. 151, no. 6, pp. 691-698, 2004.

[12] X. Su, P. Shi, L. Wu, and Y. D. Song, “A novel control design on discrete-time Takagi-Sugeno fuzzy systems with time-varying delays,” IEEE Trans. Fuzzy Syst., vol. 21, no. 4, pp. 655-671, 2013.

[13] X. Li, and C. E. De Souza, “Criteria for robust stability and stabilization of uncertain linear systems with state delay,” Automatica, vol. 33, no. 9, pp. 1657-1662, 1997.

[14] Y. Y. Cao, Y. X. Sun, and C. Cheng, “Delay-dependent robust stabilization of uncertain systems with multiple state delays,” IEEE Trans. Autom. Control, vol. 43, no. 11, pp. 1608-1612, 1998.

[15] M. V. Kothare, V. Balakrishnan, and M. Morari, “Robust constrained model predictive control using linear matrix inequalities,” Automatica, vol. 32, no. 10, pp. 1361-1379, 1996.

[16] W. H. Kwon, Y. S. Lee, and S. H. Han, “General receding horizon control for linear time-delay systems,” Automatica, vol. 40, no. 9, pp. 1603-1611, 2004.

[17] S. C. Jeong, and P. Park, “Constrained MPC algorithm for uncertain time-varying systems with state-delay,” IEEE Trans. Autom. Control, vol. 50, no. 2, pp. 257-263, 2005.

[18] M. Reble, R. M. Esfanjani, S. K. Y. Nikravesh, and F. Allgower, “Model predictive control of constrained non-linear time-delay systems,” IMA J. Math. Control Inf., vol. 28, no. 2, pp. 183-201, 2011.

[19] M. Reble, F. D. Brunner, and F. Allgower, “Model predictive control for nonlinear time-delay systems without terminal constraint,” In Proc. Of the 18th IFAC World Congress, pp. 9254-9259, Milan, Italy, 2011.

[20] W. Yang, G. Feng, and T. Zhang, “Robust Model Predictive Control for Discrete-Time Takagi-Sugeno Fuzzy Systems With Structured Uncertainties and Persistent Disturbances,” IEEE Trans. Fuzzy Syst., vol. 22, no. 5, pp. 1213-1228, 2014.

[21] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. Scokaert, “Constrained model predictive control: Stability and optimality,” Automatica, vol. 36, no. 6, pp.789-814, 2000.

[22] J. C. Geromel, P. D. Peres, and J. Bernussou, “On a convex parameter space method for linear control design of uncertain systems,” SIAM J. Control Optim., vol. 29, no. 2, pp. 381-402, 1991.

[23] Y. Y. Cao, and P. M. Frank, “Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach,” IEEE Trans. Fuzzy Syst., vol. 8, no. 2, pp.200-211, 2000.